

The Extended Equivalence Principle

Constant speed through a density gradient is gravity:
one pattern, two readings, and the flow of the geometry

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Fundamental Density Theory

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Abstract

Einstein's equivalence principle makes a freely falling laboratory blind to the field it falls through. Fundamental Density Theory extends it (core 0.1c): moving at constant speed, on a pattern's own clocks and rulers, through a density gradient is gravity. This paper derives the principle's identities from the chain $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$ and reads them in every window where gravity is measured. Clocks and rulers are closed patterns, and their kept time and thread length rescale together with the density they cross by a clock factor $\mathbf{m}$, in the gravitational window the same for every pattern at a point. The rescaling is the conformal map $g \mapsto \mathbf{m}^2 g$, which keeps $d/t = c$, every ratio of the chain and the null cone, so no reading on a pattern's own clocks and rulers registers its value (Theorem 1). A pattern free of every force in $\mathbf{m}^2 g$ is read by a distant bookkeeper accelerating by $\nabla_{\mathcal{U}} \mathcal{U}^\mu = -(c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu) \partial_\nu \ln \mathbf{m}$, toward slower clocks, and since in the gravitational window $\mathbf{m}$ belongs to the point, every pattern falls alike (Theorem 2). In any static geometry a held pattern reads $c^2 D \ln N$ on its accelerometer and a stroke runs on the optical metric h/N^2 ; the exterior of a source is exactly the clock factor $N^2 = 1 - \alpha_{\text{ext}}$ times the optical spacetime of generalized Snell's law (Theorem 3). A pattern released from rest falls in the optical geometry at exactly GM/r^2 ; a held pattern reads the hold, $GM/(r^2 N)$; and a stroke, blind to the clock factor, is bent and delayed by the optical metric alone, twice the slow reading: the factor of two in the bending and the delay of light is the principle read on a stroke. The bookkeeper's reading is a flow (Theorem 4): patterns falling from rest far away move inward at $v = c\sqrt{\alpha_{\text{ext}}}$, keep the flow's time on flat slices, and fall at exactly GM/r^2 . Every clock keeps the flow's time at the inverse of its Lorentz factor against the flow passing it (Theorem 5). A held clock moves through the flow at v ; its strand runs $v^2 + u^2 = c^2$, and it keeps time at $u/c = \sqrt{1 - \alpha_{\text{ext}}}$, the lapse. Gravitational time dilation is the kinematic dilation of a clock's speed through the flow, and an orbiting clock's two dilations are one, $\sqrt{1 - 3\alpha_{\text{ext}}/2} = 1/(\gamma_{\text{flow}}\gamma_{\text{orbit}})$: $+38.6 \mu\text{s}$ per day for the clocks of GPS. Curvature survives only between neighbours, as the tide $(c^2/2)(\alpha_{\text{ext}}/r_f^2) \text{diag}(2, -1, -1)$ (Theorem 6). In a medium of perfect form the clock factor is the temperature, and Tolman and Ehrenfest's $TN = \text{const}$ follows. The record agrees: free fall is universal to $\eta = (-1.5 \pm 2.3 \pm 1.5) \times 10^{-15}$; the redshift of clock comparisons holds from a millimetre to 10 000 km and on eccentric Galileo orbits to 2.5×10^{-5} ; GPS runs at its predicted rate; and light bends with $\gamma = 1$ to 1.2×10^{-4} by VLBI and is delayed with $\gamma = 1$ to 2.3×10^{-5} by the Cassini link. A receipt script checks the displayed identities and numbers: 47/47 pass.

1 Einstein's principle, extended

Galileo and Newton found that every body falls alike, and Eötvös measured it with torsion balances [5]. In 1907 Einstein read the fact as an equivalence: a laboratory in uniform acceleration and a laboratory at rest in a uniform field cannot be told apart by any experiment inside them [6]. From it he predicted that clocks run slower lower in a field and that light bends near the Sun [7], and the field equations of 1915 made gravity the geometry of spacetime, with a deflection twice the value of 1911 [8]. The principle Einstein left is local. In a small enough laboratory the field vanishes; what remains, the tide between neighbouring falls, is curvature.

Fundamental Density Theory extends the principle (core 0.1c):

Moving at constant speed, on the pattern's own clocks and rulers, through a density gradient is gravity.

The extension rests on the chain

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t} \right)^2, \quad (1)$$

one identity read by quantum mechanics, electromagnetism and general relativity (0.1). Its terms adjust together, and clocks and rulers are closed patterns of the one medium whose kept time and thread length move together so that d/t measured on the spot stays c (0.1a). General relativity is the chain kept, state by state: the metric's response to its source is the pattern adjusting its own dimensions, and there is no background metric that clocks and rulers merely read. A pattern crossing a density gradient therefore carries no reading of it. The distant bookkeeper reads the pattern accelerating, slowing down and bending.

This paper derives the identities of the principle from the chain and reads them against the record, on the locked core v6.2 [1] and the companion derivation of c as the strand rate [2]. Section 3 proves the pattern's reading, Section 4 the bookkeeper's. Section 5 splits every static geometry into a clock factor and an optical geometry, and the exterior of a source exactly, which separates slow patterns from strokes and gives the factor of two in the bending and the delay of light. Section 6 writes the bookkeeper's reading as a flow and shows that every clock's rate is its speed through it. Section 7 reads the curvature between neighbours, Section 8 the clock factor of a medium, and Section 9 the strand beneath it all. The record, the tests, the declined readings and the convergence statement close the paper. The receipt script of Appendix B checks the displayed identities and numbers; check labels appear in brackets, [G05a].

2 Premises, charts and symbols

The derivation uses the locked core v6.2 [1]; each premise is quoted by section.

- (P1) *One substance, only patterns.* There is one substance, the omnium, and there are only its patterns; every physical length and time belongs to a pattern, to a pattern-to-probe measurement, or to the floor (0.1, 0.1b).
- (P2) *One state.* The terms of (1) adjust together, with the bridges $E_q = m_q c^2 = \hbar \omega_q$, $T_q = h/E_q$ and $\ell_q = cT_q$. Clocks and rulers are closed patterns whose kept time and thread length move together so that d/t on the spot stays c ; general relativity is the chain kept, state by state (0.1a).

- (P3) *The strand rate.* Clocks and rulers are closed strokes, so the rate measured on the spot is a ratio of two of their scales. A closed pattern's strand runs at c on a helix, translating v and circulating u with $v^2 + u^2 = c^2$, and keeps time by circulating, $c d\tau = u dt$ (0.1d).
- (P4) *Two readings.* The geometry of a pattern adjusts to the density gradient it crosses, and all terms of the chain adjust collectively to keep its ratios. The distant bookkeeper reads the pattern accelerating, slowing down and bending; on the pattern's own clocks and rulers the physics is unchanged. Moving at constant speed, on them, through a density gradient is gravity. In the gravitational window the clock factor belongs to the density crossed and the geometry is one for every pattern, so every pattern at a point, whatever it is made of, rescales by the same $\mathfrak{m}$ (0.1c, I-3); a medium's own clock factor, its temperature, is read in the medium's window (§8).
- (P5) *The Alpha Lock.* On the exterior branch $\alpha_{\text{ext}}(r_f) = R_s/r_f$, with $R_s = 2G\mathcal{M}/c^2 = \mathcal{M}c^2/(2F_{\text{max}})$ and $F_{\text{max}} = c^4/4G$; $0 < \alpha_{\text{ext}} < 1$, the ends limits and never states; on the A-1 chart the seam is $r_f = 2R_s$ (0.2, 0.3, 0.3a, I-6).
- (P6) *The geometry.* $G_{\mu\nu} = \kappa T_{\mu\nu}$ stands unchanged, $\kappa = 8\pi G/c^4$, $\kappa F_{\text{max}} = 2\pi$; the exact optical metric of the exterior is $d\ell_{\text{opt}}^2 = dr_f^2/(1 - \alpha_{\text{ext}})^2 + r_f^2 d\Omega^2/(1 - \alpha_{\text{ext}})$, and generalized Snell's law is its Fermat law. For the same source and data FDT and GR return the same curvature (I-3). Curvature is read between neighbours (I-3a).
- (P7) *One force law.* $F = F_{\text{max}} \alpha_1 \alpha_2$ with each body's exterior input at the separation, $\alpha_i(d) = R_{s,i}/d$, exactly $G\mathcal{M}_1\mathcal{M}_2/d^2$ on two separate spherical supports at monopole order (I-1, I-4).
- (P8) *Density equivalence.* Every channel of a source's energy enters $\mathcal{M}$ once and sets R_s ; temperature and pressure are also the clock factor of a medium's patterns (0.5, I-7).

Charts. One density coordinate appears: $\alpha_{\text{ext}} = R_s/r_f$, application A-1 (compactness), exterior branch $\sigma = +1$, for a static spherical source of total gravitating energy $\mathcal{M}$ at monopole order, with $x \equiv r_f/R_s$. Its branch tag is fixed throughout; the radius r of the displayed metrics is this r_f . The I-1 force inputs $\alpha_i(d) = R_{s,i}/d$ are the same chart's readings of each body at the separation. The clock factor $\mathfrak{m}$, the lapse $N = \sqrt{1 - \alpha_{\text{ext}}}$, the flow speed $v = c\sqrt{\alpha_{\text{ext}}}$ and the temperature of a medium are not an α and occupy no force, optical or density slot. The translation share v^2/c^2 equals α_{ext} for a pattern held on the exterior branch and for no other (0.1d). The flow is read on the exterior branch and never continued across the fold (I-3).

symbol	meaning
$\mathbf{m}$	clock factor: the common rescaling of a pattern's kept time and thread length (0.1c)
$\mathcal{U}, \tau$	four-velocity and proper time in the metric in use, $g_{\mu\nu}\mathcal{U}^\mu\mathcal{U}^\nu = -c^2$; in Theorem 2 that metric is the bookkeeper's g , and the pattern's kept time is $\mathbf{m}\tau$
$\mathfrak{P}^{\mu\nu}$	$c^2g^{\mu\nu} + \mathcal{U}^\mu\mathcal{U}^\nu$, the projector across the path
N, h, D	lapse and spatial metric of a static geometry; D the derivative of h ; for the exterior $N^2 = 1 - \alpha_{\text{ext}}$
$d\ell_{\text{opt}}^2$	optical metric h/N^2 ; for the exterior, that of I-3; ℓ_{opt} optical length
$v, t_{\text{own}}, \mathbf{V}$	flow speed $c\sqrt{\alpha_{\text{ext}}}$; the time kept by patterns falling from rest far away; the flow's velocity on its flat slices, $\mathbf{V} = -v \hat{\mathbf{r}}$
$\mathbf{w}, \gamma_{\text{rel}}$	a clock's velocity on the flat slices; its Lorentz factor against the flow passing it
u	circulation of a strand; for a held pattern $u = c\sqrt{1 - \alpha_{\text{ext}}}$
γ_v, b_v	Lorentz factor and speed fraction relative to held patterns
v_{orb}	orbital speed of a circular orbit as held patterns read it
γ	in the record and the tests, the post-Newtonian parameter: a stroke's bending and delay are $1 + \gamma$ times the slow reading
$\mathfrak{K}_{\text{in}}, \mathfrak{K}_{\text{out}}$	sectional curvatures of the optical metric (I-3a)
$T, P, E/V$	temperature, pressure and energy density of a medium
W_0	potential of the geoid, $6.263\,685\,60 \times 10^7 \text{ m}^2 \text{ s}^{-2}$ [23]

3 The pattern's reading

Definition 1 (Constant speed on a pattern's own clocks and rulers). A pattern moves at constant speed on its own clocks and rulers when no reading on them registers a change in its motion: its accelerometer reads zero. Its path is a geodesic of the geometry its clocks and rulers measure.

Theorem 1 (The common rescaling is invisible). *Under the common rescaling $d \mapsto \mathbf{m}d$, $t \mapsto \mathbf{m}t$, every ratio of the chain is unchanged: $d/t = c$, $E/m = c^2$ and $ET = h$. Written for the metric the rescaling is $g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}$, which keeps the null cone. No reading on a pattern's own clocks and rulers registers the value of $\mathbf{m}$.*

Proof. A pattern's kept period T and thread length ℓ rescale together (0.1a). By the bridges its energy $E = h/T$ and its mass $m = E/c^2$ both scale as $1/\mathbf{m}$, so E/m and ET are unchanged, and so is ℓ/T . [G01a, G01b] A null vector of g is null for $\mathbf{m}^2 g$. [G01c] Every reading a pattern makes is a ratio of its own scales, or of a stroke against them, and the clock is made of the thing it times (0.1d): every such ratio is unchanged. $\square$

A pattern therefore carries no reading of the density it crosses. The value of $\ln \mathbf{m}$ at a point is invisible; its gradient is removed by riding with the pattern; only its second derivatives, compared between neighbours, survive (§7).

4 The bookkeeper's reading

Theorem 2 (The acceleration law). *Let a pattern move at constant speed on its own clocks and rulers in the geometry $\mathbf{m}^2 g$. Read by the bookkeeper on g , with four-velocity $\mathcal{U}$ normalized by g and proper time τ of g , it accelerates by*

$$\nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathbf{m}, \quad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu. \quad (2)$$

Proof. Write $\varphi = \ln \mathbf{m}$. The connection of $\mathbf{m}^2 g$ is that of g plus

$$\tilde{\Gamma}_{\alpha\beta}^\mu - \Gamma_{\alpha\beta}^\mu = \delta_\alpha^\mu \partial_\beta \varphi + \delta_\beta^\mu \partial_\alpha \varphi - g_{\alpha\beta} g^{\mu\nu} \partial_\nu \varphi. \quad [\text{G02a}] \quad (3)$$

The time kept in $\mathbf{m}^2 g$ is $d\tilde{\tau} = \mathbf{m} d\tau$, so the pattern's four-velocity there is $\tilde{\mathcal{U}} = \mathbf{m}^{-1} \mathcal{U}$. Its geodesic equation, $d\tilde{\mathcal{U}}^\mu/d\tilde{\tau} + \tilde{\Gamma}_{\alpha\beta}^\mu \tilde{\mathcal{U}}^\alpha \tilde{\mathcal{U}}^\beta = 0$, multiplied by $\mathbf{m}^2$, reads

$$\frac{d\mathcal{U}^\mu}{d\tau} - (\mathcal{U} \cdot \partial \varphi) \mathcal{U}^\mu + \Gamma_{\alpha\beta}^\mu \mathcal{U}^\alpha \mathcal{U}^\beta + 2(\mathcal{U} \cdot \partial \varphi) \mathcal{U}^\mu - g(\mathcal{U}, \mathcal{U}) g^{\mu\nu} \partial_\nu \varphi = 0, \quad (4)$$

and with $g(\mathcal{U}, \mathcal{U}) = -c^2$ this is (2). A geodesic of $\mathbf{m}^2 g$ integrated numerically and read on g follows (2) to 10^{-6} . [G02b] $\square$

Corollary 1 (The extended equivalence principle). *A pattern moving at constant speed on its own clocks and rulers through a density gradient is read by the bookkeeper accelerating toward smaller $\mathbf{m}$, where its clocks run slower, and on its own clocks and rulers nothing changes. The acceleration (2) contains $\mathbf{m}$ and the pattern's four-velocity and nothing else, and in the gravitational window $\mathbf{m}$ belongs to the point (P_4), so every pattern at a point, whatever it is made of, falls alike. Moving at constant speed, on the pattern's own clocks and rulers, through a density gradient is gravity.*

The clock factor has three readings. Its value at a point is invisible: clocks and rulers adjust, and d/t on the spot stays c (Theorem 1). Its gradient is the free fall of one pattern, removed by riding with the pattern (Theorem 2). Its second derivatives are the deviation of neighbouring patterns, removed by no choice (Theorem 6).

5 Static geometries: clock factor and optical geometry

Proposition 1 (A static geometry). *Let $ds^2 = -N^2 c^2 dt^2 + h_{ij} dx^i dx^j$ be static, with N and h independent of t .*

- (a) *A pattern held at rest reads on its accelerometer $a = c^2 D \ln N$, of magnitude $c^2 |D \ln N|_h$.*
- (b) *$ds^2 = N^2 [-c^2 dt^2 + d\ell_{\text{opt}}^2]$ with $d\ell_{\text{opt}}^2 = h/N^2$: strokes run on the geodesics of $d\ell_{\text{opt}}^2$, Fermat's principle, and a held reader at x_o reads a stroke from a held emitter at x_e at $\omega_o/\omega_e = N(x_e)/N(x_o)$.*

Proof. (a) A pattern held at rest has the four-velocity $\mathcal{U} = N^{-1} \partial_t$, and its acceleration Γ_{tt}^i/N^2 is $c^2 h^{ij} \partial_j \ln N$. [G03b] (b) The factorization is an identity, and null curves are unchanged by a conformal factor, so strokes are null geodesics of the bracket, which are the geodesics of $d\ell_{\text{opt}}^2$. [G03c] The static run keeps the frequency conjugate to t , [G12e] and a held pattern keeps the fraction N of t , $d\tau = N dt$, which gives the ratio. [G12d] $\square$

Theorem 3 (The exterior split). *On the exterior branch the geometry of a static spherical source is exactly the clock factor $N^2 = 1 - \alpha_{\text{ext}}$ times the optical spacetime of I-3:*

$$ds^2 = -N^2 c^2 dt^2 + \frac{dr_f^2}{1 - \alpha_{\text{ext}}} + r_f^2 d\Omega^2 = N^2 [-c^2 dt^2 + d\ell_{\text{opt}}^2], \quad [\text{G03a}] \quad (5)$$

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\Omega^2}{1 - \alpha_{\text{ext}}}.$$

- (a) *A pattern released from rest falls in the optical geometry, in the bookkeeper's time, at exactly GM/r_f^2 .*

- (b) A pattern held at r_f reads the hold on its accelerometer, $a = GM/(r_f^2 N)$; at leading order the hold is the I-1 pair with $\alpha_1 = R_s/r_f$ and $\alpha_2 = R_{s,m}/r_f$, $F_{\max}\alpha_1\alpha_2 = GMm/r_f^2$.
- (c) A stroke is blind to the clock factor and runs on the geodesics of $d\ell_{\text{opt}}^2$: generalized Snell's law. It bends by $2R_s/b = 4GM/(bc^2)$ at leading order, twice the R_s/b of a slow pattern taken to c .
- (d) A stroke sent radially by a held clock at r_s to r_o and back returns after

$$\Delta\tau_s = \frac{2N_s}{c} \left[r_o - r_s + R_s \ln \frac{r_o - R_s}{r_s - R_s} \right] \quad (6)$$

of the clock's kept time, twice the optical length over c at the clock's rate. A stroke passing at impact parameter b between points at distances $r_1, r_2 \gg b$ runs an optical length longer than the straight line by $R_s \ln(4r_1 r_2 / b^2)$ at leading order, twice the slow reading: the time delay with $\gamma = 1$.

Proof. The split is the identity $g_{rr} = N^2 (d\ell_{\text{opt}}^2)_{rr}$, $g_{\Omega\Omega} = N^2 (d\ell_{\text{opt}}^2)_{\Omega\Omega}$, Proposition 1 with h the exterior's. [G03a] (a) Theorem 2 with $\mathbf{m} = N$ and g the bracket gives, for a pattern momentarily at rest, $|\nabla_{\mathcal{U}} \mathcal{U}| = c^2 |\nabla \ln N|$ measured with $d\ell_{\text{opt}}^2$:

$$c^2 (1 - \alpha_{\text{ext}}) \frac{d \ln N}{dr_f} = c^2 (1 - \alpha_{\text{ext}}) \frac{\alpha_{\text{ext}}}{2r_f(1 - \alpha_{\text{ext}})} = \frac{c^2 R_s}{2r_f^2} = \frac{GM}{r_f^2}, \quad (7)$$

exactly. [G04] The same number follows from the radial geodesic: $d^2 r_f / d\tau^2 = -GM/r_f^2$ exactly in proper time and areal radius; at the release $d^2 r_f / dt^2 = -(GM/r_f^2)(1 - \alpha_{\text{ext}})$, and the optical length $d\ell_{\text{opt}} = dr_f / (1 - \alpha_{\text{ext}})$ falls at $d^2 \ell_{\text{opt}} / dt^2 = -GM/r_f^2$. [G05a, G05b] (b) Proposition 1(a) with the physical radial ruler gives $c^2 N d \ln N / dr_f = GM/(r_f^2 N)$. [G06] At leading order this is GM/r_f^2 , the I-1 pair per unit mass (0.1d). (c) Proposition 1(b): the clock factor never bends a stroke (0.1c). With the isotropic index $n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2)$ the bending is $\int \nabla_{\perp} n d\ell = 2R_s/b$, while a slow pattern taken to c reads the clock factor alone, R_s/b . [G18] The exact null orbit at $b = 1000R_s$ bends by 2.003×10^{-3} rad, $2R_s/b + (15\pi/16)(R_s/b)^2$. [G19] (d) Along a radial stroke $c dt = dr_f / (1 - \alpha_{\text{ext}}) = d\ell_{\text{opt}}$, and the optical length from r_s to r_o is the bracket of (6); the held clock keeps N_s of t . [G28a] On a straight pass, with the endpoints at distances r_1, r_2 along the line, the excess optical length is $\int (n_{\text{iso}} - 1) d\ell = R_s \int d\ell / r = R_s [\text{arsinh}(r_1/b) + \text{arsinh}(r_2/b)]$, which tends to $R_s \ln(4r_1 r_2 / b^2)$ for $r_1, r_2 \gg b$; a slow pattern taken to c , reading the clock factor alone, $n = 1/N = 1 + \alpha_{\text{ext}}/2 + O(\alpha_{\text{ext}}^2)$, finds half of it. [G28b] $\square$

The factor of two is the principle read on a stroke. A slow pattern reads the clock factor, the lapse; a stroke reads the ratio of the ruler and clock adjustments, the optical metric, and the radial term $(1 - \alpha_{\text{ext}})^{-1}$ of the optical metric supplies the second half (I-3), in the bending and in the delay alike. For the Sun, $2R_s/b$ at the photosphere is $1.75''$. On its own terms the stroke is unchanged by the density it crosses, its rate, its action per cycle and its quantum of angular momentum included; the gradient bends its run, delays it and shifts its frequency for the bookkeeper [3].

Proposition 2 (Held at constant speed). *A pattern moved radially at constant speed $v = b_v c$ relative to held patterns, $\gamma_v = (1 - b_v^2)^{-1/2}$, reads on its accelerometer*

$$\|A\| = \gamma_v c^2 \frac{dN}{dr_f} = \gamma_v \frac{GM}{r_f^2 N}. \quad [\text{G07}] \quad (8)$$

The pattern of Proposition 2 keeps constant speed on the clocks and rulers of held patterns, which are themselves held. It is held too, and its accelerometer reads the hold, enlarged by γ_v ; Appendix A derives (8). The principle's constant speed is the pattern's own (Definition 1), at which the accelerometer reads zero.

6 The flow

Theorem 4 (The flow). *With $dt_{\text{own}} = dt + v dr_f / (c^2(1 - \alpha_{\text{ext}}))$ and $v = c\sqrt{\alpha_{\text{ext}}}$, the exterior reads exactly*

$$ds^2 = -c^2 dt_{\text{own}}^2 + (dr_f + v dt_{\text{own}})^2 + r_f^2 d\Omega^2. \quad [\text{G08}] \quad (9)$$

- (a) *Patterns falling from rest far away move inward at v , $dr_f/dt_{\text{own}} = -v$, and keep the flow's time: $ds^2 = -c^2 dt_{\text{own}}^2$ along them.*
- (b) *Each slice of constant t_{own} is flat, $dr_f^2 + r_f^2 d\Omega^2$: the whole curvature sits in the flow.*
- (c) *The flow falls at exactly GM/r_f^2 : $d^2 r_f / dt_{\text{own}}^2 = -GM/r_f^2$.*
- (d) *$v < c$ on the branch and $v \rightarrow c^-$ as $r_f \rightarrow R_s^+$; the flow is never continued across the fold into the interior chart.*

Proof. Substituting $dt = dt_{\text{own}} - v dr_f / (c^2(1 - \alpha_{\text{ext}}))$ into (5) gives (9). [G08] With $dr_f = -v dt_{\text{own}}$ the bracket vanishes. [G09] At constant t_{own} the line element is flat. Along $dr_f/dt_{\text{own}} = -c\sqrt{R_s/r_f}$, $d^2 r_f / dt_{\text{own}}^2 = -c^2 R_s / (2r_f^2) = -GM/r_f^2$. [G10] The limit of v at the fold is c , and the fold is not on the branch (I-6). [G11] $\square$

Equation (9) is Painlevé and Gullstrand's form of the exterior [9, 10], drawn as a river flowing into black holes [11]. The bookkeeper's reading is a flow, present at every size: a laboratory pattern's own window is small and correct (§9), and in a medium the same flow runs on the medium's clock factor (§8). A rotating source adds a swirl, and the frame drag a gyroscope reads is the flow's local rotation, half the curl of the swirl, at leading order the Lense–Thirring precession (0.1c; [4]).

Theorem 5 (Every clock is a speed through the flow). *A clock moving with velocity $\mathbf{w}$ on the flat slices of (9), where the flow's velocity is $\mathbf{V} = -v \hat{\mathbf{r}}$, keeps the flow's time at*

$$\frac{d\tau}{dt_{\text{own}}} = \sqrt{1 - \frac{|\mathbf{w} - \mathbf{V}|^2}{c^2}} = \frac{1}{\gamma_{\text{rel}}}, \quad (10)$$

exactly, γ_{rel} being its Lorentz factor against the flow pattern passing it.

- (a) *A held clock, $\mathbf{w} = 0$, moves through the flow at $v = c\sqrt{\alpha_{\text{ext}}}$, $\gamma_{\text{rel}} = 1/\sqrt{1 - \alpha_{\text{ext}}}$. Its strand runs*

$$v^2 + u^2 = c^2, \quad u = c\sqrt{1 - \alpha_{\text{ext}}}, \quad (11)$$

and it keeps time at $u/c = N$ per unit of the flow's time.

- (b) *Between two held clocks a stroke reads $\omega_o/\omega_s = N_s/N_o$, the ratio of their kept-time rates through the flow.*
- (c) *At the seam, $r_f = 2R_s$, $v = u = c/\sqrt{2}$: the held strand is the 45° strand.*

Proof. On the flat slices of (9), $ds^2 = -c^2 dt_{\text{own}}^2 + |d\mathbf{x} - \mathbf{V} dt_{\text{own}}|^2$, so a clock with $d\mathbf{x}/dt_{\text{own}} = \mathbf{w}$ has $d\tau^2 = dt_{\text{own}}^2 (1 - |\mathbf{w} - \mathbf{V}|^2/c^2)$. [G12a] The flow pattern passing it has four-velocity $(1, \mathbf{V})$, and $-g(\mathcal{U}_{\text{flow}}, \mathcal{U}_{\text{clock}})/c^2 = dt_{\text{own}}/d\tau$: the rate is the inverse of the relative Lorentz factor. [G12b] (a) With $\mathbf{w} = 0$, $|\mathbf{V}| = v$ and $\gamma_{\text{rel}} = 1/N$. The strand of 0.1d splits the rate into translation v and circulation u , and the pattern keeps time by circulating, $c d\tau = u dt_{\text{own}}$, so $d\tau/dt_{\text{own}} = u/c = N$. [G12c] (b) Proposition 1(b). [G12d] (c) At $r_f = 2R_s$, $\alpha_{\text{ext}} = 1/2$. [G13] $\square$

Gravitational time dilation of a held clock is the kinematic dilation of its speed through the flow. A held clock is slow because part of its strand rate goes into translation against the flow.

Corollary 2 (One dilation). *A clock on a circular orbit moves through the flow outward at v , against the inflow, and around at $c\sqrt{\alpha_{\text{ext}}/2}$ on the flat slices, which held patterns read as $v_{\text{orb}} = c\sqrt{\alpha_{\text{ext}}/(2(1 - \alpha_{\text{ext}}))}$. By (10) it keeps time at*

$$\frac{d\tau}{dt_{\text{own}}} = \frac{1}{\gamma_{\text{flow}} \gamma_{\text{orbit}}} = \sqrt{1 - \alpha_{\text{ext}}} \sqrt{1 - \frac{v_{\text{orb}}^2}{c^2}} = \sqrt{1 - \frac{3}{2}\alpha_{\text{ext}}}, \quad [\text{G14}] \quad (12)$$

exactly. Against clocks on the geoid, whose rate the geoid potential W_0 fixes, the Earth’s multipoles and rotation included, a GPS clock on its orbit of semi-major axis 26 561.75 km runs fast by $W_0/c^2 - 3GM_{\oplus}/(2ac^2) = 4.465 \times 10^{-10}$, 38.6 μs per day; orbiting clocks keep the geoid’s rate at $r = 3GM_{\oplus}/(2W_0) = 9546$ km. [G15a, G15b]

The “gravitational” and the “velocity” parts of an orbiting clock’s rate are one effect: the clock’s speed through the flow. Eccentric orbits and free-flying clocks keep the same law (10) along their paths, and for the same source and data their rates are those of general relativity (I-3).

7 Neighbours: the tide

Theorem 6 (Curvature between neighbours). *On the exterior branch, two neighbouring slow patterns, static or falling radially, separate along the radius and converge across it with the exact tide*

$$\nabla_{\mathcal{U}} \nabla_{\mathcal{U}} \mathcal{J} = \frac{c^2}{2} \frac{\alpha_{\text{ext}}}{r_f^2} \text{diag}(2, -1, -1) \mathcal{J} = \frac{GM}{r_f^3} \text{diag}(2, -1, -1) \mathcal{J}, \quad [\text{G20a, G20b}] \quad (13)$$

and neighbouring strokes deviate by the sectional curvatures of the optical metric,

$$\mathfrak{K}_{\text{in}} = -\frac{\alpha_{\text{ext}}}{r_f^2} \left(1 - \frac{3\alpha_{\text{ext}}}{4}\right), \quad \mathfrak{K}_{\text{out}} = +\frac{2\alpha_{\text{ext}}}{r_f^2} \left(1 - \frac{9\alpha_{\text{ext}}}{8}\right), \quad [\text{G21a, G21b}] \quad (14)$$

on planes containing the radius and planes tangent to the spheres. No choice of path removes either.

Proof. The slow tide is the Jacobi operator of the exterior (I-3a). At leading order it is $-\partial_i \partial_j$ of $-GM/r$ at $(r_f, 0, 0)$; [G20a] exactly, the Riemann tensor of the exterior in the frame of a held pattern gives $-R^i_{\hat{0}\hat{j}\hat{0}} = (GM/r_f^3) \text{diag}(2, -1, -1)$, and the same in the frame of a pattern falling radially at any speed. [G20b] The sectional curvatures follow from the Riemann tensor of $d\ell_{\text{opt}}^2$. [G21a, G21b] Along a freely falling path the connection vanishes in Fermi normal coordinates, $g_{\hat{a}\hat{b}} = \eta_{\hat{a}\hat{b}}$ and $\Gamma^{\hat{a}}_{\hat{b}\hat{c}} = 0$ on the path, and the first terms no coordinates remove are $g_{00} = -1 - R_{0i0j} Y^i Y^j + O(|Y|^3)$ [12]: the tide. $\square$

One pattern reads no curvature. A gradiometer, two accelerometers a baseline apart, reads R_{0i0j} ; an extended pattern reads it across its own extent, through its $l = 2$ moment (I-3a). The tide is the third reading of the clock factor, and the only one the principle leaves.

8 The medium’s clock factor

Proposition 3 (Temperature is a medium’s clock factor). *For a medium whose stress tensor has perfect form, at zero net conserved count, the projected conservation law $(E/V + P) \nabla_{\mathcal{U}} \mathcal{U}^{\mu} = -\mathfrak{P}^{\mu\nu} \partial_{\nu} P$ and the Gibbs–Duhem relation $dP = (E/V + P) dT/T$ give*

$$\nabla_{\mathcal{U}} \mathcal{U}^{\mu} = -\mathfrak{P}^{\mu\nu} \partial_{\nu} \ln T \quad (15)$$

for every equation of state $P(T)$. [G22] The medium's patterns move at constant speed on their own clocks and rulers in T^2g , and the pressure force is the bookkeeper's reading of their common rescaling. Held static in a static geometry, a medium accelerates by $c^2 D \ln N$ (Proposition 1), which is $-c^2 D \ln T$, so TN is constant: Tolman and Ehrenfest's equilibrium [13, 14]. [G23]

With a conserved count and a fixed entropy per count the clock factor is the enthalpy per count, $(E + PV)/N_{\text{count}}$. The medium's clock factor is the reading of its flow, never the rate of a clock the medium carries: that clock runs on the lapse where it sits, and its thermal shift is the time dilation of its own thermal motion (0.1c). In a medium the principle runs at the medium's strength; in the gravitational window it runs at the strength of α_{ext} .

9 Sampling, windows and the strand

A pattern crossing the gradient samples it at the rate $\dot{\alpha}_{\text{ext}} = \mathcal{U}^\mu \nabla_\mu \alpha_{\text{ext}}$, which in a stationary field is $\gamma_v v^i D_i \alpha_{\text{ext}}$; on the exterior branch $d \ln \alpha_{\text{ext}}/dt = d \ln \mathcal{M}/dt - d \ln r_f/dt$ (I-6). [G24] Reversing the pass reverses the sampling rate and leaves the clock factor's gradient as it is: the gradient is the gravity, and the sampling rate is the bookkeeper's count of how fast the pattern crosses it.

A pattern's own gravitational window is $\alpha_{\text{ext}} = \mathcal{M}c^2/(2F_{\text{max}}r_f)$, which needs no numerical G . For 50 GeV at 1 fm it is 1.3×10^{-37} ; [G25] for the Earth at its surface it is 1.39×10^{-9} . [G17] The small numbers are correct readings (I-1).

The strand carries the principle down to the pattern. A closed pattern's strand runs at c ; with $\mathcal{U}$ the pattern's four-velocity and e a unit vector turning about the run, orthogonal to $\mathcal{U}$, the strand's direction $K = \mathcal{U}/c + e$ is null, its average over the circulation is the four-velocity, $c\langle K \rangle = \mathcal{U}$, and read by the bookkeeper it translates v and circulates c/γ with $v^2 + (c/\gamma)^2 = c^2$. [G27a-c] A common rescaling enlarges the strand's triangle and keeps its shape, the ratios v/c and u/c of the pattern's own readings: Theorem 1 at the scale of one strand. For a pattern held on the exterior branch the triangle's angle is fixed by the flow, $v/c = \sqrt{\alpha_{\text{ext}}}$, and the translation share is α_{ext} itself (0.1d): at the seam the held strand stands at 45° , and toward the fold its circulation, its kept time, tends to zero and is never reached.

10 The record and the tests

what the record shows	the measurement	where it is returned
every pattern falls alike	MICROSCOPE: titanium and platinum alloys in orbit, $\eta = [-1.5 \pm 2.3 \text{ (stat)} \pm 1.5 \text{ (syst)}] \times 10^{-15}$ [15]	Corollary 1; (P4)
held clocks read the flow	Pound and Rebka, 22.5 m, $gh/c^2 = 2.46 \times 10^{-15}$, confirmed to 1% by Pound and Snider [16, 17]; [G16c] aluminium-ion clocks 33 cm apart [18]; [G16b] across a millimetre of one atomic sample, $(9.8 \pm 2.3) \times 10^{-20}$ per mm against $gh/c^2 = 1.09 \times 10^{-19}$ per mm [19] [G16a]	Theorem 5(a)–(b)
moving clocks read the flow	Gravity Probe A’s maser on its flight to 10 000 km, where the held-clock shift is 4.25×10^{-10} , agreeing at the 7×10^{-5} level [20]; [G16d] Galileo satellites on eccentric orbits, $(0.19 \pm 2.48) \times 10^{-5}$ [21]	Theorem 5; containment along the path (I-3)
an orbiting clock’s two dilations are one	GPS clocks run fast against the geoid by 4.465×10^{-10} , 38.6 μ s per day [22]	Corollary 2
a stroke reads the optical metric	bending: $\gamma = 0.99992 \pm 0.00012$ by VLBI [24]; delay: $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ by the Cassini link [25]	Theorem 3(c), (d)
the flow swirls about a rotating source	Gravity Probe B: frame drag -37.2 ± 7.2 mas per year against -39.2 ; geodetic precession -6601.8 ± 18.3 against -6606.1 [26]	containment (I-3); the swirl of 0.1c [4]

Each test below is the converse of a derived result: the measurement that would end it.

- T-1 *Universality.* A composition-dependent free fall, a nonzero η , ends the reading of the gravitational clock factor as the point’s, one **m** for every pattern at a point (P4), and with it Corollary 1.
- T-2 *Every clock is a speed through the flow.* A clock comparison whose rates depart from (10) along the clocks’ paths, beyond the source’s multipoles (D-02), held clocks reading other than N_s/N_o among them, ends Theorem 5.
- T-3 *One dilation.* An orbiting clock whose rate departs from $\sqrt{1 - 3\alpha_{\text{ext}}/2}$ against the flow’s time, beyond the source’s multipoles, ends Corollary 2.
- T-4 *The factor of two.* A stroke bent or delayed by other than twice the slow reading at leading order, $\gamma \neq 1$, ends Theorem 3(c)–(d).
- T-5 *The swirl.* A frame drag other than half the curl of the flow’s swirl ends the swirl reading of 0.1c.

Three pieces sit outside the derivation and leave every result in it unchanged:

- D-01 *The interior chart.* The flow reading stops at the fold; the kinematic reading of the share on the interior chart is to be derived (core O-16).
- D-02 *Beyond the monopole.* Multipole and tidal remainders for nonspherical and rotating sources, and the band between a rotating source’s horizon and R_s , are to be derived (core O-5).
- D-03 *The aperture.* Strokes crossing closed patterns bend by the aperture of the density cone, whose constitutive law is to be derived (core O-8).

11 Declined readings and convergence

The standard account makes gravity a force, or the curvature of a spacetime that clocks and rulers read as a background. FDT reads motion at constant speed, on a pattern's own clocks and rulers, through a density gradient: clocks and rulers are closed patterns adjusting with the density they cross, and the bookkeeper reads their common rescaling as acceleration (0.1a, 0.1c, Theorem 2).

The standard account confines the equivalence principle to uniform fields in small laboratories. FDT reads it along every path through every gradient, exactly; what no choice removes is the tide between neighbours, which is the curvature (Theorem 6, I-3a).

The standard account counts gravitational and velocity time dilation as two effects. FDT reads one: a clock's speed through the omnium's flow (Theorem 5, Corollary 2).

The standard account treats the flow form of the exterior as a choice of coordinates, drawn as a river flowing into black holes. FDT reads the flow of the geometry, present at every size, with the frame drag as its swirl (0.1c, Theorem 4).

The standard account pushes a fluid by pressure forces across a fixed background. FDT reads the medium's patterns moving at constant speed on clocks and rulers rescaled by its temperature, with the pressure force the bookkeeper's reading (Proposition 3, 0.1c).

Convergence. For the same sources and data FDT returns: the universality of free fall, with the MICROSCOPE bound; the gravitational redshift of every clock comparison, held or moving, from a millimetre to 10 000 km and on eccentric orbits; the rates of orbiting clocks, GPS among them; the deflection and the time delay of light with $\gamma = 1$; the tides read by gradiometers and by extended bodies; the geodetic and frame-dragging precessions of Gravity Probe B, by containment; and Tolman and Ehrenfest's thermal equilibrium. The curvature is that of general relativity for the same source and data (I-3, containment); the pair force is exact on its domain, $F_{\max}\alpha_1\alpha_2 = GM_1M_2/d^2$ (I-1, I-4 layer L1), and general relativity reduces to the same law in the weak-field, slow-motion limit (I-4 layer L2). No divergence is claimed: the paper supplies what gravity is, not a changed observable.

12 The extended equivalence principle

Clocks and rulers are made of the medium they measure. When a pattern crosses a density gradient its kept time and thread length rescale together, every ratio of the chain stays as it was, and on its own clocks and rulers nothing happens. The distant bookkeeper, reading on clocks and rulers of his own, sees the pattern accelerate toward slower clocks, the same for every pattern. That reading is a flow: around a source the omnium's patterns fall inward at $c\sqrt{\alpha_{\text{ext}}}$, and the flow falls at exactly GM/r^2 . Every clock moves through the flow and keeps time at the inverse of its Lorentz factor against it; a clock held against the flow keeps time by the share of its strand that still circulates, which is the lapse. A stroke, which keeps no time, is blind to the clock factor and is bent and delayed by the optical metric alone, twice as much. What no choice removes is the tide between neighbours. Moving at constant speed, on a pattern's own clocks and rulers, through a density gradient is gravity.

13 Status register

result	rests on	status
The common rescaling is invisible (Theorem 1)	0.1a, 0.1d	derived
The acceleration law (2) (Theorem 2)	0.1c	derived
Universality of free fall (Corollary 1)	Theorem 2, (P4)	derived
A static geometry: the held accelerometer $c^2 D \ln N$; the optical metric h/N^2 ; the frequency ratio (Proposition 1)	0.1c, I-3	derived, exact
The exterior split; fall at GM/r^2 in the optical geometry; the hold; the factor of two in bending and delay; the radar time (Theorem 3)	I-3, 0.1c	derived, exact on the exterior branch; the bending and the pass delay at leading order
Held at constant speed (Proposition 2)	I-3	derived, exact
The flow (Theorem 4)	0.1c, I-3	derived, exact on the exterior branch
Every clock is a speed through the flow; held clocks (Theorem 5)	0.1c, 0.1d	derived, exact
One dilation for an orbiting clock (Corollary 2)	Theorem 5	derived, exact; GPS from the record
The tide between neighbours (Theorem 6)	I-3a	derived, exact on the exterior branch
Temperature as a medium's clock factor; $TN = \text{const}$ (Proposition 3)	0.1c, I-7	derived for a stress tensor of perfect form
Sampling rate; branch-rate identity on the exterior branch	0.3, I-6	derived
The strand as a null direction averaging to $\mathcal{U}$	0.1d	derived
The kinematic reading of the share on the interior chart	0.1d, 0.3a	to be derived (D-01; core O-16)
Multipole and tidal remainders; the band outside a rotating horizon	I-1, 0.4	to be derived (D-02; core O-5)
The aperture of the density cone	P-4	to be derived (D-03; core O-8)

invariant	where it enters
I-1	the hold on a held pattern is the pair $F_{\max} \alpha_1 \alpha_2 = GMm/r_f^2$ at leading order, with the exterior inputs $\alpha_i = R_{s,i}/r_f$; a laboratory pattern's small window is a correct reading
I-2	the stroke, one carrier, blind to the clock factor; its bending and delay by the optical metric
I-3	the field equations unchanged; the exterior split (5); the flow form (9); generalized Snell's law on the optical metric; containment for eccentric orbits, free-flying clocks and the swirl's precessions
I-4	the pair face exact on its domain (layer L1); general relativity's weak-field, slow-motion reduction to the same law (layer L2); one statement read in different limits
I-5	clocks and rulers as closed patterns whose strands keep time by circulating
I-6	the open interval: $v < c$ on the branch, the fold a limit; the branch-rate identity
I-7	every channel of a source's energy sets R_s ; temperature and pressure as a medium's clock factor

A The accelerometer of a pattern held at constant speed

Set $c = 1$ and restrict the exterior to the radial plane, $ds^2 = -N^2 dt^2 + N^{-2} dr^2$, with $N' = dN/dr$. A pattern moving radially at constant speed b_v relative to held patterns has $\mathcal{U}^t = \gamma_v/N$ and $\mathcal{U}^r = \gamma_v b_v N$. The nonzero Christoffel symbols are $\Gamma_{tr}^t = N'/N$, $\Gamma_{tt}^r = N^3 N'$ and $\Gamma_{rr}^r = -N'/N$,

and $d/d\tau = \mathcal{U}^r d/dr$ along the path, so

$$A^t = \frac{\gamma_v^2 b_v N'}{N}, \quad A^r = \gamma_v^2 N N', \quad A_\mu A^\mu = \gamma_v^4 (1 - b_v^2) N'^2 = \gamma_v^2 N'^2. \quad (16)$$

Restoring c , $\|A\| = \gamma_v c^2 N'$, and with $N = \sqrt{1 - 2GM/(c^2 r)}$, $c^2 N' = GM/(r^2 N)$, which is (8). [G07]

B Receipt

The script `receipt_eep.py`, in the folder of this paper, checks the displayed identities symbolically with SymPy and the displayed numbers numerically, with an integrated geodesic for the acceleration law, the Riemann tensors of the exterior and of the optical metric for the tide and the sectional curvatures, and a high-precision null orbit for the deflection. Its output is `receipt_output.txt`; run it with `python3 receipt_eep.py`. Result: **47/47 PASS**.

check	content
G01a–c	the common rescaling keeps d/t , E/m and ET , and the null cone
G02a–b	the connection of $\mathfrak{m}^2 g$ (3); an integrated geodesic of $\mathfrak{m}^2 \eta$ read on η follows (2)
G03a–c	the exterior split (5); in any static geometry, with N and h general, the held accelerometer reads $c^2 h^{ij} \partial_j \ln N$, of magnitude $c^2 D \ln N _h$; the factorization $N^2 [-c^2 dt^2 + h/N^2]$ and its null cone
G04	at rest in the optical geometry the fall is exactly GM/r^2
G05a–b	$d^2 r/d\tau^2 = -GM/r^2$; at the release $d^2 r/dt^2 = -(GM/r^2)(1 - \alpha_{\text{ext}})$ and $d^2 \ell_{\text{opt}}/dt^2 = -GM/r^2$
G06	the held accelerometer reads $GM/(r^2 N)$
G07	held at constant speed: $\gamma_v c^2 N' = \gamma_v GM/(r^2 N)$
G08–G11	the flow form; the falling patterns keep the flow's time; the fall GM/r^2 ; $v \rightarrow c$ at the fold
G12a–e	every clock keeps $\sqrt{1 - \mathbf{w} - \mathbf{V} ^2/c^2}$ of the flow's time; that is the inverse of its Lorentz factor against the flow; a held clock: $\gamma = 1/N$, $v^2 + u^2 = c^2$, $u/c = N$; $\omega_o/\omega_s = N_s/N_o$; k_t conserved along an integrated null geodesic
G13	$v = u = c/\sqrt{2}$ at the seam
G14	$1/(\gamma_{\text{flow}} \gamma_{\text{orbit}}) = \sqrt{1 - 3\alpha_{\text{ext}}/2}$, from (10) on a circular orbit
G15a–b	GPS: 4.465×10^{-10} , $38.6 \mu\text{s}$ per day; the geoid's rate at 9546 km
G16a–d	1.09×10^{-19} per mm; 3.6×10^{-17} at 33 cm; 2.46×10^{-15} at 22.5 m; 4.25×10^{-10} at 10 000 km
G17	the Earth's window at its surface, 1.39×10^{-9}
G18–G19	$2R_s/b$ for a stroke from n_{iso} , R_s/b for a slow pattern at c from $n = 1/N$; the exact null orbit at $b = 1000R_s$, 2.003×10^{-3} rad
G20a–b	the slow tide $(GM/r^3) \text{diag}(2, -1, -1)$: at leading order from $-\partial\partial\Phi$; exactly from the Riemann tensor of the exterior in the held frame and in radially falling frames
G21a–b	$\mathfrak{K}_{\text{in}}$ and $\mathfrak{K}_{\text{out}}$ from the Riemann tensor of $d\ell_{\text{opt}}^2$
G22–G23	the medium's acceleration $-c^2 \nabla \ln T$; $TN = \text{const}$
G24	the branch-rate identity on the exterior branch
G25	50 GeV at 1 fm: 1.3×10^{-37}
G26	$\kappa F_{\text{max}} = 2\pi$; $R_s = 2GM/c^2 = \mathcal{M}c^2/(2F_{\text{max}})$
G27a–c	the strand direction is null, averages to $\mathcal{U}$, and runs at c
G28a–b	the radial optical length and the radar time (6), $2N_s/c$ times it; the excess optical length of a pass, $R_s \ln(4r_1 r_2/b^2)$, half of it for a slow pattern

References

- [1] Fundamental Density Theory, *Core FDT Invariants*, v6.2 (author lock 2026-10-19).

- [2] M. Alfaro, “What c is: a geometric derivation of the space–time and matter–energy conversion,” Fundamental Density Theory (18 October 2026), densitytheory.com.
- [3] M. Alfaro, “The quintuple identity: photon = graviton = gluon = phonon = magnon,” Fundamental Density Theory (24 October 2026), densitytheory.com.
- [4] M. Alfaro, “The omnium vortex,” Fundamental Density Theory (17 October 2026), densitytheory.com.
- [5] R. v. Eötvös, D. Pekár and E. Fekete, “Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität,” *Annalen der Physik* **68** (1922) 11–66.
- [6] A. Einstein, “Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen,” *Jahrbuch der Radioaktivität und Elektronik* **4** (1907) 411–462.
- [7] A. Einstein, “Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes,” *Annalen der Physik* **35** (1911) 898–908.
- [8] A. Einstein, “Die Feldgleichungen der Gravitation,” *Sitzungsberichte der Preussischen Akademie der Wissenschaften* (1915) 844–847.
- [9] P. Painlevé, “La mécanique classique et la théorie de la relativité,” *Comptes Rendus de l’Académie des Sciences* **173** (1921) 677–680.
- [10] A. Gullstrand, “Allgemeine Lösung des statischen Einkörperproblems in der Einsteinschen Gravitationstheorie,” *Arkiv för Matematik, Astronomi och Fysik* **16** (8) (1922) 1–15.
- [11] A. J. S. Hamilton and J. P. Lisle, “The river model of black holes,” *American Journal of Physics* **76** (2008) 519–532.
- [12] F. K. Manasse and C. W. Misner, “Fermi normal coordinates and some basic concepts in differential geometry,” *Journal of Mathematical Physics* **4** (1963) 735–745.
- [13] R. C. Tolman, “On the weight of heat and thermal equilibrium in general relativity,” *Physical Review* **35** (1930) 904–924.
- [14] R. C. Tolman and P. Ehrenfest, “Temperature equilibrium in a static gravitational field,” *Physical Review* **36** (1930) 1791–1798.
- [15] P. Touboul *et al.* (MICROSCOPE Collaboration), “MICROSCOPE mission: final results of the test of the equivalence principle,” *Physical Review Letters* **129** (2022) 121102.
- [16] R. V. Pound and G. A. Rebka, “Apparent weight of photons,” *Physical Review Letters* **4** (1960) 337–341.
- [17] R. V. Pound and J. L. Snider, “Effect of gravity on gamma radiation,” *Physical Review* **140** (1965) B788–B803.
- [18] C. W. Chou, D. B. Hume, T. Rosenband and D. J. Wineland, “Optical clocks and relativity,” *Science* **329** (2010) 1630–1633.
- [19] T. Bothwell *et al.*, “Resolving the gravitational redshift across a millimetre-scale atomic sample,” *Nature* **602** (2022) 420–424.
- [20] R. F. C. Vessot *et al.*, “Test of relativistic gravitation with a space-borne hydrogen maser,” *Physical Review Letters* **45** (1980) 2081–2084.
- [21] P. Delva *et al.*, “Gravitational redshift test using eccentric Galileo satellites,” *Physical Review Letters* **121** (2018) 231101.
- [22] N. Ashby, “Relativity in the Global Positioning System,” *Living Reviews in Relativity* **6** (2003) 1.

- [23] G. Petit and B. Luzum (eds.), *IERS Conventions (2010)*, IERS Technical Note 36 (Verlag des Bundesamts für Kartographie und Geodäsie, Frankfurt am Main, 2010).
- [24] S. B. Lambert and C. Le Poncin-Lafitte, “Improved determination of γ by VLBI,” *Astronomy & Astrophysics* **529** (2011) A70.
- [25] B. Bertotti, L. Iess and P. Tortora, “A test of general relativity using radio links with the Cassini spacecraft,” *Nature* **425** (2003) 374–376.
- [26] C. W. F. Everitt *et al.*, “Gravity Probe B: final results of a space experiment to test general relativity,” *Physical Review Letters* **106** (2011) 221101.