

Confinement and the Mass Gap

The common carrier, self-supported omnium patterns,
and a background-dependent neutrino minimum

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Fundamental Density Theory

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Core v6.3 (author lock 2026-10-26) · companion to *The Premises of Yang–Mills*

Abstract

In Fundamental Density Theory all mass is confinement cost. The gluon is the one carrier standing inside a closed omnium pattern, and the mass of its confined mode is the energy that configuration costs; the carrier has no intrinsic mass to add. Nuclear binding is transacted at the pattern's dense skin, and the same closed-support construction repeats at every size: nucleus, compact object, galaxy, domain. The core's mode relation gives an exact criterion for a mass gap: the stable closed sector of a background has a gap exactly when its admitted periods are bounded, and the gap is $h/(c^2 T_{\text{sup}})$. Under the neutrino-minimum premise, whose derivation from the spectrum of the confinement operator is entered as D-01, the least stable closed mode is the free two-strand neutrino, and the gap is its whole confinement cost divided by c^2 , strictly positive in every background and set by that background's cumulative standing-wave state. Free radiation carries no gap. The exterior optical metric of the core places the circular stroke at $3R_s/2$, at $\alpha_{\text{ext}} = 2/3$ on the compactness chart, radially unstable with the exponent general relativity gives the photon sphere. No dataset and no fitted mass enters a derivation. The observable the paper adds is a lightest neutrino mass strictly greater than zero, resting on the premise it names.

1 All mass is confinement cost

Fundamental Density Theory has one substance, the omnium, and the central invariant

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t} \right)^2 \quad (1)$$

is one identity read by quantum mechanics, electromagnetism and general relativity (core 0.1). A carrier is a disturbance of a medium at that medium's speed; the chain gives one speed to every reading and there is one medium to disturb, so there is one carrier: photon = graviton = gluon = phonon = magnon, one stroke read in different density and geometry regimes (I-2) [5]. Its gluon presentation is the stroke standing inside a closed pattern's own region. "Photon-type" therefore means the same carrier with different configuration and boundary data. The standing stroke and the whole braided closure play distinct roles: the carrier identity is carrier-only, and the braided closures of I-5 stand outside it. A nucleus is one closed omnium pattern, its support included in the configuration (0.1b, I-2, I-5).

The nodal architecture of P-2 supplies the standing-wave reading. A stable closed pattern is its own plate, sand and driver: support, nodal density structure and sustaining circulation, three

roles of one configuration solved together with its source and its surrounding state. None is a separate component and none is a second energy reservoir; I-4 reads the force, wave and geometric descriptions as one state.

The companion paper *The Premises of Yang–Mills* [3] takes the confinement results below and the neutrino-minimum premise of this paper as its own premises. The argument runs as one derivation. Section 2 states the premises, each quoted from the locked core v6.3 [1] except the neutrino-minimum premise, which is this paper’s. Section 3 proves that every admitted closed mode has a positive mass equal to its whole confinement cost (Theorem 1). Section 4 places the binding at the skin and the force scale on the chart. Section 5 defines the stable sector of a background, and Section 6 proves the exact criterion (Proposition 3) and the gap (Theorem 4). Section 7 keeps the energy account, and Section 8 computes the circular stroke of the exterior and its instability (Proposition 5). The record, the tests, the declined readings and the convergence statement follow. The receipt script of Appendix A checks the displayed identities and numbers; check labels appear in brackets, [CM07f].

2 Premises, charts and symbols

The derivation uses the locked core v6.3 [1]; each premise is quoted by section.

- (P1) *One substance, only patterns.* There is one substance, the omnium: matter = energy = spacetime. There are no separate particles and no boxes; a room, box or domain is a pattern’s own support with its own boundary data, never a vessel (0.1, 0.1b).
- (P2) *The chain is one state.* The four terms of (1) adjust together, and the bridges are constants of the medium: $E_q = m_q c^2 = \hbar \omega_q$, $T_q = 2\pi/\omega_q = \hbar/E_q$, $\ell_q = cT_q = \hbar/(m_q c)$. The floor $F_{\max} d_{\min} t_{\min} = \hbar$ with $d_{\min} = ct_{\min}$ gives $d_{\min} = 2\ell_P$, $t_{\min} = 2t_P$, an infimum approached and never occupied. Every step is a system-wide settlement of the state it starts from; responses are many-to-many, never particle by particle (0.1a).
- (P3) *One carrier.* Photon = graviton = gluon = phonon = magnon: the gluon is the stroke standing inside a closed pattern’s own region, the I-5 mode with $m_q c^2 = \hbar \omega_q$. There are no independent force carriers; interactions are curvature plus geodesic motion (I-2).
- (P4) *Matter is closed stroke.* A closed pattern’s strand runs at c on a helix and keeps time by circulating, and a stationary closed pattern, closure included, has inertia E/c^2 (0.1d) [4].
- (P5) *Mass is a confinement eigenvalue.* For a pattern confined to its support Ω with boundary data B, braid configuration n , orientation χ and position on the inside/outside chart, $D u_q = \omega_q u_q$ and $m_q c^2 = \hbar \omega_q$; the operator D is to be derived (core O-9). Particle identity has two axes: the helix count n fixes the class, and the confinement position, the cone-position axis of application A-4, fixes the mass inside a class. R_{box} is never the reduced Compton wavelength, and no universal mass formula $m(\alpha)$ is part of the theory (I-5).
- (P6) *The atlas and the regimes.* $\alpha_\sigma = (R_s/r_f)^\sigma$, a ratio of two curvatures, on the open interval of each branch; on the A-1 compactness chart $\alpha_\sigma < 1/2$ is the general-relativistic regime, $\alpha_\sigma = 1/2$ the seam and $\alpha_\sigma > 1/2$ the quantum-mechanical regime, with the seam at $r_f = 2R_s$ on the exterior branch and $r_f = R_s/2$ on the interior branch (0.3, 0.3a, 0.4).
- (P7) *The skin and the two charts.* A bound configuration is densest at the skin and dilute at the centre, and it carries two declared charts, the exterior atlas of its source and its own interior

chart, with no transfer map between them; the clamp and the binding are transacted at the high- α_{int} skin (0.6, I-3, P-3).

- (P8) *Stability, the ledger form.* A closed mode is stable exactly when $\langle f|\mathbf{T}|i\rangle = 0$ for every final state f at or below threshold that keeps every exact book: four-momentum, charge, fermion parity, angular momentum (I-5b).
- (P9) *The cumulative background.* The cosmic microwave background is the cosmological reading of the substrate's cumulative standing-wave state (0.8), whose derivation is the core's register item O-15. That state is the interference pattern of all the universal density waves the domain carries, past and present; read at quantum scale, it is what the standard account calls quantum fluctuations. One background is read at two sizes, the repetition of 0.3a applied to the substrate's own state.
- (P10) *The neutrino-minimum premise.* Every admitted background $\mathcal{B}$ carries a stable two-strand family $\mathcal{F}_2[\mathcal{B}]$ inside its stable sector $S[\mathcal{B}]$ (Definition 2), and a free $n = 2$ neutrino mode of that family attains the least energy over $S[\mathcal{B}]$, at positive frequency:

$$E_{C,q}[\mathcal{B}] \geq E_{2,\min}[\mathcal{B}] = \hbar \omega_{\nu,\min}[\mathcal{B}] > 0 \quad \text{for every } (C, q) \in S[\mathcal{B}]. \quad (2)$$

This premise is this paper's. Its derivation from the spectrum of D is to be derived (D-01).

- (P11) *Density equivalence.* $\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \dots$, every channel entering once, with $a = 4\sigma_{\text{SB}}/c$; gravitational binding is read from the solved geometry (0.5, I-7).

Charts. Two readings on the A-1 compactness chart appear. A source's exterior atlas carries $\alpha_{\text{ext}}(r_f) = R_s/r_f$ on $r_f > R_s$ and its reciprocal interior branch $\alpha_{\text{int}}(r_f) = r_f/R_s$ on $r_c \leq r_f < R_s$, with $R_s = 2GM/c^2$; the I-1 inputs $\alpha_{i,\text{ext}} = R_{s,i}/d$ are exterior cross-centre ratios of nonoverlapping spherical supports. A bound configuration's own interior chart is read with its measured skin at that chart's fold; a measured nuclear skin radius is never a gravitational R_s . The A-2 chart is computational and stands in no slot here. A-3 (overlap) and A-4 (confinement position) are each a range and nothing else: neither carries a threshold, an ordering, a sign or a value in this paper, and the canonical A-4 odds are the core's register item O-3. No threshold transfers between applications: the A-1 1/2 and the A-3 2/3 are separate numbers. The 2/3 of Section 8 is an A-1 value.

symbol	meaning
$C = (\Omega, \mathbf{B}, n, \chi, \tau_{\text{conf}})$	a closed configuration: support, boundary data, helix count, orientation, confinement position (τ_{conf} renames the core's position argument so that t stays a duration, I-6)
$D_C, u_q, \omega_{C,q}, m_{C,q}$	the confinement operator of I-5 for C ; its eigenmode, frequency and mode mass
$\mathcal{B}$	a declared background state: profile, boundary environment, correlations; stationary means fixed in time, not uniform
$D_C[\mathcal{B}], E_{C,q}[\mathcal{B}], T_{C,q}[\mathcal{B}]$	the operator with its background dependence displayed; a mode's confinement cost and period
$S[\mathcal{B}], \mathcal{F}_2[\mathcal{B}]$	the stable closed modes admitted in $\mathcal{B}$; its stable two-strand family
$E_{2,\min}[\mathcal{B}], \omega_{\nu,\min}[\mathcal{B}]$	the least confinement cost over $S[\mathcal{B}]$ and its frequency, attained by the free $n = 2$ neutrino under (P10)
$\Delta_m[\mathcal{B}], T_{\text{sup}}[\mathcal{B}]$	the stable-sector mass infimum and period supremum
$\xi_{nq}(\mathbf{B}), R_{\text{box}}$	dimensionless eigenvalue and geometric support radius of the scale-similar face
f, H, J_{opt}	$1 - R_s/r_f$; the angular coefficient r_f^2/f of the equatorial optical metric; the stroke's conserved optical angular quantity
T_{rad}	the radiation temperature of a channel of $\mathcal{M}$, distinct from a mode period

3 Confinement is closure; mass is its mode cost

I-5 identifies matter as braided photon helices and confinement as topological closure. For a closed configuration C ,

$$D_C u_q = \omega_{C,q} u_q, \quad m_{C,q} c^2 = \hbar \omega_{C,q}. \quad (3)$$

Theorem 1 (confined-mode mass). *For an admissible topologically closed pattern with a positive-frequency eigenmode, the standing stroke is the gluon presentation of the one carrier, and the confined mode's mass is*

$$m_{C,q} = \frac{\hbar \omega_{C,q}}{c^2} > 0. \quad (4)$$

It is the mode's whole confinement cost: the carrier adds no intrinsic mass.

Proof. (P3) identifies the standing stroke; (P5) identifies closure as confinement and supplies the mode-cost relation (3), and (P4) makes the inertia of the closed pattern, closure included, exactly its energy over c^2 . Division by $c^2 > 0$ gives (4), and $\hbar > 0$, $\omega_{C,q} > 0$ give its sign. [CM01a] The carrier is open stroke, which stands in no closure and has no rest frame (0.1d), so the closure's mode is the only mass the configuration carries. $\square$

The gluon's confined mode has mass because confinement costs energy, and that cost is its entire mass. Every admitted mode costs a positive amount. Which modes exist and which are stable is fixed by the spectrum of D_C and the ledger of (P8), and the lower bound across the whole stable sector is the criterion of Section 6.

The core's scale-similar reduction, the locally linear-dispersion face of the eigenproblem, reads

$$m_{nq} \simeq \frac{\hbar \xi_{nq}(B)}{c R_{\text{box}}}. \quad (5)$$

At fixed n , mode q , boundary type and dimensionless eigenvalue ξ_{nq} , a smaller support raises the mass: a tighter room is a more expensive mode. [CM02a, CM02b] R_{box} is the pattern's geometric support, fixed independently of the mass it returns, and never the reduced Compton wavelength, which would force $\xi = 1$ and empty the eigenproblem. [CM02c] All the geometry sits in ξ_{nq} ; there is no universal $m(\alpha)$ law.

The helix count and the confinement position are separate axes. I-5 pairs the neutrino and the neutron at $n = 2$, and the electron and the proton at $n = 3$: one closure read in the free room and in the nuclear room. Confinement moves the mass at fixed count; changing the count changes the topological class, by re-braiding. No named particle carries an absolute density address.

4 The binding skin and the force scale

On the source's exterior atlas, $R_s = 2GM/c^2$ and $\alpha_{\text{ext}}(r_f) = R_s/r_f$ on $r_f > R_s$, with the reciprocal interior branch $\alpha_{\text{int}}(r_f) = r_f/R_s$ on $r_c \leq r_f < R_s$. On this chart α reads the regime (P6); the seam lies at $r_f = 2R_s$ on the exterior branch and at $r_f = R_s/2$ on the interior branch, and both branches approach 1 at the fold, which neither reaches. [CM03a-c]

A bound configuration also carries its own interior chart, read with its measured skin at that chart's fold (P7). Both charts are declared and no transfer map joins them. P-3 separates the skin's saturation coordinate from the same nucleus's small exterior compactness. The observation-region rule places the clamp and the binding at the high- α_{int} skin: low- n closures have their support toward the dilute centre, and higher rungs of the n -ladder have footing in the skin terrain. The

strong-force reading lives in the pattern's binding geometry at the skin, and the interior is not uniformly dense.

The density that counts is the local profile ρ_{loc} , which rises toward the skin (0.6); its function is to be derived (core O-4). The enclosed mean over an interior sampling volume is a different object, a comparison proxy whose ratio to the separator density scales as α_{int}^{-3} and rises inward by construction. [CM03d] The local profile keeps its skin peak.

The closed-support construction repeats at every size. The chain is all ratios and selects no size, α_σ is invariant under $(R_s, r_f) \mapsto (\lambda R_s, \lambda r_f)$, and the support, its seam and its two phases have one form at nucleus, compact object, galaxy and domain; the floor $2\ell_P$, $2t_P$ bounds the repetition below (0.3a, 0.1a). The form is shared; each size solves its own source and data.

I-1 supplies the common force scale, and I-6 holds every input in the open interval:

$$F = F_{\text{max}} \alpha_{1,\text{ext}} \alpha_{2,\text{ext}} < F_{\text{max}}, \quad F_{\text{max}} = \frac{c^4}{4G} = 3.0256 \times 10^{43} \text{ N}. \quad (6)$$

The pair evaluation uses exterior cross-centre A-1 ratios of nonoverlapping spherical supports, where it returns $G\mathcal{M}_1\mathcal{M}_2/d^2$ exactly; skin coordinates stay out of these slots. [CM04c, CM04f] I-3 keeps $G_{\mu\nu} = \kappa T_{\mu\nu}$, with α downstream of the solved geometry, and

$$\kappa = \frac{8\pi G}{c^4} = \frac{2\pi}{F_{\text{max}}}, \quad \kappa F = 2\pi \alpha_{1,\text{ext}} \alpha_{2,\text{ext}} < 2\pi. \quad (7)$$

κ carries inverse-force units. [CM04a, CM04b, CM04d] The exact identity $\kappa F_{\text{max}} = 2\pi$ normalises the character of a nontrivial 2π -rotation class to $\exp(i\kappa F_{\text{max}}/2) = -1$ with no fitted parameter; [CM04e] the class itself is $\nu_F = B_s \bmod 2$, and the proof that it is nontrivial is the core's register item O-11 (I-5, I-5b). The normalisation acts on the rotation class; the helix count is I-5's first axis, and the matter class begins at $n = 2$.

Compactness places a configuration on the atlas; the confinement frequency comes from D_C , and the stability of a closed mode from (P8). Section 8 computes the distinction for circular propagation.

5 The cumulative background and the stable sector

The ground of every domain is the substrate's cumulative standing-wave state (P9). Its cosmological reading is the microwave background, its quantum-scale reading what the standard account names quantum fluctuations, and no variable of the chain vanishes anywhere in it (0.1). The confinement eigenproblems below are stationary in a declared background $\mathcal{B}$; a time-dependent background is tracked by the phase exchange of I-6, whose rates are the core's register item O-12. The pattern's boundary data B and the background $\mathcal{B}$ are distinct symbols.

Definition 2 (the stable sector). With the background dependence of I-5's admissibility data displayed,

$$D_C[\mathcal{B}] u_q = \omega_{C,q}[\mathcal{B}] u_q, \quad E_{C,q}[\mathcal{B}] = m_{C,q}[\mathcal{B}] c^2 = \hbar \omega_{C,q}[\mathcal{B}], \quad (8)$$

$S[\mathcal{B}]$ is the nonempty set of self-consistent, stable, positive-frequency closed modes (C, q) admitted in $\mathcal{B}$. Membership is the ledger's verdict of (P8) on the system-wide settlement (P2), never a prescription of loop and frequency.

The bridge of (P2) ties each mode's period to its confinement cost:

$$T_{C,q}[\mathcal{B}] = \frac{2\pi}{\omega_{C,q}[\mathcal{B}]} = \frac{h}{E_{C,q}[\mathcal{B}]}, \quad m_{C,q}[\mathcal{B}] = \frac{h}{c^2 T_{C,q}[\mathcal{B}]}. \quad (9)$$

[CM01b, CM01c] Every duration belongs to the configuration whose mass it fixes; none varies on its own. The background's confinement minimum is a condition on self-sustaining configurations, whose support, source, boundary data and stability are solved together.

6 The exact criterion and the mass gap

Define the stable-sector mass infimum and period supremum at fixed background,

$$\Delta_m[\mathcal{B}] = \inf_{(C,q) \in S[\mathcal{B}]} m_{C,q}[\mathcal{B}], \quad T_{\text{sup}}[\mathcal{B}] = \sup_{(C,q) \in S[\mathcal{B}]} T_{C,q}[\mathcal{B}]. \quad (10)$$

Proposition 3 (the exact criterion). *For every background,*

$$\Delta_m[\mathcal{B}] = \frac{h}{c^2 T_{\text{sup}}[\mathcal{B}]}, \quad (11)$$

which is zero exactly when $T_{\text{sup}}[\mathcal{B}] = \infty$.

Proof. By (9) the mass of a mode is the positive, continuous, strictly decreasing function $h/(c^2 T)$ of its period, with limit 0 as $T \rightarrow \infty$, so its infimum over $S[\mathcal{B}]$ is its value at the period supremum. [CM01d, CM01e, CM05a] Bounded admitted periods give a positive gap; unbounded admitted periods give zero. Attainment is not needed. [CM05c] $\square$

The criterion puts the whole gap in one place: the stable sector has a gap exactly when its admitted periods are bounded.

Theorem 4 (the mass gap). *Under the neutrino-minimum premise (P10),*

$$\Delta_m[\mathcal{B}] = \frac{E_{2,\min}[\mathcal{B}]}{c^2} = \frac{\hbar \omega_{\nu,\min}[\mathcal{B}]}{c^2} = \frac{h}{c^2 T_{\text{sup}}[\mathcal{B}]} > 0 \quad (12)$$

in every admitted background $\mathcal{B}$. The least stable closed mode is the free two-strand neutrino, and it is also the mode of longest admitted period.

Proof. Divide the energy ordering (2) by $c^2 > 0$. The neutrino mode attains equality, so its mass is the infimum, and its positive frequency makes the infimum strictly positive. The frequency bridge gives the second expression and Proposition 3 the third; the least energy is the longest period by (9). [CM05b, CM05d] $\square$

The closed sector's rest-energy threshold is $\Delta_E[\mathcal{B}] = E_{2,\min}[\mathcal{B}]$. The theorem fixes the gap's identity and its sign; its number is the neutrino's confinement cost in the given background, read through the spectral relation D-01 derives. The background enters $E_{2,\min}[\mathcal{B}]$ only through $D_C[\mathcal{B}]$: it changes which modes are admitted and what they cost. For two comparable stationary backgrounds, under (P10) in both,

$$\Delta_m[\mathcal{B}_b] - \Delta_m[\mathcal{B}_a] = \frac{E_{2,\min}[\mathcal{B}_b] - E_{2,\min}[\mathcal{B}_a]}{c^2}, \quad (13)$$

a comparison of two stationary solutions in which no derivative, rate or instantaneous response enters. [CM05e] Which of two backgrounds sets the costlier least mode is a spectral statement, entered as D-02; the confinement position of (P5) joins it once the canonical A-4 odds exist (O-3).

The gap of this paper is the least mass of the stable closed sector. It is a different object from the lower edge ϵ_* of the pure gauge problem on the physical domain, the step between the two lowest

confinement eigenvalues of a closed pattern's own region on the closure sector, whose operator and uniform estimate are D-1 of the companion paper [2]. Positivity holds background by background,

$$\Delta_m[\mathcal{B}] > 0 \text{ for every } \mathcal{B} \not\Rightarrow \inf_{\mathcal{B}} \Delta_m[\mathcal{B}] > 0, \quad (14)$$

and a bound uniform across all backgrounds is its own spectral statement, entered as D-03. [CM05f]

7 Energy accounting

I-7 counts each source channel once:

$$\mathcal{M} = M + \frac{E_{\text{other}}}{c^2} + \frac{PV}{c^2} + \frac{a T_{\text{rad}}^4 V}{c^2} + \dots, \quad a = \frac{4\sigma_{\text{SB}}}{c} = 7.5657 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}. \quad (15)$$

The thermal term is a mass with the energy-density constant a ; written with the flux constant σ_{SB} it would be a momentum. [CM06a-c] A confined mode's energy enters once, as the rest contribution of its state, and never again as an added energy of the same state. The solved mass aspect already carries gravitational binding (0.5). A mass-budget partition divides the one confinement cost into channels and adds no intrinsic mass.

$E_{2,\min}[\mathcal{B}]$ is the whole mode's cost in its environment. The background changes which mode is admitted and what it costs; the background's own energy enters $\mathcal{M}$ once, as a non-rest contribution of the one stress tensor, and is never added a second time to a mode's cost. The same accounting holds for the standing gluon mode and for the complete pattern, each at its own descriptive level.

The integral strand ledger forbids any continuous reduction of n below two inside the matter class. Count and energy are different books: one count admits many supports and boundary data, so the minimum runs over all stable admitted modes, which is what (P10) states. The gap is the statement that admitted frequencies stay bounded away from zero, which is stronger than excluding $\omega = 0$.

8 The circular stroke of the exterior

Circular propagation is exact inside the core's optical geometry. Restrict the exact optical metric of I-3 to an equatorial plane, with $f(r_f) = 1 - R_s/r_f$ on $r_f > R_s$:

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\phi^2}{1 - \alpha_{\text{ext}}} = f^{-2} dr_f^2 + H(r_f) d\phi^2, \quad H(r_f) = \frac{r_f^2}{f} = \frac{r_f^3}{r_f - R_s}. \quad (16)$$

[CM07a, CM07j]

Proposition 5 (the circular stroke). *The exterior optical metric admits one circular geodesic, at*

$$r_* = \frac{3R_s}{2}, \quad \alpha_{\text{ext}}(r_*) = \frac{2}{3} \quad (A-1), \quad (17)$$

inside the exterior seam $r_f = 2R_s$, in the quantum-mechanical regime of $0.3a$. It is radially unstable: a radial perturbation obeys

$$\frac{d^2 \delta r_f}{d\ell_{\text{opt}}^2} = \frac{f_*^2 H_*''}{2H_*} \delta r_f = \frac{4}{27R_s^2} \delta r_f, \quad f_* = \frac{1}{3}, \quad H_* = \frac{27R_s^2}{4}, \quad H_*'' = 18, \quad (18)$$

and grows as $\exp(\sqrt{4/27} \ell_{\text{opt}}/R_s)$.

Proof. A constant-radius optical geodesic requires $H'(r_*) = 0$, and $H'(r_f) = r_f^2(2r_f - 3R_s)/(r_f - R_s)^2$ vanishes on $r_f > R_s$ only at $3R_s/2$. [CM07b–d] Parameterise by optical arclength. The conserved angular quantity is $J_{\text{opt}} = H d\phi/d\ell_{\text{opt}}$, and the unit-tangent equation $f^{-2}(dr_f/d\ell_{\text{opt}})^2 + J_{\text{opt}}^2/H = 1$ gives $(dr_f/d\ell_{\text{opt}})^2 = W(r_f)$ with $W = f^2(1 - J_{\text{opt}}^2/H)$, so $d^2r_f/d\ell_{\text{opt}}^2 = W'(r_f)/2$. The circular stroke has $J_{\text{opt}}^2 = H_*$; there $W = W' = 0$ and $W''/2 = f_*^2 H''/(2H_*)$, which with the displayed values is $4/(27R_s^2) > 0$. [CM07e–g] A positive coefficient drives an exponentially growing perturbation; a direct integration of the optical geodesic returns the rate. [CM07h] $\square$

Optical arclength is c times the bookkeeper’s coordinate time along a stroke, so the rate is the photon-sphere exponent $c/(3\sqrt{3}GM/c^2)$ of general relativity in coordinate time. [CM07i] Circularity supplies a loop’s geometry; closure is I-5’s topological condition, and a standing mode’s stability is the ledger of (P8). The 2/3 here is exterior A-1 compactness. It is not I-1a’s A-3 overlap threshold and carries no confinement or gluon threshold; 0.4 admits no transfer without a derived map.

9 The record and the tests

what the record shows	the measurement	where it is read
neutrino masses are not all zero	oscillation experiments measure two nonzero mass-squared splittings, $\Delta m_{21}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$ and $ \Delta m_{32}^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2$, and leave the lightest mass free [6]	Theorem 4; (P10)
the absolute scale is bounded above	the tritium beta endpoint bounds the effective electron-neutrino mass at $m_\nu < 0.45 \text{ eV}$ (90% C.L.) [7]	Theorem 4: no number printed, every bound met
the standing carrier has an infrared mass scale	lattice computations of the Landau-gauge gluon propagator find it finite at zero momentum, a consensus computation read beside the record [8, 9]	Theorem 1
glueball candidates	the $X(2370)$ in $J/\psi \rightarrow \gamma K_S^0 K_S^0 \eta'$, with $J^{PC} = 0^{-+}$ [10]	Theorem 1: the cost of a standing mode, its spectrum O-9
the circular stroke	the shadows of M87* and Sgr A* agree with the photon ring of general relativity [11, 12]	Proposition 5, by containment (I-3)

Each test below is the converse of a derived result: the measurement that would end it.

T-1 *The confined gluon.* A free massless asymptotic state of the standing carrier ends Theorem 1’s reading of the gluon as a confined mode.

T-2 *A massive least mode.* A lightest neutrino established as massless ends the neutrino-minimum premise (P10), and with it the value of the gap in Theorem 4.

T-3 *The least stable mode.* A stable closed mode lighter than the lightest neutrino ends the identification of the gap with the neutrino’s confinement cost.

T-4 *The circular stroke.* A nonrotating source whose photon ring sits at other than $3R_s/2$, or a circular stroke that is radially stable, ends Proposition 5.

Three pieces sit outside the derivation and leave every result in it unchanged:

D-01 *The premise.* The neutrino-minimum premise (P10), from the spectrum of D : that the free $n = 2$ neutrino attains the least energy over $S[\mathcal{B}]$ at positive frequency, with the

background's temperature entering $E_{2,\min}[\mathcal{B}]$ through the same spectral relation, is to be derived (core O-9).

D-02 *Across backgrounds.* The response of $E_{2,\min}[\mathcal{B}]$ across comparable stationary backgrounds, and the confinement position of the least mode, read through the canonical A-4 odds once they exist, are to be derived (core O-3, O-9).

D-03 *Uniformity.* A gap uniform across backgrounds, $\inf_{\mathcal{B}} \Delta_m[\mathcal{B}] > 0$, is to be derived.

10 Declined readings and convergence

The standard account treats the gluon as a gauge boson distinct from the photon, massless in its Lagrangian, with an infrared mass scale generated dynamically. FDT reads the gluon as the one carrier standing in a closed pattern (I-2), whose mass is that standing mode's confinement cost (I-5, Theorem 1).

The standard account reads quantum fluctuations as virtual pairs appearing in an empty vacuum. FDT reads them as the quantum-scale reading of the substrate's cumulative standing-wave state: no variable of the chain vanishes, so there is no empty background (0.1), and there is no virtual exchange (I-2).

The standard account gives each particle an intrinsic mass, set for the leptons and quarks by a coupling to a scalar field. FDT reads every mass as confinement cost, the eigenvalue of a closed pattern's mode in its support and background (I-5), with no intrinsic property to add.

The standard account's cosmological fits bound the sum of the neutrino masses through the expansion history of its chronology. FDT reads those fits as resting on the declined chronology (0.8); the laboratory bound and the oscillation splittings bind.

Convergence. For the same source and data FDT returns the measured record. The exterior optical metric of I-3 is the Schwarzschild optical metric, and the circular radius $3R_s/2$ with its radial instability and exponent are the general-relativistic photon-sphere results, recovered exactly by containment. The relations $mc^2 = \hbar\omega$ and $T = h/E$ are the measured Planck–Einstein and Compton relations, read as the chain's one state (0.1a). The lattice computations of an infrared gluon mass scale are consensus computations read beside the record, and the measured glueball-candidate spectrum is accepted as data; FDT reads each as the confinement cost of the standing carrier, the numbers to be derived with the operator (O-9). The oscillation splittings and the laboratory bound on the absolute scale are accepted in full: Theorem 4 prints no number, so every measured bound is met. Freely propagating light carries no gap and stays massless; the gap belongs to the closed sector. One observable rests on an input this paper names: the consensus fits leave the lightest neutrino mass free down to zero, and the neutrino-minimum premise (P10) fixes it strictly positive.

11 Confinement and the mass gap

The gluon's confined mode has mass because mass is confinement cost: the carrier standing in a closed pattern's own region costs the energy of its mode, and that is all the mass it has. A stable closed sector has a mass gap exactly when its admitted periods are bounded. Under the neutrino-minimum premise the gap is the free two-strand neutrino's whole confinement cost divided by c^2 , strictly positive in every background, and the background's cumulative standing-wave state sets

that cost. Free radiation carries no gap. The circular stroke of the exterior sits at $3R_s/2$ and falls away from it, as general relativity's photon sphere does.

12 Status register

result	rests on	status
Confined-mode mass, the whole confinement cost (Theorem 1)	0.1d, I-2, I-5	derived
The scale-similar face and the vacuity guard (5)	I-5	derived, as the core's approximation
The seams, the fold and the interior proxy	0.3, 0.3a, 0.6	derived
The force scale and $\kappa F_{\max} = 2\pi$ (6), (7)	0.2, I-1, I-3	derived, exact on the pair domain
The character $\exp(i\kappa F_{\max}/2) = -1$ of a nontrivial rotation class	0.2, I-5	derived; the class's nontriviality core O-11
The stable sector and the period bridge (Definition 2, (9))	0.1a, I-5, I-5b	derived
The exact criterion (Proposition 3)	0.1a, I-5	derived
The mass gap $E_{2,\min}[\mathcal{B}]/c^2 > 0$ (Theorem 4)	(P10)	derived under (P10)
The neutrino-minimum premise (P10)	I-5, I-5b	to be derived (D-01; core O-9)
The comparison (13)	(P10)	derived; its sign D-02
The response across backgrounds; the A-4 position of the least mode	I-5, 0.4	to be derived (D-02; core O-3)
A gap uniform across backgrounds	I-5	to be derived (D-03)
The energy account (15)	0.5, I-7	derived
The circular stroke and its instability (Proposition 5)	I-3, 0.3a	derived, exact
The background's standing-wave state	0.8	core O-15
The local profile ρ_{loc}	0.6	core O-4

invariant	where it enters
I-1	the common force scale $F = F_{\max}\alpha_{1,\text{ext}}\alpha_{2,\text{ext}} < F_{\max}$, exact on the pair domain (6)
I-2	the gluon as the standing presentation of the one carrier; free radiation as the open stroke with no gap
I-3	two declared charts and the skin; containment for the circular stroke (16)–(18)
I-4	the force, wave and geometric descriptions read as one state
I-5	mass as a confinement eigenvalue, the two axes, the scale-similar face and the vacuity guard; I-5b's ledger form of stability
I-6	every coordinate on the open interval; the phase exchange of a time-dependent background
I-7	every channel of $\mathcal{M}$ entering once (15); the background's energy never added a second time

A Receipt

The script `receipt_confinement.py`, in the folder of this paper, checks the displayed identities symbolically with SymPy and the displayed numbers numerically, with a direct integration

of the equatorial optical geodesic for the instability of Proposition 5. Run it with `python3 receipt_confinement.py`; its output is `receipt_confinement.out.txt`. Result: **37/37 PASS**.

check	content
CM01a–e	$m = \hbar\omega/c^2 > 0$; $T = 2\pi/\omega = \hbar/E$; $m = \hbar/(c^2T)$; $dm/dT < 0$; $\hbar/(c^2T) \rightarrow 0$ as $T \rightarrow \infty$
CM02a–c	(5) has the units of mass; a smaller support raises it at fixed ξ ; $R_{\text{box}} = \hbar/(mc)$ forces $\xi = 1$
CM03a–d	the seams $r_f = 2R_s$ and $R_s/2$; the fold; $d\alpha_\sigma/dr_f = -\sigma\alpha_\sigma/r_f$; the interior proxy α_{int}^{-3} , rising inward
CM04a–f	$\kappa F_{\text{max}} = 2\pi$; $\kappa = 2\pi/F_{\text{max}}$; $F_{\text{max}}\alpha_1\alpha_2 = G\mathcal{M}_1\mathcal{M}_2/d^2$; $\kappa F < 2\pi$; $\exp(i\kappa F_{\text{max}}/2) = -1$; $F_{\text{max}} = 3.0256 \times 10^{43}$ N
CM05a–f	the criterion on 400 random spectra; the least energy at the longest period; zero without attainment for unbounded periods; the chain (12); the comparison (13); positivity per background without a uniform bound
CM06a–c	aT^4V/c^2 a mass; $\sigma_{\text{SB}}T^4V/c^2$ a momentum; $a = 7.5657 \times 10^{-16}$ J m ^{−3} K ^{−4}
CM07a–j	$H = r_f^3/(r_f - R_s)$; H' ; $r_* = 3R_s/2$ alone; $\alpha_{\text{ext}}(r_*) = 2/3$ inside the seam; f_* , H_* , H''_* ; the coefficient $4/(27R_s^2)$; $W = W' = 0$ and $W''/2$ at r_* ; the direct integration, rate $0.3849/R_s$; the photon-sphere exponent; the I-3 optical metric on the equator

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