

The gallium anomaly as a comparison of reclosure weights

FDT analysis

Target: M. Cadeddu, N. Cargioli, F. Dordei,
L. Ferro, C. Giunti and M. Pitzalis,
“Possible Solution to the Gallium Anomaly Moving beyond
the Leptonic Wave-Function Factorization,”
Physical Review Letters **137**, 131805 (2026).
DOI: 10.1103/5pvf-mcr1; published 24 September 2026.
An arXiv identifier for the target is not printed in the target paper.

Abstract

Cadeddu et al.[1] compare neutrino capture on gallium with electron capture in germanium through the weak transition density integrated with the lepton wave functions of each process. They find, in-sample, that the revised cross section can be significantly reduced; they state that the transition densities adopted are not unique. Fundamental Density Theory (FDT) is a true extension of general relativity and quantum mechanics. The manuscript’s reading is that capture raises the whole atom and electron capture resettles it: two events, each a many-to-many reclosure of the whole atom settled by its own global optimisation, its energy, mass, kept time and thread length changing together. The manuscript’s reading also identifies the two lepton measurements and an independently normalised hadronic measurement as three profile-weighted means, each of its own event, of the transition density the source writes is the same for electron capture and neutrino capture, “up to a trivial overall phase” (L189–L190). The fitted profile is distinct from the local density of the omnium. A finite-charge-sphere reconstruction exposes the cancellation that magnifies small differences between lepton weights. Independent ground-state charge exchange and energy-dependent capture discriminate the fitted profiles. There are zero prospective predictions beyond consensus from the five candidate divergences: each lacks its specified map from the core to an observable. One exploratory observable concerns the fitted nodes and the skin radius that the manuscript’s reading takes from the source’s box radius.

1 Framework

The source is theoretical: `source.status = theoretical`. Measured, observed and reported describe the target paper’s results in their stated scope; derived, estimated, reconstructed and illustrative-assumed distinguish the calculations here. Numerical-input identifiers use U; source loci refer to the physical lines of Cadeddu et al.[1]; the locked core[2] supplies the FDT clauses. The source’s combined fits are in-sample results, including their half-lives and cross sections. The inferred ground-state targets inherit an excited-state subtraction and its normalisation.

1.1 A settlement at every step and its dimensional bridges

Here E is energy, m a confinement-position mass, c the local invariant speed, d a pattern’s length and t its kept time. The symbols $\in$, ∞ and $O(\cdot)$ denote set membership, an unbounded limit and remainder order; mod denotes integer remainder. The response coefficients ε_0 and μ_0 belong to

the omnium; h and $\hbar = h/(2\pi)$ are the action constants, and π is the circle constant. Core 0.1 and 0.1a give

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t}\right)^2. \quad (1)$$

Matter, energy and spacetime are one substance, the omnium. Core 0.1a: “The four terms of the chain adjust simultaneously: the omnium settles matter, energy and spacetime by one global optimisation, never one term at a time.” Core 0.1b: “A nucleus is one pattern; what a probe reconstructs is that pattern’s response, not a distribution of constituents inside a boundary.” For mode label q , let $E_q, m_q, \omega_q, T_q, \ell_q$ denote its energy, mass, angular frequency, period and thread length. Upright m, s, fm, pm, d, keV, MeV, GeV, eV, N denote metre, second, femtometre, picometre, day, the indicated electronvolt units and newton. The SI speed and action conventions are U-01 and U-03[3].

$$E_q = m_q c^2 = \hbar \omega_q, \quad (2)$$

$$T_q = \frac{2\pi}{\omega_q} = \frac{h}{E_q}, \quad \ell_q = c T_q = \frac{h}{m_q c}, \quad (3)$$

$$\frac{d}{t} = c = 299792458 \text{ m s}^{-1} \text{ exactly.} \quad (4)$$

The energy equation has energy dimensions, the period equation time dimensions and the thread equation length dimensions; the frequency bridge explicitly contains $\hbar$. The Compton thread length is distinct from a confinement support fixed by geometry. Quantisation acts on configurations; the exterior chart remains continuous. Pressure is P ; the Planck subscript in the floor below is a label. Let G denote Newton’s constant, F a force and $F_{\max}$ its unattained ceiling. Let $d_{\min}, t_{\min}$ be the existence infima and ℓ_P, t_P the Planck length and time. The floor follows from core 0.2 and Planck’s quantum:

$$F_{\max} = \frac{c^4}{4G}, \quad F_{\max} d_{\min} t_{\min} = \hbar, \quad (5)$$

$$d_{\min} = c t_{\min}, \quad d_{\min}^2 = \frac{\hbar c}{F_{\max}} = 4\ell_P^2, \quad (6)$$

$$d_{\min} = 2\ell_P, \quad t_{\min} = 2t_P. \quad (7)$$

These infima bound strokes without assigning a spacing or an occupied cell. The product $F_{\max} d_{\min} t_{\min}$ has units of action.

1.2 Notation guard

The following census translates source names before their use. Legacy names in the first column are quoted names, not additional meanings of the manuscript’s symbols. Subscripts naming states, shells, neutrino sources and indices are labels, and so are EC for electron capture, IBD for neutrino capture on gallium and GT for Gamow–Teller. The radial integration variable r runs over the nuclear support; field evaluation uses r_f , configuration support uses R_b , and the gravitational radius is R_s . No density coordinate is evaluated on the integration variable r .

Source name	Manuscript symbol	Meaning
cross-section sigma	$\mathcal{S}_{\text{gs}}$	Ground-state cross section; superscripts db and exp distinguish factorisation and inferred target.
excited-state Delta	$\mathfrak{E}_{(p,n)}$	Excited-state fraction; Δ marks a finite change.
significance sigma	standard deviations	Significance written in words.
Dirac kappa	$\varkappa$	Dirac angular quantum number.
Gaussian a,b,c	$\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3$	Widths.
Gaussian centres	r_1, r_2, r_3	Centres; r_c remains a core cutoff.
amplitudes A,B,C	$\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3$	Profile amplitudes; $\mathcal{A}_{2\text{pF}}$ is the Fermi-shape amplitude.
Fermi c,a	$\mathfrak{c}, \mathfrak{d}$	Half-density radius and diffuseness.
transition rho	$\varrho_{\text{TD}}, \varrho_0$	Signed transition profile and Fermi prefactor.
branching B	$\mathcal{B}_j$	Neutrino line fraction for line index j .
neutrino-source X	src	Neutrino-source label.
chi squared	$\mathcal{C}_{\text{IBD}}^2, \mathcal{C}_{\text{EC}}^2$	Least-squares objectives.
radial I	$\mathcal{I}_1, \mathcal{I}_2$	Radial transition integrals.
transition H	$\mathcal{H}_j^{\text{IBD}}, \mathcal{H}_{1s}^{\text{EC}}$	Capture amplitudes; $\hat{\mathcal{O}}_{\text{GT}}$ is the GT operator.
nuclear M	$\mathfrak{M}_{\text{nuc}}^{\text{IBD}}, \mathfrak{M}_{\text{nuc}}^{\text{EC}}$	Nuclear matrix elements.
shell P	$\mathcal{P}_K, \mathcal{P}_L, \mathcal{P}_M$	Shell capture fractions, with their respective bound-electron weights.
Fermi G	$\mathcal{G}_F$	Fermi interaction constant.
CKM V	$\mathcal{V}_{ud}$	Cabibbo–Kobayashi–Maskawa (CKM) matrix element.
axial g	$\mathfrak{g}_A$	Axial coupling.
Fermi F	$\mathcal{F}(E_e, Z, r_0)$	Electron distortion function; E_e is electron energy, Z nuclear charge number, r_0 evaluation point.

The nuclear spin is J , with Ga and Ge labels; p_e and p_ν are electron and neutrino momenta. The half-life is $t_{1/2}$, its experimental uncertainty $\delta t_{1/2}$, and δ otherwise marks the uncertainty of its labelled quantity. The nuisance parameters are η_1, η_2, η_3 , their uncertainties $\delta\eta_1, \delta\eta_2, \delta\eta_3$, and Θ is a profile’s parameter vector.

Source name	Manuscript symbol	Meaning
EC f	$\mathfrak{f}_{\text{EC}}$	Allowed-capture phase-space factor.
neutrino q	p_ν	Momentum; the inverse length is $p_\nu/\hbar$.
figure box radius	R_{room}	Source box radius read as skin radius, the manuscript's reading; R_{ch} is charge rms radius.
atomic Q	Q	Neutral-atom mass difference as energy.
decay lambda	$\Gamma_{\text{EC}}, \bar{t}$	Capture rate and mean life.
Bohr a-zero	r_B	Bohr radius.
fine-structure alpha	$e^2/(4\pi\epsilon_0\hbar c)$	Coupling, with e the elementary charge; ς is Z times this ratio.
magnetic moment	written in words	Nuclear magnetons are the units.
reaction labels	ν_e, e^-, p, n	Reaction names, never extra scalar quantities.
shell K,L,M	K,L,M	Shell labels; l is orbital angular degree.
radial r	r	Integration radius, no atlas evaluation.
hadronic momentum	k_t	Transfer divided by $\hbar$, an inverse length.
cancellation	Υ	Absolute signed integral divided by absolute-weight integral.
hadronic form factor	Ξ_{had}	Fourier–Bessel integral of the signed profile.
probe weights	w_b, w_c	Centre-normalised bound and continuum weights.
hyperfine F	states in words	Hyperfine angular-momentum labels; F stays force.
phase and position	$\phi_h, \phi_0, t_{\text{conf}}$	Helix phase, initial phase and confinement-position argument.
final states	$\mathfrak{F}_i$	Final states of initial state i preserving every exact book (four-momentum, charge, fermion parity, angular momentum, any derived turning class) at or below threshold.

Here g_κ, f_κ are the large and small Dirac radial components, g_{1s}, f_{1s} their bound-state counterparts, and j_0, j_1 the spherical Bessel functions. Nuclear wavefunctions are $\Psi_{\text{Ga}}, \Psi_{\text{Ge}}$; the projected lepton waves are ψ_e, ψ_ν . The bare n labels strand count; σ labels an atlas branch, apart from the explicitly subscripted Stefan–Boltzmann constant σ_{SB} . The Einstein coupling is κ ; X is odds, with X_σ its branch-tagged value; D is the confinement operator; Ω is its support domain. The radiation energy-density constant is a ; B denotes a closure's boundary data. No independent gamma, W, S or L scalar is introduced.

1.3 Charts, force and curvature

The total gravitating energy divided by c^2 is $\mathcal{M}$; a subscript i labels a body. The atlas coordinate α_σ is a ratio of curvatures, with ext and int the branch names. The computational coordinate is β_σ , and α_{core} is the schematic skin-saturation diagnostic. The named overlap, confinement and mode tokens are $\alpha_{\text{ovl}}, \alpha_{\text{conf}}, \alpha_{\text{mode}}$; the chart inventory below declares each application once.

Application	Coordinate	Warrant and use
A-1	$\alpha_{\text{ext}}, \alpha_{\text{int}}$	Geometric: gravitational radius R_s , field point r_f , or declared support evaluation R_b . Exterior nuclear values appear below; interior skin is schematic.
A-2	β_σ	Geometric computational compactification, unused; excluded from force, optical and density slots.
A-3	α_{ovl}	No geometric warrant: token and open range only for overlapping closure and clamp.
A-4	α_{conf}	No geometric warrant: unused token and open range only.
A-5	α_{mode}	No geometric warrant: unused token and open range only.
skin diagnostic	$\alpha_{\text{core}} \rightarrow 1^-$	Interior saturation, the skin value of the nucleus's interior chart (P-3); schematic, with no assigned value.

All three non-geometric tokens inherit an open range without an assigned threshold, order, sign or trajectory; the clock factor of 0.1c is not α : it is no odds, no bounded output and no application of core 0.4, and it never stands in a force, optical or density slot. Core 0.3–0.4 fixes the following A-1 atlas and gap odds, with r_c the finite core boundary:

$$R_s = \frac{2GM}{c^2} = \frac{\mathcal{M}c^2}{2F_{\text{max}}}, \quad (8)$$

$$\alpha_\sigma(r_f) = \left(\frac{R_s}{r_f}\right)^\sigma, \quad \frac{d\alpha_\sigma}{dr_f} = -\frac{\sigma\alpha_\sigma}{r_f}, \quad (9)$$

$$\sigma = +1 : R_s < r_f < \infty, \quad \sigma = -1 : r_c \leq r_f < R_s, \quad (10)$$

$$X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \quad X_{\text{int}} = \frac{r_f}{R_s - r_f}, \quad (11)$$

$$\alpha_\sigma = \frac{X_\sigma}{1 + X_\sigma}, \quad 0 < \alpha_\sigma < 1. \quad (12)$$

All ratios are dimensionless and their radial derivatives have inverse-length dimensions. The exterior and interior approach the fold from their separate sides. Each density cone has a dilute rim and a high-density apex circle; the two cones meet circle to circle at the skin. The interior centre is dilute, and the local profile is densest at the skin (core 0.6). The A-1 seam on each branch is the Clifford torus at $\alpha_\sigma = 1/2$; the fold is the limiting core circle. The geometric repetition is bounded below by the existence floor.

For an A-1 odds enhancement $x \geq 0$, let $X_{\sigma,0}$ and $\alpha_{\sigma,0}$ be the starting odds and bounded coordinate. Core I-6 gives the exact finite update

$$X_\sigma = X_{\sigma,0}(1+x), \quad \alpha_\sigma(x) = \frac{\alpha_{\sigma,0}(1+x)}{1+\alpha_{\sigma,0}x}. \quad (13)$$

The increment is dimensionless. For the mean-density comparison object, define $q_\sigma = \alpha_\sigma^{3\sigma}$, $\bar{\rho}_\zeta$ the constant-shape mean, ρ_s its gravitational-radius scale and ζ its positive shape factor. The identity $\bar{\rho}_\zeta/\rho_s = q_\sigma$ orders that object; the local spatial profile ρ_{loc} is a different object. A signed transition profile belongs to neither object and is never bounded through Eq. (13).

Here α_1, α_2 (generically α_i) mean the body's exterior cross-centre evaluation at separation d . Core I-1 is universal:

$$F = \frac{c^4}{4G} \alpha_1 \alpha_2 < F_{\text{max}}. \quad (14)$$

For nonoverlapping spherical supports at monopole order, $d \geq R_{b,1} + R_{b,2}$ and $R_{b,i} \geq R_{s,i}$. Define $\lambda_i = \mathcal{M}_i/d$ on that domain. The I-4 algebraic face is

$$F = F_{\max} \frac{R_{s,1} R_{s,2}}{d^2} = G \lambda_1 \lambda_2 = \frac{G \mathcal{M}_1 \mathcal{M}_2}{d^2}. \quad (15)$$

The universal open interval, the strict force ceiling, and the pair-domain bound $F \leq F_{\max}/4$ are three distinct statements. Equality in the last requires equal gravitational radii, saturated supports and contact. I-4's first layer is exact pair algebra; its second is GR's leading weak-field, slow-motion force with relative corrections of compactness and squared speed over c^2 ; its third is the FDT source programme and each claimed divergence. Overlapping closure and clamp require the overlap complex in the status ledger, without changing I-1. A rate has inverse-time units and is not the force in Eq. (14).

For the curvature face, $G_{\mu\nu}, T_{\mu\nu}, g_{\mu\nu}$ are the Einstein, stress and metric tensors; μ, ν are tensor indices. Let $d\ell_{\text{opt}}$ denote optical length; ϑ, φ are polar and azimuthal angles, so $d\vartheta^2 + \sin^2 \vartheta d\varphi^2$ is the angular line element. Let $\mathbf{n}_1, \mathbf{n}_2$ be the exact directional optical indices at an interface, θ_1, θ_2 the incidence angles, and $\mathbf{n}_{\text{iso}}$ the isotropic weak-field index. Core I-3 states

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = \frac{8\pi G}{c^4}, \quad (16)$$

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2(d\vartheta^2 + \sin^2 \vartheta d\varphi^2)}{1 - \alpha_{\text{ext}}}, \quad (17)$$

$$\mathbf{n}_1 \sin \theta_1 = \mathbf{n}_2 \sin \theta_2, \quad \mathbf{n}_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2). \quad (18)$$

The tensor equation is unchanged; the branch-tagged coordinate is downstream of its solution. The exact radial and angular optical terms have length-squared dimensions. Both terms supply the full lensing geometry, including the factor of two relative to the slow-pattern reading. The same stress tensor, cosmological terms, initial and boundary data and gauge return the same curvature as GR.

1.4 Clocks, modes and energy channels

The positive dimensionless clock factor is $\mathbf{m}$; $\mathcal{U}$ is four-velocity in own time t_{own} , $\nabla_{\mathcal{U}}$ the covariant rate of change along its path and $\mathfrak{P}^{\mu\nu}$ the orthogonal projector with speed-squared units. Core 0.1c gives

$$d \mapsto \mathbf{m}d, \quad t \mapsto \mathbf{m}t, \quad g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}, \quad (19)$$

$$\nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathbf{m}, \quad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu. \quad (20)$$

Moving at constant speed on a pattern's own clocks and rulers through a density gradient is gravity. The bookkeeper reads acceleration toward slower clocks; the pattern keeps the chain. A conformal rescaling preserves null paths. Curvature is read between neighbouring patterns or against a reference path; an extended pattern compares its skin with its centre path through its quadrupole degree $l = 2$. With v the inward flow speed, the exterior flow obeys $v = c\sqrt{\alpha_{\text{ext}}}$ and falls at $G\mathcal{M}/r_f^2$ in own time. This flow occurs at every size, and the frame drag a gyroscope reads is the flow's local rotation, half the curl of a rotating pattern's swirl, at leading order the Lense–Thirring precession.

The rest contribution is M , pressure P , volume V and temperature T . Core I-7 and core 0.5 give the total channel book

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{aT^4 V}{c^2} + \dots, \quad a = \frac{4\sigma_{\text{SB}}}{c}. \quad (21)$$

Each term has mass dimensions; a is an energy-density constant per fourth power of temperature. Every non-rest channel enters the one stress tensor once. Gravitational binding is already carried by the solved mass aspect. For a perfect-form stress tensor at zero conserved count, temperature is the clock factor of the medium's flow; at fixed entropy per conserved count it is enthalpy per count. A carried atom's clock runs on its local lapse, with thermal motion supplying its own time dilation. Its capture rate is not read from the medium's temperature.

I-2 follows from one medium and the chain's one speed: photon, graviton, gluon, phonon and magnon are the same spin-one stroke in different density and geometry regimes. The identity applies to carriers; a closed matter braid is outside that identity. For I-5, let B be boundary data, χ orientation, u_q a mode and ω_q its angular-frequency eigenvalue. The position argument t_{conf} distinguishes confinement position from time.

$$D(\Omega, B; n, \chi, t_{\text{conf}})u_q = \omega_q u_q, \quad m_q c^2 = \hbar \omega_q. \quad (22)$$

The operator has inverse-time dimensions; its specification is O-9 in the status ledger. The geometric confinement support R_{box} has no assigned value here. It is fixed independently of a predicted mass and is never identified with a Compton wavelength. The strand count fixes class, while confinement fixes the eigenvalue within a class. No absolute density address is assigned to a named closure.

For I-4's projection, ψ denotes the projected helix and A is helix amplitude, ω angular frequency, $\mathbf{e}_1, \mathbf{e}_2$ orthonormal transverse basis vectors and $\mathbf{B} = \mathbf{e}_1 \mathbf{e}_2$ the unit bivector. The imaginary unit is i , identified with this bivector in the Clifford lift; $\mathbf{B}^2 = -1$. Cylindrical projection gives

$$\phi_h(t) = \phi_0 - \omega t, \quad \psi = A e^{i\phi_h} \longleftrightarrow A e^{\mathbf{B}\phi_h}. \quad (23)$$

The exponent is dimensionless. This single-angle phase rotor differs from a half-angle spinor rotor. Ignorance of the helical phase at observation is superposition; phase fixation is projection. The manuscript's reading identifies the sign change of a transition profile with relative reclosure phase. That identification assigns neither a local density nor a density-coordinate value to its node.

2 System chart table

The nucleus and the atom require distinct energy books. The nuclear rest energy subtracts the electrons from the atomic mass and restores their total binding once. The neutral capture step instead compares complete neutral atoms. The constants are the CODATA values (U-02, U-04–U-08, U-10)[4]; the neutral mass is U-15[8], the atomic step U-11[5], and the electronic binding convention U-14[7]. The source charge radius is $R_{\text{ch}} = 4.032 \pm 0.002$ fm (L363, L364). In I-3 a bound configuration such as a nucleus is carried by “two charts, both declared”: the exterior atlas (fold at its gravitational R_s) and “the interior chart of the configuration, read with its skin at the fold and its skin radius R_{room} taken from data”; the manuscript's reading takes the source's box radius of germanium (L389–L390) as that skin radius:

$$R_{\text{room}} = \sqrt{5/3} R_{\text{ch}} = 5.205 \text{ fm}, \quad \delta R_{\text{room}} = 0.003 \text{ fm}. \quad (24)$$

The radius retains the four significant figures of its measured input. No transfer map takes this skin radius, the manuscript's reading of the source's box radius, onto the nuclear gravitational radius. The following entries have A-1 geometric warrant only on the stated exterior evaluation.

Quantity	Value or limit	Evaluation and meaning
^{71}Ge nuclear rest energy	$\mathcal{M}c^2 = 66.05 \text{ GeV}$	Rest-energy book; electronic binding counted once.
Force ceiling	$F_{\max} = 3.026 \times 10^{43} \text{ N}$	$c^4/(4G)$, universal and unattained.
Exterior compactness	$\alpha_{\text{ext}} = 3.36 \times 10^{-38}$	$r_f = R_{\text{room}}$, exterior branch.
Atomic radial scale	$r_B/Z = 1.654 \text{ pm}$	U-10 and U-26; geometric comparison radius.
Exterior compactness	$\alpha_{\text{ext}} = 1.06 \times 10^{-40}$	$r_f = r_B/Z$, exterior branch.
Interior skin	$\alpha_{\text{core}} \rightarrow 1^-$	Schematic interior diagnostic, not either exterior number.
Overlap	$0 < \alpha_{\text{ovl}} < 1$	A-3 token and range only.

Both numerical exterior evaluations lie in the A-1 exterior general-relativistic regime $\alpha_{\text{ext}} < 1/2$. Their small gravitational window is a correct I-1 reading. The charge distortion of a bound electron by the nuclear clamp uses a charge-to-force-input dictionary distinct from that compactness calculation. The bound closure overlaps the clamp, so the disjoint-support reduction in Eq. (15) does not evaluate its binding or capture rate. The status ledger identifies both required maps.

Let $\mathcal{M}_{\text{Ga,atom}}$ denote the initial neutral atom’s mass. For the complete neutral step, I-7 and Eq. (8) imply

$$\Delta\mathcal{M} = \frac{Q}{c^2}, \quad \frac{\Delta R_s}{R_s} = \frac{Q}{\mathcal{M}_{\text{Ga,atom}}c^2} = 3.52 \times 10^{-6}. \quad (25)$$

This is an energy-to-compactness identity at fixed evaluation geometry. The step is already in the neutral mass difference; adding it again as heat or binding would recount the channel. At fixed radius the exterior input increases, while an observable’s change also requires its defined sensitivity. No sign of a capture-rate shift is inferred from this input sign alone.

3 Observation map

3.1 What each measurement weights

For a probe weight w , $\langle f \rangle_w$ denotes a weighted reading of a field f and dV the support volume element. The observation-region rule of I-3 is:

The seated low- n closures of I-5 have their support toward the low- α_{int} centre; the clamp and the binding are transacted at the high- α_{int} skin; higher rungs of the n -ladder have footing only in the skin terrain. Every consensus datum about an interior is a profile-weighted mean $\langle f \rangle_w = \int w f dV / \int w dV$ with the probe weight named—an rms radius is a skin-weighted second moment, a trace share or a mean pressure a volume mean—so a point value is never read off a mean and no single “nuclear α ” is defined by any of them.

The manuscript’s reading treats half-life, neutrino capture and forward charge exchange as three profile-weighted means, each of its own event and each with its probe weight named, of the transition density the source writes is the same for the EC process and the IBD reaction, “up to a trivial overall phase” (L189–L190). Their measured rates or cross sections involve squared amplitudes, with their respective kinematic prefactors; a signed transition amplitude is not a positive spatial average. The charge radius is instead a skin-weighted second moment of a ground-state electromagnetic reading. This distinction preserves the source’s separation of charge and weak transition densities (L310–L317, L343–L360).

In the source’s natural-unit formulas, momenta and energies share a unit and an inverse length is obtained by dividing energy by $\hbar c$ when restoring units. The source’s Eq. (6), at L170, defines

$$\varrho_{\text{TD}}(r) = \Psi_{\text{Ge}}^*(r) \hat{\mathcal{O}}_{\text{GT}} \Psi_{\text{Ga}}(r). \quad (26)$$

The source’s Eqs. (3)–(5) (L79, L92–L108) place the lepton functions inside the support integral and then display the factorisation approximation. With $dV = 4\pi r^2 dr$ after angular reduction, the corresponding amplitude statement is

$$\mathcal{H}_j^{\text{IBD}} = \int \psi_e^j(r) \psi_\nu^j(r) \varrho_{\text{TD}}(r) dV, \quad (27)$$

$$\mathcal{H}_j^{\text{IBD}} \simeq \psi_e^j(r_0) \psi_\nu^j(r_0) \mathfrak{M}_{\text{nuc}}^{\text{IBD}}, \quad |\mathfrak{M}_{\text{nuc}}^{\text{IBD}}|^2 \stackrel{\text{db}}{=} |\mathfrak{M}_{\text{nuc}}^{\text{EC}}|^2. \quad (28)$$

The point r_0 is often selected as the origin in the source (L142–L143); its preceding calculation instead used nuclear-density-averaged electron functions (L117–L118). In either construction the lepton weight is extracted from the transition-profile integral. This is the specific approximation being tested. The source then writes that, “Exploiting the detailed-balance principle, the IBD nuclear matrix element can be equated with that of the inverse reaction” (L100–L108): the two nuclear matrix elements displayed above, what the extraction leaves of the bound reading of Eq. (32) and of the continuum reading of the source’s Eqs. (7)–(9); in the manuscript’s reading those two readings are profile-weighted means with different probe weights (I-3: “Every consensus datum about an interior is a profile-weighted mean” “with the probe weight named”), and the identification is the mirror of two events, a reading the convergence statement declines. The source already distinguishes a bound initial electron from an outgoing continuum electron and states that electron capture is not the exact time-reversed process (L153–L156).

For leg-dependent p_ν , the restored Bessel argument is $p_\nu r/\hbar$. The source’s Eqs. (7)–(9), L192, L198, L206, give the continuum weights:

$$|\mathcal{H}^{\text{IBD}}|^2 = (4\pi)^2 (|\mathcal{I}_1|^2 + |\mathcal{I}_2|^2), \quad (29)$$

$$\mathcal{I}_1 = \int_0^\infty r^2 \varrho_{\text{TD}} \left[g_{-1} j_0(p_\nu r/\hbar) + \frac{f_{-1}}{3} j_1(p_\nu r/\hbar) \right] dr, \quad (30)$$

$$\mathcal{I}_2 = \int_0^\infty r^2 \varrho_{\text{TD}} \left[f_1 j_0(p_\nu r/\hbar) - \frac{g_1}{3} j_1(p_\nu r/\hbar) \right] dr. \quad (31)$$

The subscripted signs are Dirac angular labels $\varkappa = \pm 1$, not the Einstein coupling. The source’s allowed axial-spatial bound reading, Eq. (11) at L176–L185, is

$$|\mathcal{H}_{1s}^{\text{EC}}|^2 = \left| \int_0^\infty 4\pi r^2 \varrho_{\text{TD}} \left[g_{1s} j_0(p_\nu r/\hbar) + \frac{f_{1s}}{3} j_1(p_\nu r/\hbar) \right] dr \right|^2. \quad (32)$$

The source writes that the EC transition density is the same as that of the IBD reaction, “up to a trivial overall phase” (L189–L190). The shell fractions $\mathcal{P}_K, \mathcal{P}_L, \mathcal{P}_M$ weight their respective bound-shell capture channels; the displayed approximation uses the leading $1s$ matrix element with their aggregate correction. The source includes radiative and weak-magnetism corrections although it does not display them (L169–L171).

For the neutrino energy E_ν^{1s} in capture and the line energies E_e^j , the source’s Eqs. (1), (2),

(10), at L101–L129 and L160, become

$$\mathcal{S}_{\text{gs}} = \frac{\mathcal{G}_F^2 |\mathcal{V}_{ud}|^2 \mathfrak{g}_A^2}{\pi(2J_{\text{Ga}} + 1)} \sum_j p_e^j E_e^j |\mathcal{H}_j^{\text{IBD}}|^2 \mathcal{B}_j, \quad (33)$$

$$\mathcal{S}_{\text{gs}}^{\text{db}} = \frac{2\pi^2 \ln 2}{\mathfrak{f}_{\text{EC}} t_{1/2}} \frac{2J_{\text{Ge}} + 1}{2J_{\text{Ga}} + 1} \sum_j p_e^j E_e^j \mathcal{F}(E_e^j, Z, r_0) \mathcal{B}_j, \quad (34)$$

$$\frac{1}{t_{1/2}} = \frac{\mathcal{G}_F^2 |\mathcal{V}_{ud}|^2 \mathfrak{g}_A^2}{\pi \ln 2 (2J_{\text{Ge}} + 1)} (E_\nu^{1s})^2 |\mathcal{H}_{1s}^{\text{EC}}|^2 \left(1 + \frac{\mathcal{P}_L + \mathcal{P}_M}{\mathcal{P}_K} \right). \quad (35)$$

These three equations retain the source’s natural units and its wavefunction normalisation: cross section has inverse-energy-squared dimensions and inverse lifetime has energy dimensions. Converting the latter to inverse seconds divides by $\hbar$; converting the former to area multiplies by $(\hbar c)^2$. The source inputs are $\mathcal{V}_{ud} = 0.9743$ and $\mathfrak{g}_A = 1.2764$ (L107, L110), with $J_{\text{Ga}} = 3/2$ and $J_{\text{Ge}} = 1/2$ (L112, L134).

The table below fixes the epistemic and probe scopes of the principal observations.

Record	Reading and probe	Class and locus
Gallium deficit	Line-summed capture under continuum weights	Reported approximately 20% deficit; source abstract, L22–L30.
Ge half-life	Bound 1s weight and shell corrections	Reported average 11.465 ± 0.003 d; L112.
Cr ground-state target	Continuum capture after excited-state subtraction	Inferred 4.44 ± 0.33 in the area unit below; L210.
Ar ground-state target	Same construction with Ar line weights	Inferred 5.21 ± 0.72 ; L213.
Charge radius	Ground-state electromagnetic skin-weighted second moment	Reported radius above; L363–L364.
Joint profile fits	EC and IBD jointly constrain parameters	In-sample Table I, L322–L340.
Hadronic strengths	Forward charge exchange; independent normalisation required	Existing excited-state normalisation uses the EC-inferred ground state, L419–L421.

The cross-section unit in the table is 10^{-45} cm^2 , as printed at L210 and L213; it is an area, not a rate. The source adopts excited-state fractions 5.3% and 5.8% and assigns 5% uncertainty to that component (L228, L220). Whether the printed target errors already include this contribution in the least-squares denominator is not specified by those lines. The half-life and target uncertainties agree with the parenthetical uncertainties in the source table. The DG minus sign requires separate attention below.

3.2 What the fit fixes

The source uses SG, DG, mDG and mTG profiles, Eq. (12), L244–L272. Define the dimensionless Gaussian shape $\mathfrak{z}_j(r)$, with r_j its centre and $\mathfrak{s}_j$ its width; $\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3$ select the corresponding parameter entries using the widths and centres in the notation guard. Then the source forms are

$$\mathfrak{z}_j(r) = \exp[-(r - r_j)^2 / (2\mathfrak{s}_j^2)], \quad (36)$$

$$\varrho_{\text{TD}}^{\text{SG}} = \mathcal{A}_1 \mathfrak{z}_1, \quad \varrho_{\text{TD}}^{\text{DG}} = \mathcal{A}_1 \mathfrak{z}_1 - \mathcal{A}_2 \mathfrak{z}_2, \quad (37)$$

$$\varrho_{\text{TD}}^{\text{mDG}} = \mathcal{A}_1 r \mathfrak{z}_1 + \mathcal{A}_2 r^2 \mathfrak{z}_2, \quad \varrho_{\text{TD}}^{\text{mTG}} = \mathcal{A}_1 r \mathfrak{z}_1 + \mathcal{A}_2 r^2 \mathfrak{z}_2 + \mathcal{A}_3 r^3 \mathfrak{z}_3. \quad (38)$$

Widths and centres have length units; amplitudes carry the powers needed for ϱ_{TD} to have inverse-volume units. Thus amplitudes of different polynomial powers have different units. For DG, the fitted second amplitude is negative, so the explicit minus sign produces a positive second lobe. For the sign-definite benchmark, the source’s Eq. (15), L310, is

$$\varrho_{\text{TD}}^{2\text{pF}}(r) = \frac{\mathcal{A}_{2\text{pF}} \varrho_0(\mathfrak{c}, \mathfrak{d})}{1 + \exp[(r - \mathfrak{c})/\mathfrak{d}]}. \quad (39)$$

Its inputs are $\mathfrak{c} = 4.56$ fm, $\mathfrak{d} = 0.52$ fm and $\mathcal{A}_{2\text{pF}} = 0.588$ (L362). The source chooses its amplitude to match the EC nuclear matrix element within the detailed-balance-based approximation (L364–L369). It includes this charge distribution only as a conventional reference, not as its best estimate of the physical GT transition density (L351–L361).

The profile parameters used here are exactly the source’s printed values. Each tuple follows the ordering of Eq. (12): SG amplitude, centre, width; DG and mDG amplitudes, widths, centres; mTG three of each.

Form	Parameters (lengths in fm; amplitudes as in Eq. (38))
SG	(0.00094, -0.005 , 3.42); L335.
DG	(-0.0231 , -0.0802 , 3.5119, 1.855, 2.16, 2.16); L336.
mDG	(-0.1498 , 0.3462, 1.192, 1.3793, 2.6192, 0.466405); L337.
mTG	(3.514009, 4.53567, -1.5007 , 1.9457, 1.3809, 1.14085, 0.0029, 0.8595, 1.513); L338, L340.

The parameter tuples are in-sample fits to the two capture targets and the half-life. The nuisance terms in source Eqs. (13)–(14), L230–L241 and L268–L289, act on the same data used to select these parameters:

$$\mathcal{C}_{\text{IBD}}^2 = \frac{\eta_1^2}{(\delta\eta_1)^2} + \sum_{\text{src}} \left[\frac{\mathcal{S}_{\text{gs,src}}^{\text{exp}} - (1 + \eta_1)\mathcal{S}_{\text{gs,src}}(\Theta)}{\delta\mathcal{S}_{\text{gs,src}}^{\text{exp}}} \right]^2, \quad (40)$$

$$\mathcal{C}_{\text{EC}}^2 = \left[\frac{t_{1/2}^{\text{exp}} - (1 + \eta_2 + \eta_3)t_{1/2}(\Theta)}{\delta t_{1/2}} \right]^2 + \frac{\eta_2^2}{(\delta\eta_2)^2} + \frac{\eta_3^2}{(\delta\eta_3)^2}. \quad (41)$$

All terms are dimensionless. The source gives $\delta\eta_1 = 0.003\%$, $\delta\eta_2 = 0.007$ and $\delta\eta_3 = 0.005$ (L241, L283, L285). The source does not print the optimised nuisance values or a parameter covariance. The table below transcribes its fitted outputs; cross sections use the same area unit as above and half-life uses days.

Form	Cr	Ar	$t_{1/2}$	$\mathcal{C}_{\text{IBD}}^2$	$\mathcal{C}_{\text{EC}}^2$
SG	5.27	6.30	11.515	8.68	0.24
DG	4.42	5.13	11.462	0.014	0.0005
mDG	4.50	5.24	11.465	0.037	0.00003
mTG	4.39	5.10	11.463	0.044	0.0002

These are Table I entries at L335–L339; the reported small EC objectives are retained as fitted objectives, rather than reconstructed from a rounded half-life without its nuisance solution. For independent nuisance errors, define the illustrative half-life budget δt_{eff} and dimensionless nuisance budget $\delta\eta_{\text{EC}}$. Quadrature gives

$$\delta\eta_{\text{EC}} = \sqrt{(\delta\eta_2)^2 + (\delta\eta_3)^2} = 0.009, \quad (42)$$

$$\delta t_{\text{eff}} = \sqrt{(\delta t_{1/2})^2 + t_{1/2}^2 (\delta\eta_{\text{EC}})^2} = 0.1 \text{ d}. \quad (43)$$

Only one significant figure is licensed by the nuisance errors; the calculation uses their unrounded inputs. The precise half-life alone therefore does not fix a unique radial profile or its capture projection.

3.3 A controlled reconstruction of the weight difference

The reconstruction uses a uniformly charged sphere, $Z = 32$, no screening and only the large component with j_0 (U-28). It uses R_{room} , derived from the measured charge radius, and the principal Cr line 0.747 MeV; this is distinct from the source’s exact Dirac–Hartree–Fock–Slater functions and full line sum. The electron rest energy and $\hbar c$ are U-06 and U-04. The bound energies are U-12[6]; the neutrino lines are U-16[9]. Define the potential energy $\mathfrak{V}(r)$ and the radial Dirac components $\mathfrak{u}(r), \mathfrak{v}(r)$ with $\tilde{g} = \mathfrak{u}/r$ the reconstruction’s large component, distinct from the source’s $g_{\mathcal{K}}$. Here $\mathcal{E}_e$ is total electron energy, m_e its rest mass and $\varsigma = Ze^2/(4\pi\epsilon_0\hbar c)$. The U-28 reconstruction is specified by

$$\mathfrak{V}(r) = \begin{cases} -\frac{\varsigma\hbar c}{2R_{\text{room}}} \left[3 - \frac{r^2}{R_{\text{room}}^2} \right], & r < R_{\text{room}}, \\ -\frac{\varsigma\hbar c}{r}, & r \geq R_{\text{room}}, \end{cases} \quad (44)$$

$$\mathfrak{u}' = \frac{\mathfrak{u}}{r} + \frac{\mathcal{E}_e + m_e c^2 - \mathfrak{V}}{\hbar c} \mathfrak{v}, \quad \mathfrak{v}' = -\frac{\mathfrak{v}}{r} - \frac{\mathcal{E}_e - m_e c^2 - \mathfrak{V}}{\hbar c} \mathfrak{u}. \quad (45)$$

Regularity fixes $\mathfrak{u}/r \rightarrow 1$ and $\mathfrak{v}/r^2 \rightarrow -(\mathcal{E}_e - m_e c^2 - \mathfrak{V}(0))/(3\hbar c)$ at the centre. Each derivative term has its radial component divided by length; the energy-over- $\hbar c$ factors are inverse lengths. Write $\mathfrak{B}_K(\text{Ga}), \mathfrak{B}_K(\text{Ge})$ for the positive K binding energies and E_ν for incident energy. The bound total energy is $m_e c^2 - \mathfrak{B}_K(\text{Ge})$; the outgoing total energy is $m_e c^2 + E_\nu - \mathcal{Q}$. The bound neutrino energy is $\mathcal{Q} - \mathfrak{B}_K(\text{Ga})$. The centre-normalised weights are $w_b = \tilde{g}_{1s} j_0[(\mathcal{Q} - \mathfrak{B}_K(\text{Ga}))r/(\hbar c)]$ and $w_c = \tilde{g}_{-1} j_0[E_\nu r/(\hbar c)]$. The Bessel arguments explicitly retain $\hbar c$. The reconstructed large components alone at the skin are $\tilde{g}_{-1} = 0.9818$ and $\tilde{g}_{1s} = 0.9833$; the full weight difference is $w_b - w_c = 1.5 \times 10^{-3}$.

Write ϱ for a generic instance of the signed transition profile, the same for both processes, “up to a trivial overall phase”, on the source’s premise (L189–L190). Define the reading functional $\mathcal{R}[\varrho; E_\nu]$ by its ratio of squared weighted integrals at equal EC normalisation:

$$\mathcal{R}[\varrho; E_\nu] = \left| \frac{\int \varrho(r) w_c(r; E_\nu) dV}{\int \varrho(r) w_b(r) dV} \right|^2, \quad \Upsilon = \frac{|\int \varrho dV|}{\int |\varrho| dV}. \quad (46)$$

The Gaussian and Fermi tails make all integrals absolutely convergent; the signed denominator is required to be nonzero. Both diagnostics are dimensionless, and neither is an atlas coordinate. With point factorisation normalised to unity, the offsets $1 - \mathcal{R}$ are 1.9×10^{-3} for the Fermi shape and 3.6×10^{-3} for SG; the SG reading relative to the Fermi-shape reading is 0.998. For each printed fit parameter, independent uniform variation through half its last digit defines the rounding uncertainty (U-29). It is distinct from a fitted covariance and from experimental precision.

Profile	$\mathcal{R}/\mathcal{R}[\text{SG}]$	Υ	Table I Cr ratio to SG
DG	0.84 ± 0.02	0.028 ± 0.003	0.839
mDG	0.86 ± 0.02	0.012 ± 0.001	0.854

A sign-definite profile has $\Upsilon = 1$ exactly. Small differences of lepton weights multiply a strongly cancelling signed profile, making the capture comparison sensitive to radial structure. Within

the printed mTG rounding intervals, both signs occur for its unweighted integral and for its EC-weighted integral. Its printed parameters therefore supply no stable cancellation index, reading ratio or form-factor normalisation. Its principal nodes remain useful geometric summaries.

The sensitivity limit is explicit: replace the bound weight by $1 - 1.05(1 - w_b)$, keeping the continuum weight fixed. The skin perturbation is 8×10^{-4} and the DG and mDG ratios shift by 0.06 and 0.07. Every nodal reading therefore depends on the lepton functions at the level of this skin perturbation, and the DG and mDG agreement with Table I sits within that dependence. Agreement with Table I checks this reconstruction and corroborates no invariant. The tested profile precision and the lepton-weight limitation are different error sources.

DM-1: independent ground-state charge exchange. The observable is a forward ground-state $^{71}\text{Ga}(p, n)$ or $^{71}\text{Ga}(^3\text{He}, t)$ cross section divided by a unit cross section established independently of germanium EC. For the plane-wave diagnostic define

$$\Xi_{\text{had}}(k_t) = \int \varrho_{\text{TD}}(r) j_0(k_t r) dV. \quad (47)$$

At equal EC reading the squared zeroth moment relative to SG is 2.8 ± 0.4 for DG, 3.1 ± 0.4 for mDG and 0.99 for the Fermi shape. The source states that, “under the mechanism” “proposed here”, the (p,n) charge-exchange data used to normalise excited-state contributions are referenced to the ground-state matrix element inferred from EC (L418–L421); using that normalisation for this comparison is circular. An independent unit cross section is therefore part of the observable definition.

The form-factor zero solves $\Xi_{\text{had}}(k_t) = 0$ away from the origin. Its reconstructed first roots are $0.070 \pm 0.003 \text{ fm}^{-1}$ and $0.091 \pm 0.003 \text{ fm}^{-1}$ for DG and mDG. At $k_t = 0.5 \text{ fm}^{-1}$, their squared magnitude ratios to zero transfer are $1.6 \times 10^2 \pm 0.3 \times 10^2$ and $3.9 \times 10^2 \pm 0.6 \times 10^2$, compared with 0.054 for SG and 0.23 for the Fermi shape. For the illustrative 420 MeV helium beam, neglecting energy transfer, the mDG zero is at 0.6 degrees. Let p_{beam} be beam momentum, E_{beam} kinetic energy and m_{beam} projectile rest mass. Here beam momentum obeys $p_{\text{beam}} c = \sqrt{E_{\text{beam}}(E_{\text{beam}} + 2m_{\text{beam}} c^2)}$, and angle is $2 \arcsin(\hbar k_t / (2p_{\text{beam}}))$. U-21 fixes that assumed setting separately from the source’s Frekers reference[14]. The (p,n) form of the observable uses the 120 and 200 MeV proton beams of Krofcheck et al.[15]. The plane-wave zero is a reconstructed comparison; an experimental angular distribution requires its distorted-wave response.

For every independent comparison, let δ_{diff} be the uncertainty of the measured difference, Cov its covariance and z a preregistered significance multiplier. With labelled readings y_1, y_2 and their standard errors $\delta y_1, \delta y_2$, the criterion is

$$|y_1 - y_2| \geq z \delta_{\text{diff}}, \quad \delta_{\text{diff}}^2 = (\delta y_1)^2 + (\delta y_2)^2 - 2 \text{Cov}(y_1, y_2). \quad (48)$$

Both sides have the observable’s units. Corroboration scope: DM-1 corroborates no invariant: the distinction between the bound, continuum and plane-wave weights rests on the source’s Eqs. (7)–(11) and the plane wave of Eq. (47) and holds in every theory; DM-1 falsifies a fitted profile if its independently normalised strength or angular response misses Eq. (48) after the response and rounding uncertainties are propagated. Reproducing that profile’s own fitted EC and IBD inputs remains the identity map.

DM-2: energy dependence at fixed target. The observable compares capture on gallium from specified neutrino lines, after an independently normalised excited-state subtraction. The mDG-to-SG reconstruction at equal EC norm is

Neutrino source	E_ν (MeV)	mDG/SG
Cr lower line	0.427	0.95 ± 0.01
Cr principal line	0.747	0.86 ± 0.02
Ar line	0.811	0.85 ± 0.02
Zn line	1.343	0.71 ± 0.04

The ordering decreases in every rounding draw; two endpoints by themselves are a secant, not a local derivative. The zinc branch is approximately 48.5%[10]. Its excited-state share is approximately 20–30% on the EC-referenced normalisation[11]; that share must be replaced by the independent strength measurement in DM-4. The comparator is the sign-definite profile’s nearly unchanged reading, with a propagation-dependent comparator evaluated at matched geometry. Corroboration scope: DM-2 corroborates no invariant: the energy dependence of the continuum weight rests on the source’s Eqs. (7)–(9) at each line and holds in every theory; a flat measured nodal-to-reference ratio falsifies this reconstructed profile only when the prefactor, line composition and excited-state correction are fixed or removed, and Eq. (48) resolves the contrast. This is a discriminating measurement of fitted profiles, not an FDT divergence.

DM-4: recompute the ground-state targets. The observable is each excited-state GT strength relative to the independently calibrated ground-state strength of DM-1, then the residual ground-state capture target for each neutrino source. The source writes, “If the present scenario is correct, this normalization would need to be revisited in a dedicated analysis” and “we do not expect this to qualitatively affect our conclusions” (L422–L424). The comparator is its existing EC-referenced subtraction. Corroboration scope: no invariant is tested by recalculating a subtraction; the source’s conditional expectation is falsified if the independent subtraction removes the claimed profile separation under Eq. (48).

DM-5: the microscopic transition profile. The test is a microscopic fpg calculation of the transition profile, the same for both processes, “up to a trivial overall phase”, on the source’s premise (L189–L190), passed through both measured lepton responses, with EC normalisation fixed once. The source leaves that calculation “to future nuclear-structure work” (L316–L317). The target-to-SG ratios from the reported averages are 0.84 ± 0.06 for Cr and 0.83 ± 0.11 for Ar, the targets’ stated uncertainties (L210, L213) propagated through each ratio. Those are data-derived ratios, not new profile fits; the targets’ uncertainties license two figures. A microscopic profile is compared with both ratios and their correlated errors after DM-4, not just with the existence of a node. The source writes that such a reduction “is achieved only if the transition density has sufficient radial structure, in particular at least one sign change” (L396–L398). Its examples establish that two sign-definite forms fail and three nodal forms succeed in its fit; neither a sign change nor Υ alone determines the capture reading. Corroboration scope: this calculation tests the source’s open nuclear-structure item and the reconstructed response; it supplies no numerical FDT local-density law.

4 Cone-position translation

4.1 The complete atom and its two reclosures

The neutral-atom mass difference is $Q = 232.443 \pm 0.093$ keV[5]. The manuscript’s reading of 0.1a and I-5b: events do not commute, and a process run backward is never the mirror of the forward one; the omnium solves one global optimisation at every step, so a step’s expected behaviour is not necessarily the evolving system’s optimum; responses are system-wide and many-to-many, never

one-to-one and particle-wise. Capture raises the atom by this energy, and the raised configuration resettles through its one open channel; the ^{71}Ge half-life is that resettlement. They are two events, each a many-to-many reclosure of the whole atom settled by its own global optimisation: the resettlement is not the capture reversed, and the capture expected from it is not necessarily what the capture reaches; the resettlement's rate is among the rates of O-12 in the status ledger. The symbol u denotes the atomic mass unit, U-07. For this step, let $E_Q = Q$ be the step energy and $m_Q, \omega_Q, T_Q, \ell_Q$ denote the mass equivalent, angular frequency, period and thread length. The constants in Eqs. (2)–(4) give

$$E_Q = Q = m_Q c^2 = \hbar \omega_Q, \quad (49)$$

$$m_Q = 2.495 \times 10^{-4} \text{ u}, \quad (50)$$

$$T_Q = \frac{2\pi}{\omega_Q} = \frac{h}{Q} = 1.779 \times 10^{-20} \text{ s}, \quad (51)$$

$$\ell_Q = cT_Q = \frac{h}{m_Q c} = 5.334 \times 10^{-12} \text{ m}. \quad (52)$$

The mass unit is U-07; these step values retain four significant figures, below the precision of the atomic step. The thread length is the Compton length, never the support of the confinement eigenproblem. The very short step period and the long EC lifetime are different observables: one is the energy-frequency bridge, the other the transition rate.

The closure classes are the core's same- n pairings, quoted next; the manuscript's reading then lists each event's closures. “ $n = 2$ neutrino / neutron” and “ $n = 3$ electron / proton” are the pairings of I-5, each “the same closure read from the free room and from the nuclear room”; “The two members of a pairing differ in confinement, and are related by the inside/outside chart swap of the reciprocal atlas (0.3)”. Their free and seated readings occupy the exterior far region and the dilute interior centre, respectively, with no numerical address. The capture list is a free two-strand closure plus a seated two-strand closure reclosed as a seated three-strand closure and a free three-strand closure. The EC list is a bound three-strand closure plus a seated three-strand closure reclosed as a seated two-strand closure and a free two-strand closure. The lists are each event's books, never a one-to-one particle reaction: each reclosure runs over both rooms, many-to-many, and its strand counts are per-closure labels, never a sum conserved across the event. Both the bound $1s$ electron and the outgoing electron remain in the same three-strand class. Their energy labels and nuclear excited-state labels supply no confinement positions.

Let Q denote charge magnitude in closure units, ν_o, ν_i the framed outer and inner linkings, H helicity sign, ν_F the fermion class and B_s the nested turning-class degree. The exact books of I-5 distinguish

$$Q = n \bmod 2, \quad \text{sgn } Q = \text{sgn}(\nu_o \nu_i), \quad \text{sgn } H = \text{sgn } \nu_i, \quad \nu_F = B_s \bmod 2. \quad (53)$$

The magnitude formula and the sign formula refer to the magnitude and signed reading of the same charge book. Fermion class is not fixed by strand-count parity; the neutral two-strand pairing already exhibits the distinction. The linkings require declared material framing, with nonzero product; no numerical linking assignment is made here. Four-momentum, charge, fermion parity and angular momentum balance across each reclosure. The negative-parity spins are $1/2^-$ and $3/2^-$ for Ge and Ga (U-33), the allowed GT transition. The neutrino is a matter closure, never one of the carriers of I-2.

4.2 Stability is the open-channel book

Let $\mathcal{T}$ denote the transition operator of I-5b and Γ_i the rate from initial state i . This decorated operator distinguishes it from temperature T . With f a final-state label, the exact-book criterion

reads

$$\text{stable} \iff \langle f | \mathcal{T} | i \rangle = 0 \text{ for every } f \in \mathfrak{F}_i, \quad \Gamma_i = 0. \quad (54)$$

The criterion is a statement about every allowed matrix element, not an energy wall alone. For the atomic bookkeeping, define $\mathfrak{B}_e(Z)$ as total electronic binding and ΔE_{bare} as bare Ge minus bare Ga rest energy. The U-14 binding convention gives, in eV,

$$\mathfrak{B}_e(Z) = 14.4381Z^{2.39} + 1.55468 \times 10^{-6}Z^{5.35}. \quad (55)$$

Its numerical coefficients carry the energy unit, and Z is dimensionless. Using neutral masses, electron rest energy and this binding function once gives

$$\mathfrak{B}_e(31) = 53.1 \text{ keV}, \quad \mathfrak{B}_e(32) = 57.3 \text{ keV}, \quad (56)$$

$$2m_e c^2 - Q = 789.6 \text{ keV}, \quad (57)$$

$$\Delta E_{\text{bare}} = Q - m_e c^2 + \mathfrak{B}_e(32) - \mathfrak{B}_e(31) = -274.4 \text{ keV}. \quad (58)$$

The binding function's exponents limit its numerical licence; the bare ordering and positron threshold are secure at the displayed scale. The atomic electron mass subtraction is essential: the neutral positive step is not the bare-nucleus ordering.

Configuration	Exact-book channel test	Reading
Neutral gallium	No spontaneous channel at or below threshold	Stable under the specified isolated ground-state preparation.
Neutral germanium	EC open; positron emission below its threshold	Raised atomic configuration resettles by capture.
Bare germanium	No bound electron; positron channel closed by the bare energy book	Stable: no channel at or below threshold.
Hydrogen-like Ge, hyperfine zero	Lowest neutrino partial wave cannot balance the nuclear angular momentum	Allowed EC channel closed in this approximation.
Hydrogen-like Ge, hyperfine one	Angular momentum permits capture	Allowed EC channel open.

Higher partial waves and external repopulation belong to a specified measurement response; the hyperfine-zero statement concerns the allowed-capture limit.

DM-3: storage-ring charge and hyperfine preparation. The observable is the number of surviving bare or hydrogen-like germanium ions as a function of own-clock storage time, with the populated hyperfine state measured independently. The comparator uses the same energy and angular-momentum books in the conventional calculation. For an illustrative nonrelativistic estimate, the hyperfine interval scales as the hydrogen interval times Z^3 and the ratio of the nuclear magnetic moments. The inputs are the Ge spin and moment[13], the proton moment (U-18), the hydrogen interval (U-19) and its spontaneous rate (U-20). Define $\mathfrak{h}_{\text{Ge}}$, $\mathfrak{h}_{\text{H}}$ as the hyperfine energies and Γ_{H} the hydrogen spontaneous rate. Let $\bar{t}_{\text{hf}}$ denote the illustrative relaxation time. Then

$$\mathfrak{h}_{\text{Ge}} = \mathfrak{h}_{\text{H}} Z^3 \frac{0.546}{2.792847} = 0.0376 \text{ eV}, \quad (59)$$

$$\bar{t}_{\text{hf}} = \left[\Gamma_{\text{H}} \left(\frac{\mathfrak{h}_{\text{Ge}}}{\mathfrak{h}_{\text{H}}} \right)^3 \right]^{-1} = 22 \text{ min}. \quad (60)$$

Here $\bar{t}_{\text{hf}}$ is the illustrative relaxation time and the rate law has inverse-time dimensions. Positive nuclear moment places the hyperfine-zero state below the hyperfine-one state in this estimate. Corroboration scope: I-5b supplies the exact-book reading; a charge-state or hyperfine survival curve contradicting those same books falsifies the preparation or the allowed-channel calculation shared by every theory. This consistency test separates no FDT-specific prediction.

The source's neutral half-life implies $\Gamma_{\text{EC}} = \ln 2 / t_{1/2} = 0.06046 \text{ d}^{-1}$ and $\bar{t} = 1/\Gamma_{\text{EC}} = 16.540 \text{ d}$. These derived conversions keep the digits that the stated uncertainty of the five-figure half-life licenses. They do not determine the confinement operator or the decay kernel.

5 Predictions

DC-1, on SA-04, is retired because a solar capture contrast with identical flavour conversion and total capture cross section requires a fission neutrino spectrum and chirality, with the decay kernel, spectrum, chirality and rates of O-12 and the SNO computation of O-7. DC-2, on SA-05, is retired because comparing the free and seated two-strand eigenvalues with the KATRIN endpoint parameter requires the confinement operator and mass ratio of O-9. DC-3, on SA-06, is retired because the manuscript's reading of weak normalisation as an I-1 input pair requires the charge-to-input map of O-6 and its decay normalisation in O-12. DC-4, on SA-07, is retired because a constitutive prediction of the signed second-to-zeroth transition moment requires a reduction to the transition's radial weight that no core clause displays, with the decay kernel, spectrum, chirality and rates of O-12; the local-density object of O-4 is distinct. DC-5, on SA-08, is retired because a boundary-to-transition-node map requires the domains and boundary spectrum of O-9.

For the solar comparison, I-1a reads stars as fission reactors unwinding through the iron attractor. The standard solar model attributes its neutrino production to fusion. The standing counter-evidence is the SNO neutral-current total-flux agreement with that model; the required computation is listed below. The gallium response must include every excited state accessible to either proposed spectrum, using the same conversion and response dictionary on both sides. A different neutrino-source name alone supplies no quantitative capture contrast.

The angular-distribution comparison isolates a different underdetermination. For small k_t , define the signed moment ratio $\mathfrak{r}_2 = \int r^2 \varrho_{\text{TD}} dV / \int \varrho_{\text{TD}} dV$, assuming a nonzero zeroth moment; $\mathfrak{r}_{4,\text{abs}} = \int r^4 |\varrho_{\text{TD}}| dV / |\int \varrho_{\text{TD}} dV|$ is its finite fourth absolute-moment scale. Expansion of the Bessel function in Eq. (47) gives

$$\frac{\Xi_{\text{had}}(k_t)}{\Xi_{\text{had}}(0)} = 1 - \frac{k_t^2 \mathfrak{r}_2}{6} + O(k_t^4 \mathfrak{r}_{4,\text{abs}}). \quad (61)$$

The signed moment has length-squared units and the contrast is dimensionless. A forward zero at leading order measures this moment combination, not a unique node radius. Conventional configuration mixing and the source's cited weak-versus-charge surface differences both survive a comparison with a charge-density benchmark. Neither the core's skin density statement nor P-2's vanishing transverse density gradient supplies the signed transition moment. The five decisions leave the independent measurements above intact while leaving no additional observable contrast numerically assigned.

6 Cross-domain comparisons

CD-1: charge-state capture. The hydrogen-like ^{140}Pr EC constant is approximately 50% larger than its helium-like value[16]. Electron occupancy alone therefore fails as the full rate dictionary; hyperfine angular selection also enters. This external comparison supports the design of DM-3 without turning its shared selection rules into a new FDT prediction.

CD-2: the bound closure’s environment. The ${}^7\text{Be}$ half-lives are 52.68 ± 0.05 d in carbon cages and 53.12 ± 0.05 d in metal[17]. Their difference is 0.44 ± 0.07 d under the explicitly independent-error convention U-30. The shorter half-life corresponds to a fractional rate enhancement 0.008 from the ratio of half-lives. The comparison concerns environmental changes to the bound capture closure and its weight. The half-life is kept on the atom’s own clock; a host temperature is not substituted for that clock’s rate.

CD-3: heavier nuclear transitions. The source cites refined lepton functions “combined with particular weak transition densities” that decrease “transition rates by up to” 40%–50% “in heavy nuclei” (L400–L405). Those are cited heavy-nucleus results, with their original species scope. They support examining radial-weight sensitivity, without transferring their magnitude to gallium.

CD-4: the BEST spatial comparison. The BEST inner and outer ratios are 0.791 ± 0.055 and 0.766 ± 0.055 [12]. Their difference is 0.025 ± 0.078 under independence; any covariance changes the second number through Eq. (48). This comparison resolves no spatial dependence at that precision. A common capture normalisation affects both zones; the zone comparison alone does not distinguish radial nuclear profiles.

CD-5: electromagnetic and weak profiles. The source cites, “even in the case of superallowed Fermi” “transitions”, a charged weak density that “distinctively deviates from the electric charge distribution” (L317, L343–L346), and expects a surface-peaked GT baseline (L353–L355). Its inference concerns the choice of a conventional nuclear benchmark. The manuscript’s skin reading remains distinct from both charge density and a signed GT transition profile.

CD-6: counted atomic relaxation. The germanium vacancy-refilling signatures are approximately 10.4 keV for K and 1.2 keV for L counting[18]. The outgoing x rays are I-2 strokes, while Auger electrons are free three-strand closures. These deposited-energy signatures count the atomic relaxation after EC. They are not measurements of the transition profile’s sign or the neutrino’s confinement eigenvalue.

7 System-level reading (exploratory)

This section contains the manuscript’s exploratory reading of the fitted profiles only. All numerical entries re-describe the source’s in-sample parameters; no fitted profile is an FDT local-density result.

X-1: fitted nodes relative to the skin. The observable is a profile’s principal sign-change radius relative to R_{room} , the skin radius the manuscript’s reading takes from the source’s box radius, together with the fraction of its absolute radial weight outside that skin. Let r_{node} be a zero of ϱ_{TD} across which its sign changes, and $\mathbf{b}_{\text{out}}$ the absolute outer share. The defining conditions are

$$\varrho_{\text{TD}}(r_{\text{node}}) = 0, \quad \mathbf{b}_{\text{out}} = \frac{\int_{R_{\text{room}}}^{\infty} 4\pi r^2 |\varrho_{\text{TD}}| dr}{\int_0^{\infty} 4\pi r^2 |\varrho_{\text{TD}}| dr}. \quad (62)$$

The node has length units and the share is dimensionless. The comparator is the sign-definite Fermi reference and the source’s extended DG fit.

Profile	Principal nodes (fm)	Absolute outer share
mDG	3.42	0.13
mTG	2.71, 4.82	0.14
DG	5.61	0.52
Fermi	none	0.12

The compact-profile principal nodes lie inside R_{room} , while the DG node lies outside. The source says the extended DG solution “should be regarded only as an illustrative limiting case” (L396–L397). In the manuscript’s reading, the compact nodes mark surfaces between the centre-side and skin-side readings of the reclosure, and their sign change marks relative phase. The source’s fitted association of a node with reduction (SA-33) supplies that exploratory comparison, not a universal necessity theorem.

Guards. The transition profile is signed; neither its node nor its absolute outer share is a value of local omnium density. No core sentence places a weak reclosure’s transition at the skin. A zero of this profile is not an instance of P-2’s vanishing transverse density gradient without a displayed map. No density-coordinate value is assigned to a node. The mTG node summaries survive the printed rounding, while its cancellation-sensitive integral normalisation does not.

8 Convergence statement

For identical lepton functions, transition profile, line fractions, shell probabilities, corrections and nuisance values, Eqs. (26)–(41) return the source’s Eqs. (1)–(15) and Table I term for term. The limiting map is the identity on those inputs and that observable dictionary. The constant-lepton limit extracts the lepton factors, and the same nuclear matrix element with the source’s spin and phase-space factors returns its detailed-balance approximation. The sign-definite reconstructions quantify the small departures from that point limit; the nodal reconstructions preserve the signed integrals. The measured half-life, inferred capture targets and BEST zone ratios are accepted in full. In-sample recovery corroborates no invariant. For the same stress tensor and data, Eq. (16) also returns GR’s curvature; that is I-3 containment, distinct from the numerical identity map.

The following interpretations are declined while their data and mathematical projections are retained. SA-09: the conventional account of the charged-current interaction named at L41 uses W exchange; the target itself does not specify that mediator. FDT reads interactions as curvature and geodesic motion of one medium, with the carrier identity derived in I-2.

SA-10: the conventional interpretation reads the source’s transition amplitude (L113–L114), built from the wavefunctions of Eq. (3), L79, as a probability amplitude. FDT reads those complex functions as the helical projection of I-4, with superposition expressing phase ignorance and phase fixation giving the observed projection.

SA-11: the source describes the transition density as “effectively converting a neutron into a proton” (L173–L174). The manuscript’s FDT reading is the invariant-preserving many-to-many reclosure of I-5b, with a seated two-strand closure and the lepton closures changing together under the complete exact books.

SA-12: the source describes the GT transition through proton and neutron single-particle components (L295–L297) and introduces its mDG and mTG forms as they “mimic, in a minimal analytic way, the typical polynomial-Gaussian structure of products of single-particle wave functions” (L278–L284), writing that “The powers r , r^2 , and r^3 should therefore not be interpreted as a one-to-one assignment to specific shell-model orbitals, but as a compact analytic way of enforcing regular behavior near the origin” (L286–L290). FDT reads the response as one pattern on

its own support under I-5 and core 0.1b, and the manuscript’s signed-profile phase reading leaves the source’s explicitly phenomenological status intact.

SA-13, SA-19: the standard account’s time-reverse reading: “Under microscopic reversibility (time-reversal invariance)” detailed balance “relates a process and its inverse once phase-space and spin-degeneracy factors are properly included” (L60–L66), and under it the ground-state cross section “can be determined exploiting the nearly exact” “inverse process” (L117–L119); the source writes that “electron capture is not” “the exact time-reversed process of neutrino capture” (L152–L153). The manuscript’s FDT reading of 0.1a and I-5b takes the capture and the resettlement as two events, each settled by its own global optimisation and each one invariant-preserving many-to-many reclosure, neither the other’s mirror.

9 Status ledger

The table below states the outstanding theory content and the scope of every comparison; upright A_2 is the core’s name of a Cartan matrix, distinct from the application A-2.

Item	Status and binding content
O-1	Owed: the finite-core cutoff r_c , the regular core chart, or the outward phase-transfer law.
O-2	Owed: a geometrically constructed odds for the overlap-contrast application A-3.
O-3	Owed: a canonical odds for the confinement application A-4.
O-4	Owed: the local profile $\rho_{\text{loc}}(r_f)$ from the omnium constitutive law with boundary, surface and core data, reducing to the measured nuclear profile.
O-5	Owed: multipole and tidal remainders of the closed-form pair expression; the overlap complex.
O-6	Owed: the maps from a charge or a colour seat to an I-1 input; derivation of the A_2 seat table from cone geometry; the gluon octet’s dynamics.
O-7	Owed: the fission-cosmology computation against the SNO neutral-current flux.
O-9	Owed: the confinement operator D , its domains, boundary data and spectrum; every mass ratio, including the two held-out targets.
O-11	Owed: ν_F on the exact framed closures; the odd-parity absorption proof; proof that the spinorial class is nontrivial.
O-12	Owed: the phase-exchange rates and the decay kernel, spectrum, chirality and rates.
I-4, third layer	Owed: the FDT source programme, and each claimed divergence.
A-3, A-4, A-5	Provisional tokens with open ranges only; no geometric warrant.
Mass targets	Held out under I-5: proton-to-electron ratio 1836.15267343 and squared muon-to-electron ratio 4.2753×10^4 ; no output of Eq. (22) is assigned.
Five candidates	Retired for the missing maps specified in the Predictions section; no prospective observable contrast.
Profile fits	In-sample source fits; full source functions and nuisance solutions return Table I by identity.
Reconstruction	Illustrative finite-charge-sphere lepton functions, large-component weights and independent uniform printed-parameter rounding; source functions are a different input.
DM measurements	Ground-state strength, energy dependence, charge state, excited-state renormalisation and microscopic profile are comparisons of the specified readings; charge-state selection is a shared consistency check.
X-1	Exploratory, in-sample node and absolute-weight summaries; skin and relative-phase interpretation are the manuscript’s reading.

10 Conclusions

The source exposes the dependence of a capture inference on how the lepton wave functions are integrated with the nuclear transition density. Its joint fit supplies examples of a reduced cross section at the observed half-life, with the normalisation and non-uniqueness stated above. The manuscript’s reading takes the capture and the resettlement as two events, each a many-to-many reclosure settled by its own simultaneous energy, mass, time and length settlement of the omnium. The finite-charge-sphere calculation makes the cancellation mechanism quantitative and its lepton-model sensitivity explicit. The independent forward ground-state strength is the decisive additional reading of the fitted profiles; its unit cross section must be independent of EC. The all-channel energy scan and the microscopic calculation test a transition profile on the source’s premise that the EC transition density is the same as that of the IBD reaction, “up to a trivial overall phase” (L189–L190), without substituting a charge radius for a transition density. The missing constitutive maps have specific entries in the status ledger. FDT’s curvature containment, braid classes and exact books organise this comparison without promoting an in-sample fit to an additional prediction.

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