What the gap holds

The residual between two conducting plates

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H. B. G. Casimir, *On the attraction between two perfectly conducting plates*
Proc. Kon. Ned. Akad. Wetensch. **51**, 793–795 (1948)
Communicated on 29 May 1948. No arXiv identifier.

Abstract

Casimir's comparison of two positions of a conducting plate [1] selects the change of energy with separation. Its sign follows already from the first surviving boundary derivative of the mode sum. Its magnitude comes from the transverse mode density and the single standing wave at zero longitudinal index. Fundamental Density Theory (FDT) reads the pressure as the deficit in the residual of one field between two clamps. The ideal pressure, its thermal crossover and its weakening for real gold refer to three different specifications of the boundary. They agree with the conventional calculation for each specification. Of the comparisons considered here, four fall below their decision thresholds and two require an additional constitutive map. This leaves zero predictions distinguishable from the conventional readings at the specified precisions.

1 Framework

Casimir [1] sets a perfectly conducting square plate parallel to a face of a perfectly conducting cubic cavity and compares a small distance with a very large one (SC-011, L26–28 and SC-012, L29–30). The title and meeting date are those of SC-001–002, L2–4. The publication details are U-12. The source status is theoretical. Reported denotes a statement of the paper. Measured and observed denote experimental records, identified separately. Derived denotes a consequence of stated equations, reconstructed a formula recovered from the record, estimated a numerical evaluation, and illustrative-assumed a choice of apparatus or parameter.

FDT is a true extension of general relativity and quantum mechanics [2]. The question posed by the plates is particularly narrow. Two geometries fix two sets of frequencies. Taking their difference tests the boundary prescription while leaving the local propagation law the same. A test of the nature of the residual requires an observable that changes when the physical input assigned to that residual changes.

1.1 One state and the time a pattern keeps

Energy E, mass m, distance d and duration t are readings of one state. The electric and magnetic response coefficients of the omnium are ε_0 and μ_0. Their wave equation gives the strand rate c. The upright metre symbol m denotes the SI unit of length. Write h for the

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quantum of action, $\hbar$ for its reduced value and π for the circular constant. For a mode indexed by q, let $E_q, m_q, \omega_q, T_q, \ell_q$ denote its energy, mass equivalent, angular frequency, period and thread length. Core 0.1, 0.1a and 0.1d give

$$E/m = c^2 = 1/(\varepsilon_0 \mu_0) = (d/t)^2.$$

$$E_q = m_q c^2 = \hbar \omega_q, \qquad T_q = 2\pi/\omega_q = h/E_q,$$
$$\ell_q = cT_q = h/(m_q c), \qquad d/t = c = 299792458\,\mathrm{m\,s^{-1}}.$$

The last number is exact (U-01). A closed pattern reads its rest energy as its confinement mass times the squared rate. For light the period is the field period at a fixed place. An open stroke keeps no proper time. Its energy has a mass equivalent, and its invariant rest mass vanishes. The support radius R_{box} belongs to the boundary of a closed pattern. A Compton length is a run per period. Inferring R_{box} from that run would use the mass to set the geometry that should determine it.

The translational and circulating components v, u of the strand rate satisfy $v^2 + u^2 = c^2$. Time is circulation and space is translation. For a stationary closed pattern the integrated stress vanishes when the closure stress is included. Boosting its stress tensor then gives inertia per unit energy $1/c^2$ (0.1d). The share v^2/c^2 is kinematic. The photon is one unbound strand, a helix. With wave number k, the radius of the turning grades is $\hbar c/E = c/\omega$, where ω is angular frequency. The dispersion $\hbar\omega = \hbar c k$ gives equal turning and running rates. The helix stands at $\pi/4$ at every frequency. Higher energy has the smaller cylinder. The strand and the energy run along the axis. Every pattern moves as a helix. A closed pattern translates below c, and its phase rate and group rate multiply to c^2 (I-2).

The numerical evaluations use the exact definitions of h and c and the recommended gravitational constant [4]. In SI units $h = 6.62607015 \times 10^{-34}$ J s (U-02). Let $F_{\max}$ be the force ceiling, G the gravitational constant, $d_{\min}, t_{\min}$ the lower length and time bounds, and ℓ_P, t_P the Planck length and time, with P in a subscript denoting Planck units. With $G = 6.67430 \times 10^{-11}$ m^3 kg^{-1} s^{-2} (U-04), one quantum of action fixes both bounds (0.1a, 0.2):

$$F_{\max} d_{\min} t_{\min} = \hbar, \qquad d_{\min} = c t_{\min} = 2\ell_P, \qquad t_{\min} = 2t_P.$$

The ceiling is approached and never occupied. The same holds for the floor of physical strokes. The construction fixes a lower bound and leaves lengths above it continuously variable. The half quantum in the plate sum is an energy per standing wave. Its value changes with the wave.

1.2 Chart inventory and the force

Let $\mathcal{M}$ denote the total gravitating energy in mass units and R_s its gravitational radius. The field radius is r_f, the support radius R_b and the interior cutoff r_c (O-1). The branch label σ is positive unity outside and negative unity inside. The curvature ratio α_σ is A-1, with its exterior and interior readings $\alpha_{\mathrm{ext}}, \alpha_{\mathrm{int}}$. Its odds X_{ext} and X_{int} are the algebraic ratios of the bounded reading to its complement. The two radii fix this application before

any observation of the pressure (0.3–0.4):

$$R_s = 2G\mathcal{M}/c^2, \qquad \alpha_\sigma = (R_s/r_f)^\sigma,$$
$$\alpha_{\text{ext}} = R_s/r_f, \qquad X_{\text{ext}} = R_s/(r_f - R_s),$$
$$\alpha_{\text{int}} = r_f/R_s, \qquad X_{\text{int}} = r_f/(R_s - r_f).$$

The exterior uses $r_f > R_s$ and the interior $r_c \leq r_f < R_s$. An evaluation at R_b specifies a configuration. An evaluation at a companion's centre specifies a pair input. Changing the energy source changes R_s. Both coordinates have the strict open range inherited from their domains. On A-1 the odds X return the bounded reading $X/(1+X)$, and x denotes a dimensionless odds-enhancing input. This perturbation multiplies X by $1+x$ (I-6). The fold is the limit as the field radius approaches R_s. The exterior gravitational flow of a held pattern is the case where the translation share equals the exterior compactness.

Application	Warrant and role
A-1	Two geometrically fixed lengths. The exterior plate window is the numerical use here.
A-2	$\beta_\sigma = \alpha_\sigma/(1+\alpha_\sigma)$ has a geometric warrant as a computational coordinate. The force, density and optical expressions retain their own inputs.
A-3	α_{ovl} denotes overlap contrast. No geometric odds enter the calculation (O-2).
A-4	α_{conf} denotes confinement position. No canonical odds enter a mass or pressure (O-3).
A-5	α_{mode} names a mode regime. The range alone supplies no ordering of the standing waves.
Skin	$\alpha_{\text{core}} \to 1^-$ denotes the schematic saturation reading at a nuclear pinning site's interior skin (P-3).

The computational reading β_σ has no role in the pressure. The overlap label α_{ovl}, confinement label α_{conf} and mode label α_{mode} supply no threshold or trajectory here. The schematic pinning-site reading α_{core} describes a material skin and carries no numerical plate compactness. The last three applications have the open range and no geometric warrant. The interior chart of a material pinning site places its skin at the fold with skin radius R_{room} taken from data. Its exterior monopole atlas uses the source's R_s. The data radius R_{room} and the gravitational radius enter different evaluations. A measured interior profile would average over the weight of its probe. Low-rung closures have central support, binding takes place at the skin, and no numerical interior profile is used (O-4).

For the I-1 force F let $\alpha_{1,\text{ext}}$ and $\alpha_{2,\text{ext}}$ denote the A-1 exterior cross-centre inputs of two nonoverlapping spherical supports. On this domain, with mass readings $\mathcal{M}_1$, $\mathcal{M}_2$ and centre separation s, the pair expression reduces exactly:

$$F = F_{\max}\alpha_{1,\text{ext}}\alpha_{2,\text{ext}} = \frac{c^4}{4G}\alpha_{1,\text{ext}}\alpha_{2,\text{ext}} = G\mathcal{M}_1\mathcal{M}_2/s^2.$$

The universal force law holds at every scale. The open interval belongs to the map. Inputs in that interval give a force below the ceiling. Only the pair domain gives the stronger bound $F \leq F_{\max}/4$, with equality at contact of equal saturated supports. For slabs the

evaluation requires the multipole and support information of O-5. The exact pair algebra, the weak-field slow-motion Newtonian limit of GR to leading order, and an independent source calculation are the three layers of I-4.

Let $G_{\mu\nu}$ be the Einstein tensor, $T_{\mu\nu}$ the stress tensor and κ the Einstein coupling. Greek indices μ, ν label spacetime components. The optical arclength is ℓ_{opt}, d denotes a differential, and $\mathrm{d}\Omega_{\text{ang}}$ denotes angular distance on the unit sphere. I-3 gives the unchanged field equation and, for its static spherical exterior solution, the exact optical metric:

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \qquad \kappa = 8\pi G/c^4,$$
$$\mathrm{d}\ell_{\text{opt}}^2 = \frac{\mathrm{d}r_f^2}{(1-\alpha_{\text{ext}})^2} + \frac{r_f^2\,\mathrm{d}\Omega_{\text{ang}}^2}{1-\alpha_{\text{ext}}}.$$

The branch coordinate is downstream of this solution. Generalised Snell refraction is the Fermat jump law of the optical metric. Both radial and angular factors enter it. The weak-field isotropic index gives twice the leading deflection obtained from the lapse alone. A local aperture reading of P-4 enters no gap transit calculation here (O-8).

A pattern's clock factor $\mathfrak{m}$ rescales its kept time and thread length together. On its own clocks and rulers the ratio stays c. Moving at constant speed through a density gradient is gravity (0.1c). Let $g_{\mu\nu}$ denote the metric used by the bookkeeper. The pattern is free in $\mathfrak{m}^2 g_{\mu\nu}$. With four-velocity $\mathcal{U}$, orthogonal projector $\mathcal{P}^{\mu\nu}$ in squared-speed units and covariant derivative ∇, the bookkeeper reads $\nabla_{\mathcal{U}}\mathcal{U}^\mu = -\mathcal{P}^{\mu\nu}\partial_\nu \ln \mathfrak{m}$. The slow pattern reads the lapse and the stroke reads the optical metric. Deviation compares neighbouring paths or a path with a reference. The skin of an extended pattern supplies such a comparison against its centre path. Its quadrupole reads shear and the trace reads density (I-3a). In the static exterior the inward flow speed is $c\sqrt{\alpha_{\text{ext}}}$. The flow falls at $G\mathcal{M}/r_f^2$ and exists at every size. Rotation adds frame drag, half the curl of the bookkeeper's swirl at leading order. The gravitational free-fall four-velocity is irrotational.

For a medium of perfect stress form at zero net conserved count the thermal flow's clock factor is proportional to temperature T. At fixed entropy per conserved count the enthalpy per count supplies it. A clock carried by that flow reads its local lapse and its own thermal motion. A conformal factor preserves null paths. A held moment in a twist instead reads the pattern's rest energy there divided by its rest energy outside the twist. Include the work of inducing a moment in that energy. Its spatial gradient gives the slow force, its second derivative the spring. A uniform energy offset changes inertia and leaves the gradient unchanged. Copies turned alike read the twist by comparing neighbours or a reference path. Patterns with different moments per unit energy respond differently in this window. In the gravitational window monopole free fall is universal.

1.3 Matter, occupation and projection

In I-5 a plate is a closed pattern of the one omnium. A closure's braid count n fixes its class. At fixed count its boundary data B, domain Ω, orientation χ and chart position t_{pos} specify the confinement problem. Write D for its operator and u_q for its mode.

$$D(\Omega, B; n, \chi, t_{\text{pos}})u_q = \omega_q u_q.$$

Mass is a confinement eigenvalue through the mode cost in the chain. No form of the operator or its spectrum enters the gold response (O-9). A change of confinement at fixed count preserves class. Re-braiding to a different count changes class. The open stroke of I-2 belongs to the carrier identity. The chain gives one propagation rate and 0.1b gives one medium. Photon, graviton, gluon, phonon and magnon name its one excitation at different geometries.

Let M be the rest contribution, P pressure, V volume, a the radiation energy-density constant and σ_{SB} the Stefan–Boltzmann flux constant. With each independent contribution included once, the examples of core 0.5 and I-7 read

$$\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \cdots , \qquad a = 4\sigma_{\mathrm{SB}}/c.$$

The covariant parent is the single stress tensor. A mode ledger that already includes the occupied radiation includes its thermal energy. Adding a free blackbody energy to that same entry would count it twice. The gravitational binding energy is in the solved mass aspect. The negative interaction energy is a change between two configurations, both of which have positive total energy.

In cylindrical coordinates write ϕ_h for helix phase and ϕ_0 for its unknown value at the time origin. Let $\mathbf{B}$ be the oriented transverse unit bivector. Then $\phi_h(t) = \phi_0 - \omega t$ and the phase rotor is $\exp[\mathbf{B}\phi_h(t)]$. The complex oscillator is the transverse projection of this three-dimensional transport. This rotor uses the full phase in radians. A spinor rotor for rotating a frame uses half the rotation angle. Superposition reads the unknown phase at a clamp (I-4). The field identity stays fixed across the standing set.

The equal-weight pair illustrates what a clamp actually selects in I-4. Let ϕ be the shared phase, $\mathfrak{a}, \mathfrak{b}$ the clamp angles (doubled analyzer angles for polarisation), $\mathcal{A}, \mathcal{B}$ the local sign records, and $\mathfrak{p}_{\mathrm{host}}$ the phase density for a host at $\mathfrak{a}$. For opposite senses the arrivals differ by π:

$$\begin{aligned} \mathfrak{p}_{\mathrm{host}}(\phi) &= |\cos(\phi - \mathfrak{a})|/4, \\ \mathcal{A} = \operatorname{sgn}\cos(\phi - \mathfrak{a}), &\quad \mathcal{B} = \operatorname{sgn}\cos(\phi + \pi - \mathfrak{b}), \\ \langle \mathcal{A}\mathcal{B} \rangle &= -\cos(\mathfrak{a} - \mathfrak{b}). \end{aligned}$$

Projecting uniformly on the host aperture gives this mean on every record and equal single-wing frequencies at either host. Equal senses give the positive cosine. For the lock and sign records, the kernel fixes the part of the host density even under a half-turn. Flat remote frequencies remove the odd part, fixing the measure uniquely. On the sphere the square-integrable density is the absolute host-axis projection divided by 2π. A clamp-independent uniform phase yields the triangle correlation and a CHSH ceiling of two. The projection measure gives the consensus ceiling $2\sqrt{2}$. The standard account premises a hidden state independent of the clamp settings in the Bell inference. FDT accepts the inference and reads the pair and its clamps as one settlement on the host measure (0.1a, I-4).

For clamp axes $\mathbf{a}, \mathbf{b}$ and local transport rotations $\mathsf{R}_{\mathcal{A}}, \mathsf{R}_{\mathcal{B}}$, let R be their relative rotation and M its history mean. The correlation in one history, with T denoting transpose, is $-\mathbf{a}^{\mathsf{T}}\mathsf{R}\mathbf{b}$ with $\mathsf{R} = \mathsf{R}_{\mathcal{A}}\mathsf{R}_{\mathcal{B}}^{\mathsf{T}}$. Pooling replaces the rotation by M. Its signed singular values fix the four pair frequencies, purity and negativity. The two largest singular values in quadrature, doubled, give the largest CHSH value, and one plus the trace divided by four, $(1 + \operatorname{Tr}\mathsf{M})/4$,

gives fidelity. Inverting the resolved rotations restores the emitted table. With the same turns, the pooled table also agrees cell by cell. Neither a choice of host dynamics nor a measure for unequal weights or more than two patterns enters the plate problem (O-17).

1.4 Notation guard

Source letters in this table identify the original usage.

Source notation	Manuscript usage
Polarizability	$\varpi, \varpi_1, \varpi_2$ are polarizability volumes. The electromagnetic constant is $e^2/(4\pi\varepsilon_0\hbar c)$ with elementary charge e.
Distance and side	s replaces the plate distance. s_{mic} is its numerical value in microns. $\mathfrak{L}$ is the side. The atom distance is $\mathfrak{r}$. $R_{\mathrm{sph}}, R_{\mathrm{sh}}$ are sphere and shell radii.
Mode indices	j_x, j_y, j_z label standing waves. j indexes the longitudinal sum. The external mean occupation is $\mathfrak{n}$.
Function and force	$\mathfrak{F}$ and its derivatives enter Euler–Maclaurin. The force per area is a pressure. $\mathfrak{g}$ denotes plate letting-go, with second derivative $\mathfrak{g}''$.
Integration variables	$\mathfrak{u}$ is the squared transverse wave number in gap units, $\mathfrak{w}$ the total squared wave number in those units. $k_{\parallel}$ is the transverse radial wave number.
Axes and scales	x, y, z remain axis labels. k_x, k_y, k_z are components of k. k_m sets the plate cutoff and k_{fl} the stroke bound. The X in X-rays is part of the word.
Action and energy	The action glyph denotes $\hbar$. E and δE retain their energy meanings. ω_d, ω_p denote drive and plasma angular frequency.
Expansion and zeta	$\mathfrak{v} = \pi/(sk_m)$ is the expansion parameter. ζ_R is the Riemann zeta function.

2 System chart

System	Geometry fixed independently	Reading
Gap	Face and plate a distance s apart along z	The longitudinal spectrum changes with separation.
Plate	Mass and evaluation radius	The A-1 exterior ratio gives a gravitational window.
Pinning site	Interior skin of a material core	Schematic saturation fixes the clamp language.
Free comparison	Same side, very distant face	Continuum longitudinal mode density sets the reference.

The plate retains a longitudinal spacing when its transverse spacings vanish. Taking the longitudinal continuum limit on both sides at the outset would remove precisely the quantity the plates measure.

3 Observation map

Record locus	Record operation	Reading
SC-011–012, L26–30	Reported near and distant positions	Change of geometry
SC-013–016, L31–36	Reported sums and proposed interaction	Signed difference
SC-017–019, L37–46 and SC-021–022, L53–55	Reconstructed spectrum and reported multiplicity	Transverse continuum and longitudinal endpoint
SC-026–028, L77–80	Reported half weight and integral	Reference spectrum
SC-029–032, L85–90	Reported cutoff convention	Plate response
SC-035–039, L104–131	Reported expansion method and derivatives	First nonzero boundary term
SC-040–045, L134–145	Reconstructed energy and reported force coefficient	Derivative at fixed response
SC-046–050, L146–153	Reported attraction, condition and experimental prospect	Ideal boundary and real material

The gap. The possible vibrations belong to the region between the parallel surfaces (SC-017, L37–38). The gap is omnium. Its standing patterns have the face and plate as boundary data. The material plates are closed patterns and the gap carries open strokes. The same electric and magnetic responses that give the strand rate enter the propagation law across the gap. There is no vacuum (0.1d). This cluster is a reading under 0.1, 0.1b and 0.1d and corroborates no invariant (Use item 2). Its precision is the ideal boundary specification. A different local transit at a fixed proper separation tests the propagation input independently of the energy sum.

Standing waves. The saturated nuclear cores of a plate are its pinning sites (P-3). The conducting walls fix the mode geometry (P-2). The reconstructed Cartesian spectrum of SC-018–019, L40–46 is

$$k_x = \pi j_x/\mathfrak{L}, \quad k_y = \pi j_y/\mathfrak{L}, \quad k_z = \pi j_z/s, \quad k = \sqrt{k_{\parallel}^2 + k_z^2}.$$

Casimir gives two standing waves unless one of the indices is zero, when there is one (SC-021, L53–54). For very large side he writes “we may regard” the transverse wave numbers as continuous variables (SC-022, L55). The axes have zero area measure in that transverse limit. The plane of zero longitudinal index retains a whole transverse continuum. A prime on a longitudinal sum gives the zero term half weight after taking two waves as the common multiplicity (SC-026, L77–78). The two standing patterns belong to the one carrier. This cluster is a reading under P-2, P-3 and I-2 and corroborates no invariant (Use item 2). The count is exact for the ideal modes. Resolving two independent modes on the zero-index plane at fixed transverse vector would exclude that count.

The residual. The energy per standing wave in SC-013, L31–32 is $\hbar\omega/2$. A stroke delivered into a mode carries $\hbar\omega$. The chain gives the energy, time and run of the stroke together.

Core 0.1a bounds its duration below, so a finite support has a finite set of accessible wave numbers below the stroke bound. Each sum for the finite cavity is finite on this reading (0.8). The plate reads a difference. The record says it "will be shown to have a weIl defined value" (SC-015, L34) and the value "will be interpreted as the interaction between the plate and the" xy face (SC-016, L35–36). This cluster is a reading under 0.1, 0.1a and 0.8 and corroborates no invariant (Use item 2). Its per-mode normalization is the record's half quantum. Changing that normalization at fixed mode density changes the pressure in the same proportion.

The difference. For very large separation Casimir says that the last summation may be replaced by an integral (SC-027, L78–79). Casimir requires a function multiplying the integrands to obtain a finite result (SC-029, L85). It approaches unity at small argument and decays sufficiently rapidly at large argument. The scale "may be defined" by $\mathfrak{g}(1) = 1/2$ (SC-030–031, L86–87). For very short waves the plate is "hardly an obstacle at all" (SC-032, L88–90). This letting-go belongs to the plate. The stroke bound and the plate response enter different operations.

Write $U = \delta E/\mathfrak{L}^2$ for energy change per area and $k_j = \sqrt{k_\parallel^2 + (\pi j/s)^2}$ for the clamped wave number. The upper endpoint ∞ denotes an improper integral or infinite sum. Reconstruct the missing interaction display (SC-028, L80) using the positive transverse quadrant, its angular measure $\pi/2$, and the longitudinal continuum density s/π (SC-023–027, L60–79):

$$U = \frac{\hbar c}{2\pi}\Bigg[\sum_{j=0}^{\infty}{}' \int_0^\infty k_j \mathfrak{g}(k_j/k_m) k_\parallel \,\mathrm{d}k_\parallel \\ - \frac{s}{\pi}\int_0^\infty\int_0^\infty k\mathfrak{g}(k/k_m) k_\parallel \,\mathrm{d}k_\parallel \,\mathrm{d}k_z\Bigg].$$

The transverse Jacobian carries inverse area. Multiplying by $\hbar c k$ gives energy per area. The convergence condition at large wave number is integrability of $k^3|\mathfrak{g}(k/k_m)|$. Two waves times the half quantum and the transverse density $(\mathfrak{L}/\pi)^2$ give the prefactor. Interchanging the two configurations reverses the sign of the work.

In SC-033–038, L91–128, set $\mathfrak{u} = (sk_\parallel/\pi)^2$ and $\mathfrak{w} = \mathfrak{u} + j^2$. Define $\mathfrak{D}$ as the dimensionless difference of the sum and its continuum integral:

$$U = \frac{\pi^2\hbar c}{4s^3}\mathfrak{D}, \qquad \mathfrak{D} = \sum_{j=0}^{\infty}{}'\mathfrak{F}(j) - \int_0^\infty \mathfrak{F}(j)\,\mathrm{d}j,$$
$$\mathfrak{F}(j) = \int_{j^2}^\infty \sqrt{\mathfrak{w}}\,\mathfrak{g}(\mathfrak{v}\sqrt{\mathfrak{w}})\,\mathrm{d}\mathfrak{w},$$
$$\mathfrak{F}'(j) = -2j^2\mathfrak{g}(\mathfrak{v}j).$$

The argument follows from the wave number in SC-034, L92. The later arguments reconstruct as $\mathfrak{v}\mathfrak{w}$ and $\mathfrak{v}j^2$ (SC-037, L124 and SC-038, L127–128). Either gives the same quadratic coefficient in the endpoint derivative. The finite-cutoff calculation uses the wave-number argument.

For smooth rapidly decaying response the Euler–Maclaurin expansion through the fifth derivative gives

$$\mathfrak{D} = -\frac{\mathfrak{F}'(0)}{12} + \frac{\mathfrak{F}'''(0)}{720} - \frac{\mathfrak{F}^{(5)}(0)}{30240} + \cdots .$$

The prime cancels the half-value endpoint. Since $\mathfrak{g}(0) = 1$, the first derivative is zero and the third is -4 (SC-035, L104–115 and SC-039, L130–131). The fifth is $-24\mathfrak{g}''(0)\mathfrak{v}^2$. Let U_0 and P_0 be the ideal energy per area and signed normal pressure. Holding k_m fixed in differentiation gives the expansions below. The symbol O bounds the order of the remainder as the expansion ratio tends to zero.

$$\begin{aligned} U_0 &= -\pi^2\hbar c/(720 s^3), & P_0 &= -\pi^2\hbar c/(240 s^4), \\ U/U_0 &= 1 - \mathfrak{g}''(0)\mathfrak{v}^2/7 + O(\mathfrak{v}^4), \\ P/P_0 &= 1 - (5/21)\mathfrak{g}''(0)\mathfrak{v}^2 + O(\mathfrak{v}^4). \end{aligned}$$

The leading energy holds "as long as" $sk_m \gg 1$ (SC-040–042, L134–137). The finite-cutoff pressure correction has an extra factor 5/3 because the corrected energy term falls as the inverse fifth power. At $sk_m = 100$ the Gaussian $2^{-(k/k_m)^2}$ and exponential $2^{-k/k_m}$ (U-13, U-14) give energy ratios 1.00020 and 0.99993 respectively. Their curvatures at the origin have opposite signs. This cluster is a reading under P-2 and corroborates no invariant (Use item 2). The leading coefficient is an identity on the source inputs. At fixed measured cutoff a ratio outside the expansion remainder excludes that profile with those ideal boundaries.

Pressure and work. The force magnitude in SC-043–044, L137–141 is attractive (SC-046, L146–147). The integrable force is continuous at every positive lower limit s_0. I-1 recovers the work and signed normal pressure from

$$U_0(s_0) = -\int_{s_0}^{\infty} |P_0(s)|\,\mathrm{d}s, \qquad |P_0(s_0)| = \partial_{s_0} U_0(s_0), \quad P_0(s_0) = -\partial_{s_0} U_0(s_0).$$

The energy falls as the gap closes. With s_{mic} the distance in microns (SC-045, L145), the magnitude is $0.013 s_{\mathrm{mic}}^{-4}$ dyne/cm^2 (SC-044, L139). The unrounded ideal law with SI constants (U-01, U-02, U-10, U-15) gives 1.30 mPa at 1 µm, 13.0 Pa at 100 nm and 2.08 µPa at 5 µm.

An illustrative gold plate has area 1 cm^2, thickness 1 mm and density 19.3 g cm^{-3} (U-21, U-22). Its gravitational radius is 2.87×10^{-30} m. At $r_f = 1$ cm the A-1 exterior window $\alpha_{\mathrm{ext}} = \mathcal{M}c^2/(2F_{\max} r_f)$ is 2.9×10^{-28} (U-23). The small number is a correct reading. The measured pressure scale comes from the clamped spectrum. I-1 reaches a measured scale through a declared response kernel. No such kernel enters this pressure or gives a slab pair input here (O-5). This cluster is a reading under I-1 and 0.1c and corroborates no invariant (Use item 2). The cgs coefficient has two significant figures. A resolved change of sign at fixed ideal boundaries would exclude this work curve.

The atoms. For an atom or molecule with a static polarizability near a conducting plate the paper says "it is shown" in reference 1) that the interaction is given in the limit of large

distances (SC-003–004, L6–11). It gives the two-particle expression in that limit (SC-005–006, L12–17). With δE_{ap} and δE_{aa} the atom–plate and atom–atom interaction energies, the reconstructed limits are

$$\delta E_{\mathrm{ap}} = -\frac{3\hbar c \varpi}{8\pi \mathfrak{r}^4}, \qquad \delta E_{\mathrm{aa}} = -\frac{23\hbar c \varpi_1 \varpi_2}{4\pi \mathfrak{r}^7}.$$

Two polarizability volumes and $\hbar c$ require the inverse seventh power. Its exponent has no glyph in the second display (U-11, U-49). The coefficient is from reference 1), Casimir and Polder [3] (SC-020, L50). In the Paris communication the author "was able to show" that the expressions "mayalso be derived" through studying the change of electromagnetic zero point energy by means of classical electrodynamics (SC-008–009, L20–23). Casimir applies the same method to two plates (SC-010, L24–25). This cluster is a reading under I-1 and I-2 and corroborates no invariant (Use item 2). The coefficients are asymptotic. A failure of the distance powers in the retarded, static-polarizability limit tests the imported laws. An I-1 calculation at the atomic inputs requires the charge map (O-6).

4 Cone-position translation

In the plate experiment one changes s while keeping the material of the face and plate fixed. The axial standing modes then move through the cutoff in wave number. No motion on a confinement-position chart follows. In particular, a measured penetration depth fixes a length in the reflection problem. It fixes neither a nuclear skin radius nor an absolute address for a named closure. The response measured at the surface is the response of a material pattern as a whole (0.1b). Its load is taken up by interstitial packing around the pinning sites (P-3).

The same distinction governs isotope substitution. A change of mass at fixed support changes the A-1 exterior compactness before a force is measured. A change of reflectivity at fixed mass changes the clamped spectrum. If both occur, the pressure measurement alone cannot assign their separate weights. Each measured permittivity must enter the comparison for its own isotope. This is the physical reason to retain a separate gravitational estimate in the isotope comparison below.

5 Predictions

DC-1 (SA-11) is retired because the limiting floor correction at the chosen nonzero curvature lies below the pressure threshold. DC-2 (SA-17) is retired because the conventional normal-propagation excess lies below the transit threshold. DC-3 (SA-05) is retired because the distinct settlements place no numerical difference on the calibrated rate quotient. DC-4 (SA-13) is retired because the gravitational density input gives no additional ideal-clamp isotope quotient. DC-5 (SA-14) is retired because the zero-frequency magnetic reflection of a normal metal requires a circulating-charge map (O-6). DC-6 (SA-02) is retired because converting the two atomic polarizabilities to force-law inputs requires the charge map (O-6).

6 Cross-domain readings

One field at two occupations. Let k_B be Boltzmann's constant and ξ_l the imaginary angular frequency with nonnegative integer thermal index l. Here $k_B = 1.380649 \times 10^{-23}$ J K^{-1} exactly (U-03) and $\sigma_{\mathrm{SB}} = 5.670374419 \times 10^{-8}$ W m^{-2} K^{-4} (U-08). For ideal plates in thermal equilibrium the mean occupation and energy per mode are $\mathfrak{n} = [\exp(\hbar\omega/k_BT)-1]^{-1}$ and $(\mathfrak{n}+1/2)\hbar\omega$. The clamped and free occupations have the same temperature but different spectra. Use the imaginary-frequency sum of Mehra and of Brown and Maclay [5, 6] (U-18). With $\mathfrak{q}$ the positive attenuation wave number, the ideal pressure and its long-gap limit read

$$\begin{aligned} \xi_l &= 2\pi l k_B T/\hbar, \\ P(s,T) &= -\frac{2k_BT}{\pi}{\sum_{l=0}^{\infty}}' \int_{\xi_l/c}^{\infty} \frac{\mathfrak{q}^2\,\mathrm{d}\mathfrak{q}}{\exp(2s\mathfrak{q})-1}, \\ P(s,T) &\sim -\frac{\zeta_R(3)k_BT}{4\pi s^3}. \end{aligned}$$

The two polarisations give the prefactor. The zero-frequency term has half weight. Substitution of twice the separation times attenuation in the integral gives the zeta value from the sum of reciprocal cubes. The other terms decay exponentially with separation. This thermal endpoint differs from the longitudinal standing-wave endpoint in the source. Both enter their own sums once.

Let $s_T = \hbar c/(2k_BT)$ be the thermal crossover length. At $s \ll s_T$ the pressure is $P_0 - aT^4/3$ to leading thermal order. The free side presses on a gap whose modes with positive longitudinal index are too costly to populate appreciably. The zero-index waves remain accessible. At $T = 300$ K exactly (U-16), $s_T = 3.82\,\mu\mathrm{m}$, and the free thermal pressure is $2.04\,\mu\mathrm{Pa}$ (U-03, U-08). The full sum gives $P/P_0 = 1.0016, 1.55, 3.05$ at $1, 5, 10\,\mu\mathrm{m}$ respectively (U-15). The calculation holds the surfaces ideal while changing occupation. The change of radiation energy enters the stress tensor once as the difference of clamped and free configurations (I-7). This cluster is a reading under I-7 and 0.5 and corroborates no invariant (Use item 2). These quotients refer to the stated isotherm and exact ideal geometry. A pressure inconsistent with the sum for ideal reflectors in thermal equilibrium excludes that occupation prescription.

Real clamps. The paper's independence of material requires that at wavelengths comparable with the distance the penetration depth be small compared with the distance (SC-047–048, L147–150). Reflection then fixes the ideal mode set. For a real plate use its separately measured optical response. Let ξ be continuous imaginary angular frequency, ϵ relative permittivity, $\mathfrak{h}_m$ attenuation inside the metal, and $r_{\mathrm{TE}}, r_{\mathrm{TM}}$ the two reflection amplitudes. For identical gold half-spaces at zero temperature the plasma prescription and the Lifshitz stress [7, 8] give (U-17, U-19)

$$\epsilon = 1 + \omega_p^2/\xi^2, \qquad \mathfrak{q}^2 = k_\parallel^2 + \xi^2/c^2, \quad \mathfrak{h}_m^2 = k_\parallel^2 + \epsilon\xi^2/c^2,$$

$$r_{\mathrm{TE}} = \frac{\mathfrak{q}-\mathfrak{h}_m}{\mathfrak{q}+\mathfrak{h}_m}, \qquad r_{\mathrm{TM}} = \frac{\epsilon\mathfrak{q}-\mathfrak{h}_m}{\epsilon\mathfrak{q}+\mathfrak{h}_m}.$$

Writing P_{Au} for this pressure and r_b for the amplitude in polarisation channel b, the multiple-reflection series sums to

$$P_{\mathrm{Au}} = -\frac{\hbar}{2\pi^2}\int_0^\infty \mathrm{d}\xi \int_0^\infty k_\parallel \mathrm{d}k_\parallel\, \mathfrak{q} \sum_b \frac{r_b^2 \exp(-2s\mathfrak{q})}{1 - r_b^2 \exp(-2s\mathfrak{q})}.$$

The round-trip factor suppresses large attenuation at positive gap. At small attenuation the phase-space measure controls the denominator. The two perfect-reflection terms recover P_0. At finite plasma frequency the integral has its largest fractional reduction at small gaps. For $\hbar\omega_p = 9.0$ eV, the plasma wavelength $2\pi c/\omega_p$ is 1.4×10^2 nm at the two-figure precision of the input (U-05, U-19), using $1.602176634 \times 10^{-19}$ J per eV. The quotients P_{Au}/P_0 are $0.44, 0.81, 0.89, 0.98$ at 100 nm, 500 nm, 1 µm and 5 µm respectively (U-15). Raising the plasma energy by 0.1 eV changes every quotient by less than 0.003 (U-20). No form of D enters these reflection coefficients (O-9).

At room temperature the transverse-electric zero-frequency term tests a different piece of material physics. For a dissipative Drude response at a normal metal the static transverse-electric amplitude vanishes. With plasma response the product $\epsilon\xi^2$ has a nonzero limit and the amplitude survives at finite transverse wave number. Mostepanenko's precision record favours the plasma prescription and excludes the Drude one in the comparisons reviewed [8] (U-46). This difference exists at every separation and carries greater relative weight in the long-gap thermal regime. P-5 reads a static magnetic field as a macroscopic helical density gradient. Its significance here is precise: to compute reflection from a static twist one needs a material clamp law. The circulating-charge map contributes no such numerical input here (O-6). This cluster is a reading under P-2 and P-3 and corroborates no invariant (Use item 2). The quotients are quoted to hundredths. Independent permittivity and pressure measurements at the specified separation test the constitutive approximation directly.

The whole geometry. Changing the arrangement changes the standing set as a whole (0.1a). A perfectly conducting spherical shell has outward stress. In Boyer's external calculation [9], its energy is approximately $+0.09\hbar c/(2R_{\mathrm{sh}})$ (U-24). A sum over attractive element pairs would have the opposite sign. For a sphere over a plane at $s \ll R_{\mathrm{sph}}$ the leading force follows by integrating the local parallel-plane pressure over the slowly varying gap. A ring's projected area per increment of local separation is $2\pi R_{\mathrm{sph}}$ to this order. Let F_s denote the signed force toward increasing gap. Integration from the closest separation gives $F_s = 2\pi R_{\mathrm{sph}} U(s)$ (U-48). This proximity limit is a limit of one geometry's spectrum.

Casimir writes that experimental confirmation "seems not" "unfeasable and might be of a certain interest" (SC-050, L152–153). Lamoreaux measured the sphere–plate force over 0.6–6 µm, testing the force law to 5% [10] (U-25). Bressi, Carugno, Onofrio and Ruoso measured parallel plates over 0.5–3.0 µm, testing the coefficient to 15% [11] (U-26). The former experiment reads a force integrated across a varying gap. The latter reads the parallel-plate coefficient. Their precision statements therefore apply to different observables. This cluster is a reading under 0.1a and corroborates no invariant (Use item 2). The quoted experimental precisions bound the comparisons. An inward ideal-shell stress would exclude the shell sign in this example.

7 System-level reading (exploratory)

Scope: the following in-sample comparisons hold the ideal one-micron gap fixed while changing the ledger of occupation.

X-1. Residual and occupation across the gap. The observable is the occupation of the lowest branch with positive longitudinal wave number, together with its energy deficit per area. The defining condition is $k_z = \pi/s$ with transverse wave number tending to zero. The branch minimum in ordinary frequency is $c/(2s)$. At $s = 1.00\,\mu\text{m}$ (U-27), it is 150 THz. The occupation there at 300 K is 3.9×10^{-11}. For comparison, unit occupation is reached at ordinary frequency $k_B T \ln 2/h$. This is 4.33 THz at room temperature (U-16) and 39.36 GHz at 2.725 K (U-47). The gap admits zero-longitudinal-index waves below these frequencies. The ideal energy deficit in magnitude is 4.33×10^{-10} J m^{-2} at the same separation. This is the integral of the signed work curve with its magnitude taken after integration.

Guards. The thermal frequency is a comparator for appreciable occupation. It places no lower bound on the frequencies of the zero-index waves. Changing occupation does not change the definition of the stroke floor.

8 Convergence statement

Hold fixed the plate's optical response, separation, temperature and normalization of each standing wave. Subtract the distant configuration at those same inputs. In the perfectly reflecting limit the resulting pressure is the derivative of the reconstructed energy of SC-041, L136–137. At the measured boundary response one obtains the Lifshitz pressure. With sphere curvature retained one obtains the sphere observable. The external records of U-25 and U-26 enter at their own geometries and precisions. This is the identity map of Use item 2. The equal-weight clamp table and its transported images likewise agree with the conventional tables for the same rotations. I-3 gives identical curvature for identical stress tensor, data and gauge.

The standard account reads the van der Waals–London forces corrected for retardation in SC-007, L18–19 as carried by an exchanged field (SA-03). FDT reads the interaction as curvature and geodesic motion of one carrier at the finite strand rate (I-2, 0.1d).

The standard account takes the unbounded zero point sums of SC-013–016, L31–36 and obtains a finite interaction through subtraction and a cutoff (SA-06). FDT bounds each physical stroke below and reads the difference of two finite sums at finite cavity size (0.1a, 0.8).

The standard account reads the force that Casimir says "may be interpreted as a zero point pressure of" electromagnetic waves (SC-049, L150–151, SA-15) as the pressure of an empty vacuum. FDT reads the force as the residual of the one field measured between two material clamps (0.1, 0.1d, P-2, P-3).

8.1 Pressure remainder and transit

Consider the ideal plate and face with a smooth response and $|\mathfrak{g}''(0)| \leq 2$, evaluated at the bound, at 1.00 µm (U-27, U-28). A stroke of action $\hbar$ at energy $\hbar\omega$ lasts $1/\omega$. Since the floor is approached and occupied by none, this duration exceeds $t_{\min}$. The corresponding wavenumber bound is $k_m < k_{\mathrm{fl}} = 1/d_{\min}$, with $k_{\mathrm{fl}} = 3.09 \times 10^{34}$ m^{-1} (U-01, U-02, U-04). Here the letting-go approaches that bound from below. No material plate realises this specification. In particular the source's plate scarcely obstructs very short waves (SC-032, L88–90).

For the fractional pressure remainder, the limiting scale of the leading term follows from the pressure expansion,

$$(10/21)(\pi d_{\min}/s)^2 = 4.9 \times 10^{-57}.$$

At this nonzero test curvature the leading magnitude exceeds that limit before the cutoff reaches its bound. The omitted order is fourth in $\pi/(sk_m)$. The infinite-cutoff comparator has zero remainder. The relative standard error is 2.0×10^{-3} (U-29). Three such errors give 6.0×10^{-3} (U-30). The limiting difference falls below the decision scale. A larger material remainder at a lower cutoff would probe material response.

For a normal crossing of the same gap the local open stroke travels at c (I-2). The conventional low-frequency excess for propagation normal to perfectly conducting plates is the Scharnhorst–Barton value [12, 13]. Let m_e be the electron mass, δv the speed change. The comparator in U-32, with CODATA values U-06 and U-07 [4], uses $e^2/(4\pi\varepsilon_0\hbar c) = 7.2973525693 \times 10^{-3}$ and $\hbar/(m_e c) = 3.8615926796 \times 10^{-13}$ m. It gives

$$\frac{\delta v}{c} = \frac{11\pi^2}{8100}\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)^2\left(\frac{\hbar}{m_e c s}\right)^4 = 1.6 \times 10^{-32}.$$

The local-stroke excess is zero. The conventional calculation concerns a low-frequency propagation velocity. Comparing it with a local transit already assumes that the timing measurement reads that velocity. Even on this identification the excess is unresolved: the nominal transit is 3.34 fs and the fractional resolution 1.0×10^{-18} (U-27, U-31) corresponds to 3.3×10^{-33} s. No apparatus makes that reading. The design takes the fractional resolution as its noise scale, giving 3.0×10^{-18} at three times that scale (U-30). No aperture value enters (O-8).

8.2 A driven clamp and its inverse process

A superconducting coplanar waveguide terminated by a SQUID gives a driven clamp on the platform of Wilson et al. [14]. The illustrative flux drive has $\omega_d/(2\pi) = 10.0$ GHz (U-33). Read the mode pairs $(\omega_1, \omega_d - \omega_1)$, with ω_1 the selected signal frequency, at 16 detunings within ±0.50 GHz of half the ordinary drive frequency (U-34). Forward, $\mathfrak{p}_{\downarrow}$ is the pair rate per unit drive power with no input. Backward, $\mathfrak{q}_{\uparrow}$ is the up-conversion efficiency of the same mode pair, injected as two weak tones, into power at ω_d on the drive line, normalised by the input powers. Each direction has its own absolute calibration (U-35). The static plate comparison specifies two positions and neither driven rate (SC-011–012, L26–30).

The common coupling cancels only after converting mode occupancies into the particular powers and rates just defined. For a matched pair of normalised temporal packets of duration τ_g write ω_2 for the complementary frequency. Let N_d, N_1, N_2 denote incident quanta per

packet, and $\mathfrak{b}_{\mathrm{mix}}$ the resonant coupling in inverse seconds including the mode overlap. In the conventional weak-conversion calculation the squared amplitude per populated input quantum or pair is $\mathfrak{C} = |\mathfrak{b}_{\mathrm{mix}}\tau_g|^2$ on either leg. This follows by integrating the single resonant three-mode matrix element over the packet, with its conjugate on the reverse leg. Unoccupied final modes give unit spontaneous factors. Weak independent coherent input tones give the product of incident occupancies in up-conversion. The same detection bandwidth and the same conjugate packets are selected on both legs. Correct line and detection efficiencies separately back to the named ports.

The forward count per second is $\mathfrak{C}N_d/\tau_g$, and the reverse drive-line power is $\hbar\omega_d\mathfrak{C}N_1N_2/\tau_g$. The incident power of each packet is its quantum energy times its occupation divided by duration. Let $\mathcal{Q}_{\leftrightarrow}$ convert the reverse efficiency into the forward rate per power, and let $\mathfrak{y}$ be the dimensionless log quotient. These normalizations give

$$\mathfrak{p}_{\downarrow} = \mathfrak{C}/(\hbar\omega_d), \qquad \mathfrak{q}_{\uparrow} = \frac{\mathfrak{C}\,\omega_d\tau_g}{\hbar\omega_1\omega_2},$$
$$\mathcal{Q}_{\leftrightarrow} = \frac{\omega_1\omega_2}{\omega_d^2\tau_g}, \qquad \mathfrak{y} = \ln\frac{\mathfrak{p}_{\downarrow}}{\mathcal{Q}_{\leftrightarrow}\mathfrak{q}_{\uparrow}}.$$

The factor has units of reciprocal time. With one squared matrix element the conventional log quotient is zero at each detuning. Core 0.1a reads each direction as a separate settlement starting from its actual state. It gives neither rate. The measured relations behind detailed balance stand in full (Use item 6). The two settlements place no numerical departure on this quotient.

Take the common departure $\mathfrak{h}$ to have magnitude 0.30, with the same sign and size at every detuning (U-36). This is a design setting for power. The single co-varying channel is a bias of one absolute calibration against the other, common to the sixteen, bounded by $\mathfrak{c} = 0.10$ and fixed before the first quotient is read (U-38). Scope is single-channel otherwise. Independent Gaussian noise has standard deviation 0.10 per detuning (U-37). The mean $\overline{\mathfrak{y}}$ therefore has standard deviation 0.025. The common bias retains its full magnitude in the mean. Detection requires $|\overline{\mathfrak{y}}| > \mathfrak{c} + 3\,\mathrm{sd} = 0.175$ (U-30). Integrating both normal tails at the edge of the allowed bias gives false detection 0.00135. With the leak opposing the common departure $\mathfrak{h}$, power is 0.84. Replacing the bound by 0.11 gives 0.58 with the threshold raised as well (U-39). In 400000 Monte Carlo trials per setting, both nominal rates agree with the Gaussian integrals within two sampling standard errors (U-40). A detected departure would test the common-coupling comparator under the calibration bound. The design amplitude supplies no predicted departure.

8.3 Isotope and density

Read a plate enriched in ^{70}Ge and one enriched in ^{76}Ge against the same gold-coated sphere at $s = 0.50\,\mu\mathrm{m}$ (U-42). The atomic masses 69.92425 u and 75.92140 u (U-41, AME 2020 [15]) give mass ratio 1.0858. I-7 puts the added mass into the gravitational channel at fixed geometry. Its purely density-coupled equilibrium discriminator has a positive input shift there. The ideal clamped pressure depends on $\hbar, c, s$ alone. Holding the optical inputs fixed gives heavy-to-light ratio unity in both calculations. Actual permittivities enter each calculation alike. The standard error of the quotient, including its shared-reference covariance, is

5.0×10^{-3}, giving a three-error threshold of 0.015 (U-43, U-30). The shared sphere supplies no independent second error to add to that specified quotient uncertainty.

To see the scale that mass reaches, take two natural-Ge slabs, each 1 mm thick at density 5.323 g cm^{-3} (U-44, U-45). Their mass per area is 5.323 kg m^{-2}. In the weak-field, slow-motion limit and for gaps and thicknesses small against the lateral size, one sheet gives gravitational acceleration $2\pi G$ times its areal density. This follows by integrating the Newtonian divergence law over a pillbox crossing the sheet. Multiplying by the second sheet's areal density gives a pressure of 1.2×10^{-8} Pa, or 5.7×10^{-7} of the ideal pressure magnitude at the isotope-test gap. This is the leading Newtonian containment layer of I-4. The slab calculation estimates the density channel in a reference geometry. The actual observable uses a sphere and a plate.

9 Status ledger

Item	Status and extent
O-1	To be derived: finite-core cutoff, regular core chart or outward phase-transfer law.
O-2	To be derived: geometrically constructed overlap-contrast odds.
O-3	To be derived: canonical confinement odds.
O-4	To be derived: local profile from the constitutive law with boundary, surface and core data.
O-5	To be derived: multipole and tidal remainders of the pair expression, rotating-source exterior band, overlap complex and sign of a pair the many-cone settlement does not join.
O-6	To be derived: charge, circulating-charge and colour-seat input maps, seat table from cone geometry and gluon octet dynamics.
O-8	To be derived: constitutive derivation of the aperture speed and index and reconciliation with lensing.
O-9	To be derived: confinement operator, domains, boundary data, spectrum and mass ratios.
O-17	To be derived: host selection, transport of the settlement measure and its extension to unequal weights or more patterns.
Overlap, confinement, mode	Labels on the open range only; no value attaches to them.
Source readings	In sample: gap, modes, residual, sum, pressure and two atomic limits.
External readings	In sample: thermal and material pressures, gravitational window, shell, sphere and parallel-plate data.
Designs	In sample: cutoff, transit, rate quotient and isotope comparisons at the specified design points.
Occupation	In sample: lowest positive longitudinal branch, thermal crossover and energy deficit.

10 Conclusions

The attraction follows from what changes when the plate is brought near the face. The transverse continuum gives the area factor, the zero-index wave fixes the endpoint weight, and the third derivative at that endpoint fixes the energy coefficient. Differentiation at

fixed response gives the signed pressure. Finite conductivity and temperature alter the standing-wave calculation through independently specified inputs. The plate's gravitational compactness answers a different measurement question. At the separations and precisions considered here, the accepted plate-force data and their conventional values coincide with the clamped residual of the one field.

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