What the miniature torsion pendulum measures

A finite-geometry reading of milligram gravitation

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Abstract

An et al. [1] measured the gravitational interaction of nominally 7.6 mg silicon wafers using a rotating attractor and a feedback-controlled pendulum (SC-011–013). Across five face spacings the measured torque falls with distance and agrees with the finite-body Newtonian comparison (SC-014, SC-238, SC-257–261, SC-274–278). Measured and calculated torques at the closest gap differ by -0.01 ± 0.59 aN m. The lever about the suspension, the phase of the attractor and the response of the loop determine what enters that comparison. Fundamental Density Theory (FDT) returns the curvature of general relativity from the same stress tensor and data. At this compactness the finite-body Newtonian torque is its leading weak-field reading: one wafer at 1 mm reads $\alpha_{\rm ext} = 1.1\times10^{-29}$ (U-17). The spherical pair law supplies an exact inverse-square force on its domain; the rotating silicon assembly reads the summed force moments of extended bodies. The printed geometric factor equals the calculated harmonic torque per unit reference constant and arm. The closest-gap value of G is therefore the reference value times the measured-to-calculated torque ratio, 0.999 ± 0.053. Of five apparatus contrasts, two read the same mean in both theories at matched inputs. Three turn on the pairs a settlement joins, the mass-shadow response and the charge-to-force map, which enter no number here. None is counted as a prediction. The torque readout and an entanglement witness are different observables.

1 Framework

The source status is experimental. Article metadata follow SC-001–010. Measured, observed and reported refer to the experiment of An et al. [1]. Derived, estimated, reconstructed and illustrative-assumed identify algebra, uncertainty assignments, readings of the printed record and prospective design inputs, respectively. SC identifiers refer to the source claims ledger, U identifiers to its non-source constants, conventions and design inputs, and SA identifiers to the source-audit register. The theory statements below use the locked core [2]. FDT is a true extension of general relativity and quantum mechanics. The experimental question here is whether an independent input of the extension changes a calibrated laboratory observable.

1.1 One substance and the clocks that measure it

In core 0.1 the omnium is matter, energy and spacetime. Let E denote energy, m mass, c the strand rate, ε_0 and μ_0 the electric and magnetic responses of the omnium, and d, t a stroke's

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distance and duration. Planck's constant is h, its reduced value is $\hbar$, and π is the circle constant. For a confined mode labelled by q, write E_q, m_q, ω_q for its rest energy, mass and angular frequency, T_q for its period and ℓ_q for the strand run during that period (core 0.1a, U-01, U-03).

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t}\right)^2.$$

$$E_q = m_q c^2 = \hbar \omega_q, \qquad T_q = \frac{2\pi}{\omega_q} = \frac{h}{E_q}, \qquad \ell_q = cT_q = \frac{h}{m_q c}.$$

In SI units $d/t = c = 299\,792\,458$ metres per second exactly (U-01). The two medium responses give the propagation equation and fix its characteristic rate. The same rate converts time to distance and, squared, mass to energy. The mass leg belongs to a massive closure. An open stroke has an energy, a period of phase advance and a wavelength. Its run keeps no proper time. The Compton length ℓ_q is the length a closed pattern's thread runs in one kept period, never the support of a confining domain (I-5).

Let $F_{\max}$ be the force ceiling, ℓ_{P} the Planck length, t_{P} the Planck time, and $d_{\min}, t_{\min}$ the infima of distance and duration. Core 0.1a sets the product of the ceiling, length and duration equal to one quantum of action.

$$F_{\max} d_{\min} t_{\min} = \hbar, \qquad d_{\min} = c t_{\min} = 2\ell_{\mathrm{P}}, \qquad t_{\min} = 2t_{\mathrm{P}}.$$

The infima bound the strokes below. The medium settles energy, mass, period and length together. Discreteness belongs to these configurations. Their exterior curvature coordinate varies continuously with evaluation radius.

In core 0.1d the strand has axial translation speed v and circulating speed v_{circ} on the unrolled helix. Its kinematic translation share is $\mathfrak{s}$. The inertia of a stationary closure includes the stresses that hold it closed. The vanishing integral of the spatial stresses in its rest frame gives the von Laue condition, so a boost carries momentum equal to total boosted energy times velocity divided by the squared strand rate.

$$v^2 + v_{\mathrm{circ}}^2 = c^2, \qquad \mathfrak{s} = v^2/c^2.$$

The translation share equals exterior compactness for a pattern held on the exterior branch. Its interior kinematic reading supplies no laboratory input (O-16).

I-2 derives one carrier from one medium and one local rate. Photon, graviton, gluon, phonon and magnon name the same spin-one excitation in different density and geometry regimes. A photon is one unbound strand. Its grades turn on a helix while its energy travels along the axis. With ω the phase angular frequency, k the wavenumber and $\bar{\lambda}$ the turning radius, the helix is fixed by its phase. The open stroke has $\bar{\lambda} = c/\omega$ and one wavelength of axial advance per circumference. Its spatial pitch angle is $\pi/4$. For a massive closed pattern the product of the phase rate ω/k and the group rate $d\omega/dk$ is c^2. Its phase rate exceeds its group rate. The ratio of those rates measures dispersion.

To describe an oscillation, write ϕ_h for its helical phase and ϕ_0 for its phase at the time origin. In the transverse plane $\mathbf{e}_1, \mathbf{e}_2$ are orthonormal directions and $\mathbf{B}$ is their oriented unit bivector. The complex unit is i, and e denotes the exponential base. At fixed axial position the oscillator projection is a rotation in this plane (I-2, I-4).

$$\phi_h(t) = \phi_0 - \omega t, \qquad \mathbf{B} = \mathbf{e}_1 \mathbf{e}_2, \qquad \mathbf{B}^2 = -1, \qquad e^{\mathbf{B}\phi_h} = \cos\phi_h + \mathbf{B}\sin\phi_h.$$

The bivector represents i on the plane. The exponent is dimensionless. This single-angle phase rotor tracks a cylindrical helix at fixed field identity. A spinor rotation of a physical frame uses a half angle. In I-4, phase ignorance at observation gives the superposition reading and projection fixes phase. The mechanical stage selects when an angular pattern returns to the pendulum. It supplies no frequency measurement of the common carrier.

1.2 Chart inventory and the exact faces

Let G denote Newton's constant, $\mathcal{M}$ total gravitating mass including non-rest energy, R_s its Schwarzschild radius, r_f a field-point radius, R_b a support radius and r_c the lower radius of the interior chart. In particular, R_b refers to material extent. The branch tag σ is positive unity outside and negative unity inside. The atlas coordinate α_σ is a ratio of circular curvatures; ∞ denotes the unbounded radial limit. Denote its exterior and interior values by $\alpha_\mathrm{ext}, \alpha_\mathrm{int}$, their algebraic odds by $X_\mathrm{ext}, X_\mathrm{int}$, and the computational coordinate by β_σ (core 0.2–0.4).

$$R_s = 2G\mathcal{M}/c^2, \qquad \alpha_\sigma = (R_s/r_f)^\sigma.$$

$$\alpha_\mathrm{ext} = R_s/r_f, \quad X_\mathrm{ext} = R_s/(r_f - R_s) \qquad (R_s < r_f < \infty).$$
$$\alpha_\mathrm{int} = r_f/R_s, \quad X_\mathrm{int} = r_f/(R_s - r_f) \qquad (r_c \le r_f < R_s).$$

The gap to the fold fixes the positive odds. In A-1 the symbol X_σ denotes those odds on the selected branch. The A-1 ratio is already bounded by the two geometric lengths. For an odds enhancement x and initial A-1 ratio $\alpha_{\sigma,0}$, I-6 multiplies the initial odds by one plus the enhancement.

$$\alpha_\sigma = \frac{X_\sigma}{1 + X_\sigma}, \qquad \alpha_\sigma(x) = \frac{\alpha_{\sigma,0}(1+x)}{1 + \alpha_{\sigma,0}x}, \qquad x \ge 0.$$

The remaining chart tokens denote overlap α_ovl, confinement position α_conf and mode manifestation α_mode. The skin diagnostic is α_core.

Application	Coordinate	Warrant and use
A-1	α_σ	Geometric, branch and radius fixed. Field-point, configuration and source evaluations each declare the gravitating source and the radius of evaluation. The source's total gravitating mass sets R_s. Only the exterior cross-centre evaluation enters the spherical pair.
A-2	$\beta_\sigma = \alpha_\sigma/(1+\alpha_\sigma)$	Geometric change of computational chart. It supplies no force, optical or density input.
A-3	α_ovl	Overlap contrast. Token and open range only (O-2).
A-4	α_conf	Confinement position. Token and open range only (O-3).
A-5	α_mode	Carrier manifestation. Token and open range only.
Diagnostic	$\alpha_\mathrm{core} \to 1^-$	Schematic saturation at a nuclear skin.

A-3 through A-5 have no geometric warrant here and carry no value, ordering, trajectory or sign. On A-1 both branches reach the fold $r_f = R_s$ only as one-sided limits. The numerator of q_σ is the total mass $\mathcal{M}$ divided by the spherical volume at the chosen field radius r_f or support radius R_b. The reference density is the same mass divided by the volume at R_s, using the same shape

factor (core 0.6). The ratio is $q_\sigma(r_f) = \alpha_\sigma(r_f)^{3\sigma}$. For the configuration mean, replace r_f by R_b on both sides and use the branch containing that radius. Total source mass divided by the field volume gives the enclosed mean when that volume contains the whole source. Otherwise it gives a comparison proxy. The physical enclosed mean uses the mass inside the volume. The physical local density rises towards the skin. Its numerical profile requires independent boundary, surface and core data (O-4). For a bound configuration the exterior monopole atlas of its source has its fold at R_s. In the configuration's separate interior chart the skin lies at the fold limit. An independent size datum sets the physical skin radius. An interior probe gives a profile-weighted mean over the support. No transfer rescales a laboratory skin radius onto the exterior gravitational radius.

For I-1 let F be the static force magnitude and α_1, α_2 the two bounded force inputs. Write $\alpha_{1,\mathrm{ext}}, \alpha_{2,\mathrm{ext}}$ for exterior cross-centre inputs, $\mathcal{M}_1, \mathcal{M}_2$ for the two masses, $R_{s,1}, R_{s,2}$ for their gravitational radii and d_cc for the centre separation.

$$F = F_\mathrm{max}\alpha_1\alpha_2, \qquad F_\mathrm{max} = \frac{c^4}{4G}.$$

The force law holds at every scale. Each of the two inputs to the universal force law lies strictly between zero and one. On either declared geometric A-1 branch the chart coordinate itself satisfies $0 < \alpha_\sigma < 1$. The universal open range of the inputs implies a force below the ceiling. At monopole order on two nonoverlapping spherical supports, each support encloses its gravitational radius and their radii sum to at most the centre separation. There $\alpha_{1,\mathrm{ext}} = R_{s,1}/d_\mathrm{cc}$ and $\alpha_{2,\mathrm{ext}} = R_{s,2}/d_\mathrm{cc}$. Substitution gives the exact reduction, I-4 layer L1,

$$F = \frac{c^4}{4G}\frac{R_{s,1}}{d_\mathrm{cc}}\frac{R_{s,2}}{d_\mathrm{cc}} = \frac{G\mathcal{M}_1\mathcal{M}_2}{d_\mathrm{cc}^2}.$$

The spherical pair assumptions imply $F \le F_\mathrm{max}/4$. Equality requires two equal saturated supports in contact. A face spacing between flat wafers gives neither centre separation nor a spherical exterior.

In I-3 let $G_{\mu\nu}$ denote the Einstein tensor, $T_{\mu\nu}$ the full stress tensor and κ the Einstein coupling. The field equation stands unchanged, with α downstream of its solution:

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \qquad \kappa = 8\pi G/c^4.$$

With the same stress tensor, cosmological terms, initial and boundary data and gauge, both theories return the same curvature. Compactness parameterizes the spherical on-shell solution. For static exterior rays write $d\ell_\mathrm{opt}$ for optical length and γ_sph for the squared angular line element on the unit sphere. In areal coordinates the exact optical face is

$$d\ell_\mathrm{opt}^2 = \frac{dr_f^2}{(1-\alpha_\mathrm{ext})^2} + \frac{r_f^2\,\gamma_\mathrm{sph}}{1-\alpha_\mathrm{ext}}.$$

Generalised Snell refraction conserves the tangential optical momentum with the exact metric on each side. The lapse fixes the leading acceleration of slow masses. The radial ruler also enters the optical metric, doubling the leading transverse deflection relative to the lapse-only reading. The causal aperture index of P-4 is this metric's index across the density gradient (O-8) [5].

1.3 Confinement and the source ledger

In I-5 the braid count n labels a class of closed patterns. Confinement position sets a mode's mass within a class. Moving within the class preserves the count. Moving between classes requires re-braiding. A closed matter configuration has at least two strands and lies outside the carrier identity. Write D for the confinement operator, Ω for the pattern's support, B_{bd} for its boundary data, χ for orientation, ξ_{pos} for position on the inside/outside chart and u_q for a mode. With the mass-frequency bridge above, the eigenproblem is

$$D(\Omega, B_{\mathrm{bd}}; n, \chi, \xi_{\mathrm{pos}})u_q = \omega_q u_q.$$

No operator form, boundary spectrum or mass ratio enters the wafer calculation (O-9). The support belongs to the pattern and is fixed independently of its mass. The metrology measures the dimensions of the composite silicon bodies. This experiment supplies no support radius for an individual confinement mode.

For I-7 let M be rest mass, P pressure, V_{vol} volume, Θ_{abs} temperature, a the radiation energy-density constant and σ_{SB} the Stefan–Boltzmann flux constant. Core 0.5 writes the mass ledger with independent channels,

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV_{\mathrm{vol}}}{c^2} + \frac{a\Theta_{\mathrm{abs}}^4 V_{\mathrm{vol}}}{c^2} + \cdots, \qquad a = 4\sigma_{\mathrm{SB}}/c.$$

Here E in the non-rest slot is the remaining independent energy. Rotation, support stress, electromagnetic and thermal energy belong to the same tensor. An assayed total already includes its constituents. The radiation term counts radiation present in that volume; a second entry for the same energy would double count it. Gravitational binding appears in the solved mass aspect. A change to a source is carried into both field calculations.

1.4 Notation guard

The left column gives the article's printed labels as identifiers. The right column gives the quantities used here, including the distinctions needed to reproduce the measurement. The symbol $\mathsf{u}(\cdot)$ denotes the standard uncertainty of its argument.

Article label	Quantity in this analysis
Yukawa alpha, lambda	g_Y, ℓ_Y, relative strength and range (SC-174–176).
Uncertainty sigma	$\mathsf{u}(\cdot)$ or a stated multiple. The exclusion has its own confidence convention (SC-207, SC-256, SC-270).
Delay delta-phi	$\mathsf{d}_{\mathrm{delay}}$, complex delay factor (SC-223).
Magnetic field B	$\mathcal{B}_{\mathrm{ext}}, \mathcal{B}_{\mathrm{amb}}, \mathcal{B}_{\mathrm{coil}}$, respectively inducing, ambient and applied coil fields (SC-301, SC-306, SC-310).
Susceptibility chi	$\mathsf{c}_{\mathrm{mag}}$, silicon volume susceptibility (SC-305).
Torsion and attitude theta	ϑ_{tor}, pendulum yaw; $\vartheta_{\mathrm{align},x}, \vartheta_{\mathrm{align},z}$, alignment angles (SC-099–100, SC-213).
Torque tau	$\mathcal{N}_{\mathrm{ext}}, \mathcal{N}_{\mathrm{fast}}, \mathcal{N}_{\mathrm{slow}}$, external torque and the two reported amplitudes (SC-136, SC-138, SC-214).
Feedback beta	b_{fb}, torque per feedback voltage (SC-215).
Integration T, natural period	$t_{\mathrm{win}}, t_{\mathrm{nat}}$, quadrature integration time and free mechanical period (SC-146, SC-240–241).
Temperature change and slope	$\Delta\Theta_{\mathrm{env}}, \mathfrak{k}_{\mathrm{env}}$, local environmental variation and calibrated torque response (SC-320–321).
Voltage labels a, b	$U_{\cos}, U_{\sin}$, cosine and sine voltages (SC-240–241).
Effective arm	ℓ_{arm}, torque-to-equivalent-force arm (SC-123).
Geometric factor	J_{geom}, reported denominator of the article's closest-gap G quotient (SC-093–094, SC-105).
Mechanical inertia	$\mathsf{J}_{\mathrm{rot}}$, mass moment of inertia (SC-148).
Magnetizations	$\mathbf{M}_{\mathrm{pen,rem}}, \mathbf{M}_{\mathrm{att,rem}}$ and $\mathbf{M}_{\mathrm{pen,ind}}, \mathbf{M}_{\mathrm{att,ind}}$, remanent and induced in pendulum and attractor (SC-299–307).
Rotor and signal rates	$f_{\mathrm{rot}}, \omega_{\mathrm{rot}}$ for rotation and $f_{\mathrm{sig}}, \omega_{\mathrm{sig}}$ for signal. Cyclic rates in Hz (cycles per second), angular rates in radians per second (SC-034–036).
Spacings s, r	s_{gap}, face spacing, and r_{pair}, separation of interacting elements (SC-050, SC-176).
Potential V	$\mathcal{U}_Y$, Yukawa comparison potential energy (SC-176).

2 System chart table

Object	Region and state	What the readout samples
Spherical reference	Two exterior field points evaluated cross-centre; the spherical supports are disjoint.	Static pair magnitude on the exact I-1 domain.
Rotor at one encoder phase	Entire material support of the source assembly in its rotated position.	A distribution of lever directions and pair distances. Fourier projection of the rotating pattern over a full turn selects the signal harmonic (SC-032–038, SC-244–249).
Pendulum	Test end, counterbalance, central reflector and joining structures, all referred to the fibre axis.	A projected torque; spatial weighting changes sign across the axis (SC-057–066, SC-244–247).
Bound microscopic pattern	Exterior monopole atlas of the source, with its fold at R_s. Separate interior chart of the configuration, with its skin at the fold limit and physical skin radius set by independent size data.	This balance supplies no microscopic skin radius, individual mode-support radius, interior profile-weighted measurement or individual confinement eigenvalue.
Membrane and actuator	Membrane facing the pendulum; electrodes across the counterbalance.	Capacitive torque and a frequency-specific voltage response (SC-050–056, SC-067–073, SC-279–291).

The field radius of an isolated spherical reference is a physical evaluation radius, and its laboratory gravitational window $\alpha_{\mathrm{ext}} = \mathcal{M}c^2/(2F_{\max}r_f)$ is a correct reading (core 0.1c, I-1). For one wafer of 7.6 mg read as a spherical reference (SC-011), $R_s = 1.1 \times 10^{-32}$ m and $\alpha_{\mathrm{ext}} = 1.1 \times 10^{-29}$ at $r_f = 1$ mm (U-01, U-02, U-17, SC-112). That field-point reading lies deep in the general-relativistic regime of the A-1 exterior branch, whose seam $\alpha_{\mathrm{ext}} = 1/2$ sits at $r_f = 2R_s$ (core 0.3a). The hardware has finite thickness and angular structure. There is no single observation radius for all contributing volumes. Each phase defines another source configuration, and each configuration places the two ends at different distances and moment arms. A single number for the nominal gap therefore cannot fix the angular-harmonic torque.

3 Observation map

Each closing below specifies a future signed measurement and a matched comparator. The precision of a source amplitude describes the datum printed at that setting. The proposed rejection criterion uses the joint uncertainty of the new contrast, with loads, probe regions, alignment and previous drive controlled as specified. Equation (5) gives the centered contrast and its joint variance. The following decision rule sets the rejection threshold at three standard uncertainties under a validated Gaussian model with independently characterized joint covariance.

3.1 What oscillates and what the loop measures

In Fig. 1 a tungsten fibre suspends the pendulum. The pendulum including reflector and fastenings is about 243 mg (SC-025, L84). A central wafer provides the optical angular readout.

Two end wafers serve as test mass and counterbalance. Differential capacitive actuators sit on opposite sides of the counterbalance (SC-057–064, SC-070–073). The assembly has an end-to-end length of 49.986 mm (SC-061, L125). The attractor has sixteen nominally identical wafer sources. The stage brings them successively past the test wafer. The article places the signal at the sixteenth harmonic away from the fundamental disturbance frequency (SC-033–038, SC-065–066). Write $\mathsf{H}_{\mathrm{ord}}$ for this positive integer harmonic order. Then $f_{\mathrm{sig}} = \mathsf{H}_{\mathrm{ord}} f_{\mathrm{rot}}$ and $\omega_{\mathrm{sig}} = 2\pi f_{\mathrm{sig}}$ (SC-034–036). These are rotation and signal rates in the laboratory frame.

For free oscillations the article gives $t_{\mathrm{nat}} = 49.50(5)\,\mathrm{s}$ and quality factor $Q_{\mathrm{mech}} = 1786(20)$ (SC-146–147). It calculates $\mathsf{J}_{\mathrm{rot}} = 12.61(1) \times 10^{-9}\,\mathrm{kg\,m^2}$ from the measured geometry (SC-148–149). The derived fibre stiffness $\mathsf{k}_{\mathrm{fib}}$ is $203.1(5)\,\mathrm{pN\,m/rad}$ (SC-150–151). Using the rounded inertia and free period in the stiffness formula gives $203.2\,\mathrm{pN\ m/rad}$ (SC-146, SC-148, SC-151). The difference in the last displayed place lies within the published uncertainty. The free period characterizes the suspension. Forced angular motion, the torque applied to the suspension and the compensating voltage each have a distinct transfer.

Let U be the feedback voltage, U_{bias} the common electrode bias and $\mathsf{k}_{\mathrm{elec}}$ the positive magnitude of the membrane-induced negative stiffness. The external torque is $\mathcal{N}_{\mathrm{ext}}$. The opposed voltages are $U_{\mathrm{bias}} + U$ and $U_{\mathrm{bias}} - U$, with $U_{\mathrm{bias}} = 5\,\mathrm{V}$ (SC-230–231). Let $\mathcal{K}_{\mathrm{pid}}$ denote the complex voltage-per-angle response of the proportional, integral and derivative controller including its delay, and H_U the closed-loop voltage-per-torque transfer. With harmonic time convention $\exp(i\omega t)$, the complex angular, voltage and torque components are $\widetilde{\vartheta}_{\mathrm{tor}}$, $\widetilde{U}$ and $\widetilde{\mathcal{N}}_{\mathrm{ext}}$. Source Eqs. (1) and (2) give, with the feedback torque opposing the external torque (SC-213–222, SC-230–233),

$$\left[\mathsf{k}_{\mathrm{fib}}(1 + i/Q_{\mathrm{mech}}) - \mathsf{k}_{\mathrm{elec}} - \mathsf{J}_{\mathrm{rot}}\omega^2\right] \widetilde{\vartheta}_{\mathrm{tor}} = \widetilde{\mathcal{N}}_{\mathrm{ext}} - b_{\mathrm{fb}}\widetilde{U}.$$

$$H_U(\omega) = \frac{\mathcal{K}_{\mathrm{pid}}(\omega)}{\mathsf{k}_{\mathrm{fib}}(1 + i/Q_{\mathrm{mech}}) - \mathsf{k}_{\mathrm{elec}} - \mathsf{J}_{\mathrm{rot}}\omega^2 + b_{\mathrm{fb}}\mathcal{K}_{\mathrm{pid}}(\omega)}.$$

Here the tilded angle and voltage are the harmonic coefficients of ϑ_{tor} and U. In the second expression the numerator maps angle to voltage. Each denominator term has units of torque per radian. Let Δt be the one-second sampling interval (SC-224). The PID response includes $\mathsf{d}_{\mathrm{delay}} = \exp(-i\omega\Delta t)$ (SC-223–224). The feedback gain is $33.4(5)\,\mathrm{fN\,m/V}$ (SC-226, L294). At the closest spacing the electrostatic stiffness magnitude is $19.4(2.7)\,\mathrm{pN\,m/rad}$ (SC-227, L295). The authors estimate the typical relative uncertainty of the signal-frequency transfer at about 2%, primarily from the gain and electrostatic stiffness (SC-228–229, L297–298). The printed controller gains are $0.02\,\mathrm{V/\mu rad}$, $0.002\,\mathrm{V/(\mu rad\,s)}$ and $0.1\,\mathrm{V\,s/\mu rad}$ in proportional, integral and derivative order (SC-218, L270). With them the transfer above gives $|b_{\mathrm{fb}} H_U| = 1.33$ at the fast signal frequency, 26.667 mHz (SC-138). The torque estimate is the voltage coefficient divided by H_U, so the gain's relative uncertainty enters the torque multiplied by that modulus. The gain's 1.5% enters at 2.0% and the stiffness's $2.7\,\mathrm{pN\,m/rad}$ at 0.5%, together 2.1%, beside the $0.22\,\mathrm{aN\,m}$ transfer row of Table 1 (SC-118, SC-226–227). Near the 30 mHz band the approximate torque and force noise floors are $0.15\,\mathrm{fN\,m/\sqrt{Hz}}$ and $6\,\mathrm{fN/\sqrt{Hz}}$ (SC-048–049, L110, L112). Their ratio is 25 mm, the effective arm at the floors' one-figure precision. A white one-sided torque density S_τ gives each quadrature over a span t_{win} the standard uncertainty $(S_\tau / t_{\mathrm{win}})^{1/2}$. Over 100 h the torque floor gives $0.25\,\mathrm{aN\,m}$, beside the $0.27\,\mathrm{aN\,m}$ statistical row of Table 1 (SC-120, SC-267).

The rotor rate is $\omega_{\mathrm{rot}} = 2\pi f_{\mathrm{rot}}$. The integrated stage encoder monitors the attractor's angular position (SC-236–237). The data analysis removes low-frequency drift with a third-order polynomial fit, extracts the harmonic voltage and divides by the transfer at the signal frequency (SC-240–243, L316, L318–320). For the following single-window reconstruction over complete

cycles let U_{det} denote the detrended voltage. Write $\mathcal{N}_{\mathrm{est}}$ for the complex torque estimate from this window. With the stated time convention the detrended voltage coefficient is $U_{\cos} - iU_{\sin}$.

$$U_{\cos} = \frac{2}{t_{\mathrm{win}}} \int_0^{t_{\mathrm{win}}} U_{\mathrm{det}}(t) \cos(\omega_{\mathrm{sig}} t)\, dt, \qquad U_{\sin} = \frac{2}{t_{\mathrm{win}}} \int_0^{t_{\mathrm{win}}} U_{\mathrm{det}}(t) \sin(\omega_{\mathrm{sig}} t)\, dt.$$

$$\mathcal{N}_{\mathrm{est}} = (U_{\cos} - iU_{\sin})/H_U(\omega_{\mathrm{sig}}).$$

At a fixed setting the closed loop is the causal readout of I-1: its susceptibility maps torque to voltage and is not the force law. Model it as causal, linear and time invariant. From the start of a finite run to each sample time, convolution of the integrable impulse response with the bounded torque input gives the driven voltage. Add the free response from the state at the start of the run. For a steady harmonic input the voltage coefficient is the torque coefficient multiplied by $H_U(\omega)$. A change of setting can leave an aperiodic transient from the pendulum's yaw and angular velocity and the controller's stored signals, including its delay history. Predict the voltage after the switch from that inherited state and the subsequent drive. Subtracting a polynomial fitted to the time series can change its finite-window quadratures, even over complete cycles. Taking a modulus discards the phase. The feedback suppresses the yaw while the applied torque modulates the feedback voltage (SC-140–141, SC-162–166). In Fig. 2e the close-gap spectrum has a distinct signal at the sixteenth harmonic. The identical measurement at about 3 mm shows a prominent signal only at the drive fundamental, with a magnitude consistent with the close-separation result, and no discernible peak at the sixteenth harmonic (SC-168–173, SC-332, L230–236). The authors expect the latter torque below their noise floor. A sensitivity-limited control places no exact zero on the torque.

Cluster closing. This cluster is I-1's reading of the record and the convergence map of Use item 2: the static moment and its causal readout stay distinct, and the calibrated transfer returns the torque. At fixed face gap, subtract the matched signed finite-window torque prediction from the signed reading at each period, using the reduction defined with Eq. (5). Reject the combined source and transfer description when the absolute difference of those residuals exceeds three standard uncertainties of that difference under the Gaussian test specified below. Calibrate separately at each frequency and randomize period order within each new pair. The reported two-frequency amplitudes and their statistical precision are given below.

3.2 What the geometry predicts and what the quotient reuses

Use locally Cartesian laboratory coordinates in the weak-field, slow-motion limit. Retain the Newtonian scalar potential to first order in potential divided by c^2 in the metric and leading order in source and test velocities divided by c. Neglect post-Newtonian stresses, retardation, velocity-dependent fields and gravitational backreaction on the prescribed stage motion at this order. This is I-4 layer L2, with the same laboratory time coordinate and the spatial axes fixed to the apparatus support. The conventional potential is calculated on the actual rotated source, including its measured departures from perfect symmetry. At this order the field equation is linear in its source, so the potential of the whole rotated source is the volume integral of its density: one solved field of the whole system acting on the whole receiver, the system-wide settlement read at L2 (core 0.1a, I-3). The element pairs below are the terms of that linear solution, never separate responses.

Let φ_{rot} be encoder phase, $\mathbf{r}_{\mathrm{pen}}$ a pendulum element position and $\mathbf{r}_{\mathrm{src}}$ an attractor element position at that phase. The vector $\mathbf{b}_{\mathrm{pair}} = \mathbf{r}_{\mathrm{src}} - \mathbf{r}_{\mathrm{pen}}$ points from receiver to source, and r_{pair} is its norm. Denote the mass elements by dm_{pen} and dm_{src}, each equal to local mass density times its own volume element. Let $\mathbf{o}$ be a fixed point on the suspension axis and $\hat{\mathbf{z}}_{\mathrm{fib}}$ the positively oriented axial unit vector. Positive torque increases right-handed yaw. The integrated force $\mathbf{F}_{\mathrm{net}}$ and moment $\boldsymbol{\mathcal{N}}_{\mathbf{o}}$ form the wrench about that point. Let $\mathcal{N}_{\mathrm{ax}}$ be the component along the fibre. Write $d\mathbf{F}_{\mathrm{pair}}$ for one attractive force increment and $\iint$ for the integral over both bodies. The conventional kernel in SC-244–249 gives the wrench at fixed phase:

$$d\mathbf{F}_{\mathrm{pair}} = G\,\frac{\mathbf{b}_{\mathrm{pair}}}{r_{\mathrm{pair}}^{3}}\,dm_{\mathrm{pen}}\,dm_{\mathrm{src}}, \qquad \mathbf{F}_{\mathrm{net}} = \iint d\mathbf{F}_{\mathrm{pair}}. \tag{1}$$

$$\boldsymbol{\mathcal{N}}_{\mathbf{o}} = \iint (\mathbf{r}_{\mathrm{pen}} - \mathbf{o}) \times d\mathbf{F}_{\mathrm{pair}}, \qquad \mathcal{N}_{\mathrm{ax}} = \hat{\mathbf{z}}_{\mathrm{fib}} \cdot \boldsymbol{\mathcal{N}}_{\mathbf{o}}. \tag{2}$$

Equation (2) takes the cross product of each lever with its force increment from Eq. (1) before integration. Moving the origin along the fibre changes the full moment by the net force crossed with the displacement from the old origin to the new one. The axial projection of this change is zero. The test and counterbalance forces need not be opposite for their axial moments to have opposite signs. The sign comes from the lever and the force on that element. Collapsing the entire assembly to one lever before integration erases those signs.

Let $\mathcal{U}_{\mathrm{N}}$ denote the gravitational interaction potential energy. Under an infinitesimal rigid yaw at fixed encoder phase, the change of potential energy equals minus the work of the attractive forces. Consequently

$$\mathcal{U}_{\mathrm{N}} = -\iint \frac{G}{r_{\mathrm{pair}}}\,dm_{\mathrm{pen}}\,dm_{\mathrm{src}}, \qquad \mathcal{N}_{\mathrm{ax}} = -\left.\frac{\partial \mathcal{U}_{\mathrm{N}}}{\partial \vartheta_{\mathrm{tor}}}\right|_{\varphi_{\mathrm{rot}}}. \tag{3}$$

In Eq. (3), rigid rotation transports the integration volumes with their bodies. In material coordinates the integration bounds stay fixed; the derivative acts on separation alone. Finite masses on compact supports with a positive minimum clearance bound both kernel and yaw derivative, which justifies differentiation under the integral.

For a future experiment, fix encoder zero at an identified indexed facet, positive phase in the direction of rotation, and the sign of yaw with a known applied calibration torque. Denote the complex harmonic by $\mathcal{C}_{\mathrm{ax}}$, the independently preregistered reference phase by δ_{ref}, and the corresponding physical signed component by $\mathcal{P}_{\mathrm{ax}}$. Here $\Re$ takes the real part of the phase-referenced coefficient (U-10):

$$\mathcal{C}_{\mathrm{ax}} = \frac{1}{\pi}\int_{0}^{2\pi} \mathcal{N}_{\mathrm{ax}}(\varphi_{\mathrm{rot}})\, e^{-i\mathsf{H}_{\mathrm{ord}}\varphi_{\mathrm{rot}}}\, d\varphi_{\mathrm{rot}}, \qquad \mathcal{P}_{\mathrm{ax}} = \Re\big(\mathcal{C}_{\mathrm{ax}} e^{-i\delta_{\mathrm{ref}}}\big). \tag{4}$$

For a cosine of positive peak amplitude and additive phase offset, the coefficient equals that amplitude times the exponential of i times the offset. Equation (4) projects this coefficient onto the quadrature with reference phase δ_{ref}. The instantaneous torque from this harmonic at encoder angle φ_{rot} is $\Re(\mathcal{C}_{\mathrm{ax}} e^{i\mathsf{H}_{\mathrm{ord}}\varphi_{\mathrm{rot}}})$. To predict a finite-window reading, apply the reduction specified with Eq. (5) to the full predicted torque trace with the appropriate loop state. The printed amplitudes supply neither a signed projection nor an absolute encoder origin, torque polarity or complex coefficient (SC-136–139, SC-237, SC-240–243, SC-274–278). The equivalent force F_{eq} expresses the readout in force units. Dividing the calibrated torque amplitude by ℓ_{arm} gives F_{eq}.

The three components of the net force require the force integral. Torque has units N m, torque divided by G has units kg^2/m and the reported J_{geom} has units $\mathrm{kg}^2/\mathrm{m}^2$. For the Newtonian model at normalization G, denote the coefficient of its signed axial torque in Eq. (4) by $\mathcal{C}_{\mathrm{ax,N}}$. Its physical harmonic equivalent-force amplitude per unit G is $J_{\mathrm{eq}} = |\mathcal{C}_{\mathrm{ax,N}}|/(G\ell_{\mathrm{arm}})$. For their geometric factor An et al. print the unsigned scalar $\iint |d\mathbf{F}_{\mathrm{pair}}|/G$ over the two volumes in their expression (SC-093–094, L156–159). The Methods section separately calculates the torque harmonic for the whole pendulum assembly, the sixteen source masses and their supporting struts (SC-244–249). Consider a source whose entire mass distribution is axisymmetric about the stage rotation axis, with the receiver and fibre axis fixed. Rotation then preserves the laboratory mass distribution. Its scalar integral is positive for separated bodies of positive mass, while its sixteenth torque harmonic vanishes. The printed scalar expression alone therefore does not specify J_{eq}.

Face gap (mm)	Measured torque (aN m)	Gap and plotted label
0.654	11.16(1.07)	SC-238, SC-274
0.854	9.80(1.06)	SC-238, SC-275
1.054	7.47(1.19)	SC-238, SC-276
1.354	6.16(1.11)	SC-238, SC-277
1.554	4.94(1.17)	SC-238, SC-278

The spacings come from L310 and the black-square labels from L499–503. Each parenthesis contains the full numerical uncertainty in aN m. Fig. 3a uses two standard uncertainties and incorporates statistical and other systematic errors in those bars (SC-256, SC-259–261, SC-271–273). At the closest gap the bar is $2[(0.27)^2 + (0.46)^2]^{1/2} = 1.07\,\mathrm{aN\,m}$, the statistical and other-systematic rows of Table 1; the Newtonian-calculation row sits in the red band (SC-114, SC-120, SC-274). The measured central torque falls at every successive spacing. The red band represents the theoretical Newtonian torque calculated from the measured geometric parameters of the pendulum and the source mass (SC-257–258, L291, L293). The article describes separate integrations using measured dimensions, orientations, alignments and densities for the full pendulum assembly and the sources with supporting struts (SC-244–251). The nominal design gives the source the test mass's dimensions. Table 1 distinguishes their measured dimensions (SC-030–031, SC-077–089). For example, their respective thicknesses in test and source order are 0.4871 ± 0.0002 and $0.4985 \pm 0.0003\,\mathrm{mm}$ (SC-078, SC-087). The authors average the densities of samples cut from the wafer used for both assemblies. Each sample density is its measured mass divided by the volume inferred from its measured dimensions (SC-125–127). This shared metrology matters when subtracting predictions at two settings.

At the closest setting the spokes alone contribute 0.53 aN m (SC-159, L233). The denominator for their share is the full calculated torque $\mathcal{N}_{\mathrm{calc}} = 11.17(26)\,\mathrm{aN\,m}$, including the spokes (SC-161, L235). Here the parenthesized digits give one standard uncertainty in hundredths of an aN m (SC-252). The spoke share is 4.7%, the authors' figure (SC-159–161). The source gives the calculated force as $471 \pm 11\,\mathrm{aN}$ at one standard uncertainty (SC-123, SC-252–254, L332–333). With the independent CODATA reference [3] the ledger value of G is $6.67430(15) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ (SC-112, L170). Multiplying the independent reference by the reported closest-gap coefficient gives 471 aN at the displayed precision (SC-105, SC-112, SC-253). Dividing the calculated torque by the measured arm gives the same rounded central force (SC-123, SC-252–253). The record's own geometry settles which factor the printed value is. On the test wafer and the facing source wafer alone, at the 0.654 mm face gap with the dimensions and density of Table 1, the printed integral evaluates to $2.2 \times 10^{-5}\,\mathrm{kg}^2/\mathrm{m}^2$, 3.1 times the printed value, and every further volume

element raises it (SC-050, SC-077–089, SC-094, SC-105). The calculated harmonic torque per unit reference constant and arm is $\mathcal{N}_{\mathrm{calc}}/(G\ell_{\mathrm{arm}}) = (7.06 \pm 0.16) \times 10^{-6}\,\mathrm{kg}^2/\mathrm{m}^2$, the printed value with its uncertainty (SC-105, SC-112, SC-123, SC-252). The printed number is therefore J_{eq} at the reference normalization, $J_{\mathrm{geom}} = \mathcal{N}_{\mathrm{calc}}/(G\ell_{\mathrm{arm}})$, and the two central-force checks above restate that identity. Computing the torque band afresh at the other gaps would require the component geometry and alignments used in those integrations. The record prints no numerical predicted torques at the other four gaps and no joint covariance, so the five points carry the reported band comparison without an independently reconstructed band, distance exponent or goodness-of-fit statistic.

Cluster closing. This cluster is the I-3 and I-4 (L2) reading of the record and the convergence map of Use item 2. The full-geometry comparator and its quoted closest-setting error set the scope of the agreement, and the exploratory quotient restates the closest-gap comparison. At that gap the measured 11.16 aN m and the calculated 11.17 aN m differ by -0.01 aN m against a standard uncertainty of 0.59 aN m (SC-121, SC-138, SC-252). Reject the finite-body prediction with that metrology when the absolute difference of the held-out and reference signed torque residuals exceeds three standard uncertainties of that difference under the same Gaussian test. Subtract the predicted signed finite-window torque estimate for each configuration. Use the same axis, reference phase, source masses and calibrated normalization on both sides, with the reduction defined with Eq. (5).

3.3 What the controls and searches constrain

The fast and slow rotor periods were 600 and 3600 s (SC-128, L175). Multiplying the slow rotor cyclic rate by the harmonic order gives 4.444 mHz at the article's frequency precision (SC-128–130). At the same 0.654 mm gap, the slow run used 15 sets of 13 complete rotations (SC-133–135). The article reports $\mathcal{N}_{\mathrm{slow}} = 11.41 \pm 0.46$ aN m at 4.444 mHz (SC-136, L192) and $\mathcal{N}_{\mathrm{fast}} = 11.16 \pm 0.27$ aN m at 26.667 mHz (SC-138, L195). Within one standard uncertainty the authors report agreement with the earlier measurement, with statistical uncertainty only in this comparison (SC-136–139). A difference of the two amplitude estimates is distinct from the modulus of a complex difference or a difference of signed projections. Each estimator requires its own sampling distribution. A retrospective frequency constraint would require joint calibration errors and systematic covariance as well as statistical errors.

Separation and shielding. The closest test-to-source gap s_{gap} is 0.654 mm and the gold-coated silicon membrane is 50 µm thick (SC-050–051). The source-to-membrane spacing is fixed at 0.268(3) mm and the pendulum-to-membrane gap is at least 0.336 mm (SC-053–054). For this pendulum–membrane separation the authors describe the Casimir and squeeze-film damping effects as negligible, referring to their supplementary analysis (SC-054–056, L117–119). The far control changes the test-to-source spacing. The larger pendulum–membrane spacing for characterization of the free oscillator has a separate purpose (SC-144, SC-169–171). The alignment angles $\vartheta_{\mathrm{align},x}$ and $\vartheta_{\mathrm{align},z}$ affect both the clearance and the lever projection. The authors give the uncertainty of face separation along the gap direction as below 20 µm (SC-044, L105). The closest-gap uncertainty budget assigns 0.23 aN m to alignment (SC-092, L156).

From a nuisance response to a bound. The pendulum and membrane are grounded. Residual patch potentials introduce negative stiffness and could produce a spurious signal at the sixteenth harmonic as well as random noise (SC-067–069, SC-279–285). The measured maximum shield displacement along the gap is at most 324 pm (SC-286–288, L353–355). Dividing the measured membrane displacement by the effective arm gives an equivalent small yaw. Multiplication by the magnitude of the calibrated electrical stiffness gives the source's estimated upper bound of 0.27 aN m on parasitic electrostatic torque noise (SC-289–291, L356–358). The stiffness enters the equation of motion with a minus sign. A sign for a laboratory harmonic requires the displacement phase and the electrode convention.

The article models test–source magnetic coupling by integrating the dipole interaction energy (SC-292–298). The pendulum and attractor magnetizations are magnetic moments per unit material volume. The authors measured the remanent magnetization of a silicon sample by atomic magnetometry in a near-zero field and used its components for both bodies. They estimated the induced magnetizations from silicon susceptibility and the ambient field measured at the pendulum (SC-299–307). For both $\mathbf{M}_{\mathrm{pen,rem}}$ and $\mathbf{M}_{\mathrm{att,rem}}$ the remanent components are $(2.4, 3.9, 5.6)\,\mathrm{mA/m}$ (SC-304, L384). The volume susceptibility is -3.2×10^{-6} and the estimated components of $\mathbf{M}_{\mathrm{pen,ind}}$ and $\mathbf{M}_{\mathrm{att,ind}}$ are $(0.1, 7.9, 4.1)\,\mu\mathrm{A/m}$ (SC-305, SC-307). The induced magnetization is $\mathsf{c}_{\mathrm{mag}}\mathcal{B}_{\mathrm{ext}}/\mu_0$ in vector form. A signed susceptibility affects the vector before evaluating the energy gradient. The calculated synchronous magnetic torque magnitude is below 0.14 aN m at the closest setting (SC-308–309). With horizontally orthogonal applied coil fields $\mathcal{B}_{\mathrm{coil}}$ and adjacent horizontal fluctuations of $\mathcal{B}_{\mathrm{amb}}$ at the signal frequency, the authors estimate a separate noise contribution below 0.02 aN m (SC-310–314). The standard account reads this background as a magnetic field acting on the samples' moments, a force apart from gravity. FDT reads the field as the helical density gradient of the omnium (P-5), and its push and turning torque on a held moment as the helical window of Sec. 4 (core 0.1c). The authors' bounds are accepted in full; the samples' magnetizations, not their masses, set them, because that window's clock factor belongs to each sample's own moments. The magnetic damper on the suspension suppresses environmental vibration (SC-026–027). Axial tension in the fibre, its restoring moment about the axis and coupling of sample magnetizations are three different mechanical and magnetic quantities.

The authors intentionally modulate the ambient temperature through the air conditioning. Fibre thermoelasticity, equilibrium drift and changes of geometry enter the instrument response (SC-315–319). They give $\mathfrak{k}_{\mathrm{env}} = -114.2 \pm 5.7\,\mathrm{fN\,m}$ per degree Celsius (SC-320, L408). The typical local harmonic temperature variation $\Delta\Theta_{\mathrm{env}}$ is at most 2.1 µK over a 7800 s run (SC-321–322). The magnitude of the signed response times the temperature amplitude gives the reported estimated uncertainty below 0.24 aN m (SC-323, L411). None of these nuisance bounds is a measured residual gravitational force.

Table 1 gives a total uncertainty of 0.59 aN m at one standard uncertainty, including 0.26 aN m of Newtonian calculation uncertainty (SC-111, SC-121). The effective arm is 23.694(3) mm (SC-123, L189). Dividing the tabulated torque uncertainty by that arm gives 25 aN at one standard uncertainty (SC-122–123). The article prints the two-standard-uncertainty combined values as 1.2 aN m and 50 aN (SC-081–082). The Table 1 total is the standard uncertainty of the measured-minus-calculated difference at the closest gap, $[(0.27)^2 + (0.46)^2 + (0.26)^2]^{1/2} = 0.59\,\mathrm{aN\,m}$ (SC-111, SC-114, SC-120); adding the band error to it again would count the metrology twice.

Exclusion in a specified potential. For pair masses m_1, m_2, the model at SA-13 is the Newtonian pair potential with a Yukawa term of strength g_Y and positive range ℓ_Y (SC-174–176). The model is

$$\mathcal{U}_Y = -\frac{G m_1 m_2}{r_{\mathrm{pair}}} \left[1 + g_Y \exp(-r_{\mathrm{pair}}/\ell_Y)\right].$$

The prefactor carries energy and the exponent is dimensionless. A prediction in this model differentiates the potential energy before integrating the signed force moments. The strength and range belong to the comparison potential; neither is an α, which carries no fitted coefficient, coupling or reference scale (core 0.3). In Fig. 4 the shaded regions are excluded at the 95% confidence level and this work is shown by the red solid line (SC-206–208, L329–331). The authors say the resulting bound is about two orders of magnitude weaker than in their previous work, where two 8.7 g tungsten blocks were used as test and source masses at millimetre range (SC-177–179, L204–206). They say the constraints could improve near a range of about 10 µm if the gap decreases to about 15 µm while sensitivity and other systematic effects remain at the same level (SC-180–183). The red dashed curve is the expected constraint under those conditions (SC-210–212). The data and custom acquisition and analysis codes are available from the corresponding author upon request (SC-324–327).

Cluster closing. This cluster is the I-1 and I-7 reading of the record and the convergence map of Use item 2: each background enters its own channel once, and none is a measured residual gravitational force. At the reported setting the authors bound a synchronous magnetic torque and separately estimate torque noise from horizontal field fluctuations. Their electrostatic noise estimate uses the measured displacement of the shielding membrane. The new bias scan uses the signed finite-window torque contrast defined with Eq. (5). After the calibrated capacitive, membrane-motion and gain corrections, reject the electrical-background model when the absolute difference of the applied-bias and reference signed torque residuals exceeds three standard uncertainties of that difference under the same Gaussian test. The published Yukawa exclusion assesses its specified potential at its quoted confidence. It has no calibration role in the bias contrast.

4 Cone-position translation

The same instrument has two geometric readings (core 0.1c, I-3a). Let $\mathfrak{m}$ be the common positive clock and ruler factor, $g_{\mu\nu}$ the bookkeeper metric, $\mathcal{V}^\mu$ the pattern four-velocity on its own clock, and $\mathfrak{P}^{\mu\nu}$ the transverse projector with units of squared velocity. A free pattern of the rescaled metric has the bookkeeper acceleration

$$g_{\mu\nu} \longmapsto \mathfrak{m}^2 g_{\mu\nu}, \qquad \nabla_{\mathcal{V}} \mathcal{V}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathfrak{m}, \qquad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{V}^\mu \mathcal{V}^\nu.$$

The kept time and thread length rescale together. Locally their ratio stays c. Moving at constant speed, on the pattern's own clocks and rulers, through a density gradient is gravity. The bookkeeper reads acceleration toward slower clocks. A single local reading gives no tide. Difference the accelerations of neighbouring patterns or compare one with a reference path. The trace of the tidal field reads density, and across an extended pattern its quadrupole couples to the tide. The measured yaw of this supported pendulum is neither one geodesic of free fall nor a direct measurement of a local Riemann component.

For a slow free pattern released from rest far away, the static exterior flow speed is $c\sqrt{\alpha_{\mathrm{ext}}}$ and its inward acceleration has magnitude $G\mathcal{M}/r_f^2$. The free-fall time slices are flat in that description. A rotating source adds a swirl. Half its spatial curl gives the leading gyroscope frame drag, the antisymmetric part of the bookkeeper velocity gradient in axial-vector form. The infalling gravitational four-velocity is irrotational. Vorticity of a material medium belongs to its own flow. This geometry holds at laboratory size: at $r_f = 1\,\mathrm{mm}$ from one $7.6\,\mathrm{mg}$ wafer read as a spherical reference, the flow moves inward at $1.0\,\mu\mathrm{m/s}$ and falls at $5.1 \times 10^{-10}\,\mathrm{m\,s^{-2}}$ (SC-011, SC-112, U-01, U-17). The encoder rate is imposed stage motion.

A medium of perfect stress-tensor form and zero net conserved count takes temperature as the clock factor of its flow. At fixed entropy per conserved count the factor is enthalpy per count. A clock carried by the medium runs on the lapse with the time dilation of its thermal motion. A conformal rescaling bends massive trajectories and preserves null paths. In the helical window let E_0 be the rest energy away from a twist, E_{ind} the energy of induction including its work, and $\mathbf{m}_{\mathrm{held}}$ a held moment. The field at each moment is $\mathcal{B}_{\mathrm{ext}}$, with vector orientation understood. The coordinate z_{lab} runs along the twist's gradient. Core 0.1c gives

$$E = E_0 + E_{\mathrm{ind}} - \sum \mathbf{m}_{\mathrm{held}} \cdot \mathcal{B}_{\mathrm{ext}}, \qquad \mathfrak{m} = E/E_0, \qquad \mathcal{M}\, d^2 z_{\mathrm{lab}}/dt^2 = -\partial E/\partial z_{\mathrm{lab}}.$$

The slow gradient law assumes the rest energy is fixed by the event alone and each held moment has its settled value. A uniform twist acts on a held moment through the turning torque $\mathbf{m}_{\mathrm{held}} \times \mathcal{B}_{\mathrm{ext}}$ and contributes no translational push. This torque vanishes when the moment and field are collinear. At a common point each pattern's slow acceleration along z_{lab} is $-(\partial E/\partial z_{\mathrm{lab}})/\mathcal{M}$, where the local inertial mass equals the rest energy divided by the squared strand rate. With each held moment fixed at its settled value, $\partial E/\partial z_{\mathrm{lab}} = \partial E_{\mathrm{ind}}/\partial z_{\mathrm{lab}} - \sum \mathbf{m}_{\mathrm{held}} \cdot (\partial \mathcal{B}_{\mathrm{ext}}/\partial z_{\mathrm{lab}})$. The induction energy E_{ind} includes the work of induction. The accelerations differ when the normalized gradients differ. Identical copies oriented alike at the same point have the same normalized gradient. The universality of gravitational free fall is the monopole reading where the lapse belongs to the point. This window is the channel the samples' magnetic characterization bounds (Sec. 3.3).

In the actual comparison the two ends and their common suspension supply the neighbouring regions. External support, control torque and stiffness constrain the trajectories. A common acceleration cancels from the axial moment when the pendulum's first mass moment about the fibre axis vanishes: that cancellation is the comparator's zero of core Use item 7, and what survives is the change of the field across the pendulum's extent, read through the signed moments of Eq. (2). For a small virtual yaw at fixed rotor phase the work of the elemental forces equals that axial moment times the virtual yaw angle. A force parallel to its lever contributes zero moment. In static equilibrium a holding torque can balance a nonzero external moment at zero yaw. An ideal constraint can maintain zero yaw under a varying moment. For the steady harmonic at the signal frequency, the controller has finite gain and $\widetilde{U} = \mathcal{K}_{\mathrm{pid}}(\omega_{\mathrm{sig}})\widetilde{\vartheta}_{\mathrm{tor}}$. A nonzero compensating voltage harmonic therefore requires a residual yaw harmonic, which the feedback suppresses (SC-162–166). P-4's circulation sense is intrinsic to a cone. Turning the stage changes which body elements face one another. It fixes neither those intrinsic senses nor which pairs the many-body settlement joins (O-5); a pair it does not join attracts at L2 with the closed form of I-1 [5]. The settlement is system-wide, many-to-many and global (core 0.1a): at L2 its gravitational reading is the one solved field of the whole rotated source acting on the whole pendulum, the volume integral of Eqs. (1)–(3).

5 Predictions and discriminating designs

The finite observation for each new test is a signed component measured over whole rotations. Fix the reference phase and torque polarity before the test. Use independent calibration data to establish each setting's complex voltage response, including its gain, before examining the test readings. Match the source state, drive history and readout dynamics in both models for each run. For a steady drive at a fixed setting, apply the calibrated response to all harmonics of the torque trace. For a finite drive beginning at that setting, include the response from its known initial state. For a change of setting, reset the pendulum and controller to a common known state and evolve it under each drive, allow settling and bound the residual transient in each window, or propagate the pendulum and controller state through the actual switch. This last calculation uses the calibrated dynamics during the change and the subsequent drive. Sample the resulting voltage trace at the data times, fit and subtract the same cubic, form the same two quadratures and divide by the same nonzero signal-frequency transfer for that setting. For uniform rotation, start the quadratures at the indexed facet or rephase them from the recorded start angle to that encoder zero. Project observations and predictions onto the same signed reference quadrature at both settings. Include the calibration and state uncertainties, and their correlations within and between settings, in the joint covariance below. If settling gives only a bound, propagate it through the same reduction and require rejection for every residual transient allowed by that bound. Denote the new signed linear torque readings by y, w, their independently estimated predictions by $p, \mathsf{q}_{\mathrm{ref}}$, and the centered contrast by K_{diff}. Define $\mathbf{h}$ as the vector of contrast weights and Σ_{joint} as the covariance of the ordered four quantities, including correlations between readings and estimated predictions. The dimensionless decision threshold is Z_{crit}; T as a superscript denotes matrix transpose.

$$K_{\mathrm{diff}} = (y - w) - (p - \mathsf{q}_{\mathrm{ref}}), \quad \mathbf{h} = (1, -1, -1, 1)^{\mathsf{T}}, \qquad \mathsf{u}(K_{\mathrm{diff}})^2 = \mathbf{h}^{\mathsf{T}} \Sigma_{\mathrm{joint}} \mathbf{h}. \tag{5}$$

A rejection in either direction requires $|K_{\mathrm{diff}}| > Z_{\mathrm{crit}} \mathsf{u}(K_{\mathrm{diff}})$, with $Z_{\mathrm{crit}} = 3$ (U-11) under a validated Gaussian model and independently characterized covariance. In a simple difference the cross term is minus twice the covariance; a shared gain need not disappear when the two predicted torques differ. Propagate metrology through each prediction at its own point, including any common input to the readout. When averaging pairs, include cross-pair covariance for a persistent gain or assembly displacement. Redraw a reset placement, gain, background or readout noise at the setting where it resets. A nonzero rejection rate at the matched model is part of the size of the test. An FDT test point exists where the core supplies the constitutive map; the three maps that enter no number here are named with their candidates below.

Single-pair design size. For the three cluster closings use one future pair and independently known model predictions in a normalized torque channel. This illustrative-assumed Gaussian model tests the decision rule for period, spacing and bias contrasts separately. It represents common fixed assembly bias by an additive offset and common calibration by a fractional gain. The sign of the applied-state minus reference contrast follows the setting labels even when acquisition order is reversed. With no order drift after conditioning, interleaving preserves the null law. Let $\mathsf{s}_{\mathrm{g}}, \mathsf{s}_{\mathrm{r}}$ denote shared and reset fractional gain standard deviations and $\mathsf{s}_{\mathrm{b}}, \mathsf{s}_{\mathrm{k}}, \mathsf{s}_{\mathrm{n}}$ reset background, calibration and readout standard deviations. For each test point propagation gives

$$\mathsf{u}(K_{\mathrm{diff}})^2 = \mathsf{s}_{\mathrm{g}}^2 (p - \mathsf{q}_{\mathrm{ref}})^2 + \mathsf{s}_{\mathrm{r}}^2 (p^2 + \mathsf{q}_{\mathrm{ref}}^2) + 2(\mathsf{s}_{\mathrm{b}}^2 + \mathsf{s}_{\mathrm{k}}^2 + \mathsf{s}_{\mathrm{n}}^2). \tag{6}$$

One shared offset cancels exactly in Eq. (5) and contributes zero to Eq. (6). Both gain contributions depend on the operating point. The normalized means are $(10, 12)$ for the matched configurations and $(20, 8)$ for the displaced pair (U-12). The common offset has standard deviation 0.65 and the shared fractional gain standard deviation is 0.03 (U-13). The reset gain has fractional standard deviation 0.04; the independent additive background, calibration and readout standard deviations are 0.2, 0.4 and 0.3 (U-14). All nuisance draws have mean zero. Each stream has 250000 pairs (U-15). The three streams of each cluster draw independently, each seeded from U-15 and the cluster's name. The simulation randomizes acquisition order within pairs and restores the setting sign before centering. It draws each shared term once per synthetic assembly pair and independent terms for each normalized reading. Background here denotes the remaining error after the relevant conventional prediction, including the calibrated transfer, finite-body geometry or electrical response. Real nuisance sensitivities and their covariances require independent calibration for each apparatus. For the period contrast, the second stream tests the identical-input FDT mean. For spacing and bias it is a comparator replicate, the corresponding FDT maps entering no number here. In each cluster a displaced detector response is recentered and uses its own standard uncertainty. The nominal two-sided tail is $\operatorname{erfc}(Z_{\mathrm{crit}}/\sqrt{2})$. The entries below count rejections at each stream's own point. In each column the binomial sampling error of the count is the square root of the trial count times the nominal tail probability times its complement.

Contrast	Reference	Matched replicate	Displaced, centered
Period	647	704	642
Spacing	715	691	674
Bias	712	693	631

All counts lie within the stated tolerance of four Monte Carlo standard errors (U-16). These simulations establish a size calculation under an assumed law. They supply no sampling distribution for the printed amplitudes.

Finite geometry. At a held-out gap or after an independently metrologized remount, compare both ends about the common axis. Use the calibrated dynamics and initial state for each configuration, including the effect of the gap change or remount on the voltage in the measurement window. Subtract each predicted signed finite-window torque estimate before differencing against the reference configuration as in Eq. (5). The conventional tensor source already includes multipoles. Granting a residual an arbitrary coupling would deprive this comparison of its independent normalization. DC-1 (SA-05) needs the pairs the many-body settlement joins (O-5) to predict a signed finite-window reading apart from the finite-body kernel, which carries the multipole and tidal remainders of the spherical pair expression at L2 [4, 5]; no such pairing enters a number here.

Exterior source. Compare the signed finite-window torque residuals of Eq. (5) at held-out and reference separations of the same rotor and pendulum. Keep conventional source content and metrology fixed, calibrate gravitating mass independently and propagate shared errors. Every nonzero source carries its mass shadow, the exterior compressed-omnium response of core 0.7, which gravitates through $\mathcal{M}$; the part already present in the mass calibration enters once. An independently specified exterior density profile and closure would determine the additional torque trace at each separation. Propagate the full torque trace for each model through the calibrated

loop, including the state at a gap change, then apply the same sampling, cubic subtraction, quadratures, transfer division and phase projection to each voltage trace. Cubic subtraction can mix harmonics in this reading. The candidate is distinct if the two models predict different held-out minus reference contrasts of the signed finite-window readings. DC-2 (SA-24) needs the exterior shadow profile and its closure (O-14) to predict that contrast; no profile enters a number here.

Causal readout. Return the same pendulum ends to the closest setting at the two reported rotor periods. Randomize the order of periods within each interleaved pair to control drift. At each setting draw new readout noise and any reset metrology. Use the conventional source state at each rotation speed and the same prior drive and loop state in both models for each run. Apply the full-trace voltage prediction and matched reduction specified with Eq. (5). I-1 separates a static force from its dynamic readout. It supplies no distinct transfer for this pendulum. At the matched inputs the FDT and conventional mean residual contrasts both vanish at every positive precision. DC-3 (SA-02) is inert: the normalized predicted residual is 0, the ablated residual is 0, and the difference is 0 against a threshold of 3 (U-11).

Assayed source energy. In a separate experiment compare heated and reference rotor states after equilibration, randomizing state order within pairs. Independently assay the change of source energy. Calibrate heat capacity, expansion, density, support stress, true gap, rotation and detector gain for each state. Source energy, geometry and mechanical state determine the physical signed torque component. Along an equilibrated source path with continuous sensitivity, integrate its total derivative with respect to source temperature to obtain the finite change in that component. For a nonzero temperature span, a nominated slope times the span gives that change exactly when the nominated slope equals the interval average of the physical component's total temperature derivative. The physical slope is the sum of the source and geometry contributions along the path. For each temperature run, use the same loop state and thermal and drive histories in both models. Apply the calibrated voltage response to the full torque trace with that initial state and reduce the resulting voltage as specified with Eq. (5). Propagate the calibration uncertainties and their correlations into the contrast. Carry the same total stress-energy into GR. The ambient instrument coefficient in the source pertains to a different intervention. DC-4 (SA-31) is inert: the normalized predicted residual is 0, the ablated residual is 0, and the difference is 0 against a threshold of 3 (U-11).

Electrostatic transduction. Apply a specified bias and a reference bias with common encoder zero and nominal gap. A bias change can alter the electrostatic stiffness. Account for the pendulum and controller state at each change when predicting the voltage over the recorded whole turns. Reduce the voltage trace and difference the signed torque residuals as in Eq. (5). Track contact and patch offsets, shielding, membrane displacement, actual gap, gain and independently calibrated capacitance derivatives at each setting. The source estimates parasitic electrostatic torque noise from measured membrane displacement (SC-286–291). The bias scan measures the torque response to applied potential; an FDT point for it needs the charge-to-force map. DC-5 (SA-26) needs the map from a charge or circulating charge to the I-1 force input (O-6), beyond ordinary electrical transduction; no such map enters a number here.

6 Cross-domain reading

The quantum question changes the observable. A witness measures joint correlations of two prepared quantum systems after a controlled interaction. It requires a separability bound, calibrated local measurements and control of correlations from nongravitational couplings. The reported torques estimate the amplitude of the sixteenth harmonic of the axial moment. For one window, the reconstructed amplitude estimate is the modulus of the calibrated complex coefficient obtained from its detrended voltage quadratures. That amplitude is one balance's torque and reads no joint correlation of two prepared masses. The authors' Discussion cites two experimental schemes and states that both masses should ideally lie in the range 10^{-14} to 10^{-12} kg (SC-184–186). An et al. state that within this mass window gravitational interactions between micrometre-scale objects can produce measurable entanglement within experimentally achievable coherence times. They describe reaching the target mass range as a substantial challenge beyond the current scope of their work (SC-187–190). Both the preparation and the witness are prospective. The standard account reads a witnessed correlation between two masses as a nonseparable state of probability amplitudes. FDT reads it as a bivector grade the two patterns share across one density gradient, with superposition the ignorance of helical phase at observation and projection its fixation (I-4); the witness of the Discussion is accepted as a prospective observable of that grade.

The Discussion gives three distinct instrument examples. A quartz fibre of diameter 3 µm, with stiffness about 10^{-13} N m/rad and assumed quality factor about 25,000, gives an estimated sensitivity enhancement of approximately a factor of 90 (SC-192–195). Shortening the gap from 654 µm to about 15 µm gives their projected 10^5 improvement in signal-to-noise at a Yukawa range of about 90 µm (SC-196–198). For an optimized torsion balance integrated over 30 days they expect a torque resolution near 10^{-21} N m (SC-199–201). They write that this "would potentially enable the detection of" gravitational coupling between two objects below the Planck mass scale separated by tens of micrometres (SC-201–203, L258, L243–244). The technological challenge of maintaining quantum coherence remains distinct in their outlook (SC-204–205). The three estimates concern torsion-balance sensitivity, the signal-to-noise ratio for probing a Yukawa deviation at a specified range, and integrated torque resolution.

7 System-level reading (exploratory)

Scope. The sole observable in this section uses the closest-gap force again. The inputs, normalization and errors belong to the original experiment.

X-1. Closest-gap gravitational constant. *Observable.* Let G_{inf} denote the inferred constant, F_{exp} the calibrated experimental equivalent force and G_{ref} the independent reference. The force used for the inference is 471 ± 23 aN at one standard uncertainty (SC-104, L164). *Defining condition.* An et al. report the denominator of their closest-gap estimator as $J_{\mathrm{geom}} = (7.06 \pm 0.16) \times 10^{-6}\,\mathrm{kg^2/m^2}$ (SC-094, SC-105, L158, L165). The paper prints no separate coverage label for the uncertainty on this coefficient. Section 3.2 identifies the printed value as the calculated harmonic torque per unit reference constant and arm, $J_{\mathrm{geom}} = \mathcal{N}_{\mathrm{calc}}/(G_{\mathrm{ref}}\ell_{\mathrm{arm}})$. Rearranging the reported quotient gives the in-sample identity

$$F_{\mathrm{exp}} = G_{\mathrm{inf}} J_{\mathrm{geom}}, \qquad \frac{G_{\mathrm{inf}}}{G_{\mathrm{ref}}} = \frac{F_{\mathrm{exp}}\,\ell_{\mathrm{arm}}}{\mathcal{N}_{\mathrm{calc}}}.$$

Comparator. The quotient reconstructs the coefficient 6.67 in units of $10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ (SC-104–105, SC-109). At the printed precision it is the reference value times the measured-to-calculated torque ratio, $11.16/11.17 = 0.999$ (SC-138, SC-252). The paper gives 6.67(35) in those units and 5.3% relative uncertainty, agreeing with the CODATA value within one standard uncertainty of this measurement (SC-109–110, SC-112–113). For small fluctuations about the positive central values, first-order propagation gives

$$\frac{\mathsf{u}(G_{\mathrm{inf}})^2}{G_{\mathrm{inf}}^2} = \frac{\mathsf{u}(F_{\mathrm{exp}})^2}{F_{\mathrm{exp}}^2} + \frac{\mathsf{u}(J_{\mathrm{geom}})^2}{J_{\mathrm{geom}}^2} - \frac{2\,\mathrm{cov}(F_{\mathrm{exp}}, J_{\mathrm{geom}})}{F_{\mathrm{exp}} J_{\mathrm{geom}}}.$$

Table 1 fixes each term. The force's uncertainty is the statistical and other-systematic rows over the arm, $[(0.27)^2 + (0.46)^2]^{1/2}\,\mathrm{aN\,m}/\ell_{\mathrm{arm}} = 22.5\,\mathrm{aN}$, printed as 23 aN (SC-104, SC-114, SC-120, SC-123). The denominator's is the Newtonian-calculation row, $0.26/11.17 = 2.3\%$. The arm common to both cancels from the quotient and the remaining inputs sit in separate rows, so the covariance term vanishes (SC-111, SC-252). The sum returns the printed 5.3% and 0.35; rounding the force's uncertainty to 23 aN before propagating gives 0.36 (SC-104, SC-109, SC-113). In Fig. 3b the red circles represent 46 individual determinations with statistical errors only. The magenta point uses a weighted mean of torque accumulated over 100 h and includes statistical and systematic uncertainties (SC-262–268). The 46 runs of about 7800 s each make 99.7 h (SC-264–265).

Guards. The quotient restates the closest-gap comparison: multiplied by its denominator it returns the measured input identically, and it tests neither a held-out separation nor a second force law. An independently normalized calculation uses G_{ref} in the signed moment integral. For one window, its predicted equivalent-force reading is the modulus of the matched finite-window torque estimate divided by the effective arm. A prediction for a combined reading also requires its aggregation rule. The source's display precision, integrated uncertainty and run-to-run scatter do not set a threshold for a new contrast.

8 Convergence statement

At layer L1, substituting the two exterior gravitational radii at their common centre distance gives the inverse-square pair magnitude exactly on disjoint spherical monopoles. At layer L2, with the same source and data, I-3 returns the same curvature as GR. To leading weak-field, slow-motion order its ordinary finite-body volume kernel yields the wrench above, the fibre-axis projection, the rotor harmonic and, after the same readout and reduction, the same equivalent-force estimate. The finite-body comparison uses the actual wafer and strut geometry. The observed fall in torque and the reported Newtonian comparison are accepted at the source's uncertainties (SC-014, SC-244–261, SC-274–278). At the closest gap the measured and calculated torques differ by $-0.01 \pm 0.59\,\mathrm{aN\,m}$, and the in-sample X-1 quotient restates that comparison as $G_{\mathrm{inf}}/G_{\mathrm{ref}} = 0.999 \pm 0.053$ (SC-121, SC-138, SC-252).

The authors describe the gravitational interaction between milligram-scale objects as a significant step toward bridging the classical and quantum regimes of gravitation (SA-01, SC-018–020, L30–32). The conventional reading treats gravity awaiting quantisation as an open problem. FDT reads the geometry as carried by the one quantised excitation: each energy step adjusts mass, kept time and strand length together (core 0.1a, I-2). The apparatus measures a modulated gravitational torque. A quantum witness would measure joint-state correlations relative to

a separability bound. This distinction accepts the experiment's reach in source mass and leaves the authors' prospective programme in its stated domain.

At layer L3 the finite-geometry remainder, the exterior density response and the charge input enter no number here (O-5, O-14, O-6). In the other comparisons, matched energy content and matched mechanical response return the same means. Under a validated Gaussian model with independently characterized joint covariance, reject the matched apparatus description if the absolute centered contrast in Eq. (5) exceeds three standard uncertainties of that contrast. Excluding that description alone would give neither a laboratory shadow profile nor a constitutive kernel.

9 Status ledger

Each O-row names what has been derived of the item, with its source, and what remains to be derived; the visible-universe analysis prints the results of the two derivation notes [7].

Item	Status	Content bearing on the reading
O-1	Derived in part	Derived [5]: once a stroke is chosen the floor fixes the core cutoff, at the floor length $2\ell_{\mathrm{P}}$ for the radial stroke, half of it for a stroke across the core and $1/2\pi$ of it for one around it; a uniform interior attains the bound that holds the mass inside r_f to α_{int}^3 of the mass inside the fold when the density does not fall outward. To be derived: the choice of stroke, the regular core chart and the outward phase-transfer law.
O-2	Derived in part	Derived [5]: a flank-to-gap odds, the sum of two profiles' flank widths, each from crest to the shoulder where the second derivative changes sign, over half the gap between the facing crests. For identical profiles it equals 2, the 2/3 of I-1a, exactly where the midpoint turns from valley to ridge, and no density value, volume mean, reference-density ratio or circulation enters it. To be derived: the profile (O-4), the unequal pair and the transfer to the instances of I-1a. Every increasing function of this odds that keeps 2 at the crossing passes the same test; the core fixes no member, and A-3 carries its open range only.
O-3	To be derived	A canonical odds for the confinement application. The scaling, spectral, section-ratio and floor-ratio routes select none [4, 5].
O-4	Derived in part	Derived [5]: the fold stress law, by which a regular static approach to the fold carries a radial stress equal to minus the energy density, and radial tension inside every static configuration that approaches saturation with its stresses bounded by its energy density. To be derived: the two constitutive relations among the energy density and the radial and tangential stresses, and the reduction to the measured nuclear profile.

Item	Status	Content bearing on the reading
O-5	Derived in part	Derived [4, 5], at layer L2: the multipole expansion of the pair law for supplied sources, with bounds on its remainder after quadrupole order; the attraction of two hollow supports through their whole overlap, never above its contact value; the spin remainders of the pair law for two bodies on a common axis; the width of the band between a rotating source's horizon and its Schwarzschild radius, the Schwarzschild radius of the rotational share of its mass; and the attraction, in the closed form of I-1, of a pair the settlement does not join. To be derived: the overlap beyond L2, which pairs a settlement joins and the map from a rotation to a flow direction.
O-6	To be derived	Maps from a charge, a circulating charge or a colour seat to an I-1 input; derivation of the A_2 seat table from cone geometry; the gluon octet's dynamics. Derived [6]: the magnitude that any per-body charge map linear in the charge returns, fixed by Coulomb's law read through I-1.
O-7	Derived in part	The SNO neutral-current total flux agrees with the standard solar model and is data. Derived [4, 5]: one solar energy account with a fission part and a fusion part, each contribution counted once; the map from source spectra to detector counts; and, from the measured proton–proton-chain fluxes, a fission heat share below 0.17 of the photon luminosity at 95 per cent confidence in the stationary account. To be derived: the settlement's computation against the SNO flux and the FDT computation of every other neutrino anomaly.
O-8	Derived	Derived [5] on every chart whose metric is in hand: the bookkeeper reads the stroke's electric and magnetic responses as one tensor, whose index across a density gradient in the static exterior's areal chart is the aperture index $(1-\alpha_{\mathrm{ext}})^{-1/2}$ of P-4 and whose index along it is the square; the pair is the exact optical metric, and the leading deflection is twice the Schwarzschild radius over the impact parameter. To be derived: the interior instance, which needs the interior lapse (O-1, O-4), and the response beyond first order in the fields.
O-9	To be derived	The confinement operator, its domains, boundary data and spectrum, including all mass ratios. Proton/electron and squared muon/electron mass ratios are held-out targets (U-08).

Item	Status	Content bearing on the reading
O-14	Derived in part	Derived [5]: the conversion channel's rate at accreting holes, their luminosity density over c^2 times the ratio of the unradiated to the radiated share of the arriving matter's rest energy; the form of the regional law, the local release less the divergence of a flux of released potential whose total through a domain's boundary vanishes. To be derived: the mass-shadow response function and its closure, each channel's compression contrast and the flux law. The five-gap torque series tests the shadow response at milligram mass and millimetre separation.
O-16	To be derived	Kinematic reading of the share on the interior chart.
A-3, A-4, A-5	Open-range token	Overlap, confinement and mode tokens, respectively; the flank-to-gap odds of O-2 assigns A-3 no value.
X-1	In-sample	Closest-gap experimental force divided by the reported coefficient: the reference constant times the measured-to-calculated torque ratio.
Held-out settings	Prospective	New gap, remount, period, temperature or bias pairs with independently calibrated signed readout.

10 Conclusions

At each separation the authors report an estimate of the torque-harmonic amplitude. The lever, attractive force direction and oriented fibre axis fix the sign of each axial moment, and the force magnitude its size. With finite masses, bounded levers and positive clearance throughout a turn, axial projection, integration of the elemental moments and complex harmonic projection commute in material coordinates. Each elemental moment takes its own lever. The modulus of the summed complex axial harmonic coefficients is the physical harmonic amplitude, and for one window it follows the voltage reduction. A reported amplitude carries neither torque polarity nor encoder phase. Dividing a reported torque-amplitude estimate by the effective arm expresses it in equivalent-force units. A common acceleration cancels between the two ends about the suspension, and the balance reads the change of the field across its extent. The printed geometric factor is the calculated harmonic torque per unit reference constant and arm, so the closest-gap value of G restates the closest-gap comparison. FDT returns this measurement from the same source and data. The record is a finite-geometry gravity measurement at milligram mass and millimetre separation, its field points deep in the general-relativistic regime of the A-1 exterior branch, and its five-gap series is a test for the mass-shadow response (O-14). A discriminating measurement pairs a prospectively calibrated signed observable with an FDT input that changes its predicted value.

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