

# The Born Rule as the Seating Law of Helical Patterns

A derivation in Fundamental Density Theory from the locked core v5.6

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## Abstract

The Born rule is derived from the definitions and the seven invariants of Fundamental Density Theory (locked core v5.6). A pattern meeting a probe is one helix, and its projection onto each port of the probe is a point of that port's plane,  $\psi_k = A_k e^{i\varphi_k}$  (I-4). The response coefficients of the omnium make the energy the pattern brings to port  $k$  equal to the fraction  $A_k^2$  of one quantum (0.1, 0.1a), and a closed pattern seats whole in one port (I-5). Three routes with different machinery then fix the seating frequency of port  $k$ : a frame-function route (phase blindness, integral seating and Cauchy's additive equation), an energy-ledger route (the bright-stroke energy flow counted in whole quanta), and a seating route that supplies the deterministic single-event mechanism, in which the pattern seats in the first port whose closure drive overtakes the resident disturbance of the non-empty omnium in that port mode. All three return  $\text{Pr}(k) = A_k^2 = |\langle \hat{\mathbf{e}}_k, \psi \rangle|^2$  exactly, for every port count, every carrier reading and every coupling strength. The seating route also returns the time-resolved law (the seating instant is uniform along the delivered fraction of the stroke and independent of the port), the projection update, the joint law of a pattern shared by two probes with local frequencies blind to the distant frame, and the geometry of the rule: the Born weights are the volume fractions of the cells into which the pattern's share point divides the simplex of resident shares. The only statistical input is the law of the resident disturbance, which phase ignorance (I-4) and frame invariance fix. Ten numerical checks confirm every statement. At this interface FDT recovers the consensus law exactly and supplies no additional input that changes an observable; the derivation closes in a convergence statement.

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## 1 Result

Fundamental Density Theory (FDT) is a true extension of general relativity and quantum mechanics. Quantum mechanics postulates the Born rule [1]; FDT derives it. The derivation uses the definitions and the seven invariants of the locked core (v5.6, author lock 2026-10-01). The consensus record enters only as data, and the mathematics borrowed is named where it enters.

A pattern meets a probe whose ports  $\hat{\mathbf{e}}_1, \dots, \hat{\mathbf{e}}_{N_p}$  are modes of the probe's own support. The pattern is one helix (I-4). Its projection onto the plane of port  $k$  is a point of that plane,

$$\psi_k = A_k e^{i\varphi_k} = \langle \hat{\mathbf{e}}_k, \psi \rangle, \quad A_k \geq 0, \quad (1)$$

where  $A_k$  is the projected radius and  $\varphi_k$  the helical phase. The result is

$$\boxed{\Pr(\text{the pattern seats in port } k) = A_k^2 = |\langle \hat{\mathbf{e}}_k, \psi \rangle|^2} \quad (2)$$

for every port count, every carrier reading and every coupling strength. The seating instant is uniformly distributed along the delivered fraction of the pattern's stroke and is independent of the port, and after seating the pattern's projection is the seated port's mode. Three routes with different machinery reach (2):

- Route 1. The frame function: phase blindness (I-4), integral seating (I-5) and Cauchy's additive equation (Section 4).
- Route 2. The energy ledger: the bright-stroke energy flow of the omnium's linear response, counted in whole quanta (0.1, 0.1a, I-5; Section 5).
- Route 3. The seating mechanism: a deterministic single-event rule in which the pattern seats in the first port whose closure drive overtakes the resident disturbance of the non-empty omnium in that port mode (0.1, I-4, P-2; Section 6).

Route 3 derives the premise of Route 1 and the premise of Route 2, and it carries the time-resolved law, the projection update and the joint law of a shared pattern (Section 7). Section 14 reports ten numerical checks, each by a method structurally different from the proof it tests.

## 2 What the core supplies

The derivation draws on these statements of the locked core, each quoted by its locator.

- (C1) *One substance and its response (0.1)*. The chain  $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$  is one identity;  $\varepsilon_0$  and  $\mu_0$  are response coefficients of the omnium. No variable of the chain vanishes and density is nowhere uniform, so no region and no mode of the medium is empty.
- (C2) *Quantisation (0.1a)*.  $E_q = m_q c^2 = \hbar\omega_q$ , and a pattern keeps the time  $T_q = 2\pi/\omega_q$ . A pattern quantised in energy is quantised in mass, kept time and thread length by the identical stroke.
- (C3) *Only patterns (0.1b)*. A probe is a pattern, a port is a region of the probe's own support, and a probe reconstructs one pattern's response.
- (C4) *One carrier (I-2)*. Every stroke reading (light, sound, the magnon) is the same spin-1 excitation of one medium at one speed.
- (C5) *The quantum face (I-4)*.  $\psi = Ae^{i\varphi}$  is the projection of a helix onto the plane normal to the cone axis. Superposition is ignorance of the helical phase at observation; the universe is deterministic; measurement is projection and phase fixation; correlated patterns share a bivector grade.
- (C6) *Closed patterns (I-5, I-5a)*. Matter patterns are closed braids of the carrier, mass is the mode cost  $m_q c^2 = \hbar\omega_q$  of the confinement eigenproblem, and the ledger is integral.
- (C7) *Nodal locking (P-2)*. Excitations migrate to and lock into the nodal channels of the driven medium.
- (C8) *Containment (I-3)*.  $G_{\mu\nu} = \kappa T_{\mu\nu}$  is unchanged, and patterns follow its geodesics.

## 3 Patterns, probes and ports

**Definition 1** (Port set). A port set of a probe at the angular frequency  $\omega_q$  is a list of  $N_p$  modes  $\hat{\mathbf{e}}_1, \dots, \hat{\mathbf{e}}_{N_p}$  of the probe's support whose stored energies add without cross terms and which together receive the whole of the pattern's stroke. A lossy probe is completed by its loss channel, counted as one more port.

**Definition 2** (Port projection and port share). The port projection  $\psi_k = A_k e^{i\varphi_k}$  is the point at which the pattern's helix, projected onto the plane of port  $k$ , sits at a given phase of its winding:  $A_k \geq 0$  is the projected radius and  $\varphi_k$  the helical phase. The list  $\psi = (\psi_1, \dots, \psi_{N_p})$  is the pattern's state relative to the probe, an element of the port space  $\mathcal{H}_p \cong \mathbb{C}^{N_p}$ . The port share is  $\varpi_k \equiv A_k^2$ .

**Lemma 1** (Energy share). *The energy the pattern brings to port  $k$  is  $\varpi_k E_q$ , and  $\sum_k \varpi_k = 1$ . Every change of port frame that keeps stored energy acts on  $\psi$  as a unitary map of  $\mathcal{H}_p$ .*

*Proof.* Because  $\varepsilon_0$  and  $\mu_0$  are fixed response coefficients (C1), disturbances of the omnium superpose and the energy they store is a quadratic form in the disturbance. The electromagnetic reading stores the energy density  $\frac{1}{2}(\varepsilon_0 \mathbf{E}^2 + \mathbf{B}^2/\mu_0)$ , and by (C4) every other reading carries the same form. The projection onto port  $k$  circulates uniformly on a circle of radius  $A_k$  as the helical phase advances, so its stored energy is constant over the kept period and proportional to  $A_k^2$ . Ports store energy without cross terms (Definition 1), so the energies add, and the whole pattern is one quantum  $E_q$  (C2):  $\sum_k A_k^2 = 1$  in units of  $E_q$ . A passive change of frame acts alike at every instant of the kept period, so it commutes with the advance  $\psi \mapsto e^{-i\omega_q t} \psi$  and is complex-linear; keeping  $\sum_k |\psi_k|^2$ , it is unitary.  $\square$

**Lemma 2** (Integral seating). *Each seating places the whole quantum  $E_q$  in exactly one port.*

*Proof.* Seating closes the stroke into a closed pattern of one port. Closed patterns are integral (C6), and the chain fixes the energy, mass, kept time and thread length of the quantum together (C2). A quantum shared between two ports would be two non-integral closures.  $\square$

The outcome of a seating is therefore a port label  $k_* \in \{1, \dots, N_p\}$ , and the Born rule answers one question: the frequency of each label over identically prepared patterns.

*Remark 1* (Interference is the frame dependence of the shares). The shares in the probe's own frame are blind to every helical phase. The relative phases  $\varphi_k - \varphi_j$  are not lost: a probe whose ports are rotated by a unitary  $U$  reads the shares  $|\sum_j U_{kj} \psi_j|^2$ , which depend on them. Interference is this frame dependence of the shares of one helix. A common phase,  $\psi \mapsto e^{i\varphi_0} \psi$ , enters no share in any frame.

## 4 Route 1: the frame function

Let  $W_k$  denote the seating frequency of port  $k$ .

**Premise 1** (Port locality).  $W_k$  is a function of the pattern's projection onto port  $k$  alone,  $W_k = W(\psi_k)$ , and the same function serves every port of every probe.

Port  $k$  is the region of the probe's support that receives the projection  $\psi_k$  (C3), and one medium with one carrier (C4) leaves no room for a port-specific law. The helical phase is unresolved at observation (C5), so  $W(\psi_k e^{i\theta}) = W(\psi_k)$  for every  $\theta$ , and  $W_k = g(\varpi_k)$  for a function  $g$  on  $[0, 1]$ .

**Theorem 1** (Frame-function route). *Under Premise 1 and Lemma 2,  $g(x) = x$  on  $[0, 1]$ , hence  $W_k = \varpi_k = A_k^2$ .*

*Proof.* By Lemma 2 every pattern seats in exactly one port, so  $\sum_k g(\varpi_k) = 1$  for every share vector of every port set, and every point of every simplex is the share vector of some pattern. A single port gives  $g(1) = 1$ , and two ports with shares  $(1, 0)$  give  $g(0) = 0$ . Three ports with shares  $(x, y, 1 - x - y)$  and  $(x + y, 0, 1 - x - y)$  give

$$g(x) + g(y) = g(x + y), \quad x, y \geq 0, \quad x + y \leq 1. \quad (3)$$

Frequencies are nonnegative, so  $g(x + y) \geq g(x)$  and  $g$  is nondecreasing. Additivity and  $g(1) = 1$  give  $g(x) = x$  at every rational  $x$  in  $[0, 1]$ , and monotonicity squeezes every real  $x$  between rationals, so  $g(x) = x$ .  $\square$

*Remark 2.* Route 1 needs three ports for the additivity step and carries its result to two-port probes through the universality of  $g$ . Gleason's theorem [2] reaches the same quadratic form from additivity over every orthonormal frame of a space of dimension three or more, without port locality, and Busch [3] extends it to dimension two with effect-valued frames. Route 1 assumes more and reaches the rule in one line; Route 3 derives its premise (Theorem 3).

## 5 Route 2: the energy ledger

**Premise 2** (Seating independence). In a linear medium the patterns of one stroke do not act on one another at the probe, so their seatings are independent and follow one port law  $W_k$ .

**Theorem 2** (Energy-ledger route). *Under Premise 2,  $W_k = \varpi_k$ .*

*Proof.* Send a bright stroke of  $\mathcal{N}$  identically prepared patterns into one input mode of the probe. By Lemma 1 the energy that reaches port  $k$  is  $\varpi_k \mathcal{N} E_q$  to relative order  $\mathcal{N}^{-1/2}$ : this is the calorimetric split of a bright stroke at a passive network, read by the omnium's linear response. By Lemma 2 that energy arrives as  $\mathcal{N}_k$  whole quanta, so  $\mathcal{N}_k/\mathcal{N} \rightarrow \varpi_k$ . By Premise 2 and the strong law of large numbers,  $\mathcal{N}_k/\mathcal{N} \rightarrow W_k$  almost surely. Hence  $W_k = \varpi_k$ .  $\square$

The quadratic energy form and the integral ledger leave no other count. A seating law other than  $\varpi_k$  would deliver to port  $k$ , over a bright stroke, an energy different from the energy the stroke carries there.

## 6 Route 3: the seating mechanism

### 6.1 The resident disturbance

No mode of the omnium is empty (C1). Each port mode  $\hat{\mathbf{e}}_k$  carries a resident disturbance  $\Xi_k \in \mathbb{C}$ : the superposition of the medium's strokes in that mode, each with a helical phase unresolved at observation (C5). Write  $Y_k \equiv |\Xi_k|^2/E_{\text{res}}$  for its energy in units of the mean resident energy  $E_{\text{res}}$  of a port mode, and  $\vartheta_k \equiv Y_k/\sum_j Y_j$  for the resident share of port  $k$ .

**Premise 3** (Resident law). The resident amplitudes of distinct port modes are independent and identically distributed, and their joint law is unchanged by every unitary change of port frame.

Distinct ports are separate energy channels of the medium (Definition 1), and the resident disturbance belongs to the medium, not to the frame that a probe's setting selects: a setting that rotates the port frame meets the same resident disturbance. The record agrees. The quadrature statistics of an unexcited mode, read by homodyne detection, are Gaussian, independent of phase, identical from mode to mode and unchanged by every passive linear network.

**Lemma 3** (Resident statistics). *Under Premise 3 the  $\Xi_k$  are independent circular complex Gaussian amplitudes, the  $Y_k$  are independent with  $\Pr(Y_k > y) = e^{-y}$ , and the share point  $\vartheta = (\vartheta_1, \dots, \vartheta_{N_p})$  is uniformly distributed on the simplex  $\Delta_p = \{\vartheta : \vartheta_k \geq 0, \sum_k \vartheta_k = 1\}$ .*

*Proof.* Let  $\mathbf{c}(s) = \mathbb{E} \exp\{i \operatorname{Re}(\bar{s} \Xi_1)\}$ ,  $s \in \mathbb{C}$ , be the characteristic function of one amplitude. A phase rotation of one port is a unitary change of frame, so  $\mathbf{c}$  is real and depends on  $|s|$  alone,  $\mathbf{c}(s) = \mathbf{g}(|s|^2)$ . Independence makes the characteristic function of  $(\Xi_1, \Xi_2)$  the product  $\mathbf{g}(|s|^2) \mathbf{g}(|s'|^2)$ , and frame invariance makes it a function of  $|s|^2 + |s'|^2$  alone. Setting  $s' = 0$  identifies that function with  $\mathbf{g}$ , so

$$\mathbf{g}(u) \mathbf{g}(v) = \mathbf{g}(u+v), \quad u, v \geq 0, \quad (4)$$

with  $\mathbf{g}$  continuous,  $\mathbf{g}(0) = 1$  and  $|\mathbf{g}| \leq 1$ . The only such functions are  $\mathbf{g}(u) = e^{-uE_{\text{res}}/4}$  with  $E_{\text{res}} \geq 0$ , and  $E_{\text{res}} > 0$  because no mode is empty (C1). This is the Herschel–Maxwell characterisation [5]: each  $\Xi_k$  is a circular complex Gaussian amplitude of mean energy  $E_{\text{res}}$ . Its energy is then exponentially distributed,  $\Pr(Y_k > y) = e^{-y}$ , independently in  $k$ , and normalised independent exponentials are uniformly distributed on  $\Delta_p$  (the Dirichlet law with unit parameters). Equivalently, the direction  $\Xi/\|\Xi\|$  is uniform on the port sphere  $\mathbb{S}_p^{2N_p-1}$ , and the map to its shares pushes the round measure forward to the uniform measure on  $\Delta_p$ : Archimedes’ hat-box theorem in its general form [6].  $\square$

*Remark 3* (The physical origin of the law). A sum of many strokes whose helical phases are unresolved, none of them dominant, is a circular Gaussian amplitude by the central limit theorem in the plane. The resident disturbance of a port mode collects strokes from the whole medium (C1), so Premise 3 is the law that phase ignorance (C5) gives the non-empty omnium.

## 6.2 The seating rule

Let  $\eta(t) \in [0, 1)$  be the delivered fraction, the fraction of the pattern’s quantum that has reached the ports by the instant  $t$ , and let  $I_k(t)$  be the intensity delivered into port  $k$ , so that  $\sum_k I_k = \dot{\eta}$  and  $\int I_k dt = \varpi_k$ . The depletion depth and the closure drive of port  $k$  are

$$\Upsilon(t) = -\ln(1 - \eta(t)), \quad \Upsilon_k(t) = \int_{-\infty}^t \frac{I_k(t')}{1 - \eta(t')} dt'. \quad (5)$$

Ports reached together receive the stroke with one time profile,  $I_k = \varpi_k \dot{\eta}$ , and then  $\Upsilon_k = \varpi_k \Upsilon$ .

**Premise 4** (Seating rule). The pattern seats, whole, in the first port whose closure drive reaches its resident energy. With  $T_k = \inf\{t : \Upsilon_k(t) \geq Y_k\}$  the seated port is  $k_* = \arg \min_k T_k$  and the seating instant is  $t_* = T_{k_*}$ . At seating the pattern keeps its projection onto the seated port’s mode, and its projections onto the other ports return into the seated closure.

The closure drive of port  $k$  is that port’s share of each step by which the undelivered remainder of the quantum is depleted. It grows without bound as the remainder vanishes, so a pattern always seats within its own stroke, as Lemma 2 requires. The rule is the nodal locking of (C7) made quantitative in share space: the quantum locks into the port whose resident disturbance is quietest relative to the pattern’s own drive. The last sentence of the premise is the phase fixation of (C5).

## 6.3 The seating law

**Theorem 3** (Time-resolved seating law). *Under Premises 3 and 4,*

$$\Pr(k_* = k, t_* \in [t, t + dt)) = I_k(t) dt. \quad (6)$$

*Hence  $\Pr(k_* = k) = \varpi_k = A_k^2$ , the delivered fraction at seating  $\eta_* = \eta(t_*)$  is uniform on  $[0, 1]$ , and for ports reached together  $k_*$  and  $\eta_*$  are independent.*

*Proof.* By Lemma 3,  $\Pr(T_k > t) = \Pr(Y_k > \Upsilon_k(t)) = e^{-\Upsilon_k(t)}$ , independently in  $k$ . The probability that port  $k$  closes in  $[t, t + dt)$  while every other port is still open is

$$\dot{\Upsilon}_k(t) e^{-\sum_j \Upsilon_j(t)} dt. \quad (7)$$

Since  $\sum_j \Upsilon_j(t) = \int^t \dot{\eta}/(1 - \eta) dt' = -\ln(1 - \eta(t))$ , the exponential equals  $1 - \eta(t)$ , and  $\dot{\Upsilon}_k = I_k/(1 - \eta)$ , so the product is  $I_k(t) dt$ . Integrating over  $t$  gives  $\varpi_k$ . The pattern is still unseated at  $t$  with probability  $e^{-\sum_j \Upsilon_j(t)} = 1 - \eta(t)$ , so  $\Pr(\eta_* \leq \eta) = \eta$ . For ports reached together the density (6) factorises as  $\varpi_k \dot{\eta} dt$ .  $\square$

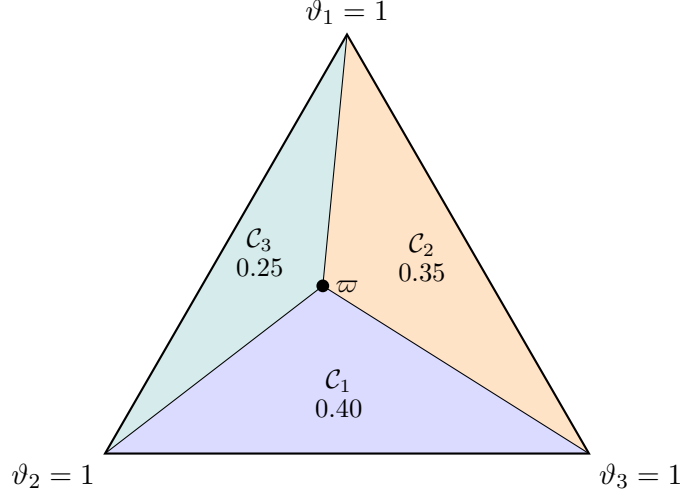


Figure 1: The seating cells for three ports. The simplex  $\Delta_p$  holds every resident share point  $\vartheta$ ; the pattern's share point  $\varpi = (0.40, 0.35, 0.25)$  cuts it into three cells. The cell  $\mathcal{C}_k$  of port  $k$  is the triangle with apex  $\varpi$  over the edge opposite the vertex  $\vartheta_k = 1$ , and its area is the fraction  $\varpi_k$  of the whole. A resident share point drawn uniformly (Lemma 3) falls in  $\mathcal{C}_k$  with frequency  $\varpi_k$ .

Equation (6) is the Born rule in time-resolved form: the seating density over ports and instants is the intensity the pattern delivers there, and its integral over instants is (2).

**Lemma 4** (Barycentric cells). *For ports reached together,  $k_* = \arg \min_k \vartheta_k / \varpi_k$ . The resident share points that seat the pattern in port  $k$  form the cell*

$$\mathcal{C}_k = \{ s \varpi + (1 - s) v : s \in [0, 1], v \in \Delta_p, v_k = 0 \}, \quad (8)$$

*the pyramid with apex at the pattern's share point  $\varpi$  over the facet of  $\Delta_p$  opposite the vertex  $\hat{e}_k$ , and its volume is the fraction  $\varpi_k$  of the volume of  $\Delta_p$ .*

*Proof.* With  $\Upsilon_k = \varpi_k \Upsilon$  the ports close in the order of  $Y_k / \varpi_k$ , and the common factor  $\sum_j Y_j$  leaves that order unchanged. If  $\vartheta \in \mathcal{C}_k$  then  $\vartheta_k = s \varpi_k$  and  $\vartheta_j \geq s \varpi_j$ , so  $\vartheta_k / \varpi_k \leq \vartheta_j / \varpi_j$ . Conversely, if  $s = \vartheta_k / \varpi_k$  is the least ratio, then  $s \leq 1$ , and  $\vartheta - s \varpi$  is nonnegative, vanishes in component  $k$  and sums to  $1 - s$ , which writes  $\vartheta$  in the stated form. In barycentric coordinates the vertices of  $\mathcal{C}_k$  are the rows  $\varpi$  and  $\hat{e}_j$  for  $j \neq k$ . The volume of a simplex relative to  $\Delta_p$  is the absolute determinant of its barycentric rows, and this determinant is  $\pm \varpi_k$ .  $\square$

The Born weights are therefore the barycentric coordinates of the pattern's share point, read as the volumes of the cells that point cuts from the simplex of resident shares (Figure 1). The cell of port  $k$  lies against the facet where port  $k$ 's resident share vanishes: the pattern seats where the resident disturbance is quiet. The same volume identity carries the hidden-measurement account of Aerts and Sassoli de Bianchi [8, 9], where the uniform point is a fluctuation of the measuring context. In FDT the point is the resident disturbance of the non-empty omnium in the port modes (Lemma 3), and the rule that cuts the cells also fixes the seating instant (Theorem 3).

**Corollary 1** (Two ports). *For  $N_p = 2$  let  $\mathfrak{h}(v) = (2 \operatorname{Re}(\bar{v}_1 v_2), 2 \operatorname{Im}(\bar{v}_1 v_2), |v_1|^2 - |v_2|^2) / \|v\|^2$  be the Hopf map of the port sphere  $\mathbb{S}_p^3$  onto the two-sphere, and let  $\mathbf{o} = \mathfrak{h}(\hat{e}_1)$  be the probe axis. Then  $\vartheta_1 = \frac{1}{2}(1 + \mathfrak{h}(\Xi) \cdot \mathbf{o})$  and  $\varpi_1 = \frac{1}{2}(1 + \mathfrak{h}(\psi) \cdot \mathbf{o})$ , and the pattern seats in port 1 exactly when*

$$\mathfrak{h}(\Xi) \cdot \mathbf{o} < \mathfrak{h}(\psi) \cdot \mathbf{o}, \quad (9)$$

that is, in the port toward which it leans further than the resident disturbance does. By Archimedes' theorem  $\mathfrak{h}(\Xi) \cdot \mathbf{o}$  is uniform on  $[-1, 1]$ , so  $\Pr(k_* = 1) = \frac{1}{2}(1 + \cos \Theta) = \cos^2(\Theta/2)$ , with  $\Theta$  the angle between  $\mathfrak{h}(\psi)$  and  $\mathbf{o}$ .

*Remark 4* (The port sphere is not the support). The port sphere  $\mathbb{S}_p^{2N_p-1} \subset \mathbb{C}^{N_p}$  is the space of port projections of one probe. It is not the three-sphere of the core's 0.3a, whose share  $|z_1|^2$  is a ratio of curvatures. No map between the two is displayed and none is used (Section 8).

## 6.4 Variants that fail, kept as evidence

*Remark 5* (Uniform thresholds). Resident energies drawn independently and uniformly from  $[0, 1]$  give  $\Pr(k_* = 1) = \varpi_1/(2\varpi_2)$  at two ports for  $\varpi_1 \leq \frac{1}{2}$ , which is 0.125 at  $\varpi = (0.2, 0.8)$ . That rule fails at the resident law.

*Remark 6* (Uniqueness of the resident law). Among laws that draw the resident energies independently from one common law, the exponential law is the only one under which the seating frequency of every port is a function of its own share alone, for every port set of three or more ports. With  $\ln(Y_k/\varpi_k)$  as the closing order this is Yellott's theorem on the double-exponential law [7]. A Gamma law of shape two and the inverse-exponential law both fail at three ports, and the inverse-exponential law passes at two (V9 of Section 14). Route 1's premise therefore fixes Route 3's resident law.

*Remark 7* (The linear share race). Comparing the normalised resident shares with the delivered fraction itself,  $\eta \varpi_k \geq \vartheta_k$ , seats the pattern in the same port as Premise 4 in every event, but at the delivered fraction  $\eta_* = \min_k \vartheta_k/\varpi_k$ , for which  $\Pr(\eta_* > \eta) = (1 - \eta)^{N_p-1}$ . Under a stroke whose delivered intensity decays with unit lifetime, its seating instants decay at the rate  $N_p - 1$ . The arrival-time record reads the seating instant along the delivered intensity, the same through one port as through many: lifetimes read through split and multiplexed detection agree with single-detector readings. That record rules the linear race out at three ports or more; at two ports the two rules coincide.

*Remark 8* (The depletion depth is forced). Call a closure law renewal-invariant when a pattern still unseated at the delivered fraction  $\eta_0$ , which is one whole quantum (Lemma 2), seats over its remainder by the law it obeys over a whole stroke. The renewal-invariant laws are exactly  $\Pr(\eta_* > \eta) = (1 - \eta)^\gamma$  with  $\gamma > 0$ , because the survival function  $S$  then satisfies  $\tilde{S}(ab) = \tilde{S}(a) \tilde{S}(b)$  for  $\tilde{S}(u) = S(1 - u)$ . Require the law to be blind to how the delivered stroke is divided among ports, and send the part delivered before  $\eta_0$  to one port and the rest to another. That probe has shares  $\eta_0$  and  $1 - \eta_0$ , the pattern seats in the first port exactly when it seats before  $\eta_0$ , and the port law of Theorems 1 and 2 gives  $1 - (1 - \eta_0)^\gamma = \eta_0$  for every  $\eta_0$ . So  $\gamma = 1$ , which is the depletion depth of (5).

*Remark 9* (A single joint race). For a pattern shared by two probes (Section 7.3), a single race over the product ports with locally generated resident energies  $Y_k^{(1)} Y_l^{(2)}$  gives  $\Pr(00) = 0.717$  for shares 0.8 on 00 and 0.2 on 11, where the port law requires 0.8. The joint law needs the phase fixation of the whole pattern, which Section 7.3 supplies.

## 7 Consequences

### 7.1 Phase fixation and repeatability

After seating in port  $k$  the pattern's projection is  $\hat{\mathbf{e}}_k$  up to a phase. A second probe with the same ports reads the share vector  $\hat{\mathbf{e}}_k$  and seats the pattern in  $k$  with certainty; a probe rotated by  $U$  seats

it in  $j$  with frequency  $|U_{jk}|^2$ . A port that is a mode space rather than one mode, with projector  $P_k$ , leaves the pattern with its normalised projection  $P_k\psi/\|P_k\psi\|$ , the update rule of Lüders [4].

## 7.2 Readings, mixtures and enlarged port spaces

If port  $k$  carries the reading  $o_k$ , the mean reading is  $\sum_k o_k A_k^2 = \langle \psi, \mathcal{O} \psi \rangle$  with  $\mathcal{O} = \sum_k o_k \hat{e}_k \hat{e}_k^\dagger$ . A preparation that issues the pattern  $\psi_{(i)}$  with frequency  $\gamma_i$  gives

$$\Pr(k) = \sum_i \gamma_i |\langle \hat{e}_k, \psi_{(i)} \rangle|^2 = \hat{e}_k^\dagger W \hat{e}_k, \quad W = \sum_i \gamma_i \psi_{(i)} \psi_{(i)}^\dagger. \quad (10)$$

A probe whose port space contains the pattern's own space through an isometry  $V$  reads  $\Pr(k) = \langle \psi, E_k \psi \rangle$  with  $E_k = V^\dagger \hat{e}_k \hat{e}_k^\dagger V \geq 0$  and  $\sum_k E_k = \mathbb{1}$ .

## 7.3 Joint patterns and the shared bivector grade

A pattern shared by two probes has projections  $\Psi_{kl}$  onto the product ports  $\hat{e}_k \otimes \hat{e}_l$ . It is one pattern (C3), and the core names its correlation the shared bivector grade (C5). Port  $k$  of probe 1 is the mode space  $\hat{e}_k \otimes \mathbb{C}_p^{N_p^{(2)}}$ , so the first seating, at probe 1, occurs in  $k$  with frequency  $\sum_l |\Psi_{kl}|^2$  (Theorem 3). Phase fixation leaves the normalised row  $\Psi_k$ , and probe 2 then seats in  $l$  with frequency  $|\Psi_{kl}|^2 / \sum_{l'} |\Psi_{kl'}|^2$ . The joint frequency is

$$\Pr(k, l) = |\Psi_{kl}|^2, \quad (11)$$

the same when probe 2 seats first, so the order of the two seatings carries no information. The marginal  $\sum_l |\Psi_{kl}|^2$  at probe 1 is unchanged by every unitary change of probe 2's frame: the distant frame never shows in local frequencies. A single seating at probe 2 depends on the port in which probe 1 seated, through the phase fixation of the one shared pattern. No influence passes between two things, because there are not two things. The seating mechanism is contextual and the joint seating is non-separable, as the theorems of Kochen and Specker [10] and of Bell [11] require of any deterministic account of the record.

## 7.4 Universality and coupling blindness

Lemma 1's energy form, Premise 3 and Premise 4 are properties of one medium and one carrier (C4), and matter patterns are closed braids of the same carrier (C6). Every carrier reading and every matter pattern therefore seats by one law, and the record shows the same Born weights for light, electrons, neutrons, atoms and molecules. The coupling between pattern and probe is the one interaction of I-1, reaching the ports through a response kernel. It sets how fast  $\eta$  advances and nothing else, because Theorem 3 is stated in  $\eta$ ; the seating law holds at every coupling strength and every scale.

# 8 The port share is not a density coordinate

The port share reaches both ends of its range. A pattern whose projection is  $\hat{e}_k$  seats in  $k$  with certainty,  $\varpi_k = 1$ , and a port orthogonal to the pattern never seats it,  $\varpi_k = 0$ . Every density coordinate lives on the open interval,  $0 < \alpha_\sigma < 1$ , with  $\alpha_\sigma \rightarrow 0^+$  and  $\alpha_\sigma \rightarrow 1^-$  as one-sided limits (I-6). The port share is therefore no output of any application A-1 to A-5 of the bounded map (core 0.4), the ratios  $Y_k/\varpi_k$  that order the closures are no odds, and the partition  $\sum_k \varpi_k = 1$  carries no geometric warrant, since it is no ratio of two lengths the geometry fixes. None of  $\varpi_k$ ,  $\vartheta_k$ ,  $Y_k$ ,  $\eta$  or  $\Upsilon$  stands in a force, optical or density slot.

## 9 Where the seven invariants enter

|     | Displayed form                                                                                                        | Role in the derivation                                                                                                                                                                                                     |
|-----|-----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| I-1 | $F = \frac{c^4}{4G} \alpha_1 \alpha_2$ , with<br>$\alpha_i(d) = R_{s,i}/d$ on<br>nonoverlapping spherical<br>supports | The pattern–probe coupling is this one interaction, reaching the ports through a response kernel. It sets the rate of $\eta$ and drops out of every seating frequency (Section 7.4).                                       |
| I-2 | photon = graviton = gluon =<br>phonon = magnon                                                                        | One energy form and one seating law for every carrier reading and every matter pattern (Lemma 1, Section 7.4).                                                                                                             |
| I-3 | $G_{\mu\nu} = \kappa T_{\mu\nu}$ , unchanged                                                                          | Strokes and port modes follow geodesics of the unchanged geometry; the derivation adds no source and no field equation.                                                                                                    |
| I-4 | $\psi = Ae^{i\varphi}$ , the projection of a<br>helix                                                                 | Port projections, phase blindness, the unresolved resident phases, phase fixation and the shared bivector grade (Sections 3–7).                                                                                            |
| I-5 | $D(\Omega, B; n, \chi, t) u_q = \omega_q u_q$ ,<br>$m_q c^2 = \hbar \omega_q$                                         | Integral seating (Lemma 2) and the quantum $E_q$ . The port modes are modes of the probe’s support, of which only orthogonality is used.                                                                                   |
| I-6 | $0 < \alpha_\sigma < 1$ , open                                                                                        | Excludes the port share from every density slot (Section 8).                                                                                                                                                               |
| I-7 | $\mathcal{M} = M + E/c^2 + PV/c^2 +$<br>$aT^4V/c^2 + \dots$ , $a = 4\sigma_{\text{SB}}/c$                             | A seating adds $E_q/c^2$ to the $\mathcal{M}$ of the seated port, once: the record of a seating is a density change of the probe. The resident disturbance is already in the probe’s $\mathcal{M}$ and is not added again. |

## 10 Chart inventory

- *Density coordinates.* No density coordinate enters a load-bearing equation of the derivation. The atlas appears only where Sections 8 and 9 name it: the exterior branch  $\alpha_{\text{ext}}(r_f) = R_s/r_f$  on  $R_s < r_f < \infty$  (branch tag  $\sigma = +1$ ) and the interior branch  $\alpha_{\text{int}}(r_f) = r_f/R_s$  on  $r_c \leq r_f < R_s$  (branch tag  $\sigma = -1$ ), application A-1, with the gap odds  $X_{\text{ext}} = \frac{R_s}{r_f - R_s}$ ; and the I-1 inputs  $\alpha_i(d) = R_{s,i}/d$  on nonoverlapping spherical supports.
- *Unused.* The computational chart  $\beta_\sigma$  (A-2), the tokens of A-3, A-4 and A-5, the saturation diagnostic, and the clock factor of 0.1c.
- *Quantities of this derivation, none an application of 0.4.* The port share  $\varpi_k$ , the resident share  $\vartheta_k$ , the resident energy  $Y_k$ , the delivered fraction  $\eta$ , the depletion depth  $\Upsilon$  and the closure drives  $\Upsilon_k$ .
- *Spaces.* The port space  $\mathcal{H}_p \cong \mathbb{C}^{N_p}$  with its port sphere  $\mathbb{S}_p^{2N_p-1}$ , and the simplex of shares  $\Delta_p$ . The three-sphere of 0.3a does not appear.

## 11 Notation guard

| Symbol                              | Meaning                                                       | Kept apart from                                                       |
|-------------------------------------|---------------------------------------------------------------|-----------------------------------------------------------------------|
| $\psi, \psi_k = A_k e^{i\varphi_k}$ | pattern state relative to a probe; port projection            | the stroke–radius angle $\psi$ of I-3a                                |
| $A_k, \varphi_k$                    | projected radius; helical phase                               | the application labels A-1 to A-5                                     |
| $\varpi_k = A_k^2$                  | port share                                                    | the density coordinate $\alpha$ ; the odds $X$                        |
| $\hat{\mathbf{e}}_k$                | port mode                                                     | the charge unit $e$ ; the lattice vectors of I-5a                     |
| $N_p, N_p^{(1)}, N_p^{(2)}$         | number of ports                                               | the count $N$ of 0.1c; the helix count $n$                            |
| $\Xi_k, E_{\text{res}}$             | resident amplitude of port mode $k$ ; its mean energy         |                                                                       |
| $Y_k, \vartheta_k$                  | resident energy in units of $E_{\text{res}}$ ; resident share | the deflection $\theta$ of I-3                                        |
| $\eta, \eta_*$                      | delivered fraction; its value at seating                      | the odds input $x$ of I-6                                             |
| $\Upsilon, \Upsilon_k$              | depletion depth; closure drive of port $k$                    |                                                                       |
| $I_k(t)$                            | intensity delivered into port $k$                             |                                                                       |
| $T_k, t_*, k_*$                     | closing instant of port $k$ ; seating instant; seated port    | the temperature $T$ ; the kept period $T_q$ ; the transition operator |
| $E_q = \hbar\omega_q$               | the pattern’s quantum                                         | the rest energy of 0.1                                                |
| $\mathbf{E}, \mathbf{B}$            | field strengths of the electromagnetic reading                | the energy $E$ ; the boundary data $B$ of I-5                         |
| $\mathcal{N}, \mathcal{N}_k$        | patterns in a stroke; patterns seated in port $k$             | the count $N$ of 0.1c                                                 |
| $\mathbf{c}, \mathbf{g}$            | characteristic function of one amplitude; its radial form     | the speed $c$ ; the constant $G$                                      |
| $\mathfrak{h}, \mathbf{o}, \Theta$  | Hopf map of the port sphere; probe axis; angle from the axis  | the involutions of 0.3a; the deflection $\theta$                      |
| $\mathcal{C}_k$                     | seating cell of port $k$                                      |                                                                       |
| $\mathbf{P}_k$                      | projector onto port $k$ ’s mode space                         | the pressure $P$                                                      |
| $o_k, \mathcal{O}$                  | reading of port $k$ ; observable                              | the owed labels                                                       |
| $\gamma_i, \mathbf{W}$              | issue frequencies; statistical operator                       | the density $\rho$                                                    |
| $\mathbf{E}_k, \mathbf{V}$          | port effects; isometry                                        | the energy $E$ ; the volume $V$                                       |

## 12 Declined readings

- Consensus postulates that  $|\psi_k|^2$  is the probability of outcome  $k$  and attaches no mechanism to it. FDT derives the same weight as the seating frequency of a deterministic helical pattern: the squared projected radius is the pattern’s energy share in the port (Lemma 1), and the seating law returns it (Theorems 1–3).
- Consensus postulates a discontinuous reduction of the state at measurement. FDT reads it as phase fixation: a closed pattern seats whole in one port and keeps its projection onto that port (Premise 4; I-4, I-5).

- Consensus reads a superposed state as a system without definite properties until it is measured. FDT reads one helix with definite projections onto every port, and what is unresolved is the helical state of the resident disturbance at the seating instant (I-4).
- Consensus reads the outcome of a single seating as irreducibly random. FDT reads it as fixed by the pattern and the resident disturbance together; the universe is deterministic (I-4).
- Consensus reads the correlated seatings of a shared pattern as an influence between two systems. FDT reads one pattern with a shared bivector grade, whose phase fixation at one probe fixes the projection the other probe receives, while local frequencies stay blind to the distant frame (0.1b, I-4; Section 7.3).
- Consensus reads the shot-noise level of an unexcited mode as the fluctuation of an empty vacuum. FDT reads it as the resident disturbance of the non-empty omnium in that mode (0.1); the measured statistics are accepted in full and are the law of Premise 3.

## 13 Status ledger

| Item                                                 | Status      | Basis; what kills it                                                                                                                                                           |
|------------------------------------------------------|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Lemma 1, energy share<br>$A_k^2$                     | DERIVED     | 0.1, 0.1a, I-2, I-4                                                                                                                                                            |
| Lemma 2, integral seating                            | DERIVED     | 0.1a, I-5, I-5a                                                                                                                                                                |
| Premise 1, port locality                             | POSITED     | Buys Route 1. Killed by a seating frequency that moves with the arrangement of the other ports at fixed $\varpi_k$ . Supplied by Theorem 3.                                    |
| Theorem 1                                            | CONDITIONAL | on Premise 1                                                                                                                                                                   |
| Premise 2, seating independence                      | POSITED     | Buys Route 2. Killed by correlations between successive seatings of identically prepared patterns. Supplied by Premise 3, a fresh resident disturbance at each seating.        |
| Theorem 2                                            | CONDITIONAL | on Premise 2                                                                                                                                                                   |
| Premise 3, resident law                              | POSITED     | Buys the exponential thresholds. Record: unexcited-mode statistics. Killed by an unexcited port mode with non-Gaussian or frame-dependent statistics.                          |
| Premise 4, seating rule                              | POSITED     | P-2 made quantitative; buys Theorem 3 and the update rule. Killed by an arrival-time profile that depends on the port count, or by a seating outside the pattern's own stroke. |
| Lemma 3, Lemma 4, Corollary 1                        | DERIVED     | mathematics, from Premise 3                                                                                                                                                    |
| Theorem 3, time-resolved law                         | CONDITIONAL | on Premises 3 and 4                                                                                                                                                            |
| Remark 8, the depletion depth                        | DERIVED     | renewal invariance, partition blindness, the port law                                                                                                                          |
| Joint law (11), local blindness to the distant frame | CONDITIONAL | on Premises 3 and 4                                                                                                                                                            |
| Port share is no density coordinate                  | DERIVED     | I-6, 0.4                                                                                                                                                                       |
| Universality and coupling blindness                  | DERIVED     | I-1, I-2, Theorem 3                                                                                                                                                            |

*Owed.* The rate kernel by which a given probe's coupling advances the delivered fraction is owed (O-12). Every frequency in this derivation is independent of it. The port modes of a given probe are modes of the confinement operator, owed (O-9). Only their orthogonality enters here.

*Tests.* The four tests below are convergence tests against the measured record; none is counted as a prediction.

**T-1** Port law: seating frequency  $A_k^2$  at every complete port set, every carrier and every coupling. The record agrees. A reproducible departure from  $A_k^2$  at a complete port set kills the derivation.

**T-2** Seating instant: uniform along the delivered fraction, independent of the port count and of the port. The record agrees: arrival-time histograms follow the delivered intensity through one port or many. A dependence of the arrival-time profile on the port count kills Premise 4.

- T-3** Joint law:  $|\Psi_{kl}|^2$ , with local frequencies blind to the distant frame. The record agrees. A local frequency that follows the distant frame kills Section 7.3.
- T-4** Coupling blindness: frequencies independent of the coupling strength and of the common resident scale. The record agrees: Born weights do not move with detector gain or bias once losses are counted as a port. A gain-dependent split at a complete port set kills Section 7.4.

## 14 Verification

Every statement above was checked numerically by a method structurally different from its proof: Monte Carlo against an analytic law, exact rational arithmetic against a geometric statement, or a distribution test against a distributional statement. The script `verify_born_fdt.py` (seed 20261006) and its output `verification-output.txt` accompany this document.

|     | Claim                                    | Result                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----|------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| V1  | Port law, Theorem 3                      | $N_p = 2, 3, 5, 8, 2 \times 10^6$ seatings each: largest $ \text{frequency} - \varpi_k  = 2.7, 4.6, 6.2, 5.5 \times 10^{-4}$ against a one-sigma scale near $3 \times 10^{-4}$ ; $\chi^2 p = 0.35, 0.16, 0.19, 0.13$ .                                                                                                                                                                                                                            |
| V2  | Barycentric cells, Lemma 4               | Exact rational determinants: cell fractions $(1/7, 2/7, 4/7)$ at $\varpi = (1/7, 2/7, 4/7)$ and $(1/10, 1/5, 3/10, 2/5)$ at $\varpi = (1/10, 1/5, 3/10, 2/5)$ , equal to $\varpi$ exactly.                                                                                                                                                                                                                                                        |
| V3  | Resident statistics, Lemma 3             | $N_p = 4, 4 \times 10^5$ draws: $\vartheta_1$ against the Beta(1, 3) law, KS $p = 0.081$ ; shares after a Haar-random frame change against the original, KS $p = 0.32$ ; $ \Xi_1 ^2$ against Exp(1), KS $p = 0.13$ .                                                                                                                                                                                                                              |
| V4  | Two ports, Corollary 1                   | $\mathfrak{h}(\Xi) \cdot \mathbf{o}$ uniform on $[-1, 1]$ , KS $p = 0.10$ . Seating frequencies 0.9777, 0.7710, 0.2927, 0.0291 against $\cos^2(\Theta/2) = 0.9777, 0.7702, 0.2919, 0.0289$ at $\Theta = 0.3, 1.0, 2.0, 2.8$ ( $10^6$ seatings).                                                                                                                                                                                                   |
| V5  | Seating instant, Theorem 3 and Remark 7  | $N_p = 3$ : $\eta_*$ uniform, KS $p = 0.027$ (ten further seeds give $p$ from 0.046 to 0.975, as a true null requires); mean $\eta_*$ per port 0.5007, 0.5008, 0.5006; the linear race gives $\eta_*$ of law Beta(1, 2) (KS $p = 0.13$ ) and the same port in every event; unit-lifetime read-out: decay rate 0.997 for Premise 4, 1.996 for the linear race. $N_p = 5$ : KS $p = 0.59$ ; Beta(1, 4), $p = 0.72$ ; decay rates 1.0005 and 3.9985. |
| V6  | Uniform thresholds fail                  | $\Pr(k_* = 1) = 0.1248$ at $\varpi = (0.2, 0.8)$ against the analytic 0.1250; the port law requires 0.2000.                                                                                                                                                                                                                                                                                                                                       |
| V7  | Joint law, Section 7.3; Remark 9         | Random shared pattern on $3 \times 3$ ports, random frames, $10^6$ events: largest $ \text{frequency} -  \Psi_{kl} ^2  = 4.4 \times 10^{-4}$ with probe 1 first and $4.5 \times 10^{-4}$ with probe 2 first; probe-1 marginals under two distant frames differ by $2.1 \times 10^{-4}$ . A single joint race with product residents gives $\Pr(00) = 0.7173$ where 0.8000 is required.                                                            |
| V8  | Ports reached at different instants, (6) | Delays (0, 1.5, 4.0): $\Pr(k) = (0.4676, 0.2705, 0.2619)$ against $\varpi = (0.4666, 0.2715, 0.2619)$ ; mean seating instant per port (0.001, 1.505, 4.000) against the delays; no pattern left unseated.                                                                                                                                                                                                                                         |
| V9  | Uniqueness of the resident law, Remark 6 | $N_p = 3$ , $\varpi = (0.2, 0.3, 0.5)$ : exponential (0.1999, 0.3001, 0.5000); Gamma of shape two (0.1398, 0.2707, 0.5895); inverse-exponential (0.1696, 0.2974, 0.5330). $N_p = 2$ , inverse-exponential at (0.3, 0.7): (0.3001, 0.6999).                                                                                                                                                                                                        |
| V10 | Phase fixation, Section 7                | Repeated seating in one frame: the same port in every one of $10^6$ events. A pattern seated in $z$ , then $x$ , then $z$ : $\Pr = 0.4999$ against 0.5000, correlation with the $x$ port +0.0019.                                                                                                                                                                                                                                                 |

## 15 Convergence statement

**Convergence.** The limiting map is the identity. The FDT seating frequency of port  $k$  is  $A_k^2 = |\langle \hat{\mathbf{e}}_k, \psi \rangle|^2$ , the consensus Born weight, with no approximation and no regime limit, for every port count, every carrier reading and every coupling strength. The seating instant is uniform along the delivered fraction of the stroke, which is the consensus arrival-time law of a single-quantum stroke; the update is the projection onto the seated port; mixtures read the statistical operator  $\mathbf{W}$ ; and shared patterns return the joint weights  $|\Psi_{kl}|^2$  with local frequencies blind to the distant frame. The derivation recovers every measured outcome frequency of the record to the precision at which that record agrees with the consensus law. FDT supplies no additional input at this interface that changes an observable, so no divergence is claimed.

**Divergence search.** The structure FDT adds here is the resident disturbance and the seating rule: a single seating is fixed by  $\psi$  and  $Y$  together. An observable departure needs a handle on  $Y$  below its law. Reading  $Y$  requires seating a pattern in that mode, which meets a fresh resident disturbance drawn by the same law; preparing  $Y$  requires injecting a known disturbance, which is a pattern and seats on its own. Premise 3 fixes the resident law in every probe, so the search finds no additional source, constitutive input or changed observable dictionary. It fails at this step: no handle on the resident disturbance exists below the law that Premise 3 fixes.

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