Time's Arrow Points Downhill

Going forward needs only the state;
going back needs the boulder rolled uphill

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Fundamental Density Theory

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Abstract

A widely shared post states Boltzmann's account of time's arrow: the microscopic laws run the same forward and backward, $S = k \log W$ makes disordered states overwhelmingly more numerous than ordered ones, a shattered glass that reassembles breaks no law and is only improbable, and the arrow is the statistical echo of a universe unfolding from a low-entropy past. Fundamental Density Theory reads the same record with the direction in every step. Each step is one system-wide settlement of the omnium, solved from the state the system is in; events do not commute, and a process run backward is never the mirror of the forward event. FDT canon states the asymmetry: going back in time requires rolling the boulder uphill; going forward does not. The hill is compression. Matter is compressed omnium, and the forward step releases it: energy a closure held in circulation leaves as open stroke, an unbound helix that runs at c, keeps no time and, being one strand, never closes by itself. A return therefore needs every released component turned and delivered back, and the push that delivers it is a forward step of a larger system. The whole never returns; a part returns only with a push, and the push returns what it reaches. Boltzmann's count stays as the bookkeeper's ledger of the step: the least work any driver spends on a return is $T_0 \Delta S$, and the forward step spends none. Every laboratory rewind, from the spin echo and the time-reversal mirror to the phase-conjugate mirror and the reset of a bit, is a driven return of this kind. The arrow needs no first frame: the hill is carried now by every compressed pattern, and the domain's released level only grows. Five tests follow, one of them a divergence: the microwave background's chemical-potential distortion is zero. A receipt script checks the displayed identities and computed numbers, 33 of 33.

1 Time's arrow points downhill

Fundamental Density Theory has one substance, the omnium: matter = energy = spacetime. The chain

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t}\right)^2 \tag{1}$$

is one identity read by quantum mechanics, electromagnetism and general relativity (core 0.1) [1]. Its four terms adjust simultaneously, never one term at a time, and every step is a system-wide settlement of the state the system is in (0.1a).

The post that prompts this paper [3] states the standard account of time's arrow in four moves. The microscopic laws that govern atoms work identically forward and backward. Boltzmann's $S = k \log W$ counts the microscopic arrangements that match a system's outward state [11], and because disordered states have vastly more arrangements than ordered ones, isolated systems drift toward higher entropy, $dS \geq 0$. No microscopic law forbids a shattered glass from

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reassembling; the reassembly is astronomically improbable. The arrow is therefore the statistical echo of a universe unfolding from a low-entropy past. The account answers Loschmidt's objection that reversed velocities retrace every motion [13] and Zermelo's that a bounded system recurs [14] with improbability, and it places the source of the arrow in a special beginning [39, 40].

FDT reads the same record with the direction in the step itself. FDT canon states it in one line [2]:

> *Going back in time requires rolling the boulder uphill; going forward does not.*

Section 3 reads the step, Section 4 the hill it runs down, and Section 5 the boulder: what a return needs and what it costs. Section 6 reads kept time, Section 7 Boltzmann's count, Section 8 the same arrow at every size, and Section 9 the past without a first frame. The record, the tests, the declined readings and the convergence statement follow. The receipt script of Appendix A checks the displayed identities and computed numbers, and its check labels appear in brackets, [TA05c].

Two working rules carry the paper. Time is kept by patterns: a clock is a closed pattern that keeps time by circulating (0.1d), and every physical length and time belongs to a pattern, to a pattern-to-probe measurement, or to the floor (0.1b). Going back in time therefore means a system returned to an earlier state; there is no background axis to travel along. And the count belongs to the bookkeeper: the step is deterministic (I-4), Boltzmann's W counts the arrangements that the bookkeeper's record leaves unresolved, and the settlement never consults it, since the core names no objective for the optimisation (0.1a).

2 Premises, charts and symbols

The readings use the locked core v6.8 [1] and FDT canon [2]; each premise is quoted by section.

(P1) *One substance, only patterns.* There is one substance, the omnium, and there are only its patterns. A room, box or domain is a pattern's own support, never a vessel; every physical length and time belongs to a pattern, to a pattern-to-probe measurement, or to the floor (0.1, 0.1b).

(P2) *Every step is a settlement.* The chain's terms adjust together by one global optimisation, solved at every step from the state the system is in; the expected behaviour is not necessarily the evolving system's optimum. Events do not commute, and a process run backward is never the mirror of the forward event. Every settlement is m-to-n and global, m and n counting its components: the closures, open radiation, recoil and medium records that enter and leave it. A response is never computed particle by particle or pair by pair, and the core names no objective for the optimisation (0.1a).

(P3) *The floor.* $F_{\max}\, d_{\min} t_{\min} = \hbar$ with $d_{\min} = c t_{\min}$ gives $d_{\min} = 2\ell_P$ and $t_{\min} = 2t_P$. The floor is a minimum, not a staircase: it bounds every stroke below and is occupied by none (0.1a).

(P4) *The strand rate and kept time.* A closed pattern's strand runs at c on a helix, $v^2 + u^2 = c^2$ with translation v and circulation u, and the pattern keeps time by circulating, $c\, d\tau = u\, dt$. Time is circulated strand and space translated strand; mass is the circulating share of the energy, $E_0 = E \sin\vartheta$, and momentum the translating share (0.1d) [6].

(P5) *The open stroke.* A photon is one strand, unbound, a helix. Nothing closes it, so its run does not circulate: it carries the whole rate as translation, $v = c$, $u = 0$, and keeps no time. Its grades turn about the run on a cylinder of radius $c/\omega = \hbar c/E$, and the helix they trace stands at $\pi/4$ at every frequency: with no closure the stroke is non-dispersive,

$\omega = ck$, so its turning rate equals its run. Light is the unbound helix running at c along its axis, and it travels on concentric cylinders, higher energy on the smaller radii. A single strand does not close; the closures of I-5 are braids of two strands or more. Photon = graviton = gluon = phonon = magnon, one carrier (I-2) [7].

(P6) *The quantum face.* $\psi = Ae^{i\varphi}$ is the projection of a helix onto the plane normal to the cone axis; superposition is ignorance of the helical phase at observation; the universe is deterministic, and there are no probability amplitudes. Of the pooled pair, the core states that nothing of the pair is lost, and the record of the history is (I-4) [9].

(P7) *Closures, swap and ejection.* Matter is braided closures, and mass is a confinement eigenvalue whose operator is to be derived (core O-9). Decay is strand ejection for every closure: a closure sheds what its new box cannot hold, every book balancing across the event. A nucleus's perpetual strand swap keeps its strand total and is its stability; an ejection lowers the total, and that fall is the decay (I-5, I-5b).

(P8) *One account.* $\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \cdots$, every channel entering once: mass ≡ energy ≡ temperature ≡ pressure → density (0.5, I-7). Temperature is the clock factor of a medium's patterns (0.1c).

(P9) *Two phases and the seam.* On the A-1 compactness chart α is the share of the three-sphere. The Clifford torus $\alpha = 1/2$ is the seam between the matter phase, $\alpha > 1/2$, compressed omnium in closed braids, and the spacetime phase, $\alpha < 1/2$, decompressed substrate. The phase flow, matter phase into spacetime phase, is passage across the seam, and the seam $\mathfrak{J} : \alpha \mapsto 1 - \alpha$ is an involution, $\mathfrak{J} \circ \mathfrak{J} = \mathrm{id}$ (0.3a). No configuration occupies either end of the open interval (I-6).

(P10) *Release.* Matter is highly compressed omnium. When a body loses mass, by fission, decay, annihilation, or radiative or gravitational-wave emission, its compression relaxes toward the less dense state and the stored potential is released, in I-1a's downhill direction. The effective cosmological term is the running total of that release, $\Lambda_{\mathrm{eff}} = (8\pi G/c^2)\rho_{\mathrm{rel}}$; its amount is regional, rising where a region expands and falling where it contracts (0.7).

(P11) *No first frame.* There is no singularity anywhere and no beginning of time. Universal domains merge gently and inherit their structure, which unwinds by fission; a black hole does not unwind, the matter accreting onto it is converted to spacetime and energy, and the hole loses no mass and grows only by merging (0.8, I-1a) [5].

(P12) *Two readings, one flow sense.* A pattern's kept time and thread length rescale together with the density it crosses; on its own clocks and rulers nothing changes, and in the gravitational window every pattern at a point falls alike (0.1c) [8]. Every density cone carries a flow direction, its own circulation sense, intrinsic and fixed once for all, and matter and antimatter are the opposite handedness of one configuration (P-4).

Charts. The A-1 compactness chart appears in two places: the exterior inputs $\alpha_{\mathrm{ext},i} = R_{s,i}/d$ of the pair law between a boulder and the Earth in Section 5.3, with the Earth's window $\alpha_{\mathrm{ext}} = 1.39 \times 10^{-9}$ at its surface, and the share of the three-sphere with its seam at $\alpha = 1/2$ in Section 4.3. No other α is evaluated, and no computational token stands in any slot. The clock factor $\mathfrak{m}$ of 0.1c and the compression contrast δ of 0.7 are not α.

symbol	meaning
S, W, k_{B}	Boltzmann's entropy, the count of arrangements the record leaves unresolved, Boltzmann's constant
T_0	the temperature of the medium a system sits in, the clock factor of its patterns (0.1c)
E_{push}	the work a driver delivers to return a system to an earlier state
η	a driver's efficiency, the share of its spent work that reaches the system
u, v, ϑ, τ	circulation and translation of a strand, the pitch of a closed pattern's strand from the axis, the time a pattern keeps
f, Γ, t_e	the share of a record's components a push reaches; the rate at which components leave beyond its reach; the time of the push
$C(t)$	the coherence of a record of N components, $N^{-1}\left\lvert \sum_k e^{i\phi_k(t)} \right\rvert$
δ, $\dot{m}$, ρ_{rel}	the compression contrast a mass loss relaxes, the mass-loss rate density, the released density (0.7)
ϖ, s_ϖ	the microwave background's chemical-potential distortion and its measurement error
a_g	the measured acceleration of antihydrogen toward the Earth

3 The step

3.1 One settlement, solved from the state it starts from

Every step is a system-wide settlement of the omnium (0.1a). The global optimisation is solved at every step from the state the system is in, and its solution is the next state. The forward step needs that state and nothing else. It is m-to-n and global: the closures, open radiation, recoil and medium records that enter it are its m components, those that leave it its n, and a settlement with $m = n = 1$ is still the solution for the whole system. Events do not commute: two settlements taken in one order leave a different state from the same two taken in the other order, and a process run backward is never the mirror of the forward event.

This is where the arrow lives. The equations the standard account writes for the parts of an atom, a gas or a glass are even in time, or, like the Schrödinger equation on its cylinder, first order with a turn that runs the other way when reversed [4]; for each of them a solution run backward is a solution too. FDT keeps every one of those equations as a reading of the settlement in its regime (I-3, I-4) and declines the inference from the symmetry of an equation written for parts to the symmetry of the step. The reversed solution starts from a state in which every component the forward step touched has been turned around: each fragment, each atom of the floor that took up heat, each stroke of sound and light already running away at c. The settlement does not reach that state from the state the system is in. It has to be made, and making it is the push of Section 5.

3.2 The damped oscillator and the loop

The damped oscillator is the smallest exhibit [4]. Its motion $x = Ae^{-\gamma t}\cos(\omega_d t + \varphi_0)$ solves $m\ddot{x} + b\dot{x} + kx = 0$; run backward it solves $m\ddot{x} - b\dot{x} + kx = 0$, not the forward equation [TA03a]. The oscillator and the medium settle together at every step, and b is the reading of that settlement on the one coordinate the bookkeeper follows: the mode posts its energy to the medium's strokes, and nothing in the medium posts it back. A magnet driven around its loop shows the same at the scale of a body: the return path is not the forward path reversed, and the area between them is the energy released to the medium in each cycle.

3.3 A reaction and its reverse

The free neutron decays by a one-room balance through a transient eight-strand state that never propagates, $8 \to p(3) + e(3) + \bar{\nu}(2)$, the strand count and the charge both balancing [TA07b] (I-5b). Its mirror would be three closures converging into one room and closing there.

The record holds other settlements instead. A proton that captures a reactor antineutrino, as Cowan, Reines and their colleagues recorded with a fission reactor as the driver [31], leaves a neutron and a positron; an atomic electron captured by a proton-rich nucleus leaves a neutrino. Each is a forward settlement solved from its own state, with its own products, and neither is the neutron's decay run backward.

Where the record compares a transition with its time-reversed partner directly, it measures two numbers. In the neutral kaons [29] and in paired neutral B mesons, whose T-conjugated transitions, compared in four pairs, differ at 14 standard deviations [30], the two rates differ. In the reactions $^{27}\mathrm{Al} + p \rightleftharpoons {}^{24}\mathrm{Mg} + \alpha$ the relative differential cross sections agree within the experimental uncertainty [28]. FDT reads each pair as two settlements and takes both numbers from the record; the kernel that computes each rate is to be derived (D-01).

4 The hill

4.1 Compression is height

Matter is highly compressed omnium (0.7). A closure holds its strands circulating, and the circulating share of its energy is its mass (0.1d); its support lies in the matter phase, compressed omnium in closed braids (0.3a). Downhill is the release: when a body loses mass its compression relaxes toward the less dense state and the stored potential is released, in I-1a's downhill direction (0.7). Uphill is compression, the omnium pushed back into closed braids. Matter is created heaviest at $\alpha \to 1^-$ and unwinds by fission, heaviest first, through the Fe-56 attractor to the terminal exhaust of hydrogen and helium (I-1a). FDT canon uses the same image for stars: the standard account's fusion-driven build-up rolls the boulder uphill, and a star runs fission downhill [2].

I-7 puts every channel on the one axis: mass, energy, temperature and pressure enter $\mathcal{M}$ once and read as density. A boulder held apart from the Earth, a hot cup, a compressed gas, a written bit and an excited nucleus are each a configuration up the hill, each in its own channel, and each has a forward step that releases and a return that needs a push.

4.2 Release unbinds the helix

The forward step at its smallest is one strand leaving a closure. A nucleus's γ decay ejects one strand from the strokes standing in its room, and it leaves unbound, as a photon, while the nucleus's strand total falls by one (I-5b). In the helix triangle of (P4) its energy was part of the closure's rest energy, the circulating share, which the closure keeps as time by circulating, $u > 0$; released, it runs off unbound, $u = 0$ and $v = c$, its strand running at c along the axis of its helix, carrying its energy along the run and keeping no time (I-2). With $v = c\cos\vartheta$ and $u = c\sin\vartheta$ for a closed pattern's strand,

$$E_0 = E\sin\vartheta = \frac{E\,u}{c}, \qquad pc = E\cos\vartheta, \qquad E^2 = E_0^2 + p^2c^2, \tag{2}$$

and the open stroke is $u = 0$: no circulating share, no mass, no kept time, all of E on the run [TA02e, TA02f]. Its helix is unbound: its grades turn about the run on a cylinder of radius c/ω, and the helix stands at $\pi/4$ at every frequency (I-2). A single strand does not close (I-2). The stroke cannot re-close by itself; only a closure it meets takes it in, in a settlement of that closure's own.

Every larger release is built from the same moves. A nucleus ejects bundles that split outside it into the three-strand and two-strand closures (I-5b); a star unwinds by fission, lighter elements fused within the fission where its many-to-many settlement calls for it (I-1a); a hot body posts its energy to the medium's strokes, which run between whole redirections [4]; and at a black hole's fold the matter accreting onto it is converted to spacetime and energy (0.8).

4.3 The seam is an involution; the flow across it is not

The seam $\mathfrak{J} : \alpha \mapsto 1 - \alpha$ exchanges the two moduli of the three-sphere at fixed phases, and $\mathfrak{J} \circ \mathfrak{J} = \mathrm{id}$: a jacket turned inside out twice is as it was, and the seam is the one locus that reads the same garment either way (0.3a). The flow across the seam is not an involution. Passage from the matter phase into the spacetime phase is the release of 0.7, and its reverse is compression, which needs a push. The support has its involution and the step has none, as the equations written for parts are even in time and the step is not.

The open interval adds that no configuration occupies either core circle (I-6): no state is fully compressed and none is fully released. There is always further to fall.

5 The boulder

5.1 What a return needs

Going back in time requires rolling the boulder uphill; going forward does not [2]. Each half of the statement rests on the step.

Going forward needs nothing but the state the system is in. The settlement is solved from that state (0.1a), and the release it settles needs no input from outside the system.

Going back needs every component the forward step released turned and delivered back, with a closure ready to take each one in. That is the boulder rolled uphill. The released strokes run at c along their axes and keep no time; nothing closes a single strand (I-2); and a medium record spread through the strokes of a floor or a room is already part of the medium's own settlements. A system's own settlement, solved from the state it is in, does not gather them.

The push that gathers them comes from a driver, a system that delivers work to the one being returned. The driver's step is itself a forward settlement: it releases.

5.2 Four consequences

(B1) *The whole never returns.* A push comes from outside the system it returns. A whole system, everything its settlements reach, has nothing outside it to push, so no settlement returns a whole system to an earlier state.

(B2) *A part returns only with a push, and the push runs downhill.* The driver releases at least what the part regains; Section 5.3 gives the price in three channels.

(B3) *A push returns what it reaches.* A part returned by a push recovers the share of the forward step's components that the push reaches. What was released beyond its reach, the strokes that left the apparatus and the records in a medium outside it, stays released (Section 5.4).

(B4) *The strand ledger runs one way by itself.* By its own settlement a nucleus keeps its strand total, in the swap, or lowers it, in an ejection (I-5b); it raises the total only by taking in strands delivered to it. Excitation and capture need a delivery; de-excitation and decay do not.

5.3 The price of a return in three channels

The gravitational channel. The pair law is exact on its declared domain of nonoverlapping spherical supports at monopole order (I-1):

$$F = F_{\max}\,\alpha_1\alpha_2, \quad \alpha_i = \frac{R_{s,i}}{d} \quad\Longrightarrow\quad F = \frac{G\mathcal{M}_1\mathcal{M}_2}{d^2}, \qquad U(s_0) = -\int_{s_0}^{\infty} F\,ds = -\frac{G\mathcal{M}_1\mathcal{M}_2}{s_0}, \tag{3}$$

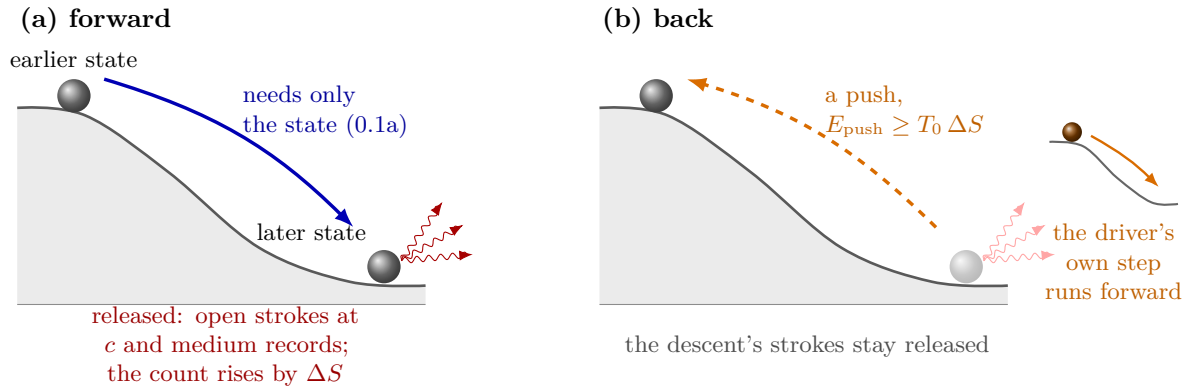


Figure 1: The boulder and the hill. (a) The forward step from the earlier state needs nothing but that state; it releases open strokes and medium records, and the bookkeeper's count rises by ΔS. (b) The return needs a push of at least $T_0\Delta S$ from a driver whose own step runs forward and releases; the strokes the descent released stay released.

with $F = \partial U/\partial s_0$ [TA04a, TA04b]. A boulder of 1000 kg held 10 m above the ground sits up the hill by the difference of two open line integrals, $U(R+h) - U(R) = G\mathcal{M}m\,h/[R(R+h)] = mgh\,[1 + O(h/R)] = 98.2\,\text{kJ}$ [TA04c]. The Earth's own window at its surface is $\alpha_{\text{ext}} = R_s/R = 1.39 \times 10^{-9}$, a small number and the correct reading, and the boulder's weight is far below $F_{\max}$ [TA04e]. The descent turns that height into the boulder's motion, then into heat and sound in the ground and the air; as the strokes leave, the pair's total gravitating energy falls by $mgh/c^2 = 1.09 \times 10^{-12}\,\text{kg}$ [TA04d]. The return needs a driver to spend $mgh/\eta \geq mgh$, and the heat of the descent stays spread: after the round trip the released total is $mgh/\eta \geq mgh$, never less than after the descent alone, and equal to it only for a driver of efficiency one [TA04f].

The thermal channel. A cup of 0.30 kg of coffee at 80 °C in a room at $T_0 = 293.15\,\text{K}$ cools to the room's temperature. The heat that leaves it is $Q = 75.3\,\text{kJ}$, so its total gravitating energy falls by $Q/c^2 = 8.38 \times 10^{-13}\,\text{kg}$ [TA05a]. The cup's entropy falls by $233.7\,\text{J K}^{-1}$ and the room's rises by $Q/T_0 = 256.9\,\text{J K}^{-1}$, so the bookkeeper's ledger records $\Delta S = +23.2\,\text{J K}^{-1}$ [TA05b]. Rolling the cup back up to 80 °C with a perfect heat pump costs at least

$$E_{\text{push}}^{\min} = mc_p\Big[(T_h - T_0) - T_0 \ln \frac{T_h}{T_0}\Big] = T_0\,\Delta S = 6.79\ \text{kJ}, \tag{4}$$

the boulder's height in the thermal channel [TA05c]. The cooling gave those 6.79 kJ of work up for nothing; no driver spent work on the forward step [TA05e].

The record channel. Resetting one bit of a record to a reference value lowers the record's count by $k_{\text{B}} \ln 2$, and the least work a driver spends on it in a medium at T_0 is $k_{\text{B}}T_0 \ln 2$, $2.87 \times 10^{-21}\,\text{J}$ or 0.0179 eV at 300 K [TA06a] [19]. Bérut and colleagues erased the bit of a colloidal bead in a modulated double well and measured the mean dissipated heat saturating at this bound in the limit of long erasure cycles [20].

The price, in general. Read in the thermal channel, the boulder is the statement of Kelvin, that heat released into a medium at one temperature drives no cycle that rolls a weight back up [15], and the statement of Clausius, that heat passes from a colder body to a hotter one only when something pushes it [16]. Clausius's derivation then gives the entropy function and its inequality [17], and with them the least work any driver spends to return a system whose forward step raised the bookkeeper's count by ΔS in a medium at T_0:

$$E_{\text{push}} \;\geq\; T_0\,\Delta S \qquad \text{(back)}, \qquad\qquad E_{\text{push}} = 0 \qquad \text{(forward)}, \tag{5}$$

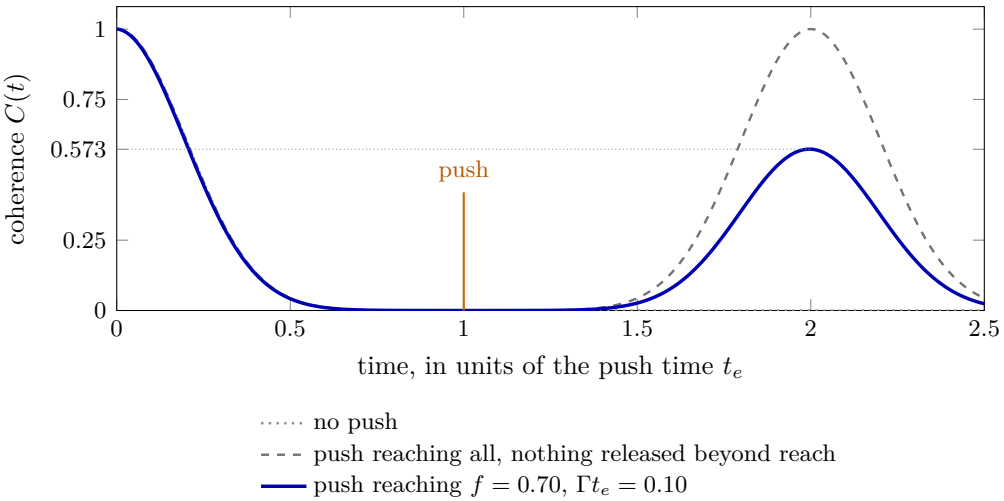


Figure 2: The reach of a push, equation (6). A record of components turning at their own rates dephases; a push at t_e reverses the phases of the share it reaches, and the record returns at $2t_e$ to $fe^{-2\Gamma t_e}$. Without the push it stays dephased. Curves for a frequency spread $\sigma_\omega t_e = 5$.

the lost work of Gouy [18]. The three channels above are its instances: mgh for the boulder, 6.79 kJ for the cup, $k_\mathrm{B}T_0 \ln 2$ for the bit. Clausius's reversible cycles are the limit in which the push of each direction falls to zero with ΔS; each direction stays a forward settlement, driven by its own small contrast.

5.4 The reach of a push: the laboratory rewinds

A record is a sum of components. Let N components turn at their own rates ω_k, so that the record's coherence $C(t) = N^{-1}\left|\sum_k e^{i\omega_k t}\right|$ falls as they dephase. At t_e a push reverses the accumulated phases of the components it reaches, a share f, and every component also loses amplitude at the rate Γ to components released beyond reach. At $2t_e$ the reached components are back in phase and the rest are not:

$$C(2t_e) = f\, e^{-2\Gamma t_e} + O(N^{-1/2}), \qquad C(2t_e)\big|_{\text{no push}} = O(N^{-1/2}). \tag{6}$$

A Monte Carlo record of 200 000 components, fully dephased at the push, returns 0.574 against $fe^{-2\Gamma t_e} = 0.573$ for $f = 0.70$ and $\Gamma t_e = 0.10$, stays below 0.01 without the push, and follows (6) as the reach and the release are varied [TA08a, TA08b, TA08c]. Figure 2 draws it. What the push returns is what it reaches, (B3).

Every laboratory rewind has this form. Hahn's spin echo returns the dephased magnetisation of nuclear spins with a second radio-frequency pulse, and the echo shrinks with the interval as the spin–spin and spin–lattice settlements carry the record beyond the pulse's reach [21]. Rhim, Pines and Waugh made the dipolar evolution of a solid's coupled spins run as if the sign of its Hamiltonian were reversed, with sequences of strong radio-frequency fields that they called a Loschmidt demon, and recovered magnetisation from states that read as decayed to equilibrium [22, 23]; the sequences return what they reach, and the echo falls short by the rest. Fink's time-reversal mirrors record an ultrasonic field on an array of transducers and re-emit it reversed, refocusing it on its source through an inhomogeneous medium [24]: the array is powered, and it refocuses what its aperture captured. The Brillouin mirror of Zel'dovich and colleagues returns a light beam with its wavefront reversed, by stimulated scattering driven by the beam itself [25]. Each rewind is a driven return of a part: the push is supplied, and it returns what it reaches.

6 Kept time runs forward

A clock is a closed pattern keeping time by circulating (0.1d):

$$v^2 + u^2 = c^2, \qquad c\,d\tau = u\,dt \qquad \Longrightarrow \qquad \frac{d\tau}{dt} = \frac{u}{c} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{\gamma} \tag{7}$$

[TA02a]. Every closed pattern circulates, $u > 0$, and its kept time grows; only the open stroke, $u = 0$, keeps none [TA02b]; its strand runs at c along the axis of its unbound helix, and the turning of that helix is its phase, never a clock (I-2). Kept time is circulated strand, accumulated, and it accumulates in either sense of circulation. The sense is the cone's flow direction, intrinsic and fixed once for all (P-4), and a pattern of the opposite sense is another configuration: the opposite handedness of one configuration is matter and antimatter (P-4), and the antimatter pattern keeps time forward by its own circulation. No closed pattern keeps time backward.

The CERN muon storage ring held muons at $\gamma = 29.33$. Their strands circulate at $u = 0.0341\,c$ and translate at $v = 0.99942\,c$ [TA02c]. Their measured lifetime, $64.419(58)\,\mu$s for positive muons, is the lifetime at rest dilated by γ, $64.44\,\mu$s, within one standard error [TA02d], and the time-dilation factor agrees with experiment to 2×10^{-3} at 95 per cent confidence [32]. The stored muon keeps 0.0341 of a second for every second of the ring's clocks: the circulating share of its strand's run.

Antihydrogen released from the ALPHA-g trap falls toward the Earth with $a_g = [0.75 \pm 0.13\,(\text{statistical+systematic}) \pm 0.16\,(\text{simulation})]\,g$ [33]: 1.2 combined errors from g, and 8.5 from $-g$ [TA12a]. In the gravitational window every pattern at a point falls alike (0.1c), the opposite-handed configuration with the rest. A published reading that carries the CPT transformation of matter into antimatter into general relativity concludes that matter and antimatter repel [34]; the record lies 8.5 errors from that value.

7 The count

7.1 Boltzmann's ledger

The universe is deterministic, and probability is ignorance of the helical phase at observation (I-4). Boltzmann's W counts the arrangements of a system, its helical phases, seat placements and stroke positions, that the bookkeeper's record leaves unresolved, and

$$S = k_{\mathrm{B}} \ln W \tag{8}$$

is the bookkeeper's ledger. The step does not consult it: the core names no objective (0.1a). The forward step spreads released content over more components, m-to-n, the strokes running outward and the medium's records multiplying, and the count compatible with the record grows. A hundred quanta shared among n components have $\ln W = \ln \binom{q+n-1}{q}$, rising strictly from 0 at $n = 1$ to 29.1 at $n = 10$ and 331.8 at $n = 1000$ [TA09a]. Nothing is lost in the spread; the record of where each component went is. The core reads the pooled pair the same way: nothing of the pair is lost, and the record of the history is (I-4).

7.2 Unpushed, not improbable

The standard account reads the reassembled glass and the re-heated cup as allowed and improbable; for the cup the odds are $e^{-\Delta S/k_{\mathrm{B}}} = e^{-1.68\times10^{24}}$ [TA05d]. FDT reads the return as unpushed. From the state the system is in, the settlement runs forward, and a return happens only when something delivers the push. With (5),

$$e^{-\Delta S/k_{\mathrm{B}}} = \exp\Big(-\frac{E^{\min}_{\mathrm{push}}}{k_{\mathrm{B}} T_0}\Big)\,: \tag{9}$$

Boltzmann's odds are the least push, 6.79 kJ for the cup, counted in units of the medium's thermal energy.

7.3 Small parts and the medium's push

Over short times a small part receives pushes from the medium it sits in. A colloidal bead dragged through water by an optical trap shows entropy-consuming trajectories at the frequencies the fluctuation theorem gives [26], and the work distributions of an RNA hairpin folded and unfolded follow Crooks's ratio of forward to reverse [27]. Those trajectories are pushes the medium delivers by its own forward steps, at frequencies that fall as $e^{-\Delta S/k_{\mathrm{B}}}$ with the size of the return. The bead returns; the medium does not; the whole does not (B1). The frequencies are accepted as data, and their computation from the measure of unread helical phases is to be derived (D-02).

7.4 Loschmidt and Zermelo

Loschmidt's reversal, every velocity of a gas turned at once [13], is a push that reaches every component. The standard account answers it with improbability; FDT answers it with the price, since the reversed state has to be made by a driver that reaches every component, the strokes that left at c included. Zermelo's recurrence [14] needs a system isolated from everything else with a bounded flow that keeps its measure. FDT has no pattern isolated from the omnium: interaction pairs are available around every point, and there are only patterns of the one medium (0.1, 0.1b). A release does not return without a push. Where nothing has been released the motion repeats: an undamped oscillator, an orbit, the turning phases of a closed pattern drifting apart and back together. The arrow is read where a settlement releases.

8 One arrow at every size

The chain is all ratios and α is scale invariant, so the support, its seam and its two phases have the same form at every size, bounded below by the floor (0.3a). The arrow repeats with them (Table 1).

size	the forward step, needing only the state	the return, needing a push
nucleus	strand ejection: a bundle, or a single strand leaving as a photon (I-5b)	excitation or capture, with the strands delivered (B4)
closure	an open stroke leaves, keeping no time (I-2)	a stroke delivered to a closure that takes it in
body	heat posted to the medium's strokes; a glass breaking; friction [4]	a heat pump; a furnace and a glassblower; the price (5)
star	fission downhill through the Fe-56 attractor, lighter elements fused within it where its settlement calls for it (I-1a)	compression into heavier matter, which took an unimaginable time [5]
black hole	the matter accreting onto it converted to spacetime and energy; a parcel reaching the fold decompresses at once (0.8) [5]	none at the hole: it loses no mass and grows only by merging (0.8)
domain	the released level accumulates, (10) (0.7)	a region's amount falls where it contracts, which an expansion elsewhere requires (0.7)

Table 1: The same arrow at every size. Each row is a forward step that needs only the state and a return that needs a push.

At the largest size the release has a ledger of its own. The effective cosmological term is the

released density of the domain Ω, accumulated over its history (0.7):

$$\rho_{\mathrm{rel}}(t) = \int^{t} dt' \, \frac{1}{V_\Omega} \int_\Omega \delta \, \dot{m} \, dV, \qquad \frac{d\rho_{\mathrm{rel}}}{dt} = \frac{1}{V_\Omega} \int_\Omega \delta \, \dot{m} \, dV \;\geq\; 0, \qquad \Lambda_{\mathrm{eff}} = \frac{8\pi G}{c^2} \, \rho_{\mathrm{rel}}, \tag{10}$$

with $\delta \geq 0$ and $\dot{m} \geq 0$, so the domain average never falls [TA10a, TA10b]. The amount rises in some regions and falls in others, an expansion here requiring a contraction elsewhere (0.7); the domain average grows with cumulative unwinding and conversion. The integral has an upper limit and no lower one.

9 No first frame

The standard account places the source of the arrow at a beginning in a state of low entropy: as Boltzmann's own fluctuation of a universe otherwise in equilibrium [12], as the past hypothesis [39], or as a smooth beginning with vanishing Weyl curvature [40]. FDT has no beginning. No variable of the chain vanishes, force is capped by $F_{\max}$ and α lives on the open interval with a regular core, so there is no singularity anywhere and the record has no first frame (0.8) [5].

The arrow needs none, because it is in the step, and the hill it runs down is carried now by every compressed pattern. Every nucleus, star and galaxy is an inherited seed holding compression, unwinding by fission (0.8, I-1a). That compression took an unimaginable time to build; a parcel reaching a black hole's fold decompresses at once, while a galaxy as a whole decompresses as slowly as it was compressed [5]. The domain's ledger (10) runs to an upper limit from no first moment. The backward look converges by perspective, toward a vanishing point that travels with the viewer [5].

Deterministic is not reversible. With perfect information about the state of a system its past can be computed, and its future with it; the three do not coexist. The past is the record of settlements already made, and events do not commute, so the record cannot be run backward like a film [2]. Going back is a rebuilding, and its price is (5).

10 The record and the tests

what the record shows	the measurement	where it is read
heat runs one way by itself	no engine turns the heat of one medium into the work that raises a weight [15]; heat passes from cold to hot only with a push [16]	(5)
the price of a reset	the mean heat of erasing a bit saturates at $k_{\mathrm{B}}T \ln 2$ in the limit of long cycles [20]	(5); TA06a
driven rewinds of spins	the spin echo and its decay with the interval [21]; magnetisation recovered by a reversed dipolar evolution [22, 23]	(6)
driven rewinds of waves	time-reversal mirrors refocus on the source [24]; a Brillouin mirror reverses a wavefront [25]	(6)
small parts, short times	entropy-consuming trajectories at the fluctuation theorem's frequencies [26]; Crooks's ratio in RNA unfolding [27]	Section 7
a reaction and its reverse	$^{27}\mathrm{Al} + p \rightleftharpoons {}^{24}\mathrm{Mg} + \alpha$ cross sections agree within the uncertainty [28]; T-conjugated kaon and B-meson transitions differ, the latter at 14 standard deviations [29, 30]	Section 3.3
kept time	muons stored at $\gamma = 29.33$ live $64.419(58)\,\mu$s; time dilation to 2×10^{-3} at 95 per cent [32]	(7)
antimatter falls	$a_g = (0.75 \pm 0.13 \pm 0.16)\, g$ [33]	Section 6; T-5
the microwave spectrum	a blackbody with $\lvert\mu\rvert < 9 \times 10^{-5}$ at 95 per cent [41]	Section 9; T-3

Each test below is the converse of a derived result: the measurement that would end it.

T-1 *The unpushed return.* A system that has released content, by radiating, decaying, cooling or breaking, found back in its earlier state with no driver, more often than $e^{-\Delta S/k_{\mathrm{B}}}$ relative to its forward step, the frequency at which the medium's own pushes deliver such a return, ends the boulder statement.

T-2 *The price.* A return of a system for less work than $T_0\Delta S$, averaged over repetitions, ends (5); a bit reset for less than $k_{\mathrm{B}}T_0 \ln 2$ on average is its smallest case.

T-3 *Zero chemical potential.* With no first moment, the microwave background's accumulated state has chemical-potential distortion $\varpi = 0$, where the dissipation history of a hot origin gives $1.57^{+0.11}_{-0.13} \times 10^{-8}$ [42], or $(2.3 \pm 0.14) \times 10^{-8}$ in the white paper's convention [43]; the test is carried from *A Day With a Yesterday*, where it is T-2 [5]. A measured $\hat{\varpi} > 3s_{\varpi}$ ends the zero. At Super-PIXIE's eight-year error, 7.7×10^{-9}, that threshold is 2.31×10^{-8}, and at zero true distortion the lower-comparator rule of that paper rejects the dissipation history with probability 0.16, rising to 0.90 at error 3.5×10^{-9} [TA11a, TA11b]. The FIRAS bound lies 5.7×10^3 times above the comparator [TA11c]. This is the paper's divergence.

T-4 *The domain's ledger.* A decrease of the domain-average released level over the domain's history ends its reading as the running total of release: (10) has $\delta \geq 0$ and $\dot{m} \geq 0$.

T-5 *Antimatter keeps time forward and falls.* A measured a_g/g other than 1 at three combined errors ends the universality of free fall in the gravitational window for the opposite-handed configuration (0.1c, P-4). The record sits 1.2 errors from 1.

Five pieces sit outside the derivation and leave every result in it unchanged:

D-01 *Rates.* The settlement kernel's rates, of an ejection, of the release to a medium that (6) writes as Γ, and of each member of a reaction and its reverse, are to be derived (core O-12).

D-02 *The count from the helix chart.* $S = k_{\mathrm{B}} \ln W$ and the fluctuation theorems' frequencies, from the measure of unread helical phases, are to be derived.

D-03 *One height.* A single height on the compression axis that orders configurations across every channel of I-7, with $T_0\Delta S$ as its thermal reading, is to be derived.

D-04 *The release function.* $\dot{m}(x)$ and $\delta(x)$, which feed Λ_{eff}, are to be derived (core O-14).

D-05 *The building of the hill.* What compressed the omnium into the matter every inherited seed carries, and the flow law of the two phases, are to be derived (core O-15).

11 Declined readings and convergence

The standard account reads the microscopic laws as running the same forward and backward, a process and its reverse as mirror events, with one matrix element for both directions in the quantum account, and detailed balance as their consequence [10, 13]. FDT reads every step as the system-wide, m-to-n settlement of the state it starts from: events do not commute, and a process run backward is never the mirror of the forward event (0.1a).

The standard account reads the arrow as statistical: the reassembly of a shattered glass breaks no law and is only improbable [11, 3]. FDT reads the return as unpushed: it needs the boulder rolled uphill at a price of at least $T_0\Delta S$, and the count is the bookkeeper's ledger of the step, never its cause or its objective (I-4, 0.1a).

The standard account places the source of the arrow in a low-entropy beginning [39, 40]. FDT has no beginning and needs none: the arrow is in every step, and the hill is carried now by every compressed pattern (0.8, 0.7).

The standard account reads a bounded isolated system as returning arbitrarily close to its initial state after a recurrence time [14]. FDT has no pattern isolated from the omnium, and a release does not return without a push, while what has released nothing repeats (0.1b, 0.1a).

The standard reading of relativity's four-dimensional geometry takes past, present and future as existing alike in one block [38]. FDT reads the past as the record of settlements already made and the future as not yet settled: perfect information computes both, and they do not coexist [2].

The standard account reads an antiparticle as a particle running backward in time [35, 36]. FDT reads it as the opposite handedness of one configuration, keeping time forward by its own circulation (P-4, 0.1d), and falling as every pattern falls in the gravitational window (0.1c).

The standard account writes radiation from time-symmetric equations and selects the outgoing wave by a boundary condition, or by the response of a universal absorber [37]. FDT reads the outgoing stroke as the release of the settlement that emits it, one strand, unbound, a helix at $\pi/4$ running at c along its axis and keeping no time, and a converging stroke as a push that a driver prepares (I-2).

Convergence. For the same source and data FDT returns the record of time's arrow as it stands. The statements of Kelvin and Clausius are the boulder read in the thermal channel; Clausius's entropy and his inequality follow by his derivation; the least work of a return is $T_0\Delta S$, with Landauer's bound as its smallest case and the measured heat of an erasure saturating there. Boltzmann's count is kept as the bookkeeper's ledger (I-4); the measured frequencies of small parts, the fluctuation theorems' and Crooks's ratio among them, are accepted as data, and their computation from unread helical phases is D-02. The equations written for parts and the field equations stand unchanged, each solution with its reversal (I-3); the settlement runs the forward one. The rewinds return what their pushes reach, as (6) and the echoes record, and the decay law is the count of whole ejections [4]. A reaction and its reverse are two settlements whose rates the record measures, and FDT takes both records as they stand: the nuclear cross sections that agree within their uncertainty and the kaon and B-meson rates that differ; the kernel that computes each is D-01. Kept time is the helix triangle exactly, $d\tau/dt = 1/\gamma$, with the stored muons' dilation, and antimatter falls as matter falls. FDT supplies one additional source: the domain has no first moment, and the microwave background's accumulated state carries $\varpi = 0$ where the dissipation history of a hot origin carries 1.57×10^{-8} (T-3).

12 Time's arrow points downhill

Every step of the omnium is one settlement, solved from the state the system is in, and events do not commute. The forward step releases compression, energy a closure held in circulation leaving as open stroke, an unbound helix running at c, and needs nothing but the state. Going back needs the boulder rolled uphill: every released component turned and delivered back, by a driver whose own step runs forward, at a price of at least $T_0\Delta S$. A part returns with a push and recovers what the push reaches; the whole never returns. Boltzmann's count is the ledger of the step and its odds are the price of the push; the clocks that keep time are closed patterns circulating forward; and the hill is carried now, by every compressed pattern, with no first frame behind it. Time's arrow points downhill.

13 Status register

result	rests on	status
Every step is the settlement of the state it starts from; a process run backward is never the mirror of the forward event	0.1a	core
Going back in time requires rolling the boulder uphill; going forward does not	FDT canon [2]	canon
Release unbinds the helix; the open stroke runs at c along the axis of its helix at $\pi/4$, keeps no time and does not close (2)	0.1d, I-2	derived
The whole never returns; a part returns only with a push; the push returns what it reaches; the strand ledger runs one way by itself (B1–B4)	the boulder, 0.1a, I-2, I-5b	derived
The price of a return, $E_{\text{push}} \geq T_0 \Delta S$, in the gravitational, thermal and record channels ((3)–(5))	the boulder, I-1, I-7, 0.1c	derived
The reach of a push (6)	the boulder, I-4	derived; the rates D-01
Kept time runs forward, antimatter with it (7)	0.1d, P-4, 0.1c	derived
The count as the bookkeeper's ledger; Boltzmann's odds as the price of the push ((8), (9))	I-4, 0.1a, (5)	derived; from the helix chart D-02
One arrow at every size (Table 1)	0.3a, I-5b, I-1a, 0.8, 0.7	derived
The domain's ledger only grows (10)	0.7	derived; the release function D-04
No first frame; deterministic is not reversible	0.8, I-4, FDT canon	core; canon
One height across every channel; the building of the hill	I-7; 0.8	to be derived (D-03; D-05)

invariant	where it enters
I-1	the pair law $F = F_{\max}\alpha_1\alpha_2 = G\mathcal{M}m/d^2$ and its open line integrals, the boulder's height in the gravitational channel (Section 5.3)
I-2	one carrier; the open stroke as one strand, unbound, a helix at $\pi/4$ on a cylinder of radius c/ω, keeping no time and never closing by itself (Sections 4, 5)
I-3	the field equations unchanged, each solution with its reversal; containment (Sections 3, 11)
I-4	determinism; superposition as ignorance of phase; the count as the bookkeeper's ledger; the pooled record (Section 7)
I-5	closures and their ejections, the strand ledger and the free neutron's one-room balance (Sections 3.3, 5.2)
I-6	the open interval: no configuration fully compressed or fully released (Section 4.3)
I-7	every channel on one axis: mass, energy, temperature and pressure as heights of the hill (Sections 4, 5.3)

A Receipt

The script `receipt_arrow_of_time.py`, in the folder of this paper, checks the displayed identities symbolically with SymPy and the computed numbers numerically with the CODATA 2022 values and the IAU 2015 nominal terrestrial $G\mathcal{M}$, with a Monte Carlo record of 200 000 components for the reach of a push. Run it with `python3 receipt_arrow_of_time.py`; its output is `receipt_arrow_of_time.out.txt`. Result: 33/33 PASS.

check	content
TA01a–b	the floor $d_{\min} = 2\ell_P = 3.23 \times 10^{-35}\,\mathrm{m}$, $t_{\min} = 2t_P = 1.08 \times 10^{-43}\,\mathrm{s}$; $E_{\mathrm{floor}} = E_P/2$
TA02a–f	kept time on the helix triangle; $d\tau/dt > 0$ for every closed pattern and zero for the open stroke; the stored muons at $\gamma = 29.33$; the circulating share as mass and the open stroke's energy on the run of its unbound helix
TA03a	the damped oscillator run backward solves $m\ddot{x} - b\dot{x} + kx = 0$, not the forward equation
TA04a–f	the pair law and its open line integrals; $mgh = 98.2\,\mathrm{kJ}$; $mgh/c^2 = 1.09 \times 10^{-12}\,\mathrm{kg}$; $\alpha_{\mathrm{ext}} = 1.39 \times 10^{-9}$ at the Earth's surface; the round trip never lowers the released total
TA05a–e	the cup: $Q = 75.3\,\mathrm{kJ}$, $Q/c^2 = 8.38 \times 10^{-13}\,\mathrm{kg}$, $\Delta S = +23.2\,\mathrm{J\,K^{-1}}$, the least work $T_0\Delta S = 6.79\,\mathrm{kJ}$ two ways, $\Delta S/k_{\mathrm{B}} = 1.68 \times 10^{24}$
TA06a	the reset of a bit, $k_{\mathrm{B}}T_0 \ln 2 = 2.87 \times 10^{-21}\,\mathrm{J}$ at $300\,\mathrm{K}$
TA07a–b	the strand ledger of the $2 + 2$ ejection and of the free neutron's one-room balance
TA08a–c	the reach of a push: dephasing, the echo $fe^{-2\Gamma t_e}$, and its scaling with the reach and the release
TA09a	the count of a release spread over more components rises strictly
TA10a–b	the domain's released density never falls; the units of Λ_{eff}
TA11a–c	the chemical-potential test: the threshold 2.31×10^{-8}, the rejection probabilities 0.16 and 0.90, the FIRAS ratio
TA12a	the antimatter fall: 1.2 errors from g, 8.5 from $-g$

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