

Dark Matter Is Heavier, Denser, Curved Spacetime

A proof from the core of Fundamental Density Theory

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Abstract

General relativity reads gravity as curvature. Fundamental Density Theory (FDT) keeps that reading and adds the one beneath it: there is one substance, the omnium, and the central invariant $E/m = c^2 = 1/(\epsilon_0\mu_0) = (d/t)^2$ makes matter, energy and spacetime three readings of it. With the FDT core as the only theory and the observational record as the data, we prove that the bulk of the dark component of galaxies is the omnium's spacetime phase held compressed around mass: heavier and denser than relaxed substrate, and curved by its own weight. Three lemmas carry the proof. (1) The exterior reading $\alpha_{\text{ext}} = R_s/r_f$ depends on the weight inside r_f and on nothing else: a galaxy, a black hole and a sphere of Sgr A*'s mass anywhere in the universe read alike wherever they hold equal weight inside the field radius, and the Schwarzschild density of a given $\mathcal{M}$ depends on $\mathcal{M}$ alone. (2) Curvature is weight: at fixed radius the enclosed $\mathcal{M}$ is $c^2 r_f \alpha_{\text{ext}} / (2G)$, and local weight sits wherever α_{ext} falls more slowly than $1/r_f$. (3) Compression is weight, as a compressed spring outweighs a relaxed one. The theorem is a phase census. The record weighs the Milky Way at 6.1 times its matter-phase census inside 39.5 kpc and 11.5 times inside 273 kpc; that weight is transparent, bends light, and wherever the record reaches is not held in closed braids. On Earth the envelope's counterpart is the atmosphere, weighed by a barometer and seen only through its refraction. Read as circular orbits, an envelope that carries a flat curve's weight in hydrostatic balance, under an isotropic barotropic pressure, is an isothermal atmosphere with $c_s^2 = c^2 \alpha_{\text{ext}} / 4$, $(165 \text{ km s}^{-1})^2$ for the Milky Way.

1 The claim and its footing

Theorem 1 (Dark matter is heavier, denser, curved spacetime). *Outside the compact-body windows of D-01 (Section 10), the bulk of the weight that the record measures around galaxies in excess of their matter-phase census is the omnium's spacetime phase held compressed around mass. It is heavier and denser than relaxed substrate, and the curvature the record attributes to it is its own weight.*

Section 0.7 of the core [1] states this identification: dark matter is heavier, denser omnium, the mass shadow that every nonzero source carries. This paper derives the identification from the founding statement and its readings (0.1, 0.1a, 0.1b), the two phases (0.3a), the exterior density profile (0.6) and the braid reading of matter (I-5), which the core states as posits; from the definitions of the constants, the Alpha Lock and $\mathcal{M}$ (0.2, 0.3, 0.5); from the seven invariants; and from the observational record. The further statement of 0.7, that the shadow is carried by its source and moves with it, stays a posit (Remark 5).

1.1 Domain declaration and chart inventory

- **Application and branch.** Every α in this paper is an A-1 compactness reading on the exterior branch, $\sigma = +1$: $\alpha_{\text{ext}}(r_f) = R_s/r_f$ with $R_s = 2G\mathcal{M}/c^2$ (0.2, 0.3, 0.4).
- **Evaluations** (0.3). *Field point*: $\alpha_{\text{ext}}(r_f)$ with the whole configuration's $\mathcal{M}$, at $r_f \geq R_b$. *Configuration*: $\alpha_{\text{ext}}(R_b)$, with R_b the support radius. *Enclosed field point*: $\alpha_{\text{ext}}(r_f) = 2G\mathcal{M}(r_f)/(c^2 r_f)$, with $\mathcal{M}(r_f)$ the total gravitating energy inside r_f , at monopole order; the multipoles of a disk belong to O-5.
- **Readings.** $\alpha_{\text{ext}}^{\text{obs}}(r_f) \equiv 2v_c^2/c^2$, the circular-speed reading (Section 6); $\alpha_{\text{ext}}^{\text{mat}}(r_f)$, the enclosed evaluation built from the matter-phase census.
- **Landmarks.** The seam $\alpha_{\text{ext}} = 1/2$ at $r_f = 2R_s$ (A-1, exterior branch; 0.3a); the fold $r_f = R_s$, approached as $\alpha_{\text{ext}} \rightarrow 1^-$ and occupied by nothing (I-6); relaxed substrate far from every mass, $\alpha_{\text{ext}} \rightarrow 0^+$.
- **Charts not used.** Every closed pattern — a nucleus, the matter of a star, the skin of a black hole — is read on its own interior chart, and no transfer map between that chart and the exterior atlas is displayed or used (I-3). On the exterior atlas of a galaxy its stars and its envelope alike read $\alpha_{\text{ext}} \approx 10^{-6}$; the phase of a constituent is fixed by what it is, closed braid or substrate, never by that reading. No β (A-2) and no A-3, A-4 or A-5 token appears.
- **Density objects** (0.6, $\zeta = 1$). The Schwarzschild scale $\rho_s = \mathcal{M}/V(R_s)$; the configuration mean $\bar{\rho}(R_b)$; the enclosed mean $\bar{\rho}(r_f)$; the local density $\rho_{\text{loc}}(r_f)$, which needs $\mathcal{M}(r_f)$.
- **Layers** (I-4). Lemmas 1–3 are exact at monopole order. The circular-speed and hydrostatic readings (Sections 6 and 7) are layer L2, weak field and slow motion, at leading order in α_{ext} ; the remainder is $O(\alpha_{\text{ext}}) \approx 10^{-6}$.

1.2 Record accepted, reading declined

FDT accepts the record of general relativity and quantum mechanics in full: the rotation curves of disk galaxies [2, 3, 4] and of the Milky Way [5, 6]; the tracer and satellite weighings of the Galaxy [7, 8]; galaxy–galaxy lensing [11]; the local dark density [6, 13]; the planetary ephemerides [14]; the microlensing and wide-binary censuses [15, 16, 17, 18]; the laboratory neutrino-mass ceiling [22]; and the null results of direct detection [23].

The standard account reads the excess weight of galaxies as an undiscovered particle species. FDT reads it as the omnium's spacetime phase held compressed around mass, with no particle content (0.7; I-2, I-5: the only patterns are strokes of the one carrier, open or closed into braids); the measured excess is accepted in full and recovered in Section 11.

The law is untouched. $F = F_{\text{max}}\alpha_1\alpha_2$ holds unconditioned at every scale (I-1); what the record corrects is the input. The $\mathcal{M}$ of a galaxy includes its envelope (0.5).

2 The chain and the syllogism

The founding statement is one identity read four ways (0.1):

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0\mu_0} = \left(\frac{d}{t}\right)^2. \quad (1)$$

Quantum mechanics reads E/m , electromagnetism reads $1/(\epsilon_0\mu_0)$, general relativity reads $(d/t)^2$. The coefficients ϵ_0 and μ_0 are response coefficients of the omnium, so c^2 is a property of the medium. Read mechanically, it is the medium's stiffness per unit inertia, the ratio that sets the speed of an elastic stroke — a string's tension over its linear density, a fluid's modulus over its density — with $1/\epsilon_0$ the stiffness and μ_0 the inertia, as in Maxwell's model of the field [31]; the medium's own stroke, the carrier of I-2, runs at c . Written $E = c^2m$, the founding statement has the form of Hooke's law $F = kx$: energy in the place of the load, mass in the place of the compression, c^2 in the place of the stiffness. The exact version, in which c^2 multiplies curvature, is Remark 4. The four terms adjust together as one state (0.1a). Matter, energy and spacetime are one substance.

The argument this paper makes exact runs in four steps.

- (i) **Gravity is curvature.** The field equation stands unchanged, $G_{\mu\nu} = \kappa T_{\mu\nu}$, and α is downstream of its solution (I-3). General relativity is the chain kept state by state: clocks and rulers are closed patterns that adjust their kept time and thread length so that d/t measured on the spot stays c , and that adjustment is the geometry (0.1a).
- (ii) **Mass is confinement.** A closed pattern's mass is the cost of its mode, $m_q c^2 = \hbar\omega_q$, the eigenvalue of its confinement problem; a tighter box is a more expensive mode (I-5).
- (iii) **Mass depends on curvature.** On the density cone, energy and density are maximal at the tip, where $\alpha_{\text{ext}} \rightarrow 1^-$, and minimal at the rim, where $\alpha_{\text{ext}} \rightarrow 0^+$ (I-3, cone orientation). The local density of an exterior rises with α_{ext} (0.6), and density is mass (I-7).
- (iv) **Therefore curved spacetime outweighs relaxed spacetime.** Spacetime held above its relaxed density carries weight that relaxed spacetime does not. Lemma 2 shows where that weight sits and how it shows in α_{ext} , Lemma 3 what compression adds, and Lemma 1 that the reading knows nothing of what made it. Theorem 1 identifies the weight the record measures.

Proposition 1 (How each reading sees compressed spacetime). *Let a region hold spacetime-phase omnium above its relaxed density, with no closed pattern in it.*

- (a) *The $(d/t)^2$ reading weighs it. Its content enters $\mathcal{M}$ once, and the orbits of test patterns around it answer to that $\mathcal{M}$.*
- (b) *The $1/(\epsilon_0\mu_0)$ reading sees it as refraction. A ray passing outside the whole configuration follows the exact exterior optical metric of I-3 and bends by $\theta = 4GM/(bc^2)$ at leading order for impact parameter b . A ray crossing the region bends by the weight projected inside b , $\theta = 4GM_{2D}(< b)/(bc^2)$ at leading order.*
- (c) *The E/m reading, which for a pattern q is $E_q = m_q c^2 = \hbar\omega_q$ (0.1a), registers nothing. The region has no line to emit or absorb.*

Proof. (a) is 0.5 with I-3 containment: for the same source and data FDT returns the curvature of general relativity, and the weight of every channel sits in the one $\mathcal{M}$. (b) is the optical statement of I-3. The deflection outside the configuration is the leading term of the exact optical metric, the same term that returns $1.75''$ at the solar limb; across the region, the same leading order sums the weight projected inside b . (c) A line is a transition between confinement eigenvalues of a closed pattern, whose energy, mass, kept time and thread length are fixed together by one stroke (0.1a, I-5). The spacetime phase is decompressed substrate, not closed braids (0.3a): it has no confinement problem of its own and no mode to relax. $\square$

Remark 1 (Transparent, not dark). A transparent medium whose density varies slowly gives sight nothing to catch on. What it offers is a gradient. The envelope is found by its gradients alone — a

lensed arc, a rotation curve — in the way the refraction of the air is found by the lift of the setting Sun (Section 7).

3 Lemma 1: the reading does not know what made it

Lemma 1 (α is blind to arrangement). *Let a configuration be spherical about its centre, and let $\mathcal{M}(r_f)$ be the total gravitating energy inside r_f , every channel counted once (0.5). At monopole order,*

$$\alpha_{\text{ext}}(r_f) = \frac{2G\mathcal{M}(r_f)}{c^2 r_f} \quad (2)$$

for every support, composition, phase and location in the universe. Any two configurations that hold equal $\mathcal{M}$ inside r_f read identically at r_f . The Schwarzschild density scale of a given $\mathcal{M}$,

$$\rho_s = \frac{\mathcal{M}}{\frac{4}{3}\pi R_s^3} = \frac{3c^6}{32\pi G^3 \mathcal{M}^2}, \quad (3)$$

depends on $\mathcal{M}$ alone.

Proof. By 0.3, α_{ext} is the ratio of two curvatures fixed by two lengths, R_s and r_f . $R_s = 2GM/c^2$ carries every form of energy exactly once (0.2, 0.5), so the semantic type of α does not change with the source (0.3, universality). By I-3 containment, a spherical configuration solves the same field equation with the same source data as in general relativity. At monopole order its field at r_f is set by the weight inside r_f , the shells outside exert no net pull, and outside all of its content the exterior solution is fixed by $\mathcal{M}$ alone. Neither expression contains a position: the chain and α are ratios and select no size and no place (0.3, 0.3a). Equation (3) is 0.6 at $\zeta = 1$. $\square$

Corollary 1 (Equal readings, equal weight). *At one field radius, the ratio of two exterior readings is the ratio of the enclosed weights, $\alpha_{\text{ext}}(r_f)/\alpha'_{\text{ext}}(r_f) = \mathcal{M}(r_f)/\mathcal{M}'(r_f)$, whatever holds either.*

Corollary 2 (The one-body mass shadow is blind to arrangement). *The mass shadow of 0.7 is the exterior compressed-omnium response that every nonzero source carries. The exterior a single spherical source presents is fixed by its own weight $\mathcal{M}_{\text{src}}$ and by r_f (Lemma 1), so at monopole order its one-body shadow depends on nothing else. Any two single spherical sources of one $\mathcal{M}_{\text{src}}$ — a compact star and a black hole, say — cast the same one-body shadow at every r_f beyond both supports. How the shadow's own weight enters the exterior, and how the shadows of a population combine, is the closure of the response function (Section 10); every channel still enters $\mathcal{M}$ once (0.5).*

Table 1 takes four configurations whose own weight is that of Sgr A*, $\mathcal{M} = 4.30 \times 10^6 M_\odot$ [12]: the black hole itself, a star cluster of radius 1 pc, a sphere of 300 pc and a sphere of 3 kpc of any composition and phase. Their compressions, read on the support as $\bar{\rho}(R_b)/\rho_s = \alpha_{\text{ext}}(R_b)^3$ (0.6), span thirty orders of magnitude. At 10 kpc their own weight reads $\alpha_{\text{ext}} = 4.12 \times 10^{-11}$ for every one of them. Whatever shadow each carries adds to the weight inside 10 kpc, and Lemma 1 reads that total with the same indifference to how it is held. Figure 1(a) draws the exterior atlas of the casters: one line, with a different starting point for each support.

Compression is measured on the support; weight is read outside it. The same holds for the whole Galaxy. The satellite population weighs $1.43 \times 10^{12} M_\odot$ inside 273 kpc [8], so $\alpha_{\text{ext}}(273 \text{ kpc}) = 5.01 \times 10^{-7}$. Any configuration that holds $1.43 \times 10^{12} M_\odot$ inside 273 kpc reads the same value there: stars and envelope, as the Galaxy holds it, or a black hole and its shadow holding it together. A hole holding all of it alone has its fold at $R_s = 0.137 \text{ pc}$, where $\rho_s = 9.0 \times 10^{-6} \text{ kg m}^{-3}$. The satellites read the weight inside their orbits and nothing else.

Table 1: Four configurations whose own weight is that of Sgr A*, $\mathcal{M} = 4.30 \times 10^6 M_\odot$: $R_s = 1.270 \times 10^{10} \text{ m} = 0.085 \text{ AU}$, seam (A-1, exterior branch) at $2R_s = 0.17 \text{ AU}$, $\rho_s = 9.97 \times 10^5 \text{ kg m}^{-3}$. The support columns read the compression of each configuration; the last column reads its weight.

configuration	support R_b	$\alpha_{\text{ext}}(R_b)$	$\alpha_{\text{ext}}(R_b)^3 = \bar{\rho}(R_b)/\rho_s$	$\bar{\rho}(R_b)$ [kg m ⁻³]	$\alpha_{\text{ext}}(10 \text{ kpc})$
black hole	$R_b \rightarrow R_s^+$	$\rightarrow 1^-$	$\rightarrow 1^-$	$\rightarrow 9.97 \times 10^5$	4.12×10^{-11}
star cluster	1 pc	4.12×10^{-7}	6.97×10^{-20}	6.95×10^{-14}	4.12×10^{-11}
dwarf-sized sphere	300 pc	1.37×10^{-9}	2.58×10^{-27}	2.57×10^{-21}	4.12×10^{-11}
diffuse sphere, any phase	3 kpc	1.37×10^{-10}	2.58×10^{-30}	2.57×10^{-24}	4.12×10^{-11}

Remark 2 (Two comparisons, one bookkeeping). Lemma 1 compares configurations of equal $\mathcal{M}$ and finds equal exterior readings, however compressed their supports. Lemma 3 compares one configuration before and after work is stored in it and finds its $\mathcal{M}$ raised by that work. Both are 0.5: the exterior reads $\mathcal{M}$, all of it, once. A body that collapses toward its fold radiates as it goes, and the solved mass aspect carries the difference; a spring compressed by an outside hand keeps what the hand put in.

4 Lemma 2: curvature is weight

Lemma 2 (Curvature is weight). *At monopole order, at every field radius,*

$$\mathcal{M}(r_f) = \frac{c^2 r_f \alpha_{\text{ext}}(r_f)}{2G}, \quad \bar{\rho}(r_f) = \frac{3c^2 \alpha_{\text{ext}}(r_f)}{8\pi G r_f^2} = \rho_s [\mathcal{M}(r_f)] \alpha_{\text{ext}}(r_f)^3, \quad (4)$$

$$\rho_{\text{loc}}(r_f) = \frac{c^2}{8\pi G r_f^2} \frac{d(r_f \alpha_{\text{ext}}(r_f))}{dr_f}. \quad (5)$$

Proof. Equation (4) inverts $\alpha_{\text{ext}} = 2G\mathcal{M}(r_f)/(c^2 r_f)$ and divides by $\frac{4}{3}\pi r_f^3$; its last form is the 0.6 identity $\bar{\rho}/\rho_s = \alpha_{\text{ext}}^3$. Equation (5) differentiates (4) with $d\mathcal{M}/dr_f = 4\pi r_f^2 \rho_{\text{loc}}$. $\square$

Equation (4) says that at a fixed size, curvature and weight are one quantity: double α_{ext} at r_f and the weight inside doubles. Equation (5) says where the weight sits. An exterior in which $r_f \alpha_{\text{ext}}$ holds constant — the falloff R_s/r_f of a caster alone — carries no local weight at monopole order. Weight sits wherever α_{ext} falls more slowly than $1/r_f$. Sections 0.6 and 0.7 place the mass shadow in every exterior, and Lemma 2 says how it shows: as $r_f \alpha_{\text{ext}}$ growing outward. Around the Sun that growth lies below the resolution of the planetary ephemerides (Section 8); around galaxies it amounts to factors of several (Section 6). An envelope of constant α_{ext} carries

$$\rho_{\text{loc}}(r_f) = \frac{c^2 \alpha_{\text{ext}}}{8\pi G r_f^2}. \quad (6)$$

Remark 3 (The weight is not the matter's field energy). The energy of the matter phase's own curvature is booked inside the solved mass aspect, enters $\mathcal{M}$ once and lowers it (0.5; I-7: binding is read from the geometry, never added as a local term). For the Milky Way's matter phase the reduction is of order α_{ext} across the disk, about 2×10^{-6} of that phase. The envelope outweighs the matter phase several times over (Table 2). Its weight is its own omnium.

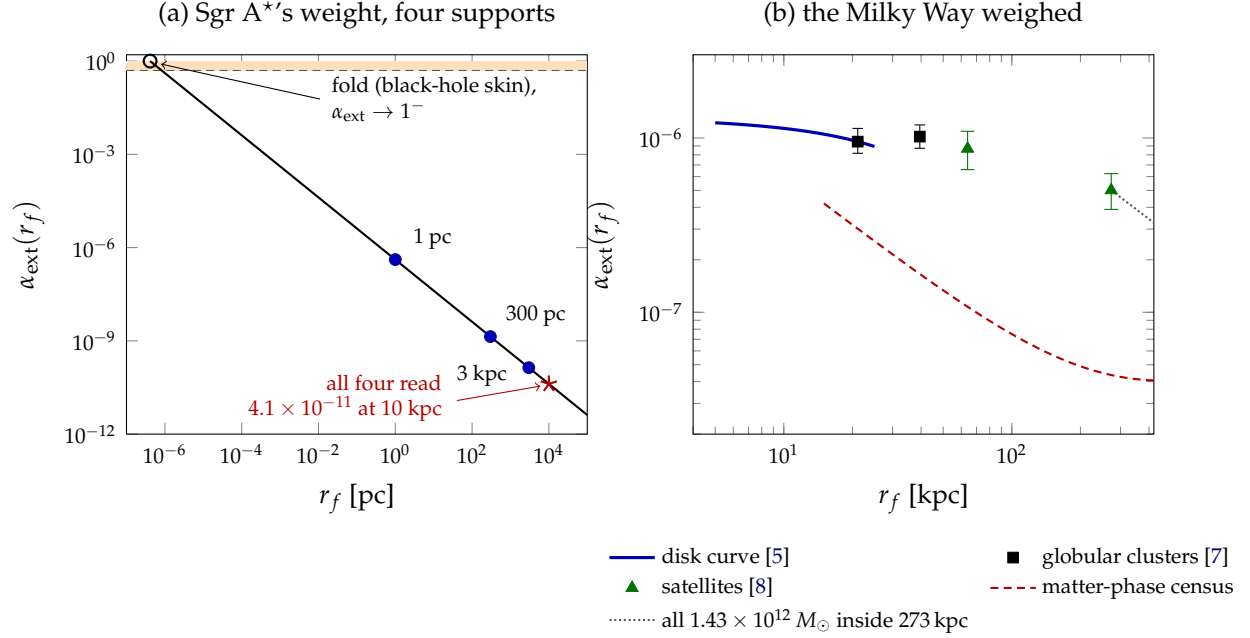


Figure 1: (a) The exterior atlas of Sgr A*'s weight, $\alpha_{\text{ext}} = R_s/r_f$. One line serves every support; each configuration of Table 1 enters it at its own support radius, and at 10 kpc all read the same value. The shaded band is the quantum-mechanical regime, $1/2 < \alpha_{\text{ext}} < 1$, and the dashed line the seam $\alpha_{\text{ext}} = 1/2$ (A-1, exterior branch; 0.3a); the line enters the band only between the fold and $2R_s$. Interiors are read on their own charts and are not drawn (I-3). (b) The Milky Way's exterior readings against the reading of its matter-phase census (Table 2), drawn from 15 kpc, inside which the census is not wholly enclosed. The gap is the envelope's weight. Beyond 273 kpc the dotted line is the reading of any configuration with all $1.43 \times 10^{12} M_\odot$ inside 273 kpc, a black hole included (Lemma 1).

Remark 4 (Curvature is a spring). Two free test patterns a small distance ξ apart accelerate relative to each other as

$$\frac{d^2 \xi^i}{dt^2} = -K^i_j \xi^j, \quad K^i_j = c^2 R^i_{0j0} \quad (x^0 = ct), \quad (7)$$

the geodesic-deviation equation [32]. It is Hooke's law, with a spring constant per unit mass equal to c^2 times the curvature. At monopole order in the weak field the spring reads the exterior coordinate directly: across the radius $K_\perp = c^2 \alpha_{\text{ext}} / (2r_f^2)$, along it $K_r = \frac{1}{2} c^2 (\alpha'_{\text{ext}} / r_f - \alpha_{\text{ext}} / r_f^2)$ with $\alpha'_{\text{ext}} = d\alpha_{\text{ext}} / dr_f$, and the trace is

$$K_r + 2K_\perp = \frac{c^2}{2r_f^2} \frac{d(r_f \alpha_{\text{ext}})}{dr_f} = 4\pi G \rho_{\text{loc}}, \quad (8)$$

which is (5). A caeter alone, $\alpha_{\text{ext}} = R_s/r_f$, squeezes across and stretches along with zero trace: a small ball of test patterns at rest changes shape and keeps its volume. In the envelope the trace is positive and the ball shrinks, $\ddot{V}/V = -4\pi G \rho_{\text{loc}}$. On a flat curve $K_r = -K_\perp$ and the trace is $K_\perp = v_c^2 / r_f^2$, the square of the orbital frequency. Around the Sun the disk's stars ride this spring through the plane: their vertical frequency obeys $\nu^2 = 4\pi G \rho_{\text{mid}} + 2(A^2 - B^2)$, with ρ_{mid} the whole density in the plane and A, B the Oort constants (the axisymmetric Poisson equation, layer L2) [33, 34], and the local determinations of the dark density rest on that vertical restoring force [13]. The dark share of the trace at the Sun is $4\pi G \rho = 5.68 \times 10^{-31} \text{ s}^{-2}$ for $\rho = 0.38 \text{ GeV cm}^{-3}$, a period of 264 Myr; the slope of the disk curve [5] adds $2(A^2 - B^2) = 1.01 \times 10^{-31} \text{ s}^{-2}$.

5 Lemma 3: compression is weight

Lemma 3 (Compression is weight). *Work stored in a configuration raises its $\mathcal{M}$ by the work divided by c^2 . Omnium held compressed outweighs relaxed omnium in the same volume by its packing and by its stored work.*

Proof. By (1), $E/m = c^2$: stored energy is mass, and it enters $\mathcal{M}$ once (0.5). A spring of stiffness k compressed by x stores $kx^2/2$ and gains $\Delta m = kx^2/(2c^2)$; for $k = 10^4 \text{ N m}^{-1}$ and $x = 0.10 \text{ m}$ that is 50 J and $5.6 \times 10^{-16} \text{ kg}$. For the omnium, I-7 adds a channel the spring does not have: compression raises the density of the one substance, and density is mass (mass $\equiv$ energy $\equiv$ temperature $\equiv$ pressure $\rightarrow$ density). The stored compression work enters as PV/c^2 (0.5, I-7). $\square$

In the galactic envelope the packing carries the weight. The envelope's stored work is

$$\frac{1}{c^2} \int p dV = \frac{c_s^2}{c^2} \mathcal{M}_{\text{env}} = \frac{\alpha_{\text{ext}}}{4} \mathcal{M}_{\text{env}} \approx 3 \times 10^{-7} \mathcal{M}_{\text{env}} \quad (9)$$

(Proposition 2). The spring's channel is present and small, because the envelope is soft: its stiffness per unit inertia, c_s^2 , is $\alpha_{\text{ext}}/4$ of the medium's c^2 (Section 2).

Held and released. The compression held around mass is weighed by orbits and lenses; that is the dark matter of the record. The compression let go is the dark energy of 0.7: the stored potential released as bodies lose mass. They are one reservoir in two conditions, moving in opposite directions along the compression axis, which is the α axis (0.3a, 0.7). The spring of Remark 4 reads both conditions. Its trace carries pressure beside density: a small ball of free test patterns at rest changes volume as $\dot{V}/V = -4\pi G(\rho + 3p/c^2)$ [35]. Held compression is pressureless to three parts in 10^7 , its pressure $c_s^2 \rho_{\text{loc}}$ with $c_s^2/c^2 = \alpha_{\text{ext},0}/4$ (Proposition 2), and it squeezes the ball. Released potential enters the field equation as $\Lambda_{\text{eff}} = 8\pi G \rho_{\text{rel}}/c^2$ (0.7), a term proportional to the metric, which is the one stress that selects no rest frame: $p_{\text{rel}} = -\rho_{\text{rel}} c^2$, a tension equal to its energy density. Then $\rho + 3p/c^2 = -2\rho_{\text{rel}}$ and the ball swells; in the Newtonian limit each test pattern feels $+\Lambda_{\text{eff}} c^2 r_f/3$, Hooke's law with its sign reversed [36]. A compressed spring pushes on the hand that holds it and adds to its own weight; only tension, $p < -\rho c^2/3$, reverses the squeeze. At the radii of Table 2 the reversed spring's share of the Galaxy's pull, $\Lambda c^2 r_f^3/(3G\mathcal{M}(r_f))$ with the measured $\Lambda = 1.11 \times 10^{-52} \text{ m}^{-2}$ [37], runs from 3.3×10^{-5} at 21.1 kpc to 1.0×10^{-2} at 273 kpc, inside every error bar: the table weighs the held reservoir. The bill comes with it. As bodies unwind, the held share falls and the released share grows, and Λ_{eff} grows with cumulative unwinding (0.7). A dark-energy history measured constant, or shrinking, ends this reading.

6 The theorem, proved

Proof of Theorem 1. Step 1: the weighing. A test pattern on a circular orbit of radius r_f inside a galaxy sits within the configuration's support, so the I-1 closed form, whose domain needs disjoint supports, is not the evaluation there. By I-3 containment the spherical solution fixes the test pattern's geodesic, and the shells outside r_f exert no monopole pull. At leading order in α_{ext} (I-4, layer L2),

$$v_c^2 = \frac{G\mathcal{M}(r_f)}{r_f} = \frac{c^2 \alpha_{\text{ext}}(r_f)}{2}, \quad \alpha_{\text{ext}}^{\text{obs}}(r_f) \equiv \frac{2v_c^2}{c^2}, \quad (10)$$

Table 2: The Milky Way weighed. $\alpha_{\text{ext}}^{\text{obs}} = 2G\mathcal{M}(r_f)/(c^2 r_f)$ from the published enclosed weights; $\alpha_{\text{ext}}^{\text{mat}}$ is the same evaluation of the matter-phase census: stars $5.43 \times 10^{10} M_\odot$ [6]; cold gas $0.80 \times 10^{10} M_\odot$ [9], raised by 1.36 for helium and counted in full at every radius; and hot gas at the upper end of [10], $5.2 \times 10^{10} M_\odot (r_f/250 \text{ kpc})^{3/2}$, the growth its two published masses imply. By Corollary 1, ratio = $\alpha_{\text{ext}}^{\text{obs}}/\alpha_{\text{ext}}^{\text{mat}}$ is the weight over the census. Floor is half the lower one-sigma weight over the census: the reading if every tracer sat at its escape speed (Section 8).

r_f [kpc]	tracer	$\mathcal{M}(r_f)$ [$10^{12} M_\odot$]	$\alpha_{\text{ext}}^{\text{obs}}$	census [$10^{10} M_\odot$]	$\alpha_{\text{ext}}^{\text{mat}}$	ratio	floor
21.1	globular clusters [7]	$0.21^{+0.04}_{-0.03}$	9.5×10^{-7}	6.65	3.0×10^{-7}	3.2	1.4
39.5	globular clusters [7]	$0.42^{+0.07}_{-0.06}$	1.02×10^{-6}	6.84	1.7×10^{-7}	6.1	2.6
64	satellites [8]	$0.58^{+0.15}_{-0.14}$	8.7×10^{-7}	7.19	1.1×10^{-7}	8.1	3.1
273	satellites [8]	$1.43^{+0.35}_{-0.32}$	5.0×10^{-7}	12.5	4.4×10^{-8}	11.5	4.5

the circular-speed reading of the enclosed weight; Newton’s $M(< r_f) = v_c^2 r_f / G$ is its face in I-4. Lensing reads the same weight with no orbit at all (Proposition 1b), and galaxy–galaxy lensing returns the excess [11]. Table 2 collects the Milky Way’s record. At the Sun $\alpha_{\text{ext}}^{\text{obs}} = 1.21 \times 10^{-6}$ for $v_0 = 232.8 \text{ km s}^{-1}$ [6]. Across 5–25 kpc the disk curve declines gently from 234 to 200 km s^{-1} [5], so $\alpha_{\text{ext}}^{\text{obs}}$ runs from 1.22×10^{-6} to 0.89×10^{-6} . Globular-cluster and satellite tracers carry the reading to 273 kpc [7, 8]. The matter-phase census counts stars, cold gas with its helium, and the hot halo at the upper end of its measured range, each as if wholly inside r_f wherever that raises the count. The weight stands at 3.2, 6.1, 8.1 and 11.5 times the census at 21.1, 39.5, 64 and 273 kpc. The hot halo accounts for at most half of the Galaxy’s missing baryons [10]; with it raised to $1.5 \times 10^{11} M_\odot$ inside 250 kpc, the weight still stands at 5.6 and 6.1 times the census inside 39.5 and 273 kpc. The orbit-shape freedom of the tracers (Section 8) lowers no ratio of Table 2 below 1.4, 2.6, 3.1 and 4.5, and neither raised-halo floor below 2.3.

Step 2: the excess is $\mathcal{M}$. By Lemma 1 and Corollary 1 the excess reading is excess $\mathcal{M}$, whatever holds it: α_{ext} counts every channel once and knows nothing of arrangement, composition or phase.

Step 3: the phase census. By 0.1b there is only omnium and its patterns. By I-2 and I-5 the strokes of the one carrier run open, as radiation, or close into braids, the matter phase (0.3a). The rest is substrate, the spacetime phase, with its own static gradients (P-5). The content of any region is exhausted by these three.

- (i) *Open strokes: radiation.* The microwave background carries $aT^4/c^2 = 4.6 \times 10^{-31} \text{ kg m}^{-3}$, 6.9×10^{-10} of the local dark density, and starlight near the Sun is of the same order.
- (ii) *Closed braids: the matter phase.* Luminous and absorbing matter is the census of Step 1. Higher rungs of the n -ladder stay closed only at higher density, with footing only in the skin terrain of a bound configuration (I-5), so the diffuse closures outside bound configurations are $n = 3$ and $n = 2$. The $n = 3$ closures are gas; the census counts it, and Step 1 bounds what it misses with the raised halo. The $n = 2$ closures (neutrinos) fail on phase space. Holding the local dark density, 0.38 GeV cm^{-3} , at the envelope’s dispersion $c_s = 165 \text{ km s}^{-1}$ needs closures of at least 19.3 eV each, or 14.7 eV with three species sharing the load (the Pauli phase-space bound of [21]). The laboratory ceiling is 0.45 eV [22], at which three species hold at most 9×10^{-7} of that density. Compact closed configurations are excluded as the bulk of the halo from 10^{-11} to $10^{-6} M_\odot$ by microlensing toward M31 [16], from 6×10^{-8} to $15 M_\odot$ by microlensing toward the Magellanic Clouds [15], and from 1.3×10^{-5} to $860 M_\odot$ by twenty years of microlensing toward the Large Magellanic Cloud, at no more than a tenth of the halo [17]. The record leaves two windows, below $10^{-11} M_\odot$ and above $860 M_\odot$, where the wide-binary limit [18] is

contested [19]. Their census is D-01, this paper's measurement item (Section 10).

- (iii) *Substrate: the spacetime phase.* Its helical gradients, the Galaxy's magnetic field of a few microgauss near the Sun [24] (P-5), weigh about 10^{-9} of the local dark density. Outside the windows of D-01, what remains of the excess is substrate.

Step 4: heavier, denser, curved. The remainder is positive $\mathcal{M}$ per unit volume carried by substrate, which is substrate above its relaxed density (I-7: density is mass). It is heavier than relaxed substrate by Lemma 3. It is denser by Lemma 2: where relaxed substrate carries no local weight at monopole order, the envelope carries $\rho_{\text{loc}} > 0$. At the Sun the measured local dark density is $0.38 \pm 0.04 \text{ GeV cm}^{-3}$ [6]; for scale, that is $(6.8\text{--}7.9) \times 10^4$ times the critical density for any Hubble constant between 67.4 and $73 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the mean inside 273 kpc is 110 to 130 times it. It is curved by its own weight: by Lemma 2 the excess $\mathcal{M}(r_f)$ sets α_{ext} at every larger radius. The census, not the reading, fixes its phase. On the exterior atlas every constituent of the Galaxy reads $\alpha_{\text{ext}} \approx 10^{-6}$, more than 4×10^5 times below the seam $\alpha_{\text{ext}} = 1/2$ (A-1, exterior branch), and the envelope, holding no closed pattern, has no chart of its own on which to read otherwise (0.3a, I-3). $\square$

Corollary 3 (What the envelope shows). *The envelope is weighed by orbits, seen by lensing, and silent in emission and absorption (Proposition 1). Every detection of it is a gradient detection.*

Remark 5 (Carried by its source). The envelope is the mass shadow of 0.7, which the core posits is carried by its source and moves with it.

7 The galaxy's atmosphere

On Earth the envelope's counterpart is the atmosphere. The air is weighed: Torricelli's column of mercury, 760 mm tall, presses $\rho_{\text{Hg}} g h = 101,325 \text{ Pa}$, the weight of $10,332 \text{ kg}$ of air over every square metre, and the whole atmosphere is $5.148 \times 10^{18} \text{ kg}$ [29], 8.6×10^{-7} of the Earth's mass. Its refraction is seen only through its gradient: near the horizon the gradient lifts the Sun by about $34'$, more than the Sun's own diameter, so the whole disc is still in view after it has set geometrically. The air is bound to the Earth and moves with it. In the isothermal approximation its profile follows from two equations, hydrostatic balance and an isothermal law, $dp/dz = -\rho g$ and $p = \rho R_{\text{gas}} T / M_{\text{air}}$, which give $\rho \propto e^{-z/H}$ with $H = R_{\text{gas}} T / (M_{\text{air}} g) = 8.43 \text{ km}$ at 288 K .

The galaxy's envelope answers the same way. It is weighed by orbits, seen only through lensing, and carried by its galaxy. Read from its flat curve, it obeys the same two equations.

Proposition 2 (A flat curve is an isothermal atmosphere). *Let a spherical envelope carry $\alpha_{\text{ext}}(r_f) = \alpha_{\text{ext},0}$, constant over an interval of r_f , and let it stand in hydrostatic balance under an isotropic barotropic pressure $p = p(\rho_{\text{loc}})$, at leading order in α_{ext} . Then $dp/d\rho_{\text{loc}} = c^2 \alpha_{\text{ext},0}/4$ throughout the interval: the envelope has constant sound speed,*

$$c_s^2 = \frac{c^2 \alpha_{\text{ext},0}}{4} = \frac{v_c^2}{2}, \quad (11)$$

and its pressure is $p = c_s^2 \rho_{\text{loc}}$ up to an additive constant. It is isothermal.

Proof. By Lemma 2, $\mathcal{M}(r_f) = c^2 \alpha_{\text{ext},0} r_f / (2G)$ and $\rho_{\text{loc}} = c^2 \alpha_{\text{ext},0} / (8\pi G r_f^2)$. The weight per unit volume is $\rho_{\text{loc}} G \mathcal{M}(r_f) / r_f^2 = \rho_{\text{loc}} c^2 \alpha_{\text{ext},0} / (2r_f)$, so balance requires $dp/dr_f = -\rho_{\text{loc}} c^2 \alpha_{\text{ext},0} / (2r_f)$. With $d\rho_{\text{loc}}/dr_f = -2\rho_{\text{loc}}/r_f$ this is $(dp/d\rho_{\text{loc}})(-2\rho_{\text{loc}}/r_f) = -\rho_{\text{loc}} c^2 \alpha_{\text{ext},0} / (2r_f)$, hence $dp/d\rho_{\text{loc}} = c^2 \alpha_{\text{ext},0} / 4$ at every density the interval spans. $\square$

Read as circular orbits, a flat rotation curve says that α_{ext} holds constant, so the enclosed weight grows in step with radius. Where the envelope carries that weight, Proposition 2 says what kind of envelope does so in balance: an isothermal atmosphere. It obeys the same two equations as the air. The one difference is whose weight holds the envelope down: the planet's, for the air, whose self-weight is negligible; its own, beyond the disk, for the galaxy's envelope. For the Milky Way, $v_0 = 232.8 \text{ km s}^{-1}$ [6] on the flat part of the curve gives $\alpha_{\text{ext},0} = 1.21 \times 10^{-6}$ and $c_s = 165 \text{ km s}^{-1}$, the value for an envelope whose density falls as r_f^{-2} . An isothermal envelope carrying all of v_0^2 at the Sun reads $v_0^2 / (4\pi G R_0^2) = 0.57 \text{ GeV cm}^{-3}$ there by (6). The measured local dark density is 0.67 of that [6], within the local and global ranges 0.4–0.6 and 0.3–0.5 GeV cm^{-3} [13], and the disk and bulge carry the rest of v_0^2 at the Sun.

Hydrostatics ties the envelope's distribution to its temperature. The temperature, the distribution and the total belong to the response function (Section 10). The mechanism transfers across scales and the number does not: the air is 8.6×10^{-7} of the Earth's mass, while the Galaxy weighs 6.1 times its matter-phase census inside 39.5 kpc. The analogy also stops at the phase. The air is matter phase — closed braids, nitrogen and oxygen — which is why it has lines and scatters sunlight. The galaxy's envelope is substrate in the spacetime phase: it has no line, and it bends light. What the analogy carries is the hydrostatic law, the transparency and the weighing, and that matter and spacetime manifest at every scale (0.3a).

8 Satellites, comets and the snapshot

The Sun holds its comets with its own weight. The Oort cloud reaches about 10^5 AU [30]. There the Sun reads $\alpha_{\text{ext}} = 2.0 \times 10^{-13}$ and a circular orbit moves at 94 m s^{-1} . Long-period comets fall inward on steep orbits and turn at perihelion within the record, because a perihelion passage lasts months even when the orbit takes a million years or more. Their orbits answer to the Sun's $\mathcal{M}$. The planetary ephemerides fix that $\mathcal{M}$ as the Sun's matter phase to within $1.7 \times 10^{-10} M_\odot$ inside Saturn's orbit, and bound any additional density at Saturn's distance below $1.1 \times 10^{-20} \text{ g cm}^{-3}$ [14]. The Galaxy's envelope passes through the Solar System at 6.2×10^{-5} of that density bound, and adds about $5 \times 10^{-3} M_\odot$ inside 10^5 AU .

The Galaxy holds its satellites with its envelope. The Large Magellanic Cloud, at 49.6 kpc [25], moves at $321 \pm 24 \text{ km s}^{-1}$ [26]. Its orbital history is debated, first passage or bound, so its speed weighs the Galaxy only if it is bound; Leo I carries the test. Leo I, at $254_{-19}^{+16} \text{ kpc}$ [27], moves at 196 km s^{-1} ($V_{\text{rad}} = 167.9$, $V_{\text{tan}} = 101.0 \text{ km s}^{-1}$) and is bound for plausible Galaxy masses [28]. With the whole census, $1.17 \times 10^{11} M_\odot$, placed inside each radius, the escape speed it supplies is 143 km s^{-1} at the Cloud and 63 km s^{-1} at Leo I: short by a factor of 3.1 at Leo I, and by 2.3 at the Cloud if it is bound. The satellite population weighs the Galaxy at $1.43 \times 10^{12} M_\odot$ inside 273 kpc (Table 2). The law is the same at both scales, $F = F_{\text{max}} \alpha_1 \alpha_2$ (I-1). The inputs differ: inside Saturn's orbit the Sun's $\mathcal{M}$ is its matter phase to within the ephemeris bound, while at the satellites the Galaxy's $\mathcal{M}$ is mostly envelope.

The snapshot. We see comets turn. We do not see the outer orbits of a galaxy turn. At Rubin and Ford's outer radii in Andromeda, about 24 kpc at 250 km s^{-1} , a lap takes about 590 Myr, and the fifty-six years since their measurements [2] are 9.5×10^{-8} of it, three seconds out of a year; at the Sun's radius a lap takes 216 Myr. Every kinematic datum is one instant of an orbit; a Doppler datum is also one component of one velocity, and circularity is supplied by the analyst. For a disk

that supply is well founded. Gas on crossing orbits collides and dissipates, and it settles onto near-circular orbits; the disk curves of external galaxies [3, 4] rest on it. The Milky Way’s disk curve rests on its stars: more than 23 000 luminous red giants with APOGEE spectra and Gaia DR2 astrometry, whose measured mean velocities and velocity dispersions enter the Jeans equation for an axisymmetric disk, so the circular speed is derived from their velocity field rather than supplied [5]. The orbit-shape freedom lives with halo stars, satellites and dwarf spheroidals, where orbit shape and enclosed weight trade against each other. A steep ellipse is an illustration of what a snapshot permits, never a claim about what the orbits are. The freedom widens the bars in both directions. For one tracer at a fixed observed speed and radius, merely staying bound needs half the enclosed weight a circle needs, since $v_{\text{esc}} = \sqrt{2} v_{\text{circ}}$. A body caught near apocentre needs more, since there $v^2 = G\mathcal{M}(1 - e)/r_f$. A line-of-sight speed is a fraction of the true one. Reading every tracer at its escape speed and taking the lower one-sigma weight, the Galaxy still weighs 2.6 times its census inside 39.5 kpc and 4.5 times inside 273 kpc (Table 2). Lensing weighs it with no orbit at all.

9 Where the seven invariants enter

invariant	use in this proof
I-1 force law	one law, unconditioned, at the Oort cloud and at 273 kpc with different inputs; its closed form needs disjoint supports, so a tracer inside the envelope is read through I-3 and I-4
I-2 one carrier	the census of patterns: strokes of one carrier, open or closed into braids; no mediator and no species beside them (Proposition 1, Step 3)
I-3 α is the field equations	α downstream of the solution; containment fixes the monopole field by the weight inside r_f (Lemma 1); the exact optical metric weighs by lensing; geodesic deviation weighs by the tidal spring (Remark 4); cone orientation puts density and energy at the tip
I-4 Newton = GR = FDT	$v_c^2 = G\mathcal{M}(r_f)/r_f = c^2\alpha_{\text{ext}}/2$ at leading order, layer L2: one statement read in three limits
I-5 braids and confinement	matter phase = closed braids; mass = confinement eigenvalue; a line needs a closed pattern, so the envelope is transparent; higher rungs need higher density; the $n = 2$ closures fail the phase-space census
I-6 open interval	the black hole’s skin at $\alpha_{\text{ext}} \rightarrow 1^-$, relaxed substrate at $\alpha_{\text{ext}} \rightarrow 0^+$, the envelope at $\alpha_{\text{ext}} \approx 10^{-6}$ strictly inside
I-7 density equivalence	packing and PV/c^2 enter $\mathcal{M}$ once; compressed omnium is heavier (Lemma 3); binding is read from the geometry

10 Status ledger

status	items
derived here	Proposition 1; Lemma 1 with Corollaries 1 and 2; Lemma 2 with Remark 4; Lemma 3; Theorem 1, outside the windows of D-01; Proposition 2
premises posited in the core	the founding statement and its readings (0.1, 0.1a, 0.1b); the two phases (0.3a); the exterior density profile (0.6); the braid reading of matter (I-5); that the shadow is carried by its source (0.7)
read from the record	$\alpha_{\text{ext}}^{\text{obs}}$ at every radius of Table 2; the weight-to-census ratios, which are read off the sky and never derived; $c_s \approx 165 \text{ km s}^{-1}$ for the Milky Way’s envelope; the local-density comparison, an in-sample reading; the reversed spring’s share of the pull at the radii of Table 2
to be derived (O-14)	the mass-shadow response function and its closure: how much compression each mass holds, how it is distributed, how the shadows of a population combine, and the envelope’s temperature c_s^2 . It must return the weighings of Table 2, the ephemeris bound around the Sun and the local dark density. With it, the release function of 0.7, $m(x)$ and $\delta(x)$: until it is derived, the size of Λ_{eff} is not the theory’s number
to be derived (O-5)	the multipoles of a disk; this proof reads every configuration at monopole order
to be measured (D-01)	the census of compact matter-phase bodies below $10^{-11} M_{\odot}$ and above $860 M_{\odot}$, where the microlensing record ends and the wide-binary limit is contested; the OGLE limit itself is disputed in [20]

11 Convergence statement

For the same sources and data, FDT returns the consensus weighings. The enclosed weights from rotation curves and tracers are the I-4 face of the one law at leading order: $v_c^2 = c^2 \alpha_{\text{ext}} / 2 = GM(r_f)/r_f$. The lensing weights are the optical statement of I-3 with the envelope inside $\mathcal{M}$: $\theta = 4GM/(bc^2)$ outside the configuration and $4GM_{2D}(< b)/(bc^2)$ across it, at leading order. The Keplerian Sun of the ephemerides and the weighed atmosphere of the Earth are returned unchanged. No divergence is claimed from α , a chart or a branch tag. The declined reading is particle dark matter (Section 1.2).

One observable commitment follows from the ontology: the direct-detection rate of a dark-matter species is zero, because there is no species. The standard account reads the null results to date [23] as weak coupling. A prediction of nothing is the cheapest kind. What it costs FDT is its exit: a confirmed direct detection of the standard account’s dark-matter species ends this reading.

References

- [1] M. Alfaro, *Core FDT Invariants*, v5.4, author lock 2026-09-13.
- [2] V. C. Rubin and W. K. Ford, Jr., *ApJ* **159**, 379 (1970).
- [3] V. C. Rubin, W. K. Ford, Jr. and N. Thonnard, *ApJ* **238**, 471 (1980).
- [4] A. Bosma, *AJ* **86**, 1825 (1981).
- [5] A.-C. Eilers, D. W. Hogg, H.-W. Rix and M. K. Ness, *ApJ* **871**, 120 (2019).
- [6] P. J. McMillan, *MNRAS* **465**, 76 (2017).
- [7] L. L. Watkins, R. P. van der Marel, S. T. Sohn and N. W. Evans, *ApJ* **873**, 118 (2019).

- [8] T. K. Fritz, A. Di Cintio, G. Battaglia, C. Brook and S. Taibi, *MNRAS* **494**, 5178 (2020).
- [9] H. Nakanishi and Y. Sofue, *PASJ* **68**, 5 (2016).
- [10] M. J. Miller and J. N. Bregman, *ApJ* **800**, 14 (2015).
- [11] T. G. Brainerd, R. D. Blandford and I. Smail, *ApJ* **466**, 623 (1996).
- [12] GRAVITY Collaboration (R. Abuter et al.), *A&A* **657**, L12 (2022).
- [13] P. F. de Salas and A. Widmark, *Rep. Prog. Phys.* **84**, 104901 (2021).
- [14] N. P. Pitjev and E. V. Pitjeva, *Astron. Lett.* **39**, 141 (2013).
- [15] P. Tisserand et al. (EROS-2), *A&A* **469**, 387 (2007).
- [16] H. Niikura et al., *Nat. Astron.* **3**, 524 (2019).
- [17] P. Mróz et al., *Nature* **632**, 749 (2024).
- [18] J. Yoo, J. Chanamé and A. Gould, *ApJ* **601**, 311 (2004).
- [19] D. P. Quinn et al., *MNRAS* **396**, L11 (2009).
- [20] M. R. S. Hawkins and J. García-Bellido, *MNRAS* **544**, 1950 (2025).
- [21] S. Tremaine and J. E. Gunn, *Phys. Rev. Lett.* **42**, 407 (1979).
- [22] KATRIN Collaboration (M. Aker et al.), *Science* **388**, 180 (2025).
- [23] LZ Collaboration (J. Aalbers et al.), *Phys. Rev. Lett.* **135**, 011802 (2025).
- [24] J. L. Han, *ARA&A* **55**, 111 (2017).
- [25] G. Pietrzyński et al., *Nature* **567**, 200 (2019).
- [26] N. Kallivayalil, R. P. van der Marel, G. Besla, J. Anderson and C. Alcock, *ApJ* **764**, 161 (2013).
- [27] M. Bellazzini, N. Gennari, F. R. Ferraro and A. Sollima, *MNRAS* **354**, 708 (2004).
- [28] S. T. Sohn et al., *ApJ* **768**, 139 (2013).
- [29] K. E. Trenberth and L. Smith, *J. Climate* **18**, 864 (2005).
- [30] J. H. Oort, *Bull. Astron. Inst. Netherlands* **11**, 91 (1950).
- [31] J. C. Maxwell, *Phil. Mag.* **23**, 12 (1862).
- [32] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
- [33] J. H. Oort, *Bull. Astron. Inst. Netherlands* **6**, 249 (1932).
- [34] J. Binney and S. Tremaine, *Galactic Dynamics*, 2nd ed. (Princeton University Press, 2008).
- [35] J. C. Baez and E. F. Bunn, *Am. J. Phys.* **73**, 644 (2005).
- [36] G. W. Gibbons and C. E. Patricot, *Class. Quantum Grav.* **20**, 5225 (2003).
- [37] Planck Collaboration (N. Aghanim et al.), *A&A* **641**, A6 (2020).