

Electronic pockets at a missing clamp in the PdAl plate of Pd₅AlI₂

FDT analysis

Target: Morgan Thinel et al., “Electronic Bound States in the Continuum in a 2D Metal,”
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Abstract

Thinel et al. report two localized defect states in the metallic continuum of Pd₅AlI₂ [1]. Fundamental Density Theory (FDT) reads a bound state as a nodal pattern of one medium. This manuscript’s reading identifies the PdAl plane as a plate, its nuclei as clamps, and a Pd vacancy as a missing clamp: the electron settles into the eigenmode that this boundary offers. The measured weight occupies the lobes of that mode. An explicit reconstruction separates the compact circulations that survive removing a Pd site from the handed combinations that cross that site. It also separates the exact unsplit pair in the ideal model from the measured pair splitting. These are identity maps of the source model and data, with no independent corroboration of an invariant. Five predictions introduce additional inputs: three conditional tests of a magnetic response to static mechanical handedness, and two conditional isotope tests of a density sensitivity at fixed lattice and electronic parameters. The former compare neighbouring mirror-related pockets under field reversal; the latter compare assayed Pd isotope substitutions against an explicit conventional bound. The twist amplitudes and isotope sensitivities are not inferred from the existing spectra. An exploratory reading of the pair maps remains an in-sample interpretation.

1 Framework

The source status is experimental. Measured, observed and reported identify source results; derived, estimated, reconstructed and illustrative-assumed identify the corresponding operations here. SC identifiers refer to claim rows, L identifiers to physical source loci, and U identifiers to stated numerical assumptions. Thinel et al. [1] supply the spectra, maps and model inputs; the locked core [2], clauses 0.1–0.6, I-1–I-7 and P-2–P-5, supplies the physical reading. FDT is a true extension of general relativity and quantum mechanics. It returns their data on their own inputs and dictionary, while its additional constitutive inputs have separate experimental consequences.

1.1 One medium, closed patterns and open strokes

The medium is the omnium; matter, energy and spacetime are its readings. Let E denote energy, m rest mass, c the strand rate, d a thread length and t time. The electric and magnetic responses are ε_0 and μ_0 . Planck’s constants are h and $\hbar$, with $\hbar = h/(2\pi)$; π is the circle

constant. For a massive mode labelled q , $E_q, m_q, \omega_q, T_q, \ell_q$ denote its energy, mass, angular frequency, period and Compton length. The units used below are SI with electronvolt energy, eV, and its millielectronvolt multiple, meV; J, kg, m, s, K and T mean joule, kilogram, metre, second, kelvin and tesla. Core 0.1a and 0.1d give

$$E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2. \quad (1)$$

$$E_q = m_q c^2 = \hbar\omega_q, \quad T_q = 2\pi/\omega_q = h/E_q, \quad \ell_q = cT_q = h/(m_q c). \quad (2)$$

$$d/t = c = 299792458 \text{ m s}^{-1}. \quad (3)$$

The last value is exact (U-01); the bridges use U-02. Energy discreteness enters the rest masses of confined modes and the periods and paths of null content at the locked ratio. The Planck length ℓ_P and time t_P set the one-sided infima $2\ell_P$ and $2t_P$, not a spatial lattice. A continuous chart describes this discrete content.

For a closed pattern, v is translation speed and u circulated speed; $\mathfrak{s}$ is its translation share. Core 0.1d states

$$v^2 + u^2 = c^2, \quad \mathfrak{s} = v^2/c^2. \quad (4)$$

Time is circulated strand and space is translated strand. The inertia E/c^2 includes the stress of closure, as required by the von Laue balance. Only for a pattern held on the exterior branch does this kinematic share equal exterior compactness. An open stroke translates straight at c , with no circulated share and no kept time. Its grades turn on a helix: with wavelength λ , reduced wavelength $\bar{\lambda}$ and angular frequency ω , its radius is $\bar{\lambda} = c/\omega = \hbar c/E = \lambda/(2\pi)$ and its inclination is $\pi/4$. Energy travels along the straight stroke. For a closed pattern with spatial angular frequency k , the phase rate ω/k and run $d\omega/dk$ multiply to c^2 ; the former exceeds the latter. Their ratio is not a helix pitch. At fixed time the phase helix has the same geometrical inclination; the equality of the two rates belongs to the open stroke (I-2).

I-2 identifies photon, graviton, gluon, phonon and magnon as the one spin-one carrier in different windows. Its identity is “carrier-only”; the electron is a braided closure of I-5 and stays outside this list. I-4 reads a wavefunction as “the projection of a helix onto the plane normal to the cone axis.” Let ψ denote that projection, ϕ_h its cylindrical phase, and ϕ_0 its initial phase. Let $\mathbf{e}_1, \mathbf{e}_2$ be orthonormal transverse Clifford vectors and $\mathbf{B}$ their oriented bivector. The lift is

$$\phi_h(t) = \phi_0 - \omega t, \quad \mathbf{B} = \mathbf{e}_1 \mathbf{e}_2, \quad \mathbf{B}^2 = -1. \quad (5)$$

The single-angle phase rotor $\exp(\mathbf{B}\phi_h)$ has a dimensionless exponent; it is distinct from a half-angle spinor rotor. Ignorance of ϕ_0 at observation is superposition. The dynamics are deterministic. A relative phase changes a pattern; an unobserved global sign does not supply a second physical state. These are phase-axis statements at fixed field identity, without a strand count assigned to a mode.

1.2 Notation guard and chart inventory

The notation guard below fixes the source-to-framework dictionary before its symbols enter calculations. Orbital labels retain their conventional coordinate subscripts x, y, z ; i is the imaginary unit. The Pauli matrix is $\hat{Y}$, and the lattice divergence below is $\mathbf{C}$. Neither is a branch tag or the confinement operator.

Source notation	Manuscript notation	Meaning and reserved distinction
a	$\mathfrak{a}$	In-plane spacing; a is the radiation constant.
t, t_1, t_2	$\mathfrak{t}, \mathfrak{t}_1, \mathfrak{t}_2$	Hoppings; t is chain time or chart position.
t_{nnn}	η_{nnn}	Next-nearest scale, with its printed unit ambiguity.
U	$\mathfrak{u}$	Scalar local energy; U is pair energy.
m	$\mathfrak{h}$	Handed local energy; m is rest mass.
Pauli matrix	$\hat{\mathbf{Y}}$	Matrix on the Pd orbital pair; σ is branch tag.
Model wavefunction	Ψ	Reconstructed orbital amplitude; ψ is the core projection.
R	$\mathfrak{r}$	Distance from vacancy; core radii retain R .
Defect numerals	type i, ii, iii	Defect labels in words, distinct from i.
Imaginary unit in phase labels	i	Upright imaginary unit; italic body indices are distinct.
Figure letters	panel D, panel E	Panel labels, distinct from D and energy E .
E_F, E_f	E_{F}	Fermi reference energy.
Zone points	$\bar{\text{M}}, \bar{\Gamma}$	Brillouin-zone labels, not mass or a width.
IPR	IPR	Reciprocal fourth moment of a normalized state.
q_{Bragg}	$\mathfrak{q}_{\text{Bragg}}$	Bragg spatial frequency, not a mode label.
Sample bias	$\mathcal{V}$	Tip-sample bias; V is volume.
dI/dV	$d\mathcal{I}/d\mathcal{V}$	Conductance; $\mathcal{I}$ is tunnelling current.
Supplement labels	Fig. S1–S9	Labels; S below denotes logarithmic sensitivity.

The source loci of these renames are NG-01–NG-18. Radius R_s is the source scale, r_f a field-point radius, r_c the interior cutoff, and R_{room} a configuration’s measured interior extent. The branch tag σ is +1 outside and -1 inside; X is the gap odds, and α is always a curvature ratio. The chart inventory is

Application	Tokens	Warrant and use
A-1	$\alpha_\sigma, \alpha_{\text{ext}}, \alpha_{\text{int}}$	Geometric atlas. Field point r_f and source scale R_s must be declared for each evaluation.
A-1	α_{core}	Schematic nuclear-skin saturation diagnostic, approached from below.
A-2	β_σ	Geometric computational coordinate; unused in force, optical or density slots.
A-3	α_{ovl}	PROVISIONAL token and open range only; unused.
A-4	α_{conf}	PROVISIONAL token and open range only; unused.
A-5	α_{mode}	PROVISIONAL token and open range only; unused.

Every curvature ratio has the open range $0 < \alpha < 1$. The last three tokens receive no value, threshold, ordering or trajectory. In A-1, core I-3 gives the branch split and gap odds

$$\alpha_\sigma(r_f) = (R_s/r_f)^\sigma, \quad X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \quad X_{\text{int}} = \frac{r_f}{R_s - r_f}. \quad (6)$$

The exterior domain is $r_f > R_s$; the interior domain is $r_c \leq r_f < R_s$. With X_σ denoting either positive gap odds,

$$\alpha_\sigma = \frac{X_\sigma}{1 + X_\sigma}. \quad (7)$$

For the bounded response of I-6, α_0 is a baseline ratio and x a nonnegative increase of its positive odds input. The exact rational map is

$$\alpha = \frac{\alpha_0(1 + x)}{1 + \alpha_0 x}, \quad 0 < \alpha_0 < 1. \quad (8)$$

Already bounded atlas ratios are not processed through this map again. Field polarity is carried separately from a positive density coordinate. No spectral weight, level, phase or fitted coefficient is an α .

1.3 Force, geometry and the two readings

Let G be Newton's constant and $F_{\max}$ the limiting force. Let F be the pair force and α_1, α_2 its two curvature-ratio inputs. I-1 holds always, at every scale, for two bodies:

$$F = F_{\max} \alpha_1 \alpha_2, \quad F_{\max} = \frac{c^4}{4G}. \quad (9)$$

The universal interval implies $F < F_{\max}$. For spherical exterior sources with total content denoted $\mathcal{M}$, and individual contents $\mathcal{M}_1, \mathcal{M}_2$, support radii $R_{b,1}, R_{b,2}$ and centre separation d , define $R_{s,i} = 2G\mathcal{M}_i/c^2$ and cross-centre inputs $\alpha_i = R_{s,i}/d$. Under nonoverlap, $d \geq R_{b,1} + R_{b,2}$ and $R_{b,i} \geq R_{s,i}$, substitution gives

$$F = G\mathcal{M}_1\mathcal{M}_2/d^2, \quad F \leq F_{\max}/4. \quad (10)$$

Equality in the second statement requires contact of equal saturated supports. Only this last bound belongs to the pair domain. The inverse square reduction is exact for this declared model and is the leading weak, slow gravitational result. The small bare force at a material's compactness is a correct prediction. A binding energy requires a declared response kernel; a measured tunnelling rate is not a force. The slab's binding is I-1 under its electromagnetic name; the maps from its charge seats to I-1 inputs are owed (O-6).

For the EFE face, $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}$ the stress tensor and κ its coupling; μ, ν are tensor indices. For the optical face, $d\mathcal{A}_{\text{ang}}^2$ is the unit-sphere angular line element and $d\ell_{\text{opt}}$ the optical path element. Core I-3 states

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G/c^4. \quad (11)$$

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\mathcal{A}_{\text{ang}}^2}{1 - \alpha_{\text{ext}}}. \quad (12)$$

The EFE are unchanged; α is downstream of their solution. The exact directional optical index is n_{opt} , with incident and outgoing ray angles θ_1, θ_2 . Generalized Snell matching reads $n_{\text{opt},1} \sin \theta_1 = n_{\text{opt},2} \sin \theta_2$. The weak isotropic index n_{iso} has the expansion $n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2)$; using it as an exact lensing relation would change the optical face. The separate cone index $n_{\text{cone}} = (1 - \alpha_\sigma)^{-1/2}$ is P-4's causal-aperture assignment, its constitutive derivation owed (O-8). It supplies no lensing coefficient.

A bound configuration has two declared charts: its source’s exterior atlas and its own interior chart, with the skin at the fold and R_{room} set by data. A source’s nuclear skin approaches $\alpha_{\text{core}} \rightarrow 1^-$ schematically, while the centre is dilute. An interior datum belongs to its observation region and to a profile-weighted mean in that region; it is not the skin value. No transfer map identifies a measured laboratory radius with a nuclear interior coordinate. The material spectra supply no interior profile.

Core 0.1c describes the clock factor $\mathbf{m}$, spacetime metric $g_{\mu\nu}$ and the two readings of a density change. A crossing pattern’s thread length and kept time rescale together under $g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}$, preserving $d/t = c$. Its own clocks and rulers register no local change. Neighbouring patterns read the gradient through relative acceleration and redshift. The clock factor is no α , no odds, no bounded output and no application of core 0.4, and stands in no force, optical or density slot. For a medium with a stress tensor of perfect form, its temperature at zero net conserved count, or its enthalpy per conserved count at fixed entropy per count, sets this factor; it does not set a clock frequency carried by the medium. The medium and the geometry move together. In an exterior flow the infall speed is $c\sqrt{\alpha_{\text{ext}}}$ and its convective acceleration is GM/r_f^2 . Rotational frame dragging is the bookkeeper’s half-curl of the geometry’s swirl, distinct from both four-velocity vorticity and material vorticity. This flow runs at every size. The twist experiments below compare neighbouring patterns.

1.4 Confinement and content

For I-5, D is the confinement operator, Ω the confinement domain, B its boundary data, n the strand count, χ the orientation, and u_q the mode field. Here the argument t names chart position; physical evolution time is distinguished by its occurrence in derivatives. The eigenproblem is

$$D(\Omega, B; n, \chi, t)u_q = \omega_q u_q. \quad (13)$$

The operator is owed (O-9). An electron has $n = 3$; changing its vacancy mode changes boundary data at fixed count. A different strand count is a different closure, not a second spectral peak. The support R_{box} is the pattern’s own region, never its Compton length. Core 0.1b calls a box “that pattern’s own support region, with the pattern’s own boundary data for a wall”.

For I-7, $\mathcal{M}$ is total content, M the rest channel, P pressure, V volume, T temperature, a the radiation energy-density constant and σ_{SB} the Stefan–Boltzmann flux constant. The ledger is

$$\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \dots, \quad a = 4\sigma_{\text{SB}}/c. \quad (14)$$

The slab’s rest content, electronic energy, pressure and thermal content each enter once. Removing a Pd site subtracts its rest content once. Gravitational binding is read from geometry and is not another local stress channel. The measured stage temperature is approximately 7 K (SC-198, L466); it sets a thermal input, not a pocket frequency. At fixed lattice geometry the isotope substitution changes this rest channel. A level response still requires the separate sensitivity stated in the isotope tests; positivity of a density input alone fixes no sign of a spectral shift.

2 System chart

Core P-2 states that “eigenstates, phonon modes and orbitals are nodal patterns of the driven density field” and that “localisation is trapping in disconnected nodal pockets”. P-3 names nuclei “fixed pinning sites”. This manuscript’s reading calls the PdAl plane a plate of the one omnium, its nuclei clamps and the Pd vacancy a missing clamp. The electron is the closed pattern that settles into the eigenmode offered by these boundary data. A pocket is that mode’s support, with no separate occupant and vessel. The drive is bias $\mathcal{V}$; e is the elementary charge (U-03). The injected energy $e\mathcal{V}$ is measured above E_F ; in this manuscript’s reading, which applies P-2’s “Energy injection . . . excites standing density waves”, the bias $\mathcal{V}$ injects a pattern at that energy.

System item	Source identification	FDT application
Layer	PdAl plane with iodine termination	Manuscript reading: one plate and its nuclear clamps (P-2, P-3).
Missing site	Type ii assigned to Pd vacancy (SC-070–SC-071)	Boundary change at fixed strand count (I-5), $n = 3$ for the electron.
Maps	Surface conductance at two resonances (SC-072–SC-075)	Pattern weight on lobes; surface projection (I-4).
Model	Pd d_{xz}, d_{yz} and Al p_z (SC-082)	Lobe field and lattice compression; source model identity map.
Pair	Adjacent vacancies and split spectra (SC-116–SC-119)	One system settling into shared modes (P-2).
Source chart	Electron rest content at a stated field radius	A-1 exterior, evaluated below (I-1, I-3).
Configuration chart	Nuclear skin and dilute interior	Schematic P-3 reading; interior profile and room extent unmeasured.

3 Observation map

3.1 Registry, spectra and spatial support

The observed square lattice has spacing $\mathfrak{a} = 4.05 \text{ \AA}$ (SC-064, L99). The iodine-plane registry places dark defects above Pd sites and bright defects above Al sites (SC-214–SC-217, L489–492). The assignment of the dark type ii site to a Pd vacancy is an identification, not an atom removed and counted during the measurement. The reported Bragg frequency is $\mathfrak{q}_{\text{Bragg}} = 2.467 \text{ nm}^{-1}$; its reciprocal is $\mathfrak{a} = 0.405 \text{ nm}$ (SC-308–SC-309, L609–610). This reciprocal conversion is an identity map. The source’s count corresponds to a per-type density of about $2.3 \times 10^{12} \text{ cm}^{-2}$, derived from approximately 70 defects in 3025 nm^2 (SC-311–SC-312, L618–619). That density is consistent with the earlier per-type interval once its power is fixed by that independent count (SC-221, L496); no combined-type density is used in an availability calculation.

Every STS datum below is surface conductance $d\mathcal{I}/d\mathcal{V}$ at the tip apex above the iodine face. Each sample state is weighted by its tunnelling matrix element through that face and the gap. Reading this conductance as LDOS also requires a flat tip density of states, an assumption not stated by the source (SC-051–SC-052, L85–86). The same probe-weight conditions apply to both families in every test. The central spectral observations are set out below. Bias is positive in all four pair rows. The source gives peak centres without centre errors; modulation widths

are not their statistical uncertainties.

Observable	Value	Tag, locus and spatial statement
Lone, four lobes	+475 mV	Reported centre, SC-069, L104; four-lobed map, SC-072–SC-073, L113–114.
Lone, circular	+620 mV	Reported centre, SC-069, L104; circular map, SC-074, L115.
Four-lobed pair, lower	+440 mV	Reported, SC-116, L165; source’s bonding label.
Four-lobed pair, upper	+510 mV	Reported, SC-117, L166; source’s antibonding label.
Circular pair, lower	+590 mV	Reported, SC-118, L167; central node.
Circular pair, upper	+650 mV	Reported, SC-119, L168; filled central region.
Extent	Within 1 nm	Estimated from line spectra, SC-168, L266; finite sensitivity.

The source reports a nonzero DOS at all bias voltages (SC-067, L102). Localized excess conductance therefore coexists with a metallic background. The map at 470 mV is a mapping setting, distinct from the quoted 475 mV centre (SC-165, L263; SC-069, L104). At two lattice spacings the source reports no observed hybridization (SC-122, L171); finite linewidth and contrast set the detection limit. A vanishing exact coupling is stronger than this observation.

The monomer average uses 150 mV, 30 pA and modulation 25 mV at 927 Hz (SC-161, L251). Line spectroscopy uses modulation 10 mV at 927 Hz (SC-170, L268). Pair spectroscopy uses −400 mV, 50 pA and modulation 5 mV at 1127 Hz (SC-187, L298; SC-349, L683). The stage is approximately 7 K, with Au(111) tip calibration before each approach (SC-198–SC-199, L466–467). These are distinct acquisition conditions, not interchangeable precision budgets.

Corroboration condition. I-5’s fixed-count confinement reading is compatible with these data at the reported spatial and spectral resolution. Returning their centres from their own spectra corroborates no invariant. A prospective same-tip map between the lone resonances tests the reconstructed additional real survivor: its absence supplies an upper weight bound, and a resolved pattern inconsistent with the calculated symmetry falsifies that ideal model assignment. This is a convergent measurement, not an extra FDT prediction; the source reports only its minus-one and positive-imaginary assignments (SC-295, L585).

3.2 Plate equation and the meaning of a node

Core P-2 says that “mode geometry is set by the boundary conditions”. This manuscript’s plate reading has a precise free-pattern limit. With Ψ the complex orbital amplitude and ∇ the spatial gradient, the Schrödinger equation for mass m_e gives, by a second time derivative,

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m_e}\nabla^2\Psi, \quad \partial_t^2\Psi + \left(\frac{\hbar}{2m_e}\right)^2\nabla^4\Psi = 0. \quad (15)$$

Each real quadrature obeys the Kirchhoff–Love plate equation. Its stiffness per areal mass, denoted $\mathfrak{k}_{\text{plate}}$, is

$$\mathfrak{k}_{\text{plate}} = (\hbar/2m_e)^2 = 3.35054 \times 10^{-9} \text{ m}^4 \text{ s}^{-2}. \quad (16)$$

U-02 and U-04 fix this derived value. This manuscript’s reading identifies the quadratures with I-4’s two transverse helical components; it does not supply a measured elastic modulus for the crystal. The lattice Hamiltonian below replaces this free equation in the solid.

All three model orbitals are odd under the mirror of the PdAl plane (SC-082, L125). Therefore their linear combination vanishes on that plane. Let $\mathbf{r}, \mathbf{\eta}$ be in-plane coordinates and $\mathcal{F}(\mathbf{r}, \mathbf{\eta})$ its leading across-plane derivative. The Taylor form and its derivative are

$$\Psi(\mathbf{r}, \mathbf{\eta}, z) = z\mathcal{F}(\mathbf{r}, \mathbf{\eta}) + O(z^3), \quad \partial_z \Psi|_{z=0} = \mathcal{F}(\mathbf{r}, \mathbf{\eta}). \quad (17)$$

Every nucleus in the plane lies on this odd-sector nodal set. The lines where $\mathcal{F}$ vanishes separate lobes with opposite sign. The source defines its nodal lines by phase shifts of π (SC-094–SC-095, L137–138). This manuscript’s reading identifies them with the plate figure’s across-plane derivative nodes. The measured weight fills the lobes between the lines. Thus “settles into the nodal lines (eigenmodes)” means settles into the eigenmode with those nodal lines, not concentration of measured intensity on a zero of amplitude.

P-2’s distinct density statement is “the loci where the transverse density gradient vanishes”, written $\nabla\alpha = 0$. This density gradient is not evaluated from Ψ , and no squared amplitude is assigned to α . The surface spectroscopic maps and the model overlays have “excellent qualitative agreement” in the source’s words (SC-180, L284). They do not measure the field in the PdAl plane. The DFT slice does show in-plane density (SC-313–SC-314, L626–627). This manuscript’s deduction is that such density includes orbitals outside the three-orbital odd sector. No equality of the DFT slice and the three-band wavefunction follows from the similar four-lobed figure.

Corroboration condition. P-2 and P-3 organize the node and clamp dictionary. The parity deduction is exact within the three-orbital mirror sector; the overlay comparison is qualitative and in-sample. A resolved nonzero in-plane amplitude within that isolated odd sector would falsify the deduction. Orbital-resolved and surface-sensitive maps at the same energy distinguish this condition from the existing multi-orbital DFT density. No invariant is corroborated by reusing the fitted overlay.

3.3 Compression, circulation and the surviving pocket

The source’s ideal model uses nearest-neighbour hopping $t = 0.395$ eV (SC-249, L532). It describes destructive Pd-to-Al interference and four compact localized states meeting at a Pd vacancy (SC-323–SC-328, L642–647). The following reconstruction fixes a real-space sign convention: Pd sites have integer coordinates in units of $\mathbf{a}$, and Al sites sit at half-integer plaquette centres. Let $\mathbf{f}$ denote the Pd lobe field with components f_x, f_y , and $\mathbf{b}_{\text{Al}}$ the Al amplitude. Lobe signs give the compression operator $\mathbf{C}$; j, l below are integer lattice indices. The binary corner indices are j_1, j_2 . Explicitly,

$$(\mathbf{C}\mathbf{f})_{jl} = t \sum_{j_1, j_2=0}^1 [(1 - 2j_1)f_x(j + j_1, l + j_2) + (1 - 2j_2)f_y(j + j_1, l + j_2)]. \quad (18)$$

Let $\mathbf{H}_0$ denote the ideal Hamiltonian and $\dagger$ its adjoint. In the Pd–Al decomposition,

$$\mathbf{H}_0 \begin{pmatrix} \mathbf{f} \\ \mathbf{b}_{\text{Al}} \end{pmatrix} = \begin{pmatrix} \mathbf{C}^\dagger \mathbf{b}_{\text{Al}} \\ \mathbf{C}\mathbf{f} \end{pmatrix}. \quad (19)$$

A circulation has $\mathbf{C}\mathbf{f} = 0$ and $\mathbf{b}_{\text{Al}} = 0$. Compression eigenstates lie in the image of $\mathbf{C}^\dagger$; the inner-product identity makes them orthogonal to every circulation. For each Bloch point the one-by-two compression row has two nonzero opposite-energy branches when its rank is nonzero and a zero branch. This reconstructs the one flat and two dispersive bands (SC-171, L275), with an energy Hamiltonian and an explicit $\hbar$ when its energies are converted to angular frequencies.

An elementary circulation has tangent Pd vectors on the four corners of one Al plaquette and zero Al weight. Their equal and opposite feeds cancel at that Al site and at every adjacent Al site. Let $\mathbf{c}_j$ be its four rotations around the central Pd site, with j the quarter-turn index and $\mathbf{c}$ the rotation eigenvalue. The combined lobe field is $\mathbf{f}_{\mathbf{c}}$, with membership $\mathbf{c} \in \{1, -1, i, -i\}$.

$$\mathbf{f}_{\mathbf{c}} = \sum_{j=0}^3 \mathbf{c}^{-j} \mathbf{c}_j, \quad \mathbf{c} \in \{1, -1, i, -i\}. \quad (20)$$

The central vector cancels for $\mathbf{c} = 1$ and $\mathbf{c} = -1$. The other two combinations cross the central Pd site and are cut by its removal. This is an algebraic distinction between survivors and cut circulations, not a different strand count.

The source specifies a scalar local term and a handed term in a three-by-three neighbourhood (SC-255–SC-258, L538–541). It does not specify which corner Al sites receive the scalar term. Three illustrative-assumed, rotation-symmetric placements put that term on the eight surrounding Pd sites and on none, the four nearest, or all sixteen surrounding Al sites (U-13). In all three placements, with $\mathfrak{h} = 0$, the two survivors $\mathbf{f}_{+1}, \mathbf{f}_{-1}$ have zero Al weight. Here $\mathbf{H}$ includes the local terms and E_{flat} denotes the ideal flat-band energy. They obey

$$\mathbf{H}\mathbf{f}_{+1} = \mathbf{u}\mathbf{f}_{+1}, \quad \mathbf{H}\mathbf{f}_{-1} = \mathbf{u}\mathbf{f}_{-1}, \quad E - E_{\text{flat}} = 0.25 \text{ eV}. \quad (21)$$

The numerical input is independently printed as an energy at SC-270, L557; the earlier local-term line prints volts (SC-257, L540). The conversion convention is U-13. The same equations hold under the periodic finite-volume checks on 9×9 and 13×13 cells (U-19).

The handed matrix on each surrounding Pd pair is defined by

$$\hat{\mathbf{Y}} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \mathbf{H}(\mathfrak{h})^* = \mathbf{H}(-\mathfrak{h}). \quad (22)$$

The dimensionless evolution $\exp(-i\mathfrak{h}\hat{\mathbf{Y}}t/\hbar)$ rotates the real lobe pair about the sheet normal; this is the manuscript's statement from the matrix, with its angular rate fixed by energy over $\hbar$. The second equality follows because all other terms are real; reversing the handed term exchanges the two hands. The source attributes the added on-site potential to iodine p orbitals (SC-092, L135), represented by $p_x \pm ip_y$ (SC-248, L530). With the illustrative $\mathfrak{h} = 1 \text{ eV}$ convention of U-13, a normalized cut handed circulation has nonzero spectral variance. Denote that variance by $\mathbf{v}_E$, the squared norm of its residual after subtracting its mean energy. For every tested placement,

$$\mathbf{v}_E = \|\mathbf{H}\mathbf{f}_{\mathbf{c}} - \langle \mathbf{f}_{\mathbf{c}}, \mathbf{H}\mathbf{f}_{\mathbf{c}} \rangle \mathbf{f}_{\mathbf{c}}\|^2 \simeq 1 \text{ eV}^2 \quad (\mathbf{c} = \pm i). \quad (23)$$

The one-significant-figure $\mathfrak{h}$ fixes the licensed precision of $\mathbf{v}_E$; $\mathfrak{t}$ has three significant figures (U-13 and SC-249). The finite-volume spectral weight is spread among multiple eigenstates, including dispersive-band weight, giving a resonance signature for the cut trial circulation. It is not the exact survivor above. Separate bound poles also occur in these finite-volume reconstructions, so spectral spreading of a trial circulation alone proves no absence of all localized handed

eigenstates. Neither placement choices nor a finite cell reproduce a unique source spectrum. In particular the source’s 400 and 700 meV model levels are referenced to E_F (SC-085, L128; SC-090–SC-091, L133–134), whereas the scalar survivor is referenced to E_{flat} ; the source gives no alignment between those axes.

The dispersion scan uses η_{nnn} from 0 to 3.5, with coefficients 0.15 and 0.038 for Pd and Al next-nearest terms (SC-250–SC-252, L533–535). The scale’s unit is absent there, while the later threshold is printed as 3.25 eV (SC-283, L570). Those statements are retained without turning them into a calibrated hopping series. The source writes that unity “corresponds to a band structure that is representative of the experimental flat band” (SC-253–SC-254, L536–537). It hypothesizes that the threshold η_{nnn} “where the BICs become trivially delocalized defect resonances is when the energy of the eigenstate begins to overlap with the flat band and can hybridize with it” (SC-271–SC-273, L558–560). Its finite-cell exponential localization is a simulated result, including the stated extent within two cells (SC-339–SC-342, L669–672). The model’s reciprocal fourth moment is a participation number: lower values select greater localization (SC-267–SC-268, L554–555).

Corroboration condition. I-4’s fixed-phase reading and I-5’s fixed-count modes give the physical dictionary; the compression algebra is the identity map of the ideal model and corroborates neither invariant. Its exact cancellation is falsified by a nonzero residual with the stated couplings and boundaries. A field normal to the sheet splits the circular state’s spin and orbital responses, while the four-lobed state’s leading response is spin-like. This is the conventional spin–orbit and Zeeman comparator. At zero field a time-reversal-even spin–orbit term retains the two circular hands as a degenerate pair. A magnetic tip’s contrast and a nonremanent tip’s field sweep distinguish this comparator from an imposed time-odd orbital term. Finite fitted errors, not the source’s modulation amplitude, set the precision of this convergent measurement.

A further convergent test moves the flat band with strain or pressure while measuring the pocket linewidth and band position independently. This manuscript’s reading is a pocket whose width turns on when a band of its own class reaches its level, applying I-5 to the source’s hybridization hypothesis (SC-271–SC-273, L558–560). A linewidth change is tested against the propagated band-position and linewidth covariance, including instrumental broadening. A resolved failure of the proposed onset relation falsifies that model hypothesis; it does not assign an α threshold to the pocket.

3.4 Pairs and simulated density

The source’s pair panels are schematics (SC-183, L294), and their phase rule is inferred from map nodes (SC-298–SC-300, L588–590). Core 0.1a states that responses are “system-wide and many-to-many”. This manuscript’s reading treats the pair as one system settling into a shared pattern. P-2 calls bonding “shared scattering-free nodal channels”. The measured order is retained without calculating a pair energy from a phase label. Changing Ψ to $-\Psi$ leaves the isolated intensity unchanged; only relative phase between the two pockets changes their union.

For two adjacent missing Pd sites there are six affected elementary circulations. Their four central-component constraints have rank four, leaving two independent survivors. If a single scalar term covers the union of both neighbourhoods, both survivors still have energy u . Let

ΔE_{ideal} be their splitting. The reconstructed defining condition gives

$$\Delta E_{\text{ideal}} = \mathbf{u} - \mathbf{u} = 0. \quad (24)$$

The source does not specify pair potentials. Its resolved splitting therefore needs coupling beyond this ideal model; the map-selected phase rule by itself supplies no numerical pair Hamiltonian.

The same ideal result extends to a straight chain. Let N be its positive integer length and K the matrix imposing zero lobe components at its removed Pd sites. The affected plaquettes number two more than twice the chain length. Successive elimination of the two new central components at each additional site supplies two independent pivots, leaving two free amplitudes at the ends. Thus the defining constraint and its solved dimension are

$$\text{rank } K = 2N, \quad \dim \ker K = 2. \quad (25)$$

With a uniform scalar term on the union of the neighbourhoods, both survivors remain at $\mathbf{u}$ for any positive chain length. This convergent chain measurement tests the ideal cancellation and bounds couplings beyond it; at two spacings the source gives only an unresolved coupling (SC-122, L171). The source envisions that Pd_5AlI_2 could provide a platform for the controllable synthesis of electronic BIC lattices (SC-143–SC-144, L197–198); it reports no measured spectrum of a chain longer than the adjacent pair. The adjacent versus separated comparison remains a measured distance constraint, not an exact zero of every longer-range term.

DFT uses a $4 \times 4 \times 1$ vacancy supercell, a 12 Å spacer, 544 eV cutoff and a $2 \times 2 \times 1$ mesh (SC-204–SC-208, L474–477). The lattice relaxes in plane with its out-of-plane vector held fixed; atomic positions relax in all directions. The simulated vacancy PDOS peak is approximately 0.55–0.6 eV, compared in the source with approximately 0.47 eV experimentally (SC-231–SC-232, L510–511). This periodic vacancy array is not a measured isolated vacancy and its peak is not the experimental centre reproduced to its quoted digits. The extra band and four-lobed density are simulated evidence for the model’s localization mechanism.

Corroboration condition. P-2’s shared-channel reading is compatible with the resolved pair maps. The ideal unsplit pair is an exact model result; the source’s pair centres have no quoted errors. Measuring the pair phase and spectrum independently of the map-selected combinations tests the proposed correspondence. Disagreement between independently resolved nodes and the assigned relative-phase pattern falsifies that correspondence. Comparing a larger DFT vacancy cell at fixed numerical convergence tests periodic-image contamination. Neither test makes the source’s in-sample phase assignment an independent corroboration of P-2.

4 Cone-position translation

Let E_e, T_e, ℓ_e denote the electron’s rest energy, period and Compton length, using the mass m_e of U-04 and constants U-01–U-03. Their bridge values are derived, not confinement eigenvalues predicted from the operator D :

$$E_e = m_e c^2 = 510998.950 \text{ eV}, \quad T_e = h/E_e = 8.09330 \times 10^{-21} \text{ s}. \quad (26)$$

$$\ell_e = cT_e = h/(m_e c) = 2.42631 \times 10^{-12} \text{ m}. \quad (27)$$

In the period expression the energy is converted to joules. A pocket level relative to E_F is a difference of mode costs. Its $eV/\hbar$ is a beat frequency, not ω_e . The support estimated from the line spectra belongs to R_{box} and carries no rest-mass radius identity.

For A-1's electron exterior evaluation take the illustrative field point $r_f = 1$ nm from the estimated decay scale (SC-168, L266). With $\mathcal{M} = m_e$, core 0.1c gives

$$R_s = \frac{\mathcal{M}c^2}{2F_{\text{max}}} = \frac{2G\mathcal{M}}{c^2}, \quad \alpha_{\text{ext}}(r_f) = \frac{R_s}{r_f} \simeq 1 \times 10^{-48}. \quad (28)$$

The one-significant-figure scale fixes the licensed precision of this estimate (U-05 and SC-168). It is a correct exterior compactness, not the binding strength of the pocket and not an address for an electron.

Observable	Translation	What remains separate
Bias centre	Difference of confinement costs	Electron rest bridge and support.
Lobe intensity	Projected mode weight	Curvature ratio and density gradient.
Nuclear pinning	Schematic skin saturation	Dilute interior and its profile mean.
Spatial decay	Estimated pocket support	Source exterior radius evaluation.
Pair splitting	Shared-mode energy separation	Static pair force and its response kernel.

For a proposed pair energy, let s be separation along a specified path and s_0 its lower end-point. An integrable continuous kernel-weighted force would give $U(s_0) = -\int_{s_0}^{\infty} F(s) ds$, with $dU/ds_0 = F(s_0)$. Under local integrability alone the last identity holds almost everywhere. No such binding kernel is inferred from the small bare compactness here. The tunnelling current vanishes at zero bias and the lock-in signal at zero modulation; differential conductance is a response coefficient and need not vanish there. I-1's force law is never recovered by turning off that drive.

5 Predictions

5.1 Shared magnetic observable and precision rules

P-5 states that a magnetic field is “a macroscopic helical density gradient” and that “mechanical vorticity and magnetic moments are different grade projections of one density-gradient topography”. The extra input tested below is the manuscript's reading that a static mechanical handedness of the plate produces a normal magnetic field. The clause “Imposing a physical twist (torsion, helical geometry) on a material is establishing a magnetic field” supplies the existence warrant; neither a magnitude nor an absolute sign follows from it. A handedness label by itself supplies no new force in the conventional calculation. The following three tests are conditional by construction. They test a field read by neighbouring patterns, after cancelling the hand-even elastic and electronic response.

Let $\mathbf{p} = \pm 1$ label matched mechanical hands, and $b = \pm 1$ the polarity of an applied normal field of positive magnitude $\mathcal{B}$. Let $\mathcal{B}_{\text{tw}}$ be the additional normal field for positive hand. Let $\mathcal{S}_j(\mathbf{p}, b)$ be the positive fitted energy separation of a doublet in family j , with $j = \text{cir}$ for circular

and $j = \text{lob}$ for four-lobed. Their measured same-configuration field slopes are $\mathbf{g}_j$, specifically $\mathbf{g}_{\text{cir}}, \mathbf{g}_{\text{lob}}$, in energy per tesla. For $\mathcal{B} > |\mathcal{B}_{\text{tw}}|$, linear response gives

$$\mathcal{S}_j(\mathbf{p}, b) = \mathbf{g}_j |b\mathcal{B} + \mathbf{p}\mathcal{B}_{\text{tw}}|. \quad (29)$$

This is a response model to be checked by a field sweep, not a value of $\mathbf{g}_j$ inferred from the source's peaks. Define $\mathcal{D}_j$ as the hand-and-field-odd combination and $\bar{\mathcal{S}}_j$ as the four-setting mean:

$$\mathcal{D}_j = \frac{\mathcal{S}_j(+, +) - \mathcal{S}_j(+, -) - \mathcal{S}_j(-, +) + \mathcal{S}_j(-, -)}{4} = \mathbf{g}_j \mathcal{B}_{\text{tw}}, \quad \bar{\mathcal{S}}_j = \mathbf{g}_j \mathcal{B}. \quad (30)$$

The circular and lobed odd readers are $\mathcal{D}_{\text{cir}}, \mathcal{D}_{\text{lob}}$; their means are $\bar{\mathcal{S}}_{\text{cir}}, \bar{\mathcal{S}}_{\text{lob}}$. The independent field reader and its four-lobed consistency residual are $\hat{\mathcal{B}}_{\text{tw}}$ and $\mathcal{R}$, defined by

$$\hat{\mathcal{B}}_{\text{tw}} = \mathcal{B} \frac{\mathcal{D}_{\text{cir}}}{\bar{\mathcal{S}}_{\text{cir}}}, \quad \mathcal{R} = \mathcal{D}_{\text{lob}} - \frac{\bar{\mathcal{S}}_{\text{lob}}}{\bar{\mathcal{S}}_{\text{cir}}} \mathcal{D}_{\text{cir}}. \quad (31)$$

The source's circular +620 mV and four-lobed +475 mV states are the actual readers (SC-069, L104; SC-072–SC-074, L113–115). A solenoid about the STM stage supplies reversed fields of a few tesla, illustratively 3 T (U-12). With the Bohr magneton $\mathbf{m}_B$ of U-08 and Boltzmann constant k_B of U-07, the conventional spin scale is $2\mathbf{m}_B \mathcal{B}$ and the thermal width estimate is $3.5k_B T$ (U-12). At that illustrative field the spin separation is approximately 0.3 meV, so the design uses a sub-kelvin stage and modulation finer than that scale. These choices support fitting; they are not statistical thresholds. The circular state also carries its orbital response. Both doublets are fitted with linewidths constrained by their own same-twist, zero-field values, propagating the width uncertainty.

For a statistic Y formed from measured settings, let $\mathbf{w}$ be its linearized coefficient vector and $\mathbf{V}$ their full covariance. The superscript T means transpose. The standard uncertainty $\mathfrak{d}_Y$ and the total criterion $\mathfrak{T}_Y$ are defined using a preregistered quantile $\mathfrak{z}$ and a bounded systematic allowance $\mathfrak{b}_Y$:

$$\mathfrak{d}_Y^2 = \mathbf{w}^{\mathsf{T}} \mathbf{V} \mathbf{w}, \quad \mathfrak{T}_Y = \mathfrak{z} \mathfrak{d}_Y + \mathfrak{b}_Y. \quad (32)$$

Equivalently the effective uncertainty $\mathfrak{d}_{Y,\text{eff}}$ is $\mathfrak{d}_Y + \mathfrak{b}_Y/\mathfrak{z}$, so every threshold is $\mathfrak{z} \mathfrak{d}_{Y,\text{eff}}$. The confidence is 95%, using a Student quantile at 0.975 with degrees of freedom estimated from the replicated groups (U-18), not a fixed Gaussian multiplier. Covariance includes repeated sweeps, fitted widths, field calibration, geometry, tip and crystal scatter and drift between reversals. Correlated differences use the full covariance, including the negative twice-covariance term. For ratios the same fitted data enter the Jacobian jointly. A bounded systematic is included in each criterion, not merely mentioned after significance is calculated.

Three separate decisions are preregistered. Detection requires $|\mathcal{D}_{\text{cir}}| > \mathfrak{T}_{\mathcal{D}_{\text{cir}}}$ and a field sweep consistent with the reader model. Exclusion reports only the finite upper bound $|\mathcal{B}_{\text{tw}}| \leq |\hat{\mathcal{B}}_{\text{tw}}| + \mathfrak{T}_{\hat{\mathcal{B}}_{\text{tw}}}$, with slope and field uncertainty included. A finite null excludes fields above that bound; it does not exclude every nonzero field in an amplitude-free existence statement. Attribution additionally requires $|\mathcal{R}| \leq \mathfrak{T}_{\mathcal{R}}$, the expected reversal between mirror-related positions, and all reference controls below. The three inequalities have their own systematic budgets.

The single magnetic-source scope and the co-varying channels are explicit:

Channel	Hand reversal	Field reversal in $\mathcal{D}_j$
Added normal field	Odd	Odd; survives.
Mirror-matched strain and scalar level shift	Even	Cancels.
Time-reversal-even spin-orbit response	Mirror paired	Cancels in the matched comparator.
Unequal nonmagnetic hands	Odd residual possible	Even contribution cancels; cross terms are bounded.
Tip remanence or transverse stray field	Apparatus dependent	Measured reversal controls and systematic allowance.

The conventional zero requires mirror-related geometry and the same nonmagnetic tip: time reversal makes each split magnitude field-even, while the vertical mirrors of the fourfold axis, kept by the cleaved face of the achiral I4/mmm crystal (SC-054, L88), map the two hands and reverse the axial normal field. Therefore its matched $\mathcal{D}_j$ vanishes. Failure of mirror matching is measured, not assumed to cancel. Before and after every hand setting, a zero-twist reference reads the field-odd residual in both families with the same tip and sweep order. Alternating order bounds drift. A nonremanent tip and reversed transverse field bound instrumental magnetism. The four-lobed state is a second field reader, not a subtraction that removes a common magnetic signal.

5.2 T-1: Static torsion of a single plate

DC-1; SA-54. The source reports crystals with lateral dimensions 1–10 mm and thickness below 100 μm (SC-195–SC-196, L461–462); it supplies no torsion measurement. This block discriminates whether the plate has a magnetic response to static elastic handedness beyond the conventional matched zero. P-5 is the warrant; the normal-field mapping is this manuscript’s reading, conditional by construction.

Grip the rim and opposite face of a cleaved crystal while leaving the central iodine-terminated face accessible to the STM. Apply equal and opposite static rotations about the normal and stop the actuator before each spectrum. Locate type ii sites on a $\langle 100 \rangle$ or $\langle 110 \rangle$ line on both sides of the torsion axis and pair their mirror-related positions. The free face has zero torsional traction and hence zero local elastic twist rate. This manuscript’s reading is that the magnetic field established by the interior static twist reaches that face. The actuator also remains at rest during spectroscopy. The defining field condition for the torsion apparatus is as follows, with superscript tor denoting this apparatus applied to the shared reader.

$$\mathcal{D}_{\text{cir}}^{\text{tor}} = \mathfrak{g}_{\text{cir}} \mathcal{B}_{\text{tw}}^{\text{tor}}, \quad \hat{\mathcal{B}}_{\text{tw}}^{\text{tor}} = \mathcal{B} \mathcal{D}_{\text{cir}}^{\text{tor}} / \bar{\mathcal{S}}_{\text{cir}}^{\text{tor}}. \quad (33)$$

The existence/sign prediction is a nonzero field whose sign reverses with the hand and with the matched opposite-side position; its absolute sign and magnitude are unspecified. The conventional comparator is $\mathcal{D}_{\text{cir}}^{\text{tor}} = 0$ at fixed field history. In this manuscript’s reading the neighbours are the two hands of this geometry; their hand-even elastic and scalar electronic responses cancel in the hand contrast.

Use the monomer setpoint 150 mV and 30 pA (SC-161, L251), with the finer modulation and lower temperature of U-12. Record each site’s zero-field width at the same applied twist. The at-rest, zero-twist reference on this same plate brackets every hand measurement; both peak

families and both opposite-side positions are retained in one covariance fit. Elastic strain, piezo creep, tilt and contact heating enter the bounded systematic allowance of the three shared decisions. Detection, finite-field exclusion and attribution use respectively $\mathfrak{T}_{\mathcal{D}_{\text{cir}}^{\text{tor}}}$, $\mathfrak{T}_{\hat{\mathcal{B}}_{\text{tw}}^{\text{tor}}}$ and $\mathfrak{T}_{\mathcal{R}^{\text{tor}}}$, with the mirror and reference conditions also required for attribution. A zero result bounds this specimen’s field; it does not determine a universal twist-to-field kernel. For the existence sign comparison the categorical encoding is FDT +1, ablated −1, threshold 1 (U-15). The encoding is not a tesla value or a confidence multiplier.

5.3 T-2: Relative twist of two flakes

DC-2; SA-43. The source proposes mechanical exfoliation, stacking and twisting (SC-149, L203); it reports no twisted-interface spectrum. This block tests the additional magnetic source of P-5 at a static interface. This manuscript’s reading identifies the relative twist as a helical boundary gradient reaching the top PdAl plate; it is conditional by construction.

Stack two exfoliated flakes at matched positive and negative relative angles, with one slab as the top flake, with its central PdAl plane half a slab above the rotated interface (SC-055–SC-057, L89–91). Probe type ii sites on its exposed iodine surface at mirror-related positions above the interface. Keep the stacking translation, separation and strain matched, and record the relaxed geometry for each hand. The normal-field observable is defined by the display below, where superscript stk denotes this stack:

$$\mathcal{D}_{\text{cir}}^{\text{stk}} = \mathfrak{g}_{\text{cir}} \mathcal{B}_{\text{tw}}^{\text{stk}}, \quad \hat{\mathcal{B}}_{\text{tw}}^{\text{stk}} = \mathcal{B} \mathcal{D}_{\text{cir}}^{\text{stk}} / \overline{\mathcal{S}}_{\text{cir}}^{\text{stk}}. \quad (34)$$

The prediction is the existence of a hand-odd normal field; the conventional mirror-matched, time-reversal-even stack has $\mathcal{D}_{\text{cir}}^{\text{stk}} = 0$. In this manuscript’s reading the neighbours are the two mirror-related stack hands; matched hand-even strain, registry and spin–orbit responses cancel in the hand contrast. Neither a moiré contrast nor a scalar resonance shift is this observable.

Read both source peak families using the monomer setpoint 150 mV, 30 pA (SC-161, L251), reversed normal fields and the sub-kelvin fit design of U-12. An aligned stack is the zero-twist reference before and after each hand; a reference away from the interface checks tip and field offsets. Each hand carries its own zero-field linewidth fits. Registry, twist inhomogeneity, interface charge transfer, thermal drift and imperfect mirror placement enter the covariance and bounded systematic allowance. Detection uses $\mathfrak{T}_{\mathcal{D}_{\text{cir}}^{\text{stk}}}$; finite exclusion uses $\mathfrak{T}_{\hat{\mathcal{B}}_{\text{tw}}^{\text{stk}}}$; attribution uses $\mathfrak{T}_{\mathcal{R}^{\text{stk}}}$ and the aligned-stack, mirror and field-reversal controls. A scalar interface shift cancels only when its matching condition is verified. The existence-sign encoding is FDT +1, ablated −1, threshold 1 (U-15); the amplitude and absolute sign are not assigned.

5.4 T-3: Handed screw cores

DC-3; SA-81. The source’s maps compare defect separation but identify no screw dislocations (SC-350–SC-352, L690–692). This block tests whether opposite screw hands produce a normal magnetic field read by nearby vacancy modes. P-5 supplies the physical-twist warrant; identifying a static helicoid as its helical density gradient is this manuscript’s reading, conditional by construction.

Search a micron-scale topographic field for screw steps of both hands, verify their Burgers sense from the step winding, and locate type ii vacancies within radius $3a$ (U-14). Compare

equal radial and angular positions at left- and right-handed cores and mirror-related positions within a core. The core-resolved reader is as follows, with superscript *scr* denoting screw-core sampling:

$$\mathcal{D}_{\text{cir}}^{\text{scr}} = \mathfrak{g}_{\text{cir}} \mathcal{B}_{\text{tw}}^{\text{scr}}, \quad \widehat{\mathcal{B}}_{\text{tw}}^{\text{scr}} = \mathcal{B} \mathcal{D}_{\text{cir}}^{\text{scr}} / \overline{\mathcal{S}}_{\text{cir}}^{\text{scr}}. \quad (35)$$

A hand-odd, position-matched field is the prediction; the matched conventional nonmagnetic comparator is $\mathcal{D}_{\text{cir}}^{\text{scr}} = 0$. In this manuscript’s reading the neighbours are the matched opposite screw hands. Their hand-even strain and chemical response cancel in the hand contrast; an unmatched nonmagnetic hand-odd shift is removed by the field contrast. A core’s chemical shift alone does not satisfy that condition.

Let ϱ_{vac} be the type ii areal density and $\mathcal{N}_{\text{near}}$ the expected number of nearby vacancies per screw. The illustrative uncorrelated placement model of U-14 gives

$$\mathcal{N}_{\text{near}} = \varrho_{\text{vac}} \pi (3\mathfrak{a})^2 \simeq 0.11. \quad (36)$$

Inputs are $\varrho_{\text{vac}} = 2.3 \times 10^{12} \text{ cm}^{-2}$ and $\mathfrak{a} = 0.405 \text{ nm}$ (SC-312, L619; SC-309, L610), hence two significant figures for the availability estimate. The source supplies no screw density. The number of cores and spectra is chosen after pilot variance and actual availability are measured, not from this mean alone.

Use both monomer families with the source’s 150 mV, 30 pA setpoint (SC-161, L251) and the finer U-12 field-reader conditions. A flat region far from the core brackets every core’s hand measurement as the zero-twist field-odd reference for both peaks. Same-core, same-twist zero-field widths constrain the doublet fits. Core chemistry, Burgers-vector variation, local tilt, mirror-position error, tip history and drift enter the covariance and the bounded systematic allowances. The three criteria are respectively $\mathfrak{T}_{\mathcal{D}_{\text{cir}}^{\text{scr}}}$, $\mathfrak{T}_{\widehat{\mathcal{B}}_{\text{tw}}^{\text{scr}}}$ and $\mathfrak{T}_{\mathcal{R}^{\text{scr}}}$, with far-core and opposite-hand controls necessary for attribution. A finite null gives only a finite field bound at the sampled cores. The existence-sign encoding is FDT +1, ablated −1, threshold 1 (U-15).

5.5 Shared isotope path and conventional comparator

I-7 changes the rest content under Pd substitution. This is an extra source input to the following tests, but a spectral response requires a constitutive derivative. I-7’s Discriminator fixes the density-channel input sign: “For purely density-coupled equilibrium observables, a heavier isotope $\rightarrow$ larger $R_s \rightarrow$ higher $\alpha \rightarrow$ larger response — opposite in sign to conventional reduced-mass scaling”; by Discipline 2, “A sign prediction competing with geometric, electronic or inertial channels is written as a sensitivity-weighted sum” with “every S_i defined (held-fixed variables stated)”, and “the observable sign is $\text{sgn}(\prod S_i)$ and stays conditional until the sensitivities are established”. Let $\mathcal{O}$ denote either the four-lobed energy above E_F or its pair coupling. Define $S_{\mathcal{M}}$ as the derivative of $\ln \mathcal{O}$ with respect to $\ln \mathcal{M}$ through I-7’s route $\mathcal{M} \rightarrow R_s \rightarrow \alpha_{\text{ext}}(r_f) = R_s/r_f$, at fixed field point r_f , along the Pd-site content path at fixed lattice spacing, hoppings, local terms and zero-point amplitudes. Define S_{zp} as the derivative of $\ln \mathcal{O}$ with respect to the logarithm of the zero-point amplitude at fixed lattice and mass-channel input, and $S_{\mathfrak{a}}$ as the derivative of $\ln \mathcal{O}$ with respect to $\ln \mathfrak{a}$ at fixed mass and zero-point amplitude. With A_{zp} denoting that amplitude, the complete comparison is

$$\Delta \ln \mathcal{O} = \int S_{\mathcal{M}} d \ln \mathcal{M} + \int S_{\text{zp}} d \ln A_{\text{zp}} + \int S_{\mathfrak{a}} d \ln \mathfrak{a}. \quad (37)$$

The path runs between assayed endpoint compositions. A pair of endpoints returns a secant, not a local derivative. The zero-point and lattice terms are conventional channels, evaluated separately with correlated errors. The illustrative FDT test point is $S_{\mathcal{M}} = 1$ along the entire path; its ablated conventional value is 0 (U-11). Neither value is inferred from the existing spectra.

At the pure-isotope test point the Pd-site content is identified with its atomic rest mass. Fixed nonrest-channel corrections are included in the assayed path if their effect exceeds the stated total criterion (U-11). Use enriched ^{104}Pd and ^{110}Pd powders; the pure isotopes' assumed atomic masses are 103.904030 and 109.905172 u (U-09). Let $\mathfrak{M}_L, \mathfrak{M}_H$ denote the two powders' assayed mean Pd masses and Λ_M their logarithmic contrast, the value the exclusion rule tests. For the pure-isotope endpoints,

$$\Lambda_M = \ln(\mathfrak{M}_H/\mathfrak{M}_L) = 0.0561502, \quad (\mathfrak{M}_H - \mathfrak{M}_L)/\mathfrak{M}_L = 0.0577566. \quad (38)$$

The fractional difference is not substituted for the logarithm. For a harmonic zero-point displacement the amplitude scales with mass to the negative quarter power at fixed force constant; an energy or frequency scales to the negative half power. Thus an estimated zero-point energy contribution uses the latter scaling, without treating it as the former amplitude. The assumed conventional total bound is $|\Delta \ln \mathcal{O}| \leq 0.005$ (U-10), including both zero-point and lattice changes. It is an investigator bound to be validated for these samples, not a measured property of Pd_5AlI_2 .

The synthesis comparator uses the same campaign and charge, with the record-powder material alongside the two enriched endpoints and at least two crystals per isotope (U-16). The source charge is Pd 100 mg, Al 7 mg and iodine 47.7 mg, heated to 600°C with a 6 h ramp and 84 h anneal, then quenched (SC-189–SC-191, L455–457; SC-193–SC-194, L459–460). Its iodine amount is molecular iodine, not an atomic-iodine mole count. Changing the isotope powder changes the molar Pd content at fixed mass; the charge must be matched by assayed stoichiometry, with the record charge retained as a comparator, before attributing a spectral change. Measure composition, lattice change, defect density and the conventional zero-point response in the same campaign. Each tip is calibrated on Au(111), and each crystal contributes to the covariance.

For either isotope contrast let Y_{iso} be the measured logarithmic difference, C_{iso} its independently estimated conventional contribution and $Z_{\text{iso}} = Y_{\text{iso}} - C_{\text{iso}}$ the residual. Detection beyond the bounded conventional account requires $|Y_{\text{iso}}| > 0.005 + \mathfrak{T}_{Y_{\text{iso}}}$ (U-10). Exclusion of the stated FDT test point requires $|Z_{\text{iso}} - \Lambda_M| > \mathfrak{T}_{Z_{\text{iso}} - \Lambda_M}$. Attribution requires consistency of the corrected record-powder intermediate point with its assayed mass-path integral and independently bounded zero-point and lattice terms, with its own covariance threshold. This distinguishes detection, exclusion of a conditional coefficient, and assignment to density. All three retain their bounded systematic allowances within $\mathfrak{T}$, including composition, energy-reference, tip, crystal and conventional-model uncertainties. For the instantiated precision target the total $\mathfrak{z}\mathfrak{d}_{\text{eff}} = 0.010$ (U-18), a design target; the measured variance determines the sampling needed to attain it.

Channel	Heavy-minus-light input	Observable sign
Added rest-content response	Positive Λ_M	Positive at the chosen S_M ; ablated zero.
Zero-point contribution	Negative energy/frequency change	Negative for positive contribution; sensitivity fitted independently.
Lattice relaxation	Measured spacing change	Sign not fixed; derivative measured at fixed mass.
Charge and defect chemistry	Assayed separately	Nuisance with correlated uncertainty, not an isotope law.

5.6 T-4: Isotope shift of the lone four-lobed level

DC-4; SA-21. The source resolves two vacancy peaks but supplies no isotope comparison (SC-069, L104–105). This block tests an additional I-7 density response of the lower level at fixed electronic parameters, using the constitutive test point U-11. It is conditional by construction.

Let $\mathcal{E}_{\text{lob}}$ be the fitted energy of the four-lobed peak above the same calibrated E_F in each crystal, and let C_E be its conventional path contribution. Let $\mathcal{E}_{\text{lob},H}, \mathcal{E}_{\text{lob},L}$ denote the heavy and light endpoint energies, and Y_E their logarithmic contrast. The defining measured contrast and the solved FDT condition are

$$Y_E = \ln(\mathcal{E}_{\text{lob},H}/\mathcal{E}_{\text{lob},L}), \quad Y_E - C_E = \int_L^H S_M d \ln \mathcal{M} = 0.0561502. \quad (39)$$

The last value instantiates the pure masses of U-09 and the test point of U-11. The conventional comparator has the density integral zero and $|C_E| \leq 0.005$ (U-10). The source’s lone centre 475 mV provides the concrete spectral referent (SC-069, L104), not the isotope-shift input.

Acquire the lone peaks at approximately 7 K (SC-198, L466) with the source monomer setting 150 mV, 30 pA and modulation 25 mV at 927 Hz (SC-161, L251). Fit centres with the actual lock-in kernel and thermal response. Independent repeated spectra, tips and crystals supply centre uncertainty despite the broad modulation. Drift and the Fermi reference are shared fit parameters. Assay and conventional-model errors enter the full contrast covariance. No fixed defect sample size is assigned before this variance is measured.

The shared isotope detection rule tests Y_E beyond the conventional bound; the exclusion rule tests the specific integrated sensitivity; the record-powder intermediate comparison supplies the attribution rule. Each uses its corresponding covariance and bounded-systematic criterion. At the illustrative total criterion 0.010 (U-18), the additional logarithmic shift exceeds the criterion. Setting only the extra density sensitivity to its conventional value returns zero residual. A measured null excludes this unit-sensitivity path at adequate precision, not all possible spectral sensitivities of I-7. The co-varying signs are those in the isotope table, with electronic hoppings and zero-point amplitudes held fixed only for the definition of the extra derivative.

5.7 T-5: Isotope shift of the adjacent-pair coupling

DC-5; SA-36. The source resolves adjacent-pair splitting but computes no pair isotope response (SC-116–SC-117, L165–166). This block discriminates a density contribution to that coupling beyond its conventional structural and vibrational changes. The I-7 input and unit-sensitivity path of U-11 are explicit; the block is conditional by construction.

Let j_{lob} be half the separation between the two fitted four-lobed pair energies, with upper and lower centres $\mathcal{E}_{\text{lob},+}, \mathcal{E}_{\text{lob},-}$. Let C_J be the independently estimated conventional isotope contribution. Let $j_{\text{lob},H}, j_{\text{lob},L}$ denote the endpoint couplings and Y_J their logarithmic contrast. The operational condition is

$$j_{\text{lob}} = (\mathcal{E}_{\text{lob},+} - \mathcal{E}_{\text{lob},-})/2, \quad Y_J = \ln(j_{\text{lob},H}/j_{\text{lob},L}). \quad (40)$$

$$Y_J - C_J = \int_L^H S_{\mathcal{M}} d \ln \mathcal{M} = 0.0561502. \quad (41)$$

The source centres 440 and 510 mV (SC-116–SC-117, L165–166) give an in-sample reference $j_{\text{lob}} = 35$ meV. The value of the integral above instantiates the pure masses of U-09 and the test point of U-11. The conventional density residual is zero, with $|C_J| \leq 0.005$ assumed and validated as in U-10. This conditional response does not repair the unsplit ideal pair by fitting its measured separation.

For availability, define the random Pd vacancy fraction $f_{\text{vac}} = \varrho_{\text{vac}} \mathfrak{d}^2$. With $\mathcal{P}_{\text{adj}}$ the conditional chance that a vacancy has an occupied nearest vacancy neighbour, U-14 gives

$$\mathcal{P}_{\text{adj}} = 1 - (1 - f_{\text{vac}})^4 \simeq 0.015. \quad (42)$$

The inputs are the two-significant-figure per-type density and the three-significant-figure spacing (SC-312, L619; SC-309, L610). This is an availability estimate, not an observed random-pair law; clustering is measured before deciding how much area to survey.

Use adjacent pairs in the same assayed crystals and campaign as the lone level, at the source’s approximately 7 K stage (SC-198, L466). The pair setting is -400 mV, 50 pA, modulation 5 mV at 1127 Hz (SC-187, L298; SC-349, L683). A joint two-centre fit retains their covariance: the separation uncertainty contains both centre variances and their negative twice-covariance. It propagates through the half-separation and both logarithms, including common tip and crystal effects. No instrument modulation is used as that uncertainty.

Detection uses Y_J against the conventional bound; exclusion tests the unit integrated sensitivity with the full $Y_J - C_J - \Lambda_M$ uncertainty; attribution requires the assayed intermediate composition and independently constrained conventional channels. Bounded systematics remain in all three criteria. At the illustrative total criterion 0.010 (U-18), the extra residual is resolved. Ablating only the added density sensitivity gives zero residual at the same geometry and conventional inputs. The sign table applies to this coupling separately from the lone-level sensitivity; agreement of the two responses is a joint empirical test, not a relation imposed by fitting.

6 Cross-domain reading

The source relates its electronic state to photonic and acoustic BICs (SC-018–SC-025, L44–53). I-2 identifies their carrier windows; P-2 states that “the same nodal structure appears in every carrier window”. The electronic pocket is a mode of a closed pattern, outside the carrier identity. Its shared architecture is the boundary-selected nodal geometry. This manuscript’s plate reading uses that architecture without transferring a photonic lifetime to an electronic vacancy. A convergent missing-site search in the Lieb and kagome lattices named by the source tests that boundary architecture (SC-136–SC-137, L190–191). Matched local spectroscopy and

spatial decay distinguish a pocket from a broad defect resonance; the measured linewidth covariance determines the comparison precision. Failure to find a mode for the stipulated boundary falsifies that lattice model, not the carrier-only identity of I-2. By analogy to the “infinite lifetime” of photonic BICs, the source writes that electronic BICs “could support lossless transport” (SC-138–SC-139, L192–193): a prospect, not a measured decay bound. The proposed magnetic impurity and Kondo pinning remain proposals (SC-146–SC-147, L200–201). In P-5 the impurity moment is a grade projection of the same density-gradient topography, not an added entity.

7 System-level reading (exploratory)

The scope of this section is in-sample re-description of the existing pair spectra and maps. It supplies no additional prediction or density coordinate. The manuscript’s reading asks how a common plate settles when two missing clamps share a channel.

X-1: Pair centroids, coupling and nodal order. The observable is the centroid and half-splitting of each measured pair family. Let $\bar{\mathcal{E}}_{\text{lob}}, \bar{\mathcal{E}}_{\text{cir}}$ denote those centroids and j_{cir} the circular half-splitting. Their defining conditions use the measured pair centres (SC-116–SC-119, L165–168):

$$\bar{\mathcal{E}}_{\text{lob}} = (440 + 510)/2 = 475 \text{ meV}, \quad j_{\text{lob}} = (510 - 440)/2 = 35 \text{ meV}. \quad (43)$$

$$\bar{\mathcal{E}}_{\text{cir}} = (590 + 650)/2 = 620 \text{ meV}, \quad j_{\text{cir}} = (650 - 590)/2 = 30 \text{ meV}. \quad (44)$$

The comparator is the lone centre in each family, respectively 475 and 620 mV (SC-069, L104); the arithmetic centroids coincide at the printed precision. The coupling ratio is derived as $j_{\text{cir}}/j_{\text{lob}} \simeq 0.86$, with no uncertainty inferred from the absence of quoted centre errors.

This manuscript’s reading identifies the lower four-lobed pair’s filled gap with a shared P-2 channel. It reads the lower circular pair’s gap node through P-4’s “chirality is one carrier of a flow direction” and “same flow direction $\rightarrow$ repulsion”, while retaining the measured reverse order of filled and nodal centres. No map from a quarter-turn eigenvalue to a material flow direction has been derived; the flow assignment is this manuscript’s reading of the figures, not an energy-order law. The source’s opposite-phase rule is in-sample: the combinations were selected to match the nodes (SC-298–SC-302, L588–597; SC-357–SC-358, L705–706).

Guards. The centroid coincidence is an arithmetic identity of the printed centres. The source compares a dimer and a lone defect in a common field and separately compares two spacings (SC-343–SC-354, L677–694), with finite modulation and no centre-error table. The ideal model’s two survivors are unsplit; its missing pair potential supplies no quantitative support for the flow assignment. A global phase flip of an isolated mode changes no intensity, and the relative-phase order is not an independent observation of chirality. None of these quantities is an α , a new electron species or a held-out fit.

8 Convergence statement

On the source’s own spectra, topography and dictionary the limiting map is the identity: it retains the reported lone and pair centres, finite-resolution maps and line spectra, the lattice

constant, the defects' sites on the iodine lattice, the defect counts and the per-type interval as measured, the sites' Pd and Al assignment as identified, the per-type density as derived from those counts, the extent as estimated and the pair phase rule as in-sample. On its model inputs, with the hoppings, the next-nearest scale and the local terms as chosen, it retains its simulated levels, the “excellent agreement with experiment” of its spectrum around the vacancy, where two states emerge a few hundreds of meV above E_F (SC-084–SC-085, L127–128), as in-sample, and its simulated finite-cell profiles; on its DFT inputs it retains the periodic-cell PDOS. The real-space reconstruction makes the ideal cancellations explicit under its own stated placements; the source specifies neither which corner Al sites receive the scalar term nor the model's flat-band-to-Fermi-level alignment. The DFT cell, molecular iodine charge convention and printed next-nearest units retain their distinct scopes. Returning these inputs and their derived outputs corroborates no invariant. For the same stress tensor, source and data, I-3 contains the same curvature as general relativity. The nonrelativistic free-mode equation is the slow-motion expansion of a closed pattern's phase relation $\omega^2 = c^2 k^2 + (m_e c^2 / \hbar)^2$ (I-2): its angular frequency above its rest value is $\omega - m_e c^2 / \hbar = \hbar k^2 / (2m_e)$ to leading quadratic order in momentum; the free-mode equation's time derivative gives the plate equation without changing its data.

The following declined readings are stated once at their loci. For SA-01 the source says BICs “defy conventional wave theories” (SC-007, L27). FDT reads their localization as P-2's disconnected nodal pockets selected by boundary data. For SA-05, the standard account reads a gap-bound defect state as an electron held in a potential (SC-026–SC-028, L56–58). FDT reads the electron as the closed pattern with its own support and boundary data, applying P-2 and core 0.1b. For SA-09, the standard account reads hopping as particles moving between sites (SC-036, L66). FDT reads the same hopping matrix as couplings of a closed pattern's nodal field (core 0.1b: a “particle” is a closed pattern of the one medium; I-5). For SA-33, the source says the states can bind when in proximity, forming molecular bound states (SC-109–SC-110, L157–158). FDT reads the pair as one shared nodal channel settling system-wide, with its energy order measured rather than assigned by the name (P-2: “bonding and Cooper pairing are shared scattering-free nodal channels”; 0.1a). For SA-56, the standard account calls the DFT spacer vacuum (SC-205, L475). FDT reads that computational separation as a low-density region of the same omnium (core 0.1: “Density is nowhere uniform”; 0.1d: there is no vacuum). For SA-69, the standard account reads squared wavefunction weight as probability density (SC-280–SC-282, L567–569). FDT reads that weight as the intensity of a deterministic helical projection (I-4: “there are no probability amplitudes”).

The additional source for the three twist tests is P-5's physical magnetic twist, with the apparatus mapping labelled as this manuscript's reading. Setting that source to its conventional absent value restores the matched field-odd zero. The two isotope tests add the declared I-7 density sensitivity at fixed conventional parameters; setting that sensitivity to zero restores the conventional isotope contribution. A chart relabelling or branch tag alone changes neither comparator.

9 Status ledger

The table below gives the physical status of the predictions, the touched owed items, the cone index and the unused provisional coordinates. An amplitude-free twist test has a finite field

exclusion limit; a conditional isotope test excludes its specified integrated response, not every possible constitutive sensitivity.

Item	Status	Convergence value or unresolved object
T-1	Prospective; conditional by construction	Torsion magnetic source absent: matched odd reader zero. P-5's quantitative twist kernel is unspecified.
T-2	Prospective; conditional by construction	Stack magnetic source absent: matched odd reader zero. Interface-to-reader amplitude unspecified.
T-3	Prospective; conditional by construction	Screw magnetic source absent: matched odd reader zero. Core density and amplitude unspecified.
T-4	Prospective; conditional by construction	Added density sensitivity zero: conventional lone-level isotope response.
T-5	Prospective; conditional by construction	Added density sensitivity zero: conventional pair-coupling isotope response.
O-1	Owed	Finite-core cutoff r_c , the regular core chart, or the outward phase-transfer law.
O-2	Owed	Geometrically constructed odds for the overlap-contrast application.
O-3	Owed	Canonical odds for the confinement application.
O-4	Owed	Local density profile from the omnium constitutive law with boundary, surface and core data.
O-5	Owed	Multipole and tidal remainders of the pair expression; overlap complex; signing a pair that settlement does not join.
O-6	Owed	Maps from charge or colour seats to I-1 inputs; seat-table derivation and gluon dynamics.
O-8	Owed	Constitutive derivation of n_{cone} and effective speed; reconciliation with lensing.
n_{cone}	POSITED	P-4's causal-aperture assignment.
O-9	Owed	Confinement operator D , domains, boundary data and spectrum; every mass ratio, including the two held-out targets, the proton-to-electron mass ratio and the squared muon-to-electron mass ratio (core I-5).
A-3, A-4, A-5	PROVISIONAL; unused	Token and open range only.
Pair-map reading	In-sample, exploratory	Centroids, half-splittings and node order; no independent phase-to-flow law.
Ideal reconstruction	Derived identity map	Compact survivors and unsplit ideal pair; handed trial spectral spreading.
Rest bridge	Derived from assumed constants	Electron mass is an input; no absolute confinement address.

10 Conclusions

The vacancy changes the boundary of a plate whose modes carry the measured lobes. This manuscript's reading follows P-2 and P-3 while keeping the measured surface weight, the odd-orbital node, and the unevaluated density gradient distinct. The ideal compression algebra isolates the compact survivors and shows why the measured pair splitting requires more than the union of two scalar neighbourhoods. The existing spectra fix level differences and figures, not twist magnetism or a density-to-spectrum sensitivity. The five prospective tests separate those additional FDT inputs from their conventional comparators with neighbouring references, field reversal, isotope assays and covariance-based decisions.

References

- [1] M. Thinel et al., “Electronic Bound States in the Continuum in a 2D Metal,” arXiv:2410.19227v2, main text and supplementary materials; version date not stated in the source record.
- [2] *Core FDT invariants*, locked version 6.2, clauses 0.1–0.6, I-1–I-7, P-2–P-5 and Use items 1–7.