

Cavity-less Brillouin strong coupling: a density-field analysis

FDT analysis

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Cavity-less Brillouin strong coupling in a solid-state continuous system
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Abstract

Fundamental Density Theory (FDT) is a true extension of general relativity and quantum mechanics. Blázquez Martínez, Zhu and Stiller report single-pass, cavity-less strong coupling in steady-state spectra of one highly nonlinear fibre in a 4 K cryostat at one pump wavelength [1]. Their criterion is coupling above the average optical and mechanical linewidth. The spectra, including the forced-detuning avoided crossing, are accepted in full. The optical and acoustic modes are read here as two regimes of the one carrier, while the source's coupled-mode calculation supplies the identity map for its spectra. Independent arithmetic separates the reported propagation loss from the optical dissipation parameter, the nearly unchanged gain–linewidth product from the falling threshold on cooling, and the source's cooperativity error from independent-input propagation. Five prospective predictions test additional inputs: two handed-geometry magnetic existence readings and three isotope responses conditional on independently established density sensitivities. Their comparators retain reciprocal leakage, composition-lot effects, cladding constraint and isotope-induced overlap changes. One uncounted exploratory observable reads phase matching as driven nodal selection.

1 Framework

The source status is experimental. Measured, observed and reported designate source results; derived, estimated, reconstructed and illustrative-assumed designate the corresponding calculations or assumptions here. Estimated couplings and simulated curves retain those narrower classes. SC identifiers refer to individual source claims; their accompanying L identifiers locate the paper's text. U identifiers denote explicit assumptions. The target's spectral experiment [1] is read through the locked framework [2]; each geometric assertion below uses its stated chart.

1.1 One substance, quantisation and the Alpha Lock

Omnium is the one substance: matter, energy and spacetime are its density configurations. Let E be energy, m equivalent mass, c the chain speed, ε_0 and μ_0 the omnium response coefficients, and d, t a kept path and time. Planck's constant is h , its reduced form is $\hbar$, and q labels a

quantised excitation with angular frequency ω_q , energy E_q , equivalent mass m_q , period T_q and path ℓ_q . Here π is the circular constant; i is the complex unit, and m,s denote metre and second. The medium's bridges, core 0.1–0.1a, are

$$E/m = c^2 = \frac{1}{\varepsilon_0 \mu_0} = (d/t)^2, \quad (1)$$

$$E_q = m_q c^2 = \hbar \omega_q, \quad \hbar = h/(2\pi), \quad (2)$$

$$T_q = 2\pi/\omega_q = h/E_q, \quad \ell_q = cT_q = h/(m_q c), \quad (3)$$

$$d/t = c = 299792458 \text{ m s}^{-1}. \quad (4)$$

The last number is exact (U-01). The quantised massive leg gives confinement eigenvalues; the null leg gives discrete periods and paths at locked ratio. The Planck length ℓ_P and Planck time t_P set the one-sided infima $2\ell_P$ and $2t_P$, never a lattice. The support radius R_{box} is distinct from the Compton path. A continuous curvature chart describes this discrete content.

Let $\mathcal{M}$ be effective gravitating mass, G Newton's constant, R_s the source radius of the gravitational atlas, and r_f the field-point radius. The branch tag σ takes exterior and interior signs; r_c is the finite interior cutoff. The force ceiling is F_{max} ; α_σ denotes the A-1 branch ratio, with α_{ext} and α_{int} its two evaluations. Core 0.2–0.3 gives

$$F_{\text{max}} = \frac{c^4}{4G}, \quad R_s = \frac{2G\mathcal{M}}{c^2} = \frac{\mathcal{M}c^2}{2F_{\text{max}}}, \quad (5)$$

$$\alpha_\sigma(r_f) = (R_s/r_f)^\sigma, \quad \sigma = +1 \text{ exterior}, \quad -1 \text{ interior}, \quad (6)$$

$$\alpha_{\text{ext}} = R_s/r_f, \quad R_s < r_f, \quad \alpha_{\text{int}} = r_f/R_s, \quad r_c \leq r_f < R_s. \quad (7)$$

Let X denote positive gap odds, X_σ its branch evaluation and $X_{\text{ext}}, X_{\text{int}}$ the exterior and interior cases. The bounded map is b , and β_σ is its computational reapplication to a ratio:

$$X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \quad X_{\text{int}} = \frac{r_f}{R_s - r_f}, \quad (8)$$

$$b(X) = \frac{X}{1+X}, \quad \alpha_\sigma = b(X_\sigma), \quad 0 < \alpha_\sigma < 1, \quad (9)$$

$$\beta_\sigma = \frac{\alpha_\sigma}{1+\alpha_\sigma}. \quad (10)$$

Only the A-1 ratios occupy physical force, optical and density slots. The bounded ratio is never compactified again in those slots. The chart inventory below declares all applications at first use. Configuration and source evaluations, $\alpha_\sigma^{\text{cfg}}$ and $\alpha_\sigma^{\text{src}}$, mean the same atlas evaluated on a specified configuration or source; they introduce no extra map. The token α_{core} is the schematic skin-saturation diagnostic.

Application	Warrant	Meaning and domain
A-1: $\alpha_\sigma(r_f)$, $\alpha_\sigma^{\text{cfg}}$, $\alpha_\sigma^{\text{src}}$	Geometric	Branch-tagged radius ratio, open interval; source and field point always declared.
A-2: β_σ	Geometric	Computational compactification only, excluded from physical slots.
A-3: α_{overlap}	None	Token and range only; no value, threshold, ordering, sign or trajectory.
A-4: α_{conf}	None	Token and range only; no value, threshold, ordering, sign or trajectory.
A-5: α_{mode}	None	Token and range only; no value, threshold, ordering, sign or trajectory.
$\alpha_{\text{core}} \rightarrow 1^-$	Schematic	Interior skin saturation; no numerical address.

The A-1 seam has $\alpha_{\text{ext}} = 1/2$ at $r_f = 2R_s$, and $\alpha_{\text{int}} = 1/2$ at $r_f = R_s/2$. It is distinct from the fold approached at the open endpoint. The Clifford torus is the equal-share seam of the support; its Hopf core circles are distinct from that torus. No coupling threshold or spectral energy share supplies this geometric seam. At the interior arithmetic point U-08, the exterior ratio $1/4$ gives gap odds $1/3$ and returns $1/4$ under b ; the reciprocal interior field point returns the same bounded ratio with the opposite branch tag.

1.2 Notation guard

The following census applies to every source equation transcribed here. Source tokens in the first column are names being renamed, not additional meanings of the manuscript symbols. Upright labels distinguish the optical and acoustic modes at fixed field identity.

Source token	Manuscript quantity	Reserved meaning retained
gamma(opt), gamma(dB) Gamma(m)	Υ_{opt} , optical dissipation; η_{dB} , propagation loss Υ_{m} , mechanical dissipation	γ names the photon. Γ is the core curve/rate family, unused here.
Omega(B) n	ω_{Br} , Brillouin angular frequency n, material index	Ω is a confinement domain. n is strand count; named optical indices keep their subscripts.
B label	Br, Brillouin label	B is boundary data; $\mathbf{B}$ is a phase bivector.
G(B), g(B)	$\mathcal{G}_{\text{Br}}$, peak gain; $\mathbf{g}_{\text{Br}}(\omega)$, gain spectrum	G is Newton's constant.
g(om)	$\mathbf{g}_{\text{om}}$, pump-enhanced coupling; superscript max its maximum	$g_{\mu\nu}$ is the metric.
a, b	$\hat{\mathbf{a}}_{\text{aS}}$, $\hat{\mathbf{b}}_{\text{ac}}$, envelope operators; $\hat{\mathbf{a}}(k, t)$, $\hat{\mathbf{b}}(k, t)$, mode operators	a is radiation energy-density constant; b is the bounded map.
z	z, distance along the fibre	Support coordinates retain their own names.
H(int)	$\mathcal{H}_{\text{int}}$, source Hamiltonian	H is the core helicity label, unused here.
P(pump), P(probe)	$\mathcal{P}_{\text{pump}}$, $\mathcal{P}_{\text{probe}}$, optical powers	P is pressure.
lambda(pump)	$\mathcal{L}_{\text{pump}}$, pump wavelength	Core λ symbols are not reused.
v(ac), upsilon(opt)	$\mathbf{v}_{\text{ac}}$, acoustic speed; $\mathbf{v}_{\text{opt}}$, optical group speed	v is gravitational flow speed.
delta(split)	$\mathfrak{s}_{\text{split}}$, half the peak separation	Core δ symbols are not reused.

Source token	Manuscript quantity	Reserved meaning retained
T	T , temperature	Same meaning as the mass ledger.
micro	μ , upright prefix	μ is a tensor index; μ_0 is a medium coefficient.
k	k , wave number, with upright mode labels	No strand count on a carrier mode.
omega	ω , angular frequency; upright pump, probe, aS and LO labels	ω_q is the confinement eigenvalue.
Delta(probe), Delta(det)	Δ_{probe} , pump-relative detuning; Δ_{det} , Brillouin-relative detuning	$\Delta \ln$ denotes a finite contrast.
C(max)	C_{max} , cooperativity	Not a curvature ratio.
c, hbar	$c, \hbar$, medium constants	Same meanings as the chain.
e	e , exponential base	$\mathbf{e}_1, \mathbf{e}_2$ are Clifford basis vectors.
Added occupation	$\bar{N}_{\text{th}}$, thermal occupation; k_B , Boltzmann constant	Neither is strand count or wave number.
Added handed geometry	$t_{\text{tw}}, t_{\text{sp}}$, twist and frozen spin rates; I_{null} , null current; N_c , turns	Distinct from branch and seam involutions.

1.3 The force law and the one carrier

For force magnitude F , let α_1, α_2 be exterior A-1 evaluations of the two sources; core I-1 holds always and at every scale:

$$F = \frac{c^4}{4G} \alpha_1 \alpha_2 = F_{\text{max}} \alpha_1 \alpha_2, \quad 0 < \alpha_1, \alpha_2 < 1, \quad F < F_{\text{max}}. \quad (11)$$

For nonoverlapping spherical monopole supports, let $R_{b,1}, R_{b,2}$ be their support radii and d their separation. Source index i labels $\mathcal{M}_i, R_{s,i}, \alpha_i$, with $\mathcal{M}_1, \mathcal{M}_2$ the masses and $R_{s,1}, R_{s,2}$ their atlas radii. The pair domain and its reduction are

$$d \geq R_{b,1} + R_{b,2} \geq R_{s,1} + R_{s,2}, \quad (12)$$

$$\alpha_i = R_{s,i}/d, \quad F = \frac{G\mathcal{M}_1\mathcal{M}_2}{d^2} \leq F_{\text{max}}/4. \quad (13)$$

The last bound belongs only to that pair domain. Equality is the limiting contact of saturated equal supports. Let $\mathbf{u}_{12}$ be the two sources' relative speed. The exact pair algebra is the L1 reduction; GR's weak-field, slow-motion L2 limit has relative corrections of order $(R_{s,1} + R_{s,2})/d$ and $\mathbf{u}_{12}^2/c^2$. The source programme is L3. A small laboratory gravitational window remains a correct evaluation of I-1.

The fibre and its travelling grating are one nonspherical dynamical configuration. A response kernel maps force to an observable; it is not the force law. For a drive $\mathfrak{h}$, let $\mathfrak{h}_0$ be a positive drive scale and $\mathfrak{S}$ the unnormalised bounded drive, and let $\mathfrak{K}_{\mathcal{O}}$ be a causal susceptibility for a driven observable $\mathcal{O}$. Here $-\infty$ means integration over the full past, with the preparation having finite support. With an absolutely integrable time kernel,

$$\mathfrak{S} = \frac{\mathfrak{h}}{\mathfrak{h}_0 + \|\mathfrak{h}\|}, \quad (14)$$

$$\mathcal{O}(t) = \int_{-\infty}^t \mathfrak{K}_{\mathcal{O}}(t, t') F(t') \mathfrak{S}(t') dt', \quad \mathcal{O}|_{\mathfrak{h}=0} = 0. \quad (15)$$

The kernel includes the conversion units needed for $\mathcal{O}$. This equation defines the response class; it assigns no kernel to the fibre. The zero-drive value does not recover a static force.

By I-2 a carrier is a disturbance of a medium at that medium's speed. Together with the one-substance chain this gives photon = graviton = gluon = phonon = magnon: the same spin-one excitation in different density and geometry regimes. Matter braids lie outside this carrier identity. This manuscript reads the anti-Stokes light and the acoustic grating as its two regime modes hybridizing. Their glass speeds v_{opt} and v_{ac} are distinct and neither is the chain's c . Wave number, mode occupation and thermal occupation never change field identity or strand count.

1.4 Three curvature faces and two readings

For the Einstein tensor $G_{\mu\nu}$, stress tensor $T_{\mu\nu}$ and coupling κ , the symbols μ, ν are tensor indices. I-3 retains

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G/c^4. \quad (16)$$

The same stress tensor and boundary data give the same curvature, with the atlas downstream of that solution. This is containment. It is not a new fitted correction to the source's spectrum.

Let $d\ell_{\text{opt}}$ be optical distance and $d\mathbf{q}_{\text{ang}}$ the angular line element, distinct from the confinement domain. The static exterior optical metric of I-3 is

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\mathbf{q}_{\text{ang}}^2}{1 - \alpha_{\text{ext}}}. \quad (17)$$

The named isotropic weak-field index is n_{iso} ; the aperture index of P-4 is n_{cone} , with effective, chart-dependent aperture speed c_{eff} , never the invariant c of local null-cone kinematics; the computational index is n_{dens} . The symbol O denotes asymptotic remainder order. These indices remain separate:

$$n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2), \quad (18)$$

$$n_{\text{cone}} = (1 - \alpha_{\text{ext}})^{-1/2}, \quad c_{\text{eff}} = c/n_{\text{cone}}, \quad (19)$$

$$n_{\text{dens}} = (1 - \beta_{\sigma})^{-1/2}. \quad (20)$$

The aperture index is not a lensing index; the computational index enters no physical slot. For two optical segments, let n_1, n_2 denote their local material indices and θ_1, θ_2 their ray angles to the interface normal. Fermat matching is the exact relation $n_1 \sin \theta_1 = n_2 \sin \theta_2$. The gravitational optical metric contains radial and clock contributions; its first-order deflection has the factor of two absent from a lapse-only reading.

Let $\mathbf{m}$ be the common clock-and-ruler rescaling, $\mathcal{U}^\mu$ a unit timelike tangent and $\mathfrak{P}^{\mu\nu}$ its orthogonal projector. Core 0.1c gives the bookkeeper's reading

$$g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}, \quad \nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathbf{m}. \quad (21)$$

Moving at constant speed on a pattern's own clocks and rulers through a density gradient is gravity. Its kept ratio remains $d/t = c$. For a perfect stress tensor with zero net conserved count, the medium's clock factor follows its temperature; with conserved count and fixed entropy per count it follows enthalpy per count. These are conditions on the medium's flow. The frequency

of an oscillator carried by that medium follows its own dynamics and thermal-motion time dilation, never a substituted temperature ratio. The conformal factor bends massive paths and leaves strokes unbent. Strokes respond to the aperture and, in the static exterior, the optical metric. This manuscript reads Brillouin reflection from the travelling density grating as aperture response at the medium's strength.

For a pattern released from rest far away, define v as inward flow speed and $\mathbf{a}_{\text{fall}}$ as its inward acceleration. Let $\mathbf{v}_{\text{sw}}$ be the rotating source's swirl and $\boldsymbol{\varpi}_{\text{drag}}$ its gyroscope precession. The gravitational window gives

$$v = c\sqrt{\alpha_{\text{ext}}}, \quad \mathbf{a}_{\text{fall}} = GM/r_f^2, \quad \boldsymbol{\varpi}_{\text{drag}} = \frac{1}{2}\nabla \times \mathbf{v}_{\text{sw}}. \quad (22)$$

The last expression gives Lense–Thirring precession at leading order. The flow and its local rotation run at every size. Curvature, a tide and a density gradient are comparisons between neighbours or with a reference path. The skin against a centre path is such a comparison; an isolated pointwise clock is not.

1.5 Confinement, bounded response and the mass ledger

Let Ω be the confinement domain, B its boundary data, n integer strand count, χ orientation, and ϑ confinement position, renamed from the core's position coordinate to distinguish it from time. Let D be the confinement operator and u_q an eigenmode. I-5 states the eigenproblem

$$D(\Omega, B; n, \chi, \vartheta)u_q = \omega_q u_q. \quad (23)$$

The same strand class has different confinement positions and masses. A different strand class requires rebraiding; the generation fork stays open. No mass formula, anchor radius or absolute address for a glass nucleus follows. Isotope substitution changes the matter configuration whose mode is probed, not a strand number attached to the acoustic wave.

For a dimensionless odds-enhancing input $x \geq 0$, let α_0 be the initial A-1 ratio on a declared branch. The bounded I-6 response is

$$\alpha(x) = \frac{\alpha_0(1+x)}{1+\alpha_0 x}, \quad \alpha(x) - \alpha_0 = \frac{\alpha_0(1-\alpha_0)x}{1+\alpha_0 x}. \quad (24)$$

Here x enhances the geometric odds, not mass on an unnamed branch. At fixed exterior field point, raising source mass raises the exterior ratio. For simultaneous geometric change, the mass and radius terms are both retained.

Let M be material rest mass, V the corresponding volume, P pressure, T temperature, a the radiation energy-density constant and σ_{SB} the radiation flux constant. I-7 gives

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{aT^4 V}{c^2} + \cdots, \quad a = \frac{4\sigma_{\text{SB}}}{c}. \quad (25)$$

The pump energy held in optical and acoustic fields and heated glass enters E/c^2 , the heated glass's pressure enters PV/c^2 , and thermal radiation held in its volume enters $aT^4 V/c^2$, each once. Binding is read from the solved geometry and is never an extra local entry. The rounded constants are $F_{\text{max}} \simeq 3.03 \times 10^{43}$ N, $a \simeq 7.566 \times 10^{-16}$ J m⁻³ K⁻⁴ and $\sigma_{\text{SB}} \simeq 5.67 \times 10^{-8}$ W m⁻² K⁻⁴ (U-05, U-06, U-07). Their independent rounding is retained; the exact bridge is Eq. (25).

For observable $\mathcal{O}$, let x_i be independently fixed channel inputs and S_i their partial logarithmic sensitivities with all other listed inputs fixed. The operator δ is a local variation; Δ is a finite difference and $\mathcal{C}$ the enrichment path. They give

$$\delta \ln \mathcal{O} = \sum_i S_i \delta \ln x_i, \quad \Delta \ln \mathcal{O} = \int_{\mathcal{C}} \sum_i S_i d \ln x_i. \quad (26)$$

The local product of slope and span is not a finite amplitude unless the slope is constant on that path. Two endpoints define a secant. For I-7's density discriminator, the observable sign is $\text{sgn}(\prod_i S_i)$ and stays conditional until the sensitivities are established. The total multi-channel response in Eq. (26) is separately evaluated. Cooling supplies no direction of total compactness from a temperature partial alone.

2 System chart

Let R_{room} denote a configuration's observed support scale and ρ_{loc} its interior density profile. For a nonnegative response weight $w_{\mathcal{O}}$, define the profile mean by the weighted integral below. Its domain $\mathcal{V}$ includes every part of the sampled configuration.

$$\langle \rho_{\text{loc}} \rangle_{\mathcal{O}} = \frac{\int_{\mathcal{V}} w_{\mathcal{O}} \rho_{\text{loc}} dV}{\int_{\mathcal{V}} w_{\mathcal{O}} dV}, \quad 0 < \int_{\mathcal{V}} w_{\mathcal{O}} dV < \infty. \quad (27)$$

The exterior gravitational atlas uses the source mass and field point. The separate interior configuration chart has its skin at the fold, its local density ρ_{loc} densest at the skin and dilute toward the centre. There is no transfer map between these charts. Observed fibre radii and winding radii are support sizes, never inferred interior addresses.

System element	Chart or axis	Operational weight
Glass, fibre core and cladding	A-1 exterior $\alpha_{\text{ext}} = \mathcal{M}c^2/(2F_{\text{max}}r_f)$; separate interior schematic	Independent mass and geometry; no frequency-to-mass inversion.
Pump-probe spectrum	Carrier phase and wave number	Probe overlap with pumped grating through the whole fibre, including warm sections.
Forced-detuning map	Same carrier, changed probe detuning	Spontaneous backscatter plus transmitted probe, anti-Stokes filtering and heterodyne detection, including warm-part resonances.
Gain and linewidth	Whole-fibre response	Their method and normalising fibre length are unstated in the paper.
Polarization null	Neighbouring opposite launch paths	Coil, splices and both leads between polarimeters, including warm sections.
Isotope contrasts	A-1 fixed-geometry mass perturbation plus separate strain channels	Independently reconstructed optical-acoustic overlap across fibre core and cladding and along the whole fibre.

The spectral weights follow the source's transmitted-power reading, port signal and warm-section qualification (SC-021-07, L211; SC-021-08, L213; SA-21, SC-037-06, L381; SC-037-07,

L382; SC-037-08, L383; SA-37, SC-027-07, L249; SC-027-09, L394; SA-27). The interior weight is an observation-region statement of I-3, not a numerical profile recovered from a single spectral curve.

3 Observation map

3.1 Source equations and settings

The gain spectrum in source Eq. (1) is transcribed with the peak, width and centre names of the notation guard (SC-055-28, L134; SC-055-29, L136). The detached powers and fraction are reconstructed as the Lorentzian squares; the even, finite peak fixes this reading independently of their placement. The source transcription is

$$\mathfrak{g}_{\text{Br}}(\omega) = \mathcal{G}_{\text{Br}} \frac{(\Upsilon_{\text{m}}/2)^2}{(\omega - \omega_{\text{Br}})^2 + (\Upsilon_{\text{m}}/2)^2}. \quad (28)$$

The phase-matching and pump-enhanced coupling relations are source relations (SC-013-05, L149; SC-016-02, L153):

$$\omega_{\text{Br}} = \frac{4\pi n v_{\text{ac}}}{\mathcal{L}_{\text{pump}}}, \quad \mathfrak{g}_{\text{om}} \simeq \sqrt{\frac{\mathcal{G}_{\text{Br}} \Upsilon_{\text{m}} \mathcal{P}_{\text{pump}} c}{4n}}. \quad (29)$$

The radical is reconstructed over the whole product; that dimensionally closed choice returns the reported strong-setting pair. The paper leaves the factor-of- 2π convention of its MHz rates unstated (SC-029-02, L253; SC-029-03, L253; SA-29). Arithmetic below uses those printed numerals without silently changing that convention. Angular dispersion has energy $\hbar\omega$; a rate in reciprocal seconds requires $\hbar$ to become energy.

The following reported parameters describe the whole fibre at the nominal temperatures; the paper states no normalising fibre length for its per-length gain or propagation loss.

Reported parameter	Value	Claim and source locus
Υ_{opt}	13 ± 2 MHz	SC-007-03, L116
$\mathcal{G}_{\text{Br}}$ at 293 K	1.41 ± 0.06 W ⁻¹ m ⁻¹	SC-008-03, L119
$\mathcal{G}_{\text{Br}}$ at 4 K	6 ± 1 W ⁻¹ m ⁻¹	SC-008-03, L119
Υ_{m} at 293 K	46.3 ± 0.2 MHz	SC-008-05, L121
Υ_{m} at 4 K	9.93 ± 0.12 MHz	SC-008-06, L122
η_{dB}	0.001 dB m ⁻¹	SC-055-27, L156
n	1.48	SC-055-27, L156
$\mathcal{L}_{\text{pump}}$	1550 nm	SC-037-01, L373
$\omega_{\text{Br}}/(2\pi)$	9.5997 ± 0.0001 GHz	SC-019-04, L253

The source says the sample *can be spliced to single-mode fibre with low loss, resulting in* the optical dissipation rate, and that *More information about the sample can be found in the Supplementary Table 1* (SC-007-01, L114; SC-007-02, L115; SC-007-03, L116; SC-007-04, L117). The optical dissipation rate is not derived here from the propagation loss.

3.2 Optical observations, estimates and simulations

Each row below uses the whole-fibre weight appropriate to its technique in the system chart. The comparison is steady state, at the nominal 4 K cryostat setting (SC-017-02, L153) and the pump wavelength above.

Source location	Record and verb class	Scope
Pump-probe	Estimated $g_{\text{om}} = 8 \pm 3$ MHz; a single Lorentzian response	SC-028-01, L251; SC-029-01, L252
Pump-probe	Estimated $g_{\text{om}} = 61 \pm 2$ MHz and 75 ± 2 MHz; split optical response	SC-028-03, L254; SC-028-04, L257
Maximum split	Reported $2s_{\text{split}} = 26.7$ MHz at estimated $g_{\text{om}} = 76 \pm 2$ MHz	SC-032-03, L258
Forced detuning	Estimated $g_{\text{om}} = 68 \pm 2$ MHz at $\mathcal{P}_{\text{pump}} = 1.55 \pm 0.1$ W	SC-054-03, L367
Weak forced detuning	Estimated $g_{\text{om}} = 4 \pm 3$ MHz at reported $\mathcal{P}_{\text{pump}} = 0.004 \pm 0.1$ W	SC-036-02, L362
Readout	Reported $\mathcal{P}_{\text{probe}} = 0.6$ μ W	SC-054-02, L366
Optical crossing	Source: <i>An avoided crossing is successfully observed</i>	SC-040-01, L385
Theory curves	Simulated; source: <i>showing good agreement with the experiment</i>	SC-024-02, L231

The source does not state whether each coupling estimate comes from its gain formula or a spectral fit. The acoustic displacement spectrum is theoretical; the optical anti-Stokes spectrum is the measured response (SC-031-02, L255; SC-031-05, L258). The paper’s threshold says that splitting becomes observable when coupling exceeds the average linewidth (SC-030-02, L253; SC-030-03, L254). This is the criterion used throughout, not a geometric endpoint. The source assigns time-domain Rabi oscillations to pulsed schemes (SC-015-02, L148; SC-015-03, L150; SC-015-04, L152). No exchange time is measured by these steady-state curves.

The identity map recovers the source Lorentzian and its coupled-mode comparison at their own input precision. It corroborates no invariant. This manuscript’s I-2 carrier reading of these spectra takes the source’s coupled-mode algebra as that identity map and adds no observable to it; the prospective comparisons test P-5’s magnetic source and I-7’s density-response constitutive channel, while I-3’s containment is the equality of curvature from equal stress data, not a reproduced fit; a failure of the source’s coupled-mode algebra to return these spectra on the source’s own inputs falsifies that identity map.

3.3 Arithmetic separating the record from its reading

Let $\mathcal{Q} = \mathcal{G}_{\text{Br}} \Upsilon_{\text{m}}$ be the gain–linewidth product, $\mathcal{F}_{\text{th}}$ the average-linewidth threshold, and $\mathcal{F}_{\text{loss}}$ the propagation-loss decay estimate using the illustrative group-speed substitution c/n . The source relations imply

$$\mathcal{Q} = \mathcal{G}_{\text{Br}} \Upsilon_{\text{m}}, \quad \mathcal{F}_{\text{th}} = (\Upsilon_{\text{opt}} + \Upsilon_{\text{m}})/2, \quad (30)$$

$$\mathcal{F}_{\text{loss}} = (\ln 10/10)\eta_{\text{dB}}c/n, \quad \mathcal{P}_{\text{pump}} = \frac{4ng_{\text{om}}^2}{c\mathcal{G}_{\text{Br}}\Upsilon_{\text{m}}}. \quad (31)$$

For any function of measured inputs, uncertainty is the gradient contracted with their covariance matrix. The following comparison uses zero covariance only as the explicit U-73 convention. It is not a reconstruction of the authors' fit covariance. Central derived numbers carry their propagated uncertainty, itself at the figures of the least precise input uncertainty, and are rounded at its last digit; a value without an uncertainty keeps the figures of its least precise printed input: one significant figure for the cold gain and loss, and three for the material index.

Derived comparison	Result	Limiting input
Propagation decay $\mathcal{F}_{\text{loss}}$	0.05 MHz	Loss, one significant figure
Cold gain–linewidth product	$60 \pm 10 \text{ W}^{-1} \text{ m}^{-1} \text{ MHz}$	Cold-gain uncertainty, one figure
Warm gain–linewidth product	$65 \pm 3 \text{ W}^{-1} \text{ m}^{-1} \text{ MHz}$	Warm-gain uncertainty, one figure
Cold threshold	$11 \pm 1 \text{ MHz}$	Optical-linewidth uncertainty, one figure
Warm threshold	$30 \pm 1 \text{ MHz}$	Optical-linewidth uncertainty, one figure
Cold coupling at the strong-setting power	$68 \pm 6 \text{ MHz}$	Cold-gain uncertainty, one figure
Warm coupling at the strong-setting power	$72 \pm 2 \text{ MHz}$	Warm-gain uncertainty, one figure
Cold coupling at the weak-setting central power	$3.5 \pm 0.3 \text{ MHz}$	Cold-gain uncertainty, one figure
Cold coupling at the weak-setting upper power	$18 \pm 1 \text{ MHz}$	Cold-gain uncertainty, one figure
Acoustic-speed inversion	$5.03 \times 10^3 \text{ m s}^{-1}$	Index, three figures; U-74
Pump inversion for the single-peak estimate	0.02 W	Cold gain, one figure
Pump inversion for the lower split-trace estimate	1 W	Cold gain, one figure
Pump inversion for the upper split-trace estimate	2 W	Cold gain, one figure

The coupling uncertainties in this table propagate gain and mechanical linewidth only; the pump setting is fixed for that comparison. The weak-setting upper power is its central value plus its printed uncertainty, not a new measurement. Its uncertainty-to-value ratio is 25, derived from the printed pair. That unresolved input supports no new physical conclusion. The two temperature products agree within their propagated errors, while the average-linewidth threshold falls. I-7 reads the temperature dependence through the complete mass and geometric channels; neither cooling nor these spectra fixes a direction of total compactness. A future independently weighted channel sum with the wrong sign outside its full uncertainty falsifies that constitutive reading.

For cooperativity the source's reconstructed square gives

$$C_{\text{max}} = \frac{(2g_{\text{om}}^{\text{max}})^2}{\Upsilon_{\text{opt}} \Upsilon_{\text{m}}}. \quad (32)$$

The detached square is restored from the numerator and the printed central estimate (SC-045-03, L476). The source estimate is 179 ± 2 (SC-045-03, L476). The independent-input calculation gives approximately 180 ± 30 , rounded at the last digit of its propagated uncertainty, which the optical-linewidth and coupling uncertainties set. The central arithmetic rounds to

the reported estimate before this precision restriction. The larger propagated error identifies missing covariance or estimation information, not a replacement error bar for the source.

The reported maximum split equals neither the derived $2g_{\text{om}} = 150$ MHz nor the derived $2g_{\text{om}}/(2\pi) = 24$ MHz at two significant figures. No relation between that pump–probe split and coupling is printed. The forced-detuning split axis spans 30–60 MHz, without printed numerical data values (SC-056-02, L434; SC-056-01, L411; SC-056-03, L423; SA-42). It is not a second transcription of the pump–probe maximum.

For the Bose occupation, use the reported frequency and nominal temperature with exact unit-definition inputs $h = 6.62607015 \times 10^{-34}$ J s and $k_B = 1.380649 \times 10^{-23}$ J K⁻¹ (U-02, U-03). Let $\mathcal{P}_{\text{dec}}$ denote this manuscript’s power criterion, coupling equal to thermal occupation times mechanical linewidth (U-72).

$$\bar{\mathcal{N}}_{\text{th}} = \left[\exp\left(\frac{h\omega_{\text{Br}}}{2\pi k_B T}\right) - 1 \right]^{-1}, \quad (33)$$

$$g_{\text{om}} = \bar{\mathcal{N}}_{\text{th}} \Upsilon_{\text{m}}, \quad \mathcal{P}_{\text{dec}} = \frac{4n\bar{\mathcal{N}}_{\text{th}}^2 \Upsilon_{\text{m}}}{c\mathcal{G}_{\text{Br}}}. \quad (34)$$

The occupation is approximately 8. This criterion gives approximately 2 W, at the single-figure precision of the nominal temperature or cold gain. Using the source’s order-of-ten estimate as a specified occupation instead gives approximately 3 W. The source estimates the maximum cooperativity *achieved in this experiment* and says that *The thermal phonon occupation in this experiment is of the order of ten phonons, given their high acoustic frequency and low sample temperature*, and that *Under these conditions, the coupled system has the potential to enter the quantum coherent coupling regime over the whole bandwidth of acoustic modes addressed in the scattering process when increasing the pump power to around 5 W* (SC-045-01, L472; SC-045-02, L473; SC-047-01, L476; SC-047-02, L477; SC-047-03, L478; SC-048-01, L479; SC-048-02, L480; SC-048-03, L481; SC-048-04, L482). Its criterion is not printed. The difference is between stated criteria, not an observed discrepancy. The warm sections also prevent the nominal cryostat temperature from defining an independently measured uniform occupation.

3.4 Detuning, peak heights and the limits of the record

The pump-relative definition adopted here is the first line below; the second records the source’s other convention at its own locus. The independent wave-number mapping is the last line (SC-022-01, L204; SC-022-02, L242; SC-022-04, L366; SC-022-03, L260).

$$\omega_{\text{probe}} = \omega_{\text{pump}} + \Delta_{\text{probe}}, \quad \Delta_{\text{det}} = \Delta_{\text{probe}} - \omega_{\text{Br}}, \quad (35)$$

$$\omega_{\text{probe}} = \omega_{\text{pump}} + \omega_{\text{Br}} + \Delta_{\text{probe}} \quad (\text{source’s other convention}), \quad (36)$$

$$\omega_{\text{probe}} - \omega_{\text{pump}} = k\mathbf{v}_{\text{opt}}. \quad (37)$$

The second convention is not used to assign settings below. The source calls its phonons counter-propagating without naming the reference wave (SC-014-02, L144). This manuscript reads its backward phonon label relative to the pump in the dispersion sketch; that directional assignment is its own.

The forced-detuning map combines spontaneous backscatter and transmitted probe, filtered around the anti-Stokes response and heterodyne detected (SC-037-06, L381; SC-037-07, L382;

SC-037-08, L383). The yellow overlay is called *eigenvalues* in the caption and *eigenvectors* in the text; it is a theoretical visual aid in both places (SC-040-09, L361; SC-040-07, L392). The simulated spectrum omits the probe diagonal because it contains no probe (SC-055-42, L399; SC-055-43, L400). The source studies *the energy distribution between hybrid modes* at detunings -4.1 , -0.1 and 3.9 MHz: *more energy* in the higher-frequency mode, *the same* amount in the two modes, then energy *transferred to the lower-frequency mode* (SC-039-02, L381; SC-039-03, L382; SC-039-05, L384; SC-039-06, L385; SC-039-07, L386; SC-039-08, L389; SC-039-09, L390). Its statement that this exchange follows the expected behaviour of an adiabatic transition remains a reading of those spectra (SC-039-10, L391; SC-039-11, L392; SC-039-12, L393). An energy share has no geometric warrant for an alpha application.

The paper's simulation agreement is in-sample and has no printed goodness metric (SC-024-02, L231; SC-024-08, L266; SA-24). Its spatial-domain criterion *can also be defined*; the supporting details are supplementary-only (SC-033-02, L264; SC-033-03, L265; SC-033-06, L268). The source says that the shape of its two split peaks *is asymmetric* and that *This line profile might resemble the one measured in electromagnetically induced transparency (EIT)-like effects in Brillouin amplifiers, but it is unrelated*, and that *A comparison between these EIT-like effects and the strong coupling mode splitting can be found in Supplementary Information* (SC-034-01, L258; SC-034-02, L259; SC-034-03, L260; SC-034-04, L261; SC-034-05, L262; SC-034-06, L263; SC-034-07, L264; SA-34). These observations retain the steady-state scope of one highly non-linear fibre in a 4 K cryostat (SC-017-02, L153) at one pump wavelength. They corroborate no invariant through the identity map; a prospective failure to recover the same spectra under the same inputs would instead falsify that convergence map.

4 Cone-position translation

For the source envelope construction let $\mathbf{z}$ be fibre position and $\mathbb{R}$ the real wave-number line. The dagger $\dagger$ denotes Hermitian adjunction. The mode operators have the Fourier convention shown below. Equations (2)–(4) of the paper are reconstructed with their Fourier square roots and operator daggers in place (SC-055-30, L240; SC-055-32, L271; SC-055-31, L248). In the source's undepleted-pump regime its Hamiltonian can be linearized to a beam-splitter interaction (SC-025-03, L244; SC-025-04, L245):

$$\hat{\mathbf{a}}_{\text{aS}}(\mathbf{z}, t) = (2\pi)^{-1/2} \int_{\mathbb{R}} \hat{\mathbf{a}}(k, t) e^{-ikz} dk, \quad (38)$$

$$\hat{\mathbf{b}}_{\text{ac}}(\mathbf{z}, t) = (2\pi)^{-1/2} \int_{\mathbb{R}} \hat{\mathbf{b}}(k, t) e^{-ikz} dk, \quad (39)$$

$$\mathcal{H}_{\text{int}} = \hbar g_{\text{om}} [\hat{\mathbf{a}}^\dagger(k) \hat{\mathbf{b}}(k) + \hat{\mathbf{b}}^\dagger(k) \hat{\mathbf{a}}(k)]. \quad (40)$$

The Fourier integrals are understood for square-integrable wave packets, with the continuum operator expressions their distributional limits. For dimensionless normalized single-mode operators the Hamiltonian has energy units with the explicit $\hbar$ and an angular-rate convention. Under continuum delta normalization, the same bilinear is a Hamiltonian density in wave number; its integral over wave number has energy units. Its Hermitian algebra is retained as the source model, not made into a particlewise ontological exchange.

Define ϕ_{h} as helical phase, ϕ_0 its initial value, $\mathcal{A}$ the scalar mode amplitude, ψ its complex projection, and $\mathbf{r}$ a cylindrical amplitude radius. Let $\mathbf{e}_1, \mathbf{e}_2$ be orthonormal basis vectors and

$\mathbf{B}$ their oriented phase plane. The symbol $\leftrightarrow$ denotes the identification of the complex phase plane with its Clifford representation. The I-4 lift is

$$\phi_h(t) = \phi_0 - \omega t, \quad \psi = \mathcal{A}e^{i\phi_h}, \quad (41)$$

$$(\mathbf{r} \cos \phi_h, \mathbf{r} \sin \phi_h, \mathbf{z}) \mapsto \text{helical transport}, \quad (42)$$

$$i \longleftrightarrow \mathbf{B} = \mathbf{e}_1 \mathbf{e}_2, \quad \mathbf{B}^2 = -1, \quad e^{\mathbf{B}\phi_h}. \quad (43)$$

The exponent is dimensionless. This is a single-angle phase rotor, distinct from a half-angle spinor rotor. Superposition is ignorance of ϕ_0 at observation. This manuscript reads the source's *quantum coherent coupling regime* (SC-048-02, L480), *non-classical states* (SC-052-03, L494) and *entanglement distribution* (SC-055-52, L425) through this lift: each mode state is the projection ψ of a helix onto the plane normal to the cone axis, and the correlation that entanglement distributes is a shared bivector grade. The lift moves on the phase axis only; it changes neither strand count nor field identity. The glass's matter patterns supply confinement and boundary data, while the carrier modes supply the acoustic and optical phase coordinates.

I-3 keeps exterior curvature, interior confinement and probe-weighted response distinct. The medium aperture reading belongs to the light's passage across a neighbouring density grating, not to a curvature inferred at one isolated pattern. The identity map of Use item 2 recovers the retained coupled-mode spectra at the order of the linearized, undepleted-pump model. An independently measured violation of that order's spectral response falsifies the map; the source's own fitted curves add no out-of-sample corroboration.

5 Prospective predictions

The extra inputs tested here are P-5's magnetic source and I-7's density-response constitutive channel. Neither a bounded ratio nor a branch label alone changes an observable. Each comparison uses neighbouring paths or matched preparations so that common conventional response cancels. All numerical apparatus and material inputs below are illustrative-assumed test points, not properties measured for the source fibre. The significance level is 0.05, two-sided (U-46). The simulation seed is 3458, incremented by the block number, with 20000 independent experiments per block (U-50, U-51). A false rejection means rejection of the comparator or exclusion of a true specified alternative; attribution uses the same threshold as detection.

5.1 T-1: magnetic existence under applied twist

DC-1 discriminates SA-17 by testing the additional magnetic source of P-5 on the source's nonlinear fibre. The source describes its highly nonlinear fibre in a 4 K cryostat, without a reciprocity or magnetic-null measurement (SC-017-02, L153). This block is conditional by construction on reading P-5's twist-established magnetic field along the fibre axis and reversing with twist hand; that axis and reversal are this manuscript's reading. The standard account reads the calibrating current's magnetic field and the residual magnetic fields as independent entities acting on the glass's Faraday response, and assigns a static twist no magnetic field. P-5 reads each, and the magnetic field a twist establishes, as a macroscopic helical density gradient of the omnium, the bivector grade of the density field coherently aligned along an axis, which for the twist is the fibre axis of this manuscript's reading; the calibrated Faraday response is

accepted in full. P-5 fixes neither the magnetic field's magnitude nor its absolute sign, and no twist-to-moment kernel is assigned.

The same fibre is rewound on a nonmagnetic mandrel, with winding sense fixed. The selected radius is 0.1 m, with 20 turns and 1 m lead per end (U-09, U-10, U-11). The twist settings are +1 and -1 turn m^{-1} (U-12). Pump power is 0 W; each launch uses 0.6 μW at 1550 nm in the nominal 4 K cryostat (U-14, U-15, U-16, U-17). These support sizes are not interior radii. The weight is the guided path through the coil, splices and warm leads between both end polarimeters.

Let Θ_{nr} be the nonreciprocal rotation, defined by the difference of forward and backward polarization readings after expressing both in one calibrated propagation convention. Let I be the axial calibration current, K its independently measured rotation slope and $\Theta_{\text{h}_{\text{hand}}}$ the zero-current rotation for hand h_{hand} . A straight conductor passes through the mandrel with its return outside the coil. The scan initially spans -0.1 to $+0.1$ A (U-18); a null outside it requires an extended scan. The defining condition and its fitted solution are

$$\Theta_{\text{nr}}(I, \text{h}_{\text{hand}}) = K_{\text{h}_{\text{hand}}} I + \Theta_{\text{h}_{\text{hand}}} = 0, \quad I_{\text{null}}(\text{h}_{\text{hand}}) = -\Theta_{\text{h}_{\text{hand}}} / K_{\text{h}_{\text{hand}}}. \quad (44)$$

Each hand and each polarimeter placement has its own nonzero calibrated slope. For a path-dependent Verdet response V and magnetic field B , the physical rotation is the path integral $\Theta = \int V B \cdot d\ell$. The illustrative constant response is $0.5 \text{ rad T}^{-1} \text{ m}^{-1}$ (U-19); the optional conventional magnetic conversion is $4\pi \times 10^{-7} \text{ N A}^{-2}$ (U-21). Neither fixes the P-5 magnetic amplitude. Actual current-to-rotation calibration supplies the measurement independently of that comparator.

Define $\mathcal{I}_j$ as the half-difference of nulls for independent winding pair j , $\bar{\mathcal{I}}$ its mean, and $\mathcal{I}_{\text{conv}}$ the conventional contrast. The conventional comparator assumes time-reversal reciprocity of static photoelastic twist. Its reciprocal hand-odd rotation cancels in the ideal nonreciprocal combination, so its null-current contrast is zero:

$$\mathcal{I}_j = \frac{I_{\text{null},j}(+) - I_{\text{null},j}(-)}{2}, \quad \mathcal{I}_{\text{conv}} = 0. \quad (45)$$

This time-reversal premise belongs to the standard account. This manuscript's many-to-many reading of core 0.1a and I-5b does not identify opposite operations as mirror events.

A uniform residual magnetic field has zero unweighted line integral around the closed coil. A hand-independent residual response cancels in the contrast. Residual weighting by reciprocal birefringence requires the following controls. Forward-backward-backward-forward launches cancel first-order drift within a scan. Both polarimeter placements cancel scale mismatch; the remaining launch-drift and placement terms are bounded from measured drift and placement differences. A magnetic-field map at each winding, with illustrative uncertainty 0.000000001 T (U-20), multiplied by the measured hand-dependent slope change bounds the nonuniform-magnetic-field cross term. An untwisted winding, fibre released from the stage while the stage takes both hands, bounds the stage's own hand-odd magnetic field. There is one such control per winding pair. These four bounds enter once, with their own errors.

Let $\mathbf{b}_\ell$ be the four measured bound magnitudes, $\mathbf{e}_\ell$ their standard errors and $\mathbf{f}_\ell$ their degrees of freedom. Let $\mathbf{e}_d$ be the standard error of the mean pair contrast, $\mathbf{f}_d$ its degrees of freedom, and $\mathbf{f}_W$ the Welch-Satterthwaite combination. Let $Z_t(\mathbf{f})$ denote the two-sided Student quantile

at the chosen level and H the full decision threshold.

$$f_W = \frac{(e_d^2 + \sum_{\ell} e_{\ell}^2)^2}{e_d^4/f_d + \sum_{\ell} e_{\ell}^4/f_{\ell}}, \quad (46)$$

$$H = \sum_{\ell} b_{\ell} + Z_t(f_W) \sqrt{e_d^2 + \sum_{\ell} e_{\ell}^2}. \quad (47)$$

Detection and attribution require $|\bar{I}| > H$. For a specified signed alternative $\mathcal{I}_{\star}$, exclusion requires $\text{sgn}(\mathcal{I}_{\star})(\mathcal{I}_{\star} - \bar{I}) > H$. This excludes only the specified magnitude; a finite null never excludes an amplitude-free existence reading. The illustrative exclusion alternative is 0.01 A, not an FDT magnetic prediction (U-59).

There are 12 independent winding pairs and 4 scans per winding, with hands balanced in repeated $+, -, -, +$ order (U-47). The preparation scatter is 0.001 A and scan scatter 0.0002 A (U-52, U-53). Each bound is 0.0001 A with standard error 0.00002 A and 11 degrees of freedom (U-54, U-55, U-56, U-57, U-58). The scans are nested, never independent windings. The resulting design threshold is approximately 0.0009 A at the single-figure input precision. The seeded simulation draws preparations, scans and measured-bound errors separately; the least-favourable signed bounded offset sits at its envelope (U-77). Its false detection and attribution rate is 0.025; its worst-sign false exclusion rate is 0.025, each with binomial standard error 0.001. These are seeded simulation rates at their binomial standard error, not calibrated experimental error rates. The existence contrast uses the prescribed logical encoding: FDT presence $+1$, conventional absence -1 , threshold 1. It is distinct from the current threshold above.

5.2 T-2: magnetic existence in a frozen helix

DC-2 discriminates SA-05 through P-5's helical-geometry magnetic source in independent spun draws of the same fibre design. The source cites a chiral photonic crystal fibre for the structural diversity of available optoacoustic platforms, without measuring its handed magnetic response (SC-005-01, L52; SC-005-02, L53; SC-005-03, L542; SC-005-04, L543). This block is conditional by construction on the same axial-magnetic-field and hand-reversal reading as the applied-twist block. Each structure is frozen during drawing, then wound untwisted on the same mandrel in the same winding sense.

The selected spin rates are $+1$ and -1 turn m^{-1} (U-13). All readout powers, wavelength, temperature, path weights, current calibration and polarimeter placements are those defined above. The null solves Eq. (44) separately for each fibre, with its own slope. The observable is half the difference of the two hand-specific sample means of these nulls. The conventional comparator is zero under its static-helix time-reversal reciprocity premise. The additional source is the frozen helical geometry of P-5, not an extra refractive-index fit.

There are 12 independent spun draws per hand and 4 scans per fibre, with one fibre per draw (U-48). Draw order and reading order both follow the balanced hand sequence. A preparation retains its draw offset across scans. The same preparation and scan scatters apply (U-52, U-53). Different reciprocal birefringence changes the calibrated slope; each hand-odd reciprocal rotation receives the balanced-launch and placement controls. Each fibre's linear birefringence crossed with the nonuniform residual magnetic field receives its own magnetic-field-map-times-slope bound. Stage controls use untwisted windings as above. Thus none of the four systematic bounds is removed by assuming identical fibres.

For the two independent hand samples, the variance of their half-difference is the sum of their mean variances divided by four. Each sample variance contributes its own degrees-of-freedom term to Eq. (47). Detection, attribution and signed-alternative exclusion use the same full threshold and the same specified exclusion current. The derived design threshold is approximately 0.0008 A, at single-figure input precision. The separately seeded simulation gives false detection and attribution rate 0.025 and worst-sign false exclusion rate 0.025, each with binomial standard error 0.001. It draws at the spun-draw and scan levels, with all four uncertain bounds. The existence encoding is again FDT presence +1, conventional absence -1, threshold 1. No amplitude or absolute magnetic sign is assigned.

5.3 Isotope comparator: constrained elasticity and finite paths

The next three blocks share a comparator whose inputs are independent of the Brillouin output. The fibre core is a bonded cylinder surrounded by cladding, with free outer radial traction and zero net axial force. Let r_c, r_s be its inner and outer support radii, E_j Young moduli, ν_j Poisson ratios, and ϵ_j^* free isotropic vibrational eigenstrains, for region $j = c, s$. The star $\star$ labels the free eigenstrain; ϵ_j denotes the region-indexed strain family, while the axial strain ϵ_z is common by bonding. Let A_j and D_s be displacement coefficients, L_j, M_j Lamé moduli and $T_{rr,j}, T_{zz,j}$ stresses. In the generic regional symbols, the c,s labels denote the fibre core and cladding: A_j, D_j include A_c, A_s, D_c, D_s ; the stress components $T_{rr,c}, T_{rr,s}, T_{zz,c}, T_{zz,s}$ are their radial and axial values. For radial coordinate r and radial displacement u_j , with u_c, u_s its regional values, linear isotropic elasticity gives

$$u_c = A_c r, \quad u_s = A_s r + D_s / r, \quad (48)$$

$$L_j = \frac{E_j \nu_j}{(1 + \nu_j)(1 - 2\nu_j)}, \quad M_j = \frac{E_j}{2(1 + \nu_j)}, \quad (49)$$

$$T_{rr,j} = 2M_j(A_j - D_j/r^2 - \epsilon_j^*) + L_j(2A_j + \epsilon_z - 3\epsilon_j^*), \quad (50)$$

$$T_{zz,j} = 2M_j(\epsilon_z - \epsilon_j^*) + L_j(2A_j + \epsilon_z - 3\epsilon_j^*), \quad D_c = 0. \quad (51)$$

The four unknowns are fixed by the four boundary conditions

$$u_c(r_c) = u_s(r_c), \quad T_{rr,c}(r_c) = T_{rr,s}(r_c), \quad (52)$$

$$T_{rr,s}(r_s) = 0, \quad r_c^2 T_{zz,c} + (r_s^2 - r_c^2) T_{zz,s} = 0. \quad (53)$$

This is a constrained-cylinder derivation, not an equal-strain partition. Define $\mathbf{v}_j$ as the logarithmic volume contrast and $\mathcal{J}_j^{\text{iso}}$ as the logarithmic isotope mass contrast at fixed atom counts. Here ρ_j denotes regional mass density, and $\mathbf{v}_c, \mathbf{v}_s$ are the two regional log-volume contrasts. For this small-strain model the geometric endpoint ratios are

$$\mathbf{v}_c = 2 \ln(1 + A_c) + \ln(1 + \epsilon_z), \quad \Delta \ln \rho_j = \mathcal{J}_j^{\text{iso}} - \mathbf{v}_j. \quad (54)$$

The cladding annulus has $\mathbf{v}_s = \ln[(1 + A_s)^2 - D_s^2/(r_c^2 r_s^2)] + \ln(1 + \epsilon_z)$; the displacement cross terms cancel between its inner and outer areas. Here ρ_j is mass density, not the unknown interior omnium profile. The model's neglected nonlinear elastic terms are quadratic in strain; the logarithms retain endpoint geometry exactly within that displacement solution.

Let c_{Ge} be the independently assayed germanium fraction, $m_{\text{Ge}}, m_{\text{Si}}, m_{\text{O}}$ integer atomic-mass benchmarks and w the fibre-core overlap weight. The assumed formula-unit mass is

$(1 - c_{\text{Ge}})m_{\text{Si}} + c_{\text{Ge}}m_{\text{Ge}} + 2m_{\text{O}}$ in the fibre core, and $m_{\text{Si}} + 2m_{\text{O}}$ in the cladding. Heavy-to-light ratios define each $\mathcal{J}_j^{\text{iso}}$. Define the weighted mass contrast $\mathcal{J}_{\text{iso}} = \mathbf{w}\mathcal{J}_c^{\text{iso}} + (1 - \mathbf{w})\mathcal{J}_s^{\text{iso}}$ and weighted volume contrast $\mathbf{v} = \mathbf{w}\mathbf{v}_c + (1 - \mathbf{w})\mathbf{v}_s$. These are a specified geometric-mean comparator, not a mass inferred from a frequency.

The numerical comparator inputs are listed below; all are assumptions (U rows), including integer isotope masses rather than evaluated masses.

Quantity	Assumed value	Provenance
Germanium fraction c_{Ge}	0.10	U-22
Fibre-core radius r_c	0.000002 m	U-23
Cladding radius r_s	0.0000625 m	U-24
Young modulus E_c	70000000000 Pa	U-25
Young modulus E_s	72000000000 Pa	U-26
Poisson ratios ν_c, ν_s	0.17, 0.17	U-27, U-28
Germanium masses	70, 74	U-29
Silicon mass	28	U-30
Oxygen masses	16, 18	U-31
Pure-isotope endpoint fraction	1	U-34
Germanium free eigenstrain in fibre core	-0.00001	U-35
Oxygen free eigenstrain in each region	-0.00001	U-36
Direct elastic, index, photoelastic contrasts	0, 0, 0	U-37, U-38, U-39
Index sensitivity to volume	-0.3	U-40
Photoelastic sensitivity to volume	0	U-41
Longitudinal-modulus sensitivity to volume	-4	U-42
Fibre-core overlap weight $\mathbf{w}$	0.8	U-43
Reference effective area	0.000000000012 m ²	U-44
Additional density sensitivity	3	U-45

Let $\mathbf{C}$ be the longitudinal elastic modulus, $\mathbf{p}$ the photoelastic coefficient and S_{eff} effective overlap area. The additional constitutive sensitivity is S_{den} . The three strain sensitivities in the table are $S_{\text{ind}}, S_{\text{pe}}, S_{\text{el}}$, respectively, at fixed composition and electronic state. The effective density ρ has $\Delta \ln \rho = \mathcal{J}_{\text{iso}} - \mathbf{v}$. The assumed conventional scalar gain model (U-71) has fixed wavelength and constant prefactor:

$$\nu_{\text{ac}} = (\mathbf{C}/\rho)^{1/2}, \quad \mathcal{Q} \propto \frac{n^7 p^2}{c \mathcal{L}_{\text{pump}}^2 \rho \nu_{\text{ac}} S_{\text{eff}}}, \quad (55)$$

$$\Delta \ln n = S_{\text{ind}} \mathbf{v}, \quad \Delta \ln \mathbf{p} = S_{\text{pe}} \mathbf{v}, \quad \Delta \ln \mathbf{C} = S_{\text{el}} \mathbf{v}, \quad (56)$$

$$\Delta \ln S_{\text{eff}} = 2 \ln(1 + A_c) - 2 S_{\text{ind}} (\mathbf{v}_c - \mathbf{v}_s). \quad (57)$$

The omitted numerical prefactor is dimensionless and cancels from ratios. The last line is the independently assumed finite-path area closure U-75: area scales as fibre-core radius squared and as the inverse square of the fibre-core-to-cladding index ratio. It retains the isotope's own overlap reshaping. The scalar gain law is an explicit conventional comparator assumption, not an equation attributed to the unseen contents of the source's tutorial reference.

Define $\mathfrak{d}_{\omega}^{\text{conv}}$ and $\mathfrak{d}_Q^{\text{conv}}$ as the conventional finite log contrasts, and $\mathfrak{d}_{\omega}^{\text{FDT}}, \mathfrak{d}_Q^{\text{FDT}}$ as the additional-channel predictions. Integrating the declared constant sensitivities along the en-

richment path gives

$$\mathfrak{d}_{\omega}^{\text{conv}} = \Delta \ln n + \frac{1}{2} \Delta \ln C - \frac{1}{2} \Delta \ln \rho, \quad (58)$$

$$\mathfrak{d}_Q^{\text{conv}} = 7\Delta \ln n + 2\Delta \ln p - \frac{1}{2} \Delta \ln \rho - \frac{1}{2} \Delta \ln C - \Delta \ln S_{\text{eff}}, \quad (59)$$

$$\mathfrak{d}_{\omega}^{\text{FDT}} = \mathfrak{d}_{\omega}^{\text{conv}} + S_{\text{den}} \mathcal{J}_{\text{iso}}, \quad \mathfrak{d}_Q^{\text{FDT}} = \mathfrak{d}_Q^{\text{conv}} + S_{\text{den}} \mathcal{J}_{\text{iso}}. \quad (60)$$

The extra term is a conditional constitutive input licensed in sign by I-7, with its magnitude explicitly assumed in U-45. It is not obtained from the chart alone. For each region the fixed-geometry exterior ratio changes in direct proportion to independently counted mass; both endpoint ratios remain inside the open interval. The exact odds change determines $x = X_{\text{heavy}}/X_{\text{light}} - 1$ in Eq. (24). Vibrational volume is then a separate geometric channel, never silently held fixed and changed at once. Ablation sets S_{den} to its conventional value zero and retains every conventional channel.

Channel	Frequency / product sensitivity	Sign at the specified enrichment path
Mass density at fixed elastic modulus	$-1/2, -1/2$	Positive mass contrast lowers both conventional responses.
Index at fixed other channels	$+1, +7$	Negative vibrational volume with negative index sensitivity raises both.
Elastic modulus at fixed density	$+1/2, -1/2$	Contraction raises modulus at this benchmark; frequency rises, product falls.
Photoelastic coefficient	$0, +2$	Zero specified contrast; retained for independent calibration.
Effective area	$0, -1$	Isotope-induced area reduction raises the product.
Additional I-7 density response	$S_{\text{den}}, S_{\text{den}}$	Positive specified sensitivity raises both; sign remains conditional on its calibration.

At the single-figure precision of the eigenstrain and overlap inputs, germanium substitution gives fibre-core volume contrast approximately -0.00001 , cladding volume contrast approximately -0.00000002 , and weighted mass contrast approximately 0.005 . Whole-glass oxygen substitution gives volume contrast approximately -0.00003 in both regions and weighted mass contrast approximately 0.06 . The fibre-core-only and whole-glass solutions are distinct even with equal free eigenstrains.

5.4 Isotope statistics and independent netting

Each composition has 12 independent preforms, with 2 fibres per preform and 4 spectra per fibre (U-49). Preforms are drawn alternately; each fibre is spliced, wound and cooled independently, with cooldowns alternating between compositions. The preform is the replication unit. Electron-probe microanalysis supplies the germanium profile, polarimetry and strain mapping the residual stress, and optical near-field plus acoustic-overlap reconstruction the effective area and weights (U-68, U-69, U-70). Profile, stress and baseline area differences are netted using independently determined sensitivities. The isotope's own vibrational strain and area reshaping stay in the comparator.

Let η be the observed difference of preform means after those nuisance corrections, less the conventional finite contrast. For the heavy (+) and light (−) composition group mean variances e_+^2, e_-^2 with degrees of freedom f_+, f_- , let e_{com} be the common comparator-plus-netting uncertainty and b_{lot} the unmeasured composition-lot contrast bound. Let Z_n be the two-sided normal quantile at the same significance level. Then

$$f_W = \frac{(e_+^2 + e_-^2)^2}{e_+^4/f_+ + e_-^4/f_-}, \quad (61)$$

$$H_{\text{iso}} = b_{\text{lot}} + \sqrt{Z_t(f_W)^2(e_+^2 + e_-^2) + Z_n^2 e_{\text{com}}^2}. \quad (62)$$

Shared sensitivities enter e_{com}^2 through their full covariance contraction. For correlated spectral log-gain and log-linewidth estimates, their sum variance includes twice their covariance. For a paired comparison the variance of a difference is the sum of marginal variances minus twice the covariance. Thus repeated spectra cannot shrink a shared preform or lot term.

Detection and attribution require $|\eta| > H_{\text{iso}}$. For a specified additional change η_* , exclusion requires $\text{sgn}(\eta_*)(\eta_* - \eta) > H_{\text{iso}}$. The lot bound enters each rule once. For the conditional predictions, $\eta_* = S_{\text{den}} \mathcal{J}_{\text{iso}}$. The experimental threshold, including the lot bound, is also the ablation threshold.

Nuisance level	Frequency log scatter	Product log scatter
Preform, U-60	0.0002	0.002
Draw, U-61	0.0001	0.001
Splice, U-61	0.0001	0.001
Cooldown, U-62	0.0001	0.001
Spectral fit, U-63	0.00005	0.0005
Shared comparator and netting, U-64	0.0002	0.002
Composition-lot bound, U-65	0.0005	0.005

The gain–linewidth correlation is -0.5 (U-66); equal marginal fit scatters are chosen so their correlated sum has the specified product scatter (U-76). Independent nuisance levels have correlation 0 in this simulation convention (U-67). Each experiment draws preform offsets shared by its fibres, separate draw, splice and cooldown offsets shared by that fibre’s scans, joint spectral fit noise, and one common comparator offset. The composition-lot term is placed at its worst signed bound, not reduced by replication (U-78). The rates below report the seeded null simulations at their binomial standard error. They are separate from the single-figure precision of the benchmark amplitudes at the declared test points.

5.5 T-3: germanium isotope shift of the Brillouin centre

DC-3 discriminates SA-19 through the I-7 constitutive input in germanium-enriched fibre cores. The source reports a higher germanium doping and a weak-coupling Lorentzian centre but no isotope response (SC-019-03, L155; SC-029-01, L252; SC-019-04, L253). This block is conditional by construction on the independently established sensitivities of Eq. (60). Its own reading of I-7’s larger density response is a higher Brillouin frequency. The observable sign is $\text{sgn}(\prod_i S_i)$ and stays conditional until the sensitivities are established.

The referents are ^{74}Ge and ^{70}Ge fibre-core enrichments, both spin zero, with residual ^{73}Ge of spin 9/2 assayed independently (U-29, U-32). The matched concentration profile and preparations follow the isotope protocol. Use the source pump-probe apparatus at the assumed nominal temperature and wavelength (U-16, U-17), with a weak undepleted pump; a power extrapolation returns the unsplit line centre independently of gain or coupling inference. The observable is $\Delta \ln \omega_{\text{Br}}$, from heavy and light fitted line centres. Each line-centre fit includes the whole acoustic-optical overlap, cladding and warm sections.

The defining condition is the maximum of Eq. (28): its first frequency derivative vanishes and its second is negative at the fitted centre. For independently fitted centres $\omega_{\text{Br},+}$, $\omega_{\text{Br},-}$ this gives

$$\partial_{\omega} \mathfrak{g}_{\text{Br}}|_{\omega_{\text{Br},\pm}} = 0, \quad \Delta \ln \omega_{\text{Br}} = \ln(\omega_{\text{Br},+}/\omega_{\text{Br},-}) = \mathfrak{d}_{\omega}^{\text{FDT}}. \quad (63)$$

The comparator is the independently predicted conventional frequency contrast, including cladding-constrained volume and index response. At the declared test point the conventional contrast is approximately -0.002 . The conditional FDT contrast is approximately 0.01 . The additional-channel difference is approximately 0.01 . The full frequency threshold is approximately 0.0009 . All four rounded values use one significant figure: the two contrasts are limited by the assumed overlap, eigenstrain and response sensitivities, their difference by the assumed overlap and density sensitivity, the threshold by the assumed frequency scatters and lot bound. Unrounded path values determine the criterion. The additional channel exceeds that full threshold. A measured residual with the opposite sign outside the threshold rejects this conditional input; exclusion uses the specified-change rule above.

The separately seeded frequency simulation gives false detection and attribution rate 0.025 and worst-sign false exclusion rate 0.025 , each with binomial standard error 0.001 . It redraws each nuisance where the apparatus resets it and retains the enriched-oxide lot bound. The source's single centre leaves this isotope response undetermined; the matched isotope comparison supplies precisely that missing observable.

5.6 T-4: oxygen isotope shift of the gain–linewidth product

DC-4 discriminates SA-09 through the I-7 constitutive input in whole-glass oxygen enrichment. The source reports its cold gain and linewidth without an isotope composition or measurement method (SC-008-03, L119; SC-008-06, L122). This block is conditional by construction on Eq. (60)'s independently determined channel sensitivities. Its own reading of larger density response is a larger gain–linewidth product. The observable sign is $\text{sgn}(\prod_i S_i)$ and stays conditional until the sensitivities are established.

The referents are ^{18}O and ^{16}O in the silica of both fibre core and cladding, both spin zero; residual ^{17}O of spin 5/2 is assayed (U-31, U-33). The germanium profile, draw, splice, temperature and wavelength follow the shared protocol. The gain peak is obtained from calibrated weak-pump probe depletion per independently measured pumped length. The full width comes from the same weak-coupling Lorentzian fit, with covariance retained. Neither quantity is inferred from the comparator's density or from an estimated strong-coupling rate.

Let ω_{-} , ω_{+} , collectively $\omega_{\pm}$, denote the two half-maximum frequencies of a single trace. Its defining condition solves

$$\mathfrak{g}_{\text{Br}}(\omega_{\pm}) = \mathcal{G}_{\text{Br}}/2, \quad \omega_{+} - \omega_{-} = \Upsilon_{\text{m}}, \quad \mathcal{Q} = \mathfrak{g}_{\text{Br}}(\omega_{\text{Br}})(\omega_{+} - \omega_{-}). \quad (64)$$

The isotope observable is the finite logarithm of the heavy/light product ratio. Its weight is the full acousto-optic overlap, including the cladding and warm sections. The conventional comparator carries oxygen mass in both regions, both vibrational volume changes, index, photoelastic response and area reshaping. Only the independently measured baseline area mismatch is netted out.

At the declared oxygen test point the conventional product contrast is approximately -0.03 . The conditional FDT contrast is approximately 0.2 . The additional-channel difference is approximately 0.2 . The full product threshold is approximately 0.009 . As in the germanium benchmark, the two contrasts take their single figure from the assumed overlap, eigenstrain and response sensitivities, their difference from the assumed overlap and density sensitivity, the threshold from the assumed product scatters and lot bound. The finite path, not a local sensitivity times an assumed span, gives these endpoint values. Ablation removes only the extra density channel and lies beyond the stated threshold. Detection, attribution and specified-change exclusion use Eq. (62), with the oxygen-source lot bound.

The separately seeded oxygen simulation gives false detection and attribution rate 0.023 and worst-sign false exclusion rate 0.023 , each with binomial standard error 0.001 . Its joint gain–linewidth errors preserve the stated negative correlation. The contrast supplies the isotope dependence absent from the source’s two cold parameters.

5.7 T-5: germanium isotope shift of the gain–linewidth product

DC-5 discriminates SA-12 through the I-7 constitutive input for the same germanium-enriched fibre cores as the frequency test. The source’s automatically phase-matched subset and its gain model specify no germanium isotope derivative (SC-012-02, L127; SC-012-03, L128; SC-011-03, L129; SC-011-04, L130; SC-008-03, L119). This block is conditional by construction on the complete channel sensitivities and the independent area reconstruction. Its own reading of larger density response is a larger gain–linewidth product. The observable sign is $\text{sgn}(\prod_i S_i)$ and stays conditional until the sensitivities are established.

The concrete referents, spin assays, weak-pump preparation and temperature are those of the germanium frequency test. The independently fitted peak and full width solve Eq. (64); their heavy/light product ratio returns the observable. The comparator changes the fibre core alone, with displacement and traction matched to the cladding. It retains the isotope’s vibrational strain, refractive-index and photoelastic changes and the resulting area reshaping. Its weight remains the full fibre optical–acoustic overlap, including warm sections.

At the declared germanium test point the conventional product contrast is approximately -0.002 . The conditional FDT contrast is approximately 0.01 . The additional-channel difference is approximately 0.01 . The full product threshold is approximately 0.009 . These are single-figure values at the declared test point: the two contrasts are limited by the assumed overlap, eigenstrain and response sensitivities, their difference by the assumed overlap and density sensitivity, the threshold by the assumed product scatters and lot bound. The unrounded added response is larger than the full threshold; a smaller independently calibrated density sensitivity is judged by the same criterion, not automatically counted as a detectable prediction. The distinction from the frequency test is the separately measured gain–linewidth observable with its own area, photoelastic and covariance channels.

Detection, attribution and specified-change exclusion use the product uncertainties and

enriched-oxide lot bound in Eq. (62). The separately seeded germanium-product simulation gives false detection and attribution rate 0.024 and worst-sign false exclusion rate 0.024, each with binomial standard error 0.001. A failure of this specified constitutive input is a failure of its conditional amplitude, not of a coordinate chart. The isotope product response resolves the underdetermination of the source’s phase-matched gain description without reusing its fitted curve as a prediction.

6 Cross-domain reading

The twist comparisons and isotope comparisons test different entries of the same density description. P-5 supplies an additional magnetic source for handed geometry, while I-7 places mass, stored energy, pressure and radiation in one effective mass ledger. I-1 fixes the underlying force law and requires a response kernel for a driven laboratory signal. The core supplies no numerical magnetic strength for the chosen glass. The isotope amplitudes therefore declare their own constitutive sensitivity and preserve its conventional limit explicitly.

The constrained-cylinder calculation keeps a material support separate from a confinement address. Its mechanical boundary conditions fix an ordinary elastic response; they do not fix a strand class or a nuclear eigenvalue. I-5 enters when isotope substitution changes a matter configuration, while I-2 keeps the probing carrier’s identity fixed. I-6 keeps every density input in the bounded A-1 chart on its named branch. The zero of each comparator is a comparison of neighbouring paths or matched preparations after conventional channels are included. No isolated resonance is treated as a direct measurement of curvature.

7 System-level reading (exploratory)

The scope is one in-sample reinterpretation of the source’s phase-matched subset. No new numerical prediction is made in this section.

X-1: driven nodal selection. The observable is the weak-pump gain spectrum. Its defining condition is Eq. (28), with peak at the phase-matched Brillouin frequency and half-maximum offsets fixed by its reported width. Its comparator is the source’s same Lorentzian on the same inputs. The source calls the phase-matched acoustic subset the one given by the Lorentzian Brillouin gain spectrum (SC-011-02, L127; SC-012-03, L128; SC-011-03, L129; SC-011-04, L130). This manuscript reads it, under P-2, as nodal selection driven by the pump–anti-Stokes beat in the glass’s density field. Mode geometry follows boundary conditions, the clamps of P-3. The source’s continuum and quasi-continuum language remains its account of accessible modes.

Guards. The gain peak, full width and centre remain source inputs. Neither a mode label nor a bounded spectral share is a curvature ratio. The mode is not a braid class. All numerical evaluations here are the identity map; the exploratory observable is excluded from the prospective prediction count.

8 Convergence statement

Set the additional isotope sensitivity to its conventional value and remove the extra handed-geometry magnetic source. Keep the source's reported gain, linewidths, pump settings, phase matching and undepleted-pump coupled-mode dictionary. The gain Lorentzian, the threshold, the spectral estimates and the source's simulated comparisons then return on exactly those inputs. This is the identity map of Use item 2. The source's values return with their tags: the couplings $g_{\text{om}} = 8 \pm 3$, 61 ± 2 and 75 ± 2 MHz (SC-028-01, L251; SC-028-03, L254; SC-028-04, L257; SA-28), 68 ± 2 MHz (SC-054-03, L367; SA-54), 76 ± 2 MHz (SC-032-03, L258; SA-32) and 4 ± 3 MHz (SC-036-02, L362; SA-36), the cooperativity 179 ± 2 (SC-046-03, L476; SA-46), the order-of-ten occupation (SC-047-02, L477; SA-47) and the pump power of around 5 W at which, under the conditions of that occupation, *the coupled system has the potential to enter the quantum coherent coupling regime over the whole bandwidth of acoustic modes addressed in the scattering process* (SC-048-01, L479; SC-048-02, L480; SC-048-03, L481; SC-048-04, L482; SA-48) are estimated, and the theory curves (SC-024-02, L231; SA-24) and the acoustic displacement spectrum (SC-031-02, L255; SC-031-03, L256; SA-31) are simulated. The source's numbers stand apart from its own inputs in six places, five of them beside this manuscript's recomputation: $\Upsilon_{\text{opt}} = 13 \pm 2$ MHz (SC-007-03, L116) against the propagation decay $\mathcal{F}_{\text{loss}} = 0.05$ MHz of the printed loss (SA-07); the gain–linewidth products 60 ± 10 and 65 ± 3 W⁻¹ m⁻¹ MHz at 4 and 293 K (SC-008-03, L119; SC-008-05, L121; SC-008-06, L122), which agree within their propagated errors while the average–linewidth threshold falls from 30 ± 1 to 11 ± 1 MHz (SA-08); the reported maximum split $2s_{\text{split}} = 26.7$ MHz (SC-032-02, L257; SC-032-03, L258) against the derived $2g_{\text{om}} = 150$ MHz and $2g_{\text{om}}/(2\pi) = 24$ MHz (SA-32); the forced-detuning split axis spanning 30–60 MHz, above that reported split, without printed numerical data values (SC-056-02, L434; SC-056-01, L411; SA-42); the reported $\mathcal{P}_{\text{pump}} = 0.004 \pm 0.1$ W (SC-036-02, L362), whose uncertainty-to-value ratio is 25, with the cold coupling 3.5 ± 0.3 MHz at its central value and 18 ± 1 MHz at its central value plus its printed uncertainty (SA-36); and the cooperativity's ± 2 (SC-046-03, L476) against 180 ± 30 from independent inputs (SA-46). Beside the source's estimate of around 5 W (SC-048-04, L482; SA-48), this manuscript's own criterion $\mathcal{P}_{\text{dec}}$, coupling equal to thermal occupation times mechanical linewidth, a criterion the source does not print, gives approximately 2 W at the occupation 8 and approximately 3 W at the order-of-ten occupation (U-72). The source's optical spectra are accepted in full.

The standard account reads the source's *two systems*, its *optical photons and travelling acoustic phonons*, as one of the source's *cases of light–matter interaction* (SC-001-01, L17; SC-001-05, L36; SC-001-06, L53; SC-001-07, L54; SC-006-01, L114; SA-01), and reads that interaction as mediated by force carriers. By I-2, photon and phonon are the same spin-one excitation of the density field at different density and geometry regimes, and there are no independent force carriers, so carrier mediation is declined and interactions are curvature plus geodesic motion; this manuscript reads these spectra as that one carrier's two regime modes.

The source calls c the *speed of light in vacuum* (SC-018-02, L154; SA-18). FDT reads c^2 as the omnium's response property by core 0.1, with no empty background.

The source names annihilation operators and says that in the undepleted-pump regime its Hamiltonian can be linearized to a beam-splitter interaction (SC-025-01, L242; SC-025-02, L243; SC-025-03, L244; SC-025-04, L245; SA-25). FDT reads QM as helical geodesics on truncated density cones, with superposition the ignorance of the helical phase at observation, and declines

probability amplitudes and collapse by I-4.

The source's effective pump-enhanced coupling generates *state swapping between the two photon and phonon modes*, under its undepleted-pump premise (SC-026-01, L250; SC-026-02, L251; SC-026-03, L244; SA-26). This manuscript's R-g reading of core 0.1a and I-5b is invariant-preserving many-to-many reclosure under global optimisation: events are noncommutative, and forward and backward responses are never particlewise mirror events. R-g is labelled as this manuscript's reading of those clauses, not their verbatim statement. It also governs the source's expected adiabatic-transition reading of the detuning sweep (SC-039-10, L391; SC-039-11, L392; SC-039-12, L393). A reverse sweep is a new preparation and trajectory. R-g also governs the source's statement that *In the strong coupling regime, energy can be exchanged coherently between two systems before the coupled system decoheres* (SC-001-03, L33; SC-001-04, L34; SA-01): this manuscript reads that exchange as invariant-preserving many-to-many reclosure under global optimisation, never a particlewise one-to-one response, and reads its forward and backward responses as never particlewise mirror events.

The source's optical and acoustic speeds remain the glass's two response speeds. Its Hermitian mode algebra and the standard account's time-reversal reciprocity premise remain available comparators. Their algebraic time reversal imposes no mirror-event history on the density field, whose responses this manuscript reads as many to many. The phase lift preserves the measured linear response at the source's undepleted-pump order; the unchanged Einstein equation supplies I-3 containment for the same stress tensor, boundaries and curvature data. The first is the identity map of Use item 2 and the second the I-3 containment; neither re-evaluates an estimator.

The source's broader claims retain their stated scope. Its priority statement is introduced by *So far* and rests on its cited literature, not an independent priority claim here (SC-004-04, L48; SC-004-07, L51; SA-04). Its active-cooling example belongs to another experiment (SC-003-02, L34; SA-03). Its broadband resonance is the auto-phase-matching reading within the optical transparency window; the data here use one wavelength (SC-013-02, L143; SC-013-03, L145; SA-43). Its temperature-series details and spatial-domain comparison are supplementary-only (SC-041-08, L401; SA-41, SC-033-06, L268; SA-33). The source describes the quantum-signal-processing prospects with *may pave*, not an implemented quantum device (SC-010-03, L130; SC-010-05, L133; SA-10). The source says that the shape of its two split peaks *is asymmetric* and that *This line profile might resemble the one measured in electromagnetically induced transparency (EIT)-like effects in Brillouin amplifiers, but it is unrelated*; the supporting comparison is supplementary-only (SC-034-01, L258; SC-034-02, L259; SC-034-03, L260; SC-034-04, L261; SC-034-05, L262; SC-034-06, L263; SC-034-07, L264; SA-34). These statements add no measured bandwidth, exchange time or non-classical state.

The source's two-level-system limitation retains *might be a limiting factor*, and its thermal-decoherence outlook retains *could exceed* (SC-049-01, L482; SC-049-02, L483; SC-049-03, L484; SC-049-04, L485; SA-49). Its heating-shift neglect is argued from auto-phase matching, without printed high-power line-centre values (SC-050-01, L485; SC-050-02, L486; SC-050-03, L487; SA-50). No pump-depletion measurement accompanies these spectra (SA-51). The source says that *The onset of pump depletion could limit the access to the quantum coherent coupling regime, but this can be circumvented by preparing a sample with lower coupling losses, and thus lower optical dissipation rate, or working at a temperature where the mechanical dissipation rate is lower than at $T = 4$ K*, and that *This means that an optical waveguide with a higher Brillouin gain or lower acoustic loss, such as chalcogenide fibres, is capable of realizing non-*

classical states transfer between photons and phonons at a moderate cryogenic temperature of several kelvins; this is an outlook, with no chalcogenide parameters in the record (SC-051-01, L488; SC-051-02, L489; SC-051-03, L490; SC-051-04, L491; SC-051-05, L492; SC-052-01, L492; SC-052-02, L493; SC-052-03, L494; SC-052-04, L408; SC-052-05, L409; SA-52). The source says its experiment results *can establish waveguide platforms as important candidates for novel quantum technologies*; its *our findings may allow to explore a new class of quantum neural networks controlled by acoustic vibrations* retains the premise *Coupled with the recent advances in neuromorphic computing* (SC-053-06, L426; SC-053-08, L430; SC-053-13, L439; SC-053-14, L440; SC-053-15, L441; SA-53). The estimates of quantum coherence and thermal occupation retain the nominal-temperature and warm-section limitations. No temperature ratio is substituted for an oscillator's frequency shift.

9 Status ledger

The following single ledger names the programme dependencies and the status of every prospective and exploratory reading. The owed descriptions are those of the locked core; the additional conditional amplitude is an explicitly assumed material input.

Item	Statement	Status or convergence value
O-1	The finite-core cutoff r_c , the regular core chart, or the outward phase-transfer law	Owed; interior endpoint domain.
O-2	A geometrically constructed odds for the overlap-contrast application (A-3)	Owed.
O-3	A canonical odds for the confinement application (A-4)	Owed.
O-4	The local profile $\rho_{\text{loc}}(r_f)$ from the omnium constitutive law with boundary, surface and core data, reducing to the measured nuclear profile	Owed; no inferred fibre interior profile.
O-5	Multipole and tidal remainders of the closed-form pair expression; the overlap complex	Owed for the nonspherical dynamical fibre.
O-6	The maps from a charge or a colour seat to an I-1 input; derivation of the A_2 seat table from cone geometry; the gluon octet's dynamics	Owed; no optical coupling substituted for a force input.
O-8	The constitutive derivation of $c_{\text{eff}} / n_{\text{cone}}$ and its reconciliation with lensing	Owed; n_{cone} the posited causal-aperture assignment of P-4; medium-aperture reading.
O-9	The confinement operator D , its domains, boundary data and spectrum; every mass ratio, including the two held-out targets	Owed; isotope masses are independent assumptions, not eigenproblem outputs.
A-3, A-4, A-5	Overlap, confinement and mode tokens	PROVISIONAL; range only.

Item	Statement	Status or convergence value
T-1	Applied-twist magnetic existence	Prospective; conditional by construction; conventional hand contrast zero.
T-2	Frozen-helix magnetic existence	Prospective; conditional by construction; conventional hand contrast zero.
P-5 kernel	Quantitative twist-to-moment forms are derived in-manuscript through I-1/I-2 with a declared kernel	These two existence blocks declare none; no magnitude or absolute sign.
T-3	Germanium frequency contrast	Prospective; conditional by construction; converges to $\mathfrak{d}_\omega^{\text{conv}}$.
T-4	Oxygen product contrast	Prospective; conditional by construction; converges to its whole-glass $\mathfrak{d}_Q^{\text{conv}}$.
T-5	Germanium product contrast	Prospective; conditional by construction; converges to its constrained-fibre-core $\mathfrak{d}_Q^{\text{conv}}$.
Density sensitivity	Positive observable response and its finite-path calibration	Conditional constitutive input; amplitude U-45, channel model U-71 and U-75.
X-1	Driven nodal selection of the gain subset	In-sample; uncounted.
Source arithmetic	Gain, threshold, coupling, cooperativity and thermal comparison	In-sample identity map; source uncertainty and criterion retained beside independent calculations.
Source theory	Simulated optical/acoustic spectra and overlay	In-sample; no independent goodness metric.
R-g	Noncommutative many-to-many event reading	Manuscript interpretation of core 0.1a and I-5b.

10 Conclusions

The single-pass optical spectra establish the source's strong-coupling criterion, that *the coupling strength exceeds the average linewidth* (SC-030-03, L254), in steady-state spectra of one highly nonlinear fibre in a 4 K cryostat (SC-017-02, L153) at one pump wavelength. The source's estimated couplings and simulated acoustic response keep their respective epistemic classes. The identity map accepts those data without turning an in-sample calculation into a prediction. This manuscript reads the hybrid modes as one carrier's two regime modes (I-2), the material supports as confined matter patterns and the driven optical response through its own susceptibility.

The prospective comparisons isolate an extra handed-geometry magnetic source and an explicitly conditional isotope density response. Independent calibration, full path weights, constrained strain and hierarchical uncertainty determine their observable tests. An amplitude-free magnetic reading has a finite-magnitude exclusion rule, while each isotope benchmark has a specific finite-path amplitude and a full experimental threshold. Removing those additional inputs recovers the conventional comparators. FDT is a true extension of general relativity and quantum mechanics, with the same data recovered by the stated limiting map.

References

- [1] L. Blázquez Martínez, C. Zhu and B. Stiller, *Cavity-less Brillouin strong coupling in a solid-state continuous system*, Nature Physics, doi:10.1038/s41567-026-03458-2. Received 29 July 2025; accepted 19 August 2026.
- [2] *Core FDT Invariants*, locked version 5.9, clauses 0.1–0.5, I-1–I-7, P-2–P-5 and Use.