

Fixed Euclidean Space, Newtonian Time, and Omnium Flow

Evolving Geometry and Navier–Stokes Continuation

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Abstract

The OpenAI Navier–Stokes construction [18] evolves a forced fluid on fixed Euclidean space with Newtonian time, reads its pressure as a force across that background, and presents its finite-time blowup as Fefferman’s alternative (C). Fundamental Density Theory (FDT), a true extension of general relativity and quantum mechanics, couples matter, energy, rulers, clocks and geometry in one omnium state, and reads the same pressure gradient as the bookkeeper’s reading of the medium’s clock factor, its enthalpy per count: the source’s equation is the slow limit of the medium’s free fall in the rescaled metric. On the fixed flat incompressible domain, a maintained Euclidean velocity bound $\|u(t)\|_\infty \leq C_E$ and positive viscosity give $\|\nabla u(t)\|_2^2 \leq \|\nabla u_0\|_2^2 \exp(C_E^2 t/\nu)$ and global strong continuation: the classical velocity criterion of Leray, Prodi and Serrin [9, 10, 11], proved here in full. The helical average of the omnium supplies $C_E = c$ wherever the fluid velocity is the resolved transport in one maintained flat frame, and finite-time breakdown is excluded in that class. The construction’s blowup continues the slow limit past $|u| \sim c$ while its rulers, clocks and differential operator stay fixed; FDT declines that fixed-background reading and is under no obligation to reproduce an unbounded velocity. The physical continuation evolves rulers and clocks with the fluid: the identity $F = (c^4/4G)\alpha_1\alpha_2 = F_{\max}\alpha_1\alpha_2$ ties the pair inputs to source energy and geometry, and proper intervals and the conserved total stress describe the rulers, clocks, fluid and drive in one evolving metric. A finite integrated norm bounds no local density, in the construction as in a singular Gross–Pitaevskii branch [5]. The force floor, the seam between the omnium’s two phases, the transmission of a mode across that seam and Cooper pairing are calculated on their declared domains, and experiments and simulations of vortex recoil, knot fusion, rotor pairing and turbulent decay show local patterns taking part in the response of the coupled medium, drive and surroundings [25, 26, 27, 28, 29].

1 The fixed background is the central physical assumption

The OpenAI manuscript [18] gives a theoretical construction. Its Theorem 1.1 states that for every viscosity $\nu > 0$ there are a force $f \in C_c^\infty(\mathbb{R}^3 \times (0, \infty))$ and smooth velocity and pressure fields on $\mathbb{R}^3 \times [0, 1)$, starting from rest and supported in a fixed compact set, with $\sup_{t < 1} \|u(t)\|_2 < \infty$ and $\limsup_{t \uparrow 1} \|u(t)\|_\infty = \infty$ [18, Theorem 1.1, p. 1]. The source presents this as Fefferman’s alternative (C), and the periodic alternative (D) through its Corollary 10.6 [8]. Its spatial gradient, Laplacian, volume measure, and norm are Euclidean; its time coordinate is Newtonian. The pressure constraint couples the fluid throughout this domain, but the geometry is prescribed independently of the stress.

Let δ_{ij} denote the fixed Euclidean metric, π the pressure per constant inertial density, and f the prescribed acceleration. In the source’s fixed background the mathematical problem is

$$\partial_t u + (u \cdot \nabla)u = -\nabla \pi + \nu \Delta u + f, \quad \nabla \cdot u = 0, \quad \partial_t \delta_{ij} = 0. \quad (1)$$

There is no dynamical lapse, spatial metric, or spacetime source equation in this system. Increasing velocity, rearranging pressure, and performing work leave its rulers and time parameter fixed by

definition. Thus the forcing evolves the fluid against a prescribed spacetime background. This fixed background defines the mathematical problem and the physical approximation examined here.

The source writes the fluid as a medium pushed by pressure forces across a fixed Euclidean background. FDT reads the same pressure gradient as the bookkeeper's reading of the medium's common rescaling: the fluid's patterns move free of every force in $\mathfrak{m}^2 g_{\mu\nu}$, whose clock factor $\mathfrak{m} = \mathfrak{h}_N / \mathfrak{h}_{N,0}$ is the enthalpy per count $\mathfrak{h}_N = (E + PV)/N$ [1, 0.1c, I-7]. For a stress tensor of perfect form with a conserved count N at fixed entropy per count, with $\mathbf{U}$ the fluid's four-velocity (Section 4.2), $n = N/V$ and $\mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathbf{U}^\mu \mathbf{U}^\nu$,

$$\left(\frac{E}{V} + P\right) \nabla_{\mathbf{U}} \mathbf{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu P, \quad dP = n d\mathfrak{h}_N \implies \nabla_{\mathbf{U}} \mathbf{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathfrak{h}_N. \quad (2)$$

For a liquid of constant inertial density $\rho_0 = nm_0$, $\mathfrak{h}_N = m_0 c^2 (1 + \pi/c^2)$ up to an additive constant, and on the flat laboratory chart the slow limit of (2) is

$$\partial_t u + (u \cdot \nabla) u = -\nabla \pi \quad \text{up to relative corrections of order } |u|^2/c^2 \text{ and } \pi/c^2. \quad (3)$$

The source's pressure gradient is therefore the medium window of 0.1c at the medium's own strength. Its viscous term is the first-order non-perfect stress of a causal relativistic fluid [4], and its force is the drive's exchange $\mathcal{Q}_{\text{drv}}^\nu$ of (24).

The force is inverse-designed from the constructed velocity and pressure. At viscosity one, the source defines it by the residual in its Eq. (10.5), p. 118, and extends the same force smoothly beyond the blowup time in Lemma 10.3, p. 120. It is then one prescribed function for the comparison solutions in Lemma 10.5: every competitor receives the same prescribed force. The source's Eq. (10.14), p. 121, is its energy identity, and its Lemma 10.4 bounds the kinetic energy and the integrated dissipation by $\mathcal{F}(1)^2$, with $\mathcal{F}(t) = \int_0^t \|f(s)\|_2 ds$ (Eq. (10.13)). Its claimed blowup therefore occurs with finite total forcing work.

FDT's central invariant (Section 5) identifies matter, energy, and spacetime as one omnium state [1, 0.1–0.1c, I-3, I-7]. The fluid, its source, and its measuring patterns participate in that same state, and the geometric response runs in two windows. In the medium window the fluid's own clock factor carries it at the medium's strength, and (3) is its slow limit; the source keeps that limit and continues it past $|u| \sim c$ and $|\pi| \sim c^2$, where the medium's patterns keep $|u_{\text{loc}}| < c$ (Proposition 5.3, 0.1d). In the gravitational window the conserved total stress sources the metric through the Einstein equation (Section 4.3); for a laboratory fluid that window is small, and the small number is a correct reading (I-1). Newtonian material elements already stretch, rotate, and change their separations, and pulling the fixed Euclidean metric back to material coordinates produces time-dependent coefficients while preserving its flat ambient geometry. The physical extension adds the medium's clock factor and the evolution of spacetime itself.

The continuation proof follows first in Sections 2 and 3. Its inputs are the declared incompressible equation, positive viscosity, strong initial data, and the maintained velocity bound. The subsequent pairwise, metric, phase, and spectral comparisons support the physical critique of the construction. Section 10.1 records their distinct results and inputs.

2 The fixed-domain Navier–Stokes comparison

Let $\Omega = \mathbb{R}^3$ or a fixed flat three-torus $\mathbb{T}_L^3$, and let $\nu > 0$ be the specified kinematic viscosity. The velocity u and every norm, spatial derivative, and measure in this section use the fixed Euclidean metric and coordinate time of the comparison PDE. The periodic option specifies the boundary condition on the flat three-torus, distinct from the S^3 and Clifford-torus reference geometries used later. The unforced incompressible momentum balance is

$$\partial_t u + (u \cdot \nabla) u = -\nabla \pi + \nu \Delta u, \quad \nabla \cdot u = 0, \quad u(0) = u_0. \quad (4)$$

When $\pi = p/\rho_0$ is used, $\rho_0 > 0$ is a constant inertial reference coefficient. This incompressible coefficient is distinct from the spatially varying omnium state and its relativistic energy density.

Take divergence-free $u_0 \in H^s(\Omega)$, with integer $s \geq 3$. The finite-energy and Sobolev conditions concern u in this fixed geometry. Standard local theory gives a maximal strong solution on $[0, T_*)$. On the torus fix $\int_{\Omega} \pi \, dx = 0$; on $\mathbb{R}^3$ use the finite-energy pressure reconstruction. The theorem below states the maintained speed hypothesis on this entire maximal interval.

3 The continuation criterion on the fixed flat domain

The maintained Euclidean velocity bound closes the viscous derivative estimate and yields global strong continuation. The $C_E = c$ reconstruction appears in the corollary immediately following the proof.

Theorem 3.1 (Maintained Euclidean bound for viscous incompressible flow). *Let u be the maximal strong solution of (4) with the domain, viscosity, and initial data specified above. Suppose there is a constant $C_E > 0$, with velocity units, such that*

$$\|u(t)\|_{L^\infty(\Omega)} \leq C_E \quad (0 \leq t < T_*). \quad (5)$$

Then $T_* = \infty$. The solution is unique in the strong-solution class, belongs to $C([0, \infty); H^s)$, and is smooth for positive times. If u_0 belongs to every Sobolev space, smoothness extends through the initial time. Moreover,

$$\|u(t)\|_2^2 + 2\nu \int_0^t \|\nabla u(\tau)\|_2^2 \, d\tau = \|u_0\|_2^2, \quad (6)$$

and

$$\|\nabla u(t)\|_2^2 \leq \|\nabla u_0\|_2^2 \exp(C_E^2 t / \nu). \quad (7)$$

Theorem 3.1 is the classical L^∞ continuation criterion. Leray proved lower bounds on $\|u(t)\|_{L^p}$, $3 < p \leq \infty$, as t approaches a putative blowup time [9]; Fefferman's problem description records that the velocity becomes unbounded near any finite blowup time [8, pp. 2–3]; and the hypothesis lies in the endpoint $q = \infty$, $p = 2$ of the Prodi–Serrin class $\int_0^T \|u\|_{L^q}^p \, dt < \infty$, $2/p + 3/q = 1$ [10, 11]. The derivative estimate (8) is the estimate in the proof of [7, Theorem 4.14, Eq. (4.4), pp. 31–32]. The proof is given in full because Corollary 3.2 uses its constant.

Proof. Energy. Pair (4) with u . Periodicity or the stated whole-space Sobolev conditions justify integration by parts. Incompressibility cancels the pressure and transport contributions, giving (6).

The derivative estimate. Set $Y(t) = \|\nabla u(t)\|_2^2$ and $Z(t) = \|\Delta u(t)\|_2^2$. Pair (4) with $-\Delta u$:

$$\begin{aligned} \frac{1}{2}Y'(t) + \nu Z(t) &= \int_{\Omega} (u \cdot \nabla) u \cdot \Delta u \, dx \\ &\leq \|u\|_{\infty} \|\nabla u\|_2 \|\Delta u\|_2 \\ &\leq C_E \sqrt{Y(t)} \sqrt{Z(t)} \\ &\leq \frac{\nu}{2} Z(t) + \frac{C_E^2}{2\nu} Y(t). \end{aligned} \quad (8)$$

The pressure pairing vanishes because $\nabla \cdot \Delta u = 0$. The final inequality is Young's inequality. Hence

$$Y'(t) + \nu Z(t) \leq \frac{C_E^2}{\nu} Y(t). \quad (9)$$

Multiplication by $e^{-C_E^2 t / \nu}$ and integration gives the stronger estimate

$$e^{-C_E^2 t / \nu} Y(t) + \nu \int_0^t e^{-C_E^2 \tau / \nu} Z(\tau) \, d\tau \leq Y(0), \quad (10)$$

which implies (7). The exponent is dimensionless because $[C_E^2 t] = [\nu] = \text{m}^2 \text{s}^{-1}$.

Continuation. Suppose $T_* < \infty$. Equations (6) and (7) bound $\|u(t)\|_{H^1}$ by a single finite constant for all $t < T_*$. The three-dimensional viscous H^1 local strong-solution theorem has a positive existence time $T \geq C(\nu)\|\nabla u\|_2^{-4}$ [7, Theorem 4.14]; see also [6]. Restart the solution sufficiently close to T_* . The resulting lifespan extends beyond T_* , and local uniqueness identifies it with the previous solution on the overlap. This contradicts maximality. Higher Sobolev regularity persists under strong continuation; parabolic smoothing gives the asserted positive-time smoothness.

Uniqueness. For strong solutions v, u with the same initial data, put $w = v - u$. The difference equation and incompressibility give

$$\frac{1}{2} \frac{d}{dt} \|w\|_2^2 + \nu \|\nabla w\|_2^2 = - \int_{\Omega} (w \cdot \nabla) u \cdot w \, dx = \int_{\Omega} (w \cdot \nabla) w \cdot u \, dx.$$

Bound the last integral by $C_E \|w\|_2 \|\nabla w\|_2$ and use Young's inequality. Then

$$\frac{d}{dt} \|w\|_2^2 + \nu \|\nabla w\|_2^2 \leq \frac{C_E^2}{\nu} \|w\|_2^2.$$

Gronwall and $w(0) = 0$ prove uniqueness. $\square$

3.1 The maintained flat-frame specialization

Corollary 3.2 (The calibrated $C_E = c$ case). *If a maximal solution of the fixed flat equation (4) admits Definition 5.2 throughout its evolution, with its Euclidean coordinate velocity identified with u_{loc} in one maintained flat orthonormal frame, then $C_E = c$ is admissible and*

$$\|\nabla u(t)\|_2^2 \leq \|\nabla u_0\|_2^2 \exp(c^2 t / \nu), \quad T_* = \infty. \quad (11)$$

Proof. Proposition 5.3 supplies the Euclidean bound under the stated identification. Theorem 3.1 then applies. $\square$

The maintained average supplies the speed bound, and viscosity supplies the gradient control.

3.2 The Euclidean bound closes the continuation condition

The same argument is the $L_t^2 L_x^\infty$ case of the classical velocity continuation criterion [9, 10, 11]: with $\|u(t)\|_\infty^2$ in place of C_E^2 the computation in (8) gives $Y(t) \leq Y(0) \exp(\nu^{-1} \int_0^t \|u(\tau)\|_\infty^2 \, d\tau)$. For every finite candidate endpoint T ,

$$\int_0^T \|u(t)\|_\infty^2 \, dt \leq C_E^2 T < \infty. \quad (12)$$

The proof excludes finite-time breakdown of this fixed-domain strong solution. The exponential estimate is a finite-time continuation bound. Positive viscosity is essential to this argument; setting $\nu = 0$ removes the dissipative term used in (8).

3.3 A regular forcing extension

For a specified acceleration f added to (4), with $f \in L^2(0, T; L^2)$ on every finite interval and the same maintained Euclidean velocity bound, the gradient calculation gives

$$Y'(t) + \nu Z(t) \leq \frac{2C_E^2}{\nu} Y(t) + \frac{2}{\nu} \|f(t)\|_2^2.$$

The kinetic-energy estimate also stays finite on finite intervals, so the same continuation argument applies. The regularity of f determines the corresponding solution regularity; a smooth global solution requires suitably smooth forcing. An internal forcing is included under the same stated $L^2(0, T; L^2)$ condition on its resolved acceleration.

4 Force and geometry in the coupled evolution

The following comparisons examine the physical response to driving. They connect the central invariant to pair inputs, proper measurements, and the evolving total source.

4.1 The force law and pairwise geometry belong to the same evolution

The two notations for the FDT force law are one identity:

$$\boxed{F = \frac{c^4}{4G} \alpha_1 \alpha_2 = F_{\max} \alpha_1 \alpha_2}, \quad F_{\max} = \frac{c^4}{4G}. \quad (13)$$

The prefactor is the force ceiling. FDT applies this law to the same omnium whose energy, geometry, and proper intervals satisfy the central invariant. I-3's source equation and I-4's containment statement belong to that account throughout its evolution [1, I-1, I-3, I-4]. General relativity belongs to the physical evolution throughout. A fixed-background calculation neglects specified geometric changes at a stated accuracy.

For the explicit pair evaluation, label two patterns by $i \neq j$. Let $\mathcal{M}_i > 0$ be the assigned total gravitating monopole mass, $E_i^{\text{src}} = \mathcal{M}_i c^2$ its matched source-energy reading, and $d_{ij} = d_{ji}$ the declared pair separation. The supports are spherical and nonoverlapping, with $R_{b,i} \geq R_{s,i}$ and $d_{ij} \geq R_{b,i} + R_{b,j}$. The exterior cross-centre inputs and scalar pair-force magnitude are

$$\alpha_{i|j,\text{ext}} = \alpha_{i,\text{ext}}(d_{ij}) = \frac{R_{s,i}}{d_{ij}} = \frac{2G\mathcal{M}_i}{c^2 d_{ij}}, \quad R_{s,i} = \frac{2G\mathcal{M}_i}{c^2}, \quad (14)$$

$$F_{ij} = F_{\max} \alpha_{i|j,\text{ext}} \alpha_{j|i,\text{ext}} = \frac{G\mathcal{M}_i \mathcal{M}_j}{d_{ij}^2} = \frac{GE_i^{\text{src}} E_j^{\text{src}}}{c^4 d_{ij}^2}, \quad (15)$$

$$\kappa F_{ij} = 2\pi \alpha_{i|j,\text{ext}} \alpha_{j|i,\text{ext}}, \quad \kappa = \frac{8\pi G}{c^4}. \quad (16)$$

The substitution is exact on the stated monopole domain. It links the Newtonian force expression, the FDT product, and Einstein's coupling. The tensor field equation supplies the metric evolution. The GR force comparison additionally assumes the weak-field, slow-motion reduction, independently of the nonoverlap condition [2, Ch. 4]. Section 6 gives the sharper pair-domain ceiling.

Computing pairwise α values is consequently a computation on the joint state. One pattern has different cross-centre inputs for different partners. For differentiable variations within this evaluation, at fixed c, G ,

$$\begin{aligned} \delta \ln \alpha_{i|j,\text{ext}} &= \delta \ln E_i^{\text{src}} - \delta \ln d_{ij}, \\ \delta \ln F_{ij} &= \delta \ln \alpha_{i|j,\text{ext}} + \delta \ln \alpha_{j|i,\text{ext}} \\ &= \delta \ln E_i^{\text{src}} + \delta \ln E_j^{\text{src}} - 2\delta \ln d_{ij}. \end{aligned} \quad (17)$$

Here $\delta \ln X$ denotes the dimensionless relative variation $\delta X/X$. The energy and separation responses enter together. Holding d_{ij} fixed imposes a separation constraint and removes its variation term. Material motion changes separations even on a fixed Euclidean background; metric backreaction changes the spacetime geometry. The source-energy assignments follow I-7: each reading retains its specified rest, confinement, or observer frame, and each binding contribution enters once.

A vector evolution specifies direction and inertia in addition to the scalar force magnitude. For comparison, take a Newtonian weak-field, slow-motion reduction with constant assigned monopole masses and Euclidean positions $\mathbf{x}_i$. Its leading gravitational pair evolution uses

$$\mathcal{M}_i \ddot{\mathbf{x}}_i = \sum_{j \neq i} F_{ij} \hat{\mathbf{e}}_{ij}, \quad \hat{\mathbf{e}}_{ij} = \frac{\mathbf{x}_j - \mathbf{x}_i}{d_{ij}}, \quad d_{ij} = |\mathbf{x}_j - \mathbf{x}_i|. \quad (18)$$

This leading Newtonian gravitational reduction exhibits its directional and inertial choices. Every pair in (18) attracts, as the measured record has it, with I-1's closed-form magnitude on its

domain; how the system-wide settlement signs a pair it does not join is the core's register item O-5. Relativistic evolution uses the causal matter and apparatus equations, conserved total stress, and tensor geometry. Its pair evaluations refer to that same solution and observation chart. The extensions to variable source masses, phase exchange, and nonspherical or overlapping patterns are listed in Section 10.1.

This gives the force law an active role in the OpenAI comparison. The physical forcing source, the fluid, and their measuring patterns enter one system. Changes to their pair inputs, stress, and separations are part of its coupled response. Computing a prescribed acceleration on fixed Euclidean space defines the reduced fluid problem. The coupled source equations determine the joint geometry. The bosonic response-kernel lesson in Section 4.5 applies here as well: an exact interaction magnitude and a conservation account require a specified response law to determine the ensuing stable pattern or time evolution.

4.2 What the fluid's rulers and clocks measure

Use a Lorentzian metric g of signature $(-, +, +, +)$ and a spacelike foliation with time coordinate t in seconds and spatial coordinates x^i in metres. Define the dimensionless lapse $N > 0$, the positive dimensionless spatial metric h_{ij} , and the shift b^i in velocity units. Their line element is

$$ds^2 = -N^2 c^2 dt^2 + h_{ij}(dx^i + b^i dt)(dx^j + b^j dt). \quad (19)$$

Repeated spatial indices are summed. This is the standard lapse-shift kinematics with an explicit dimensional convention [3]. The time coordinate is a label; elapsed clock time is the proper time computed along the clock's worldline [2, Eq. (3.48)]. Define W as the future unit timelike velocity of the observers normal to these slices, normalized by $g(W, W) = -c^2$. For a timelike fluid trajectory, define its velocity relative to these observers by

$$v^i = \frac{dx^i/dt + b^i}{N}, \quad v_{\text{phys}}^2 = h_{ij}v^i v^j < c^2. \quad (20)$$

Substitution in (19), with $ds^2 = -c^2 d\tau^2$, gives the clock relation

$$d\tau = N dt \sqrt{1 - v_{\text{phys}}^2/c^2}, \quad \frac{dx^i}{dt} = N v^i - b^i. \quad (21)$$

The lapse and metric enter the measured intervals, while coordinate velocity also depends on the shift. Converting the local bound by c to a Euclidean coordinate bound therefore uses all three frame quantities.

The required frame control can be stated quantitatively. Let $h_{\min} > 0$ be a uniform lower eigenvalue bound for h relative to the Euclidean coordinate norm, let $N_{\max}$ bound the lapse above, and let $B_{\max}$ bound the Euclidean shift magnitude. On the interval and region where those bounds hold, write $u_{\text{coord}} = dx/dt$. Equation (20) gives

$$|u_{\text{coord}}|_E \leq N_{\max} \frac{c}{\sqrt{h_{\min}}} + B_{\max} \equiv C_E. \quad (22)$$

Here C_E has velocity units; the metric lower bound means $h_{ij}z^i z^j \geq h_{\min}|z|_E^2$ for every spatial vector z . In the special calibration $h = \delta$, $N = 1$, $b = 0$, the bound is $C_E = c$. The stated uniform frame bounds define the domain of this estimate. The curved fluid operator uses the same evolving metric.

For a coordinate-independent formulation, let U be the fluid's unit timelike four-velocity. Define its Lorentz factor relative to W and its rest-space velocity v by

$$\begin{aligned} \gamma_W &= -\frac{g(U, W)}{c^2}, & U &= \gamma_W(W + v), \\ g(W, v) &= 0, & \gamma_W &= \frac{1}{\sqrt{1 - g(v, v)/c^2}}. \end{aligned} \quad (23)$$

Normalization of U proves the last equality. Relative to its own comoving observer, the fluid's bulk velocity is zero. Fluid clocks can still compare elapsed proper times, neighboring elements can measure relative motion and strain, and curvature is read between neighbouring elements, or between an element and a reference path [1, I-3a]; one element carries none at a point. For a separation dX orthogonal to W , the squared proper ruler distance is $g(dX, dX)$; on these normal observers' simultaneity slices it is $d\ell^2 = h_{ij}dx^i dx^j$. A rotating fluid generally has no single global comoving simultaneity slice, so its spatial comparison is specified locally or by an explicit measurement protocol.

In FDT, clocks and rulers are closed omnium patterns whose adjustment constitutes the geometry [1, 0.1a]. Operationally, their proper intervals, internal state, and comparisons with other patterns must be calculated in the coupled solution. The locally measured null speed remains c . A null helical excitation has no timelike comoving rest clock; the clock relations above describe massive fluid patterns and observers. Lapse, shift, spatial strain, and material response retain their distinct tensor and constitutive roles. The identity $E_q T_q = h$ relates the local energy and period of the specified quantum mode; laboratory kinetic energy is a separate observer-dependent reading.

4.3 Causal matter and the dynamical spacetime source

Relativistic hydrodynamics must supply a causal matter evolution with constitutive relations, thermodynamic variables, initial and boundary data, and the equations for its drive. Causal propagation is a property of that complete constitutive system [4]; covariant notation specifies its geometric form. On a prescribed Minkowski metric, such a model enforces special-relativistic kinematics while leaving spacetime fixed. A model on a prescribed curved metric also leaves the source-to-geometry response outside its evolution.

For spacetime backreaction, define $T_{\text{fluid}}^{\mu\nu}$ as the fluid stress tensor, $T_{\text{drv}}^{\mu\nu}$ as the apparatus and other channel contribution, and $\mathcal{Q}_{\text{drv}}^\nu$ as four-force density into the fluid. Write $T_{\text{tot}} = T_{\text{fluid}} + T_{\text{drv}}$ with each channel counted once. The required source architecture is

$$\nabla_\mu T_{\text{fluid}}^{\mu\nu} = \mathcal{Q}_{\text{drv}}^\nu, \quad \nabla_\mu T_{\text{drv}}^{\mu\nu} = -\mathcal{Q}_{\text{drv}}^\nu, \quad (24)$$

$$\nabla_\mu T_{\text{tot}}^{\mu\nu} = 0, \quad G_{\mu\nu}[g] = \frac{8\pi G}{c^4} T_{\text{tot},\mu\nu}. \quad (25)$$

Here ∇ is the connection of the same evolving g . The contracted Bianchi identity [2, Eq. (3.96)] requires the conserved total source in (25), including the channel supplying the fluid's force. In an orthonormal frame $[T] = \text{J m}^{-3}$, $[\nabla T] = \text{J m}^{-4}$, and the coupling times T has units m^{-2} , matching curvature. A coordinate tensor with a time coordinate measured in seconds carries the corresponding component conversion factors. Gravitational binding belongs to the solved mass aspect; adding an independent local gravitational-energy tensor would double-count that account.

The operational feedback is the simultaneous evolution

$$T_{\text{tot}} \longleftrightarrow g \longleftrightarrow \text{proper intervals and gradients} \longleftrightarrow \text{fluid and driving state} \longleftrightarrow T_{\text{tot}}. \quad (26)$$

The diagram specifies simultaneous dependencies within a causal evolution. The Einstein equation supplies the geometric part; the matter and apparatus closures supply stress, exchange, and propagation. Section 7 gives this analysis's variational reading of the system-wide settlement (0.1a).

The standard account reads the Schwarzschild interior as ending in a curvature singularity at its centre, with the horizon a removable coordinate singularity [2, pp. 171–172]. FDT has no singularity anywhere (0.8): no variable of the chain vanishes, force and pressure are capped by F_{max} , and the density coordinate lives on the open interval, so a compact object is a hollow, double-sided density boundary, dilute at its centre and saturated at its skin, with no central singularity to regulate; the finite core, regular core chart or outward phase transfer that carries its centre is to be derived (D-1).

4.4 The additional terms in an evolving-metric estimate

The distinction enters the derivative calculation itself. On a fixed coordinate region $\mathcal{O}$ of a spatial slice, let $w^k(t, x)$ be a smooth spatial vector field with velocity units. Let D and Γ_{ij}^k be the connection

and its coefficients for $\mathbf{h}(t)$, and set $\mathbf{S}_i^k = \mathbf{D}_i \mathbf{w}^k$. Define the metric gradient integral and its local density by

$$\mathcal{Y}_{\text{geom}}(t) = \int_{\mathcal{O}} \mathcal{I}_{\text{geom}} dV_{\mathbf{h}}, \quad \mathcal{I}_{\text{geom}} = \mathbf{h}^{ij} \mathbf{h}_{kl} \mathbf{S}_i^k \mathbf{S}_j^l, \quad dV_{\mathbf{h}} = \sqrt{\det \mathbf{h}} d^3x. \quad (27)$$

A dot now means a derivative at fixed coordinates. Differentiating every factor, using $\partial_t(\mathbf{D}_j \mathbf{w}^l) = \mathbf{D}_j \dot{\mathbf{w}}^l + \dot{\Gamma}_{jm}^l \mathbf{w}^m$, gives the exact identity

$$\begin{aligned} \dot{\mathcal{Y}}_{\text{geom}} &= 2 \int_{\mathcal{O}} \mathbf{h}^{ij} \mathbf{h}_{kl} \mathbf{S}_i^k \mathbf{D}_j \dot{\mathbf{w}}^l dV_{\mathbf{h}} \\ &\quad + \int_{\mathcal{O}} (\dot{\mathbf{h}}^{ij} \mathbf{h}_{kl} + \mathbf{h}^{ij} \dot{\mathbf{h}}_{kl}) \mathbf{S}_i^k \mathbf{S}_j^l dV_{\mathbf{h}} \\ &\quad + 2 \int_{\mathcal{O}} \mathbf{h}^{ij} \mathbf{h}_{kl} \mathbf{S}_i^k \dot{\Gamma}_{jm}^l \mathbf{w}^m dV_{\mathbf{h}} \\ &\quad + \frac{1}{2} \int_{\mathcal{O}} \mathcal{I}_{\text{geom}} \mathbf{h}^{ab} \dot{\mathbf{h}}_{ab} dV_{\mathbf{h}}. \end{aligned} \quad (28)$$

The inverse metric satisfies $\dot{\mathbf{h}}^{ij} = -\mathbf{h}^{ia} \mathbf{h}^{jb} \dot{\mathbf{h}}_{ab}$; the last line is the volume-measure derivative. Each term has dimensions $\text{m}^3 \text{s}^{-3}$, whereas $[\mathcal{Y}_{\text{geom}}] = \text{m}^3 \text{s}^{-2}$. A moving integration boundary adds its transport flux. The connection variation involves spatial derivatives of $\dot{\mathbf{h}}$; these are quantities entering an evolving-metric continuation estimate, alongside the pointwise bound on $|\mathbf{w}|_{\mathbf{h}}$. The identity locates the geometric terms that enter that separate evolution problem.

4.5 The bosonic ground-state lesson: norm, domain, and response

The singular-potential review of Sakaguchi and Malomed [5] shows the same fixed-model distinction for a stationary matter field. In its dimensionless three-dimensional Gross–Pitaevskii (GP) variables, denote the radius by ϱ_{B} , the attractive inverse-square strength by $\mathfrak{U}_{\text{B}} > 0$, the radial field by ψ_{B} , and a small positive radius by ϵ_{B} . Let $\mathcal{N}_{\text{B}}$ be the dimensionless enclosed GP norm. The singular branch has

$$\begin{aligned} \psi_{\text{B}}(\varrho_{\text{B}}) &\sim \frac{\sqrt{\mathfrak{U}_{\text{B}}/2}}{\varrho_{\text{B}}}, & |\psi_{\text{B}}|^2 &\sim \frac{\mathfrak{U}_{\text{B}}}{2\varrho_{\text{B}}^2}, \\ \mathcal{N}_{\text{B}}(< \epsilon_{\text{B}}) &= 4\pi \int_0^{\epsilon_{\text{B}}} |\psi_{\text{B}}|^2 \varrho_{\text{B}}^2 d\varrho_{\text{B}} = 2\pi \mathfrak{U}_{\text{B}} \epsilon_{\text{B}} + o(\epsilon_{\text{B}}). \end{aligned} \quad (29)$$

The norm is finite while the local density diverges. The gradient and quartic energy integrals diverge at the origin, so this branch lies outside the ordinary finite-energy GP domain. A normalizable stationary branch, its stability under a specified perturbation, and a constrained energy minimum require different statements and domains: the source’s stationary branches stand, and a ground-state reading needs its declared energy domain.

A short Euclidean vector example isolates the corresponding fluid issue. Take a nonzero smooth compactly supported divergence-free dimensionless profile $\mathbf{w}_{\text{test}}$, positive reference scales V_{test} (velocity) and L_{test} (length), and a dimensionless concentration factor $\lambda_{\text{test}} \geq 1$. Define

$$\begin{aligned} u_{\text{test},\lambda}(x) &= V_{\text{test}} \lambda_{\text{test}}^{3/2} \mathbf{w}_{\text{test}}(\lambda_{\text{test}} x / L_{\text{test}}), \\ \|u_{\text{test},\lambda}\|_2^2 &= V_{\text{test}}^2 L_{\text{test}}^3 \|\mathbf{w}_{\text{test}}\|_2^2, \\ \|u_{\text{test},\lambda}\|_{\infty} &= V_{\text{test}} \lambda_{\text{test}}^{3/2} \|\mathbf{w}_{\text{test}}\|_{\infty}. \end{aligned} \quad (30)$$

The change of variables $y = \lambda_{\text{test}} x / L_{\text{test}}$ proves both identities. Every member is smooth and divergence-free; its L^2 norm stays fixed while its supremum grows without bound. This function-space family is indexed by concentration, independently of any evolution equation. It isolates the distinction between the source’s finite integrated kinetic energy and its divergent pointwise velocity.

The singular GP potential also holds its driver fixed while the matter profile responds. Selecting a finite source core or finite apparatus changes the potential, its domain, or its boundary data and defines the completed physical model. Likewise, the coupled relativistic fluid, driver, and metric extend (1) with source and response dynamics. The seam Hessian and induced-interaction calculation in Section 8 state the restoring-response data explicitly. The carrier identity of I-2 concerns the open stroke; molecular matter and fluid constituents are closed patterns with their own constitutive and exchange laws (I-5).

4.6 Vortices within the response of the whole system

In FDT each vortex is a density cone carrying its flow direction, the circulation sense of its helical geodesics (P-4), and the measured vorticity is the swirl of the medium's flow on its clock factor (0.1c, I-3a); the settlement of the coupled configuration signs the pairs it joins. A vortex pair identifies local structure within an evolving coupled system. Its motion reorganizes the surrounding fields, whose response feeds back on the patterns. The studies below resolve parts of that response through a mobile impurity, density and sound, orientational order, active rotors, and boundaries. Their coupled dynamics determine how the response develops through time.

Within FDT, injected work enters one omnium state comprising the patterns, medium, apparatus, surroundings, and spacetime, and every step is the system-wide settlement of that state (0.1a). The work changes the medium's enthalpy per count, and with it the clock factor $\mathbf{m} = \hbar_N / \hbar_{N,0}$ on which the medium's patterns keep time and run thread: the value of $\ln \mathbf{m}$ is invisible on their own clocks and rulers, its gradient is the pressure force the bookkeeper reads, and its second derivatives are read between neighbouring elements (0.1c, I-3a). The relativistic description of Sections 4.2 and 4.3 represents this response through an evolving metric. The construction already evolves full velocity and pressure fields and accounts for finite forcing work and dissipation. The physical comparison concerns the source and geometric response beyond its prescribed driving, constant-density incompressibility, and fixed Euclidean space and Newtonian time.

Quantized shedding changes the force on a mobile impurity. Baker-Rasooli et al. [25] observe an optical impurity moving upstream through a paraxial superfluid of light in hot rubidium vapour. Their experiment records the impurity trajectory and the fluid's amplitude and phase. Above the measured vortex-nucleation threshold, opposite-winding vortex pairs form downstream. Their counterflow and the associated density asymmetry accompany the impurity's upstream deflection (main Figs. 1–3). The reduced model couples the surrounding field to a mobile impurity coordinate, with the force obtained from their interaction Hamiltonian (supplement, Eqs. (1)–(7)). Allowing the impurity to respond makes recoil an explicit part of the evolution. The emitted pair, its wake, the surrounding fluid field, and the impurity participate in the same momentum exchange.

The experiment also separates vortex and acoustic contributions. Sound radiation becomes significant toward the sonic regime and competes with vortex-induced propulsion. The material response therefore changes as new excitation channels become accessible. In this fluid of light the propagation coordinate is the evolution parameter (I-2: the photon fluid and a matter fluid are disturbances of the one medium), and the optical fluid's sound speed sets the scale for the measured vortex-nucleation threshold. For the physical critique, the operative result is the coupled impurity–field response and its measurable force direction, rather than a prescribed trajectory for the source.

Fusion and fission select energetically favourable structures. Hall et al. [26] experimentally control the fusion, fission, and reconnection of vortex knots in a chiral nematic liquid crystal. Electrical pulses change which structures are favoured. Their imaging and free-energy modelling track individual heliknotons joining into larger configurations, including dimers and tetramers; larger composites occupy energy-minimizing or metastable states (Figs. 2–6). The reported transformations lower the final-state energy under the selected electrical conditions. This supplies a concrete example of energy relaxation through reconfiguration and fusion.

The material director remains continuous while the derived local helical axis becomes undefined at the vortex cores. Reconnections change the knotting of that derived field while the cumulative Hopf index of the material configuration is conserved over the studied transformations (main Eq. (1)). Their numerical relaxation includes elastic and electric energy and updates the electric potential as the director relaxes (Methods, Eqs. (2)–(4) and the numerical-relaxation procedure). Source field, material orientation, and topology therefore participate in the same response. This is a particularly direct physical example of stable patterns reorganizing under coupled constraints, with a regular material field through the reconnection.

Opposite rotation supports bound pairs and active chains. Gelvan et al. [27] study motor-driven rotors at an oil–air interface. At their small but finite Reynolds numbers, same-spin rotors orbit and repel; opposite-spin rotors approach and propagate as bound dimers. Mixed populations assemble into active chains called gyromers. Experiments, an inertial correction to the viscous-flow model, and simulations connect the collective structures to fluid motion and steric interactions (Figs. 1–3). Same-spin rotors repel and opposite-spin rotors bind: this is the selection rule of P-4, which signs the pairs a settlement joins at every Reynolds number, and the chains that mixed populations build from opposite-spin dimers are its many-cone settlement [1, P-4].

The observed order depends on the drive and surroundings. The motors maintain rotation, the container geometry redirects encounters, and dispersion in rotor speeds reduces chain stability. In the reported seven-rotor example a single assembled gyromer persists for the battery lifetime, about two hours. This provides an experimentally accessible hydrodynamic realization of orientation-dependent pairing. Rotor spin here denotes the measured mechanical rotation. The motor, fluid, boundary, and assembled structure belong in the same physical account.

Vortical organization builds larger spatial scales. Kronborg and Hoffman [28] examine developing turbulence in a stabilized incompressible Euler computation. Small counter-rotating filaments reorganize into repeating zig-zag patterns as a turbulent energy spectrum develops from high toward lower wave numbers. The authors interpret the temporal association between these events as inverse energy transfer. The reported route is recursive rearrangement with larger geometric periods while individual filament sizes remain comparable; the paper distinguishes it from the fusion of small vortices into larger individual vortices (Figs. 3–5 and accompanying discussion).

This computational example supports organization as an explicit route through the flow’s available configurations. It separates the growth of a collective spatial period from growth of an individual core. The interpreted transfer and the simulated spectral development retain their stated numerical provenance.

After driving ceases, turbulence spreads, coarsens, and decays. Matsuzawa et al. [29] create a localized turbulent blob in water using eight converging vortex generators. After the forcing is removed, particle-image velocimetry tracks its expansion and decay. Vortical structures coarsen, the integral length grows until confinement becomes important, and the late flow develops system-scale organization (Figs. 1–3). The experiment connects spatial redistribution and growing correlation length with a decaying energy reservoir. Its nonlinear-diffusion model describes the expanding turbulent region and its transition to confinement-limited decay. Coarsening is the observed result; the heliknoton study above supplies the separate direct fusion example.

These mechanisms sharpen the physical comparison: shedding, recoil, reconnection, assembly, and coarsening are local manifestations of the response of the coupled system. Their energy and momentum accounts include the surrounding medium and exchanges with the drive and boundaries. FDT reads this as one omnium response with all seven invariants operative. The continuation proof is the argument of Section 3.

5 One medium and its combined constraints

FDT's exclusion of singular physical states is a simultaneous constraint on one medium. Its central invariant, bounded interaction, physical length and time floors, and geometric support conditions describe the same state. Section 1 identifies the metric response required for physical continuation. The fluid theorem has an explicit mathematical domain: a fixed flat incompressible equation with a maintained Euclidean velocity bound.

The central invariant of the FDT framework [1] is

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t}\right)^2. \quad (31)$$

Matter, energy, and spacetime are readings of the same omnium state. The coefficients ε_0, μ_0 describe the medium's response, and c is the strand rate of the omnium: the rate at which the one stroke runs, measured on the spot, converting the time reading into the space reading and, squared, matter into energy (0.1d). The state adjusts all terms together; the quantities in the chain are not independently varied background intervals or separate substances. The nonvanishing requirements apply to these physical state quantities. The distance and duration belong to a pattern, its measurement, or its physical stroke. The core's quantum bridges are

$$E_q = m_q c^2 = \hbar \omega_q, \quad T_q = \frac{2\pi}{\omega_q} = \frac{h}{E_q}, \quad \ell_q = c T_q = \frac{h}{m_q c}, \quad h = 2\pi \hbar. \quad (32)$$

Here ℓ_q is a pattern's thread length, not its support radius.

5.1 Complementary phase tori and the density cones

The density coordinate is the share of the three-sphere (0.3a): with $|z_1|^2 = \alpha_\sigma$ and $|z_2|^2 = 1 - \alpha_\sigma$ on the unit sphere, the Clifford torus $|z_1| = |z_2|$ is $\alpha_\sigma = 1/2$, the seam, and it splits S^3 into the two solid tori of the genus-one Heegaard splitting [21]. The $\alpha_\sigma > 1/2$ solid torus is the matter phase, compressed omnium and closed braids; the $\alpha_\sigma < 1/2$ solid torus is the spacetime phase, decompressed substrate. One substance in two phases: the splitting divides the support, never the omnium, and the three-sphere is the shape of the atlas, never a container the omnium occupies (0.1, 0.1b). The two-dimensional seam is distinct from the three-dimensional phase regions.

For a reference radius $R > 0$, write

$$\begin{aligned} S_R^3 &= \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = R^2\}, \\ V_{\text{mat}} &= \{(z_1, z_2) \in S_R^3 : |z_1| \geq |z_2|\}, \\ V_{\text{sp}} &= \{(z_1, z_2) \in S_R^3 : |z_2| \geq |z_1|\}. \end{aligned} \quad (33)$$

Both regions are solid tori, diffeomorphic to $S^1 \times D^2$, and

$$\begin{aligned} S_R^3 &= V_{\text{mat}} \cup V_{\text{sp}}, \\ \mathcal{T}_{\text{Cl}} &= V_{\text{mat}} \cap V_{\text{sp}} = \partial V_{\text{mat}} = \partial V_{\text{sp}} \\ &= \{(z_1, z_2) : |z_1| = |z_2| = R/\sqrt{2}\} \cong S^1 \times S^1. \end{aligned} \quad (34)$$

For example, V_{mat} is parametrized by $z_2 = w$, $|w| \leq R/\sqrt{2}$, and $z_1 = \sqrt{R^2 - |w|^2} e^{i\theta}$; interchanging the complex coordinates gives the other solid torus. These equations fix the standard complementary embedding, rather than an arbitrary gluing of two solid tori; with $|z_1|^2/R^2 = \alpha_\sigma$ the region V_{mat} is $\alpha_\sigma \geq 1/2$ and V_{sp} is $\alpha_\sigma \leq 1/2$ (0.3a).

Crossing the seam exchanges the two regimes. The seam is the exchange of the two moduli at fixed phases (0.3a); with $z_1 = R\sqrt{\alpha} e^{i\phi_1}$ and $z_2 = R\sqrt{1-\alpha} e^{i\phi_2}$,

$$\begin{aligned} \mathfrak{J} : (R\sqrt{\alpha} e^{i\phi_1}, R\sqrt{1-\alpha} e^{i\phi_2}) &\mapsto (R\sqrt{1-\alpha} e^{i\phi_1}, R\sqrt{\alpha} e^{i\phi_2}), \\ J : (z_1, z_2) &\mapsto (z_2, z_1), \quad \mathfrak{J}^2 = J^2 = \text{id}, \quad \mathfrak{J}(V_{\text{mat}}) = J(V_{\text{mat}}) = V_{\text{sp}}. \end{aligned} \quad (35)$$

$\mathfrak{J}$ carries the torus $\{|z_1|^2 = \alpha R^2\}$ onto $\{|z_1|^2 = (1 - \alpha)R^2\}$, carries Hopf fibres to Hopf fibres and fixes $\mathcal{T}_{\text{Cl}}$ pointwise: the inside becomes the outside, as a jacket turned inside out, and the seam is the one locus that reads the same either way. The coordinate swap J is $\mathfrak{J}$ composed with the exchange of the two phases; it also carries each region onto the other, and on $\mathcal{T}_{\text{Cl}}$ it fixes only the great circle $z_1 = z_2$. Each region is the closure of the other's complement in S_R^3 . Section 7 gives the conservation account of a passage between the phases.

For the standard round reference geometry, useful coordinates are

$$z_1 = R \cos \chi e^{i\phi_1}, \quad z_2 = R \sin \chi e^{i\phi_2}, \quad 0 \leq \chi \leq \frac{\pi}{2}. \quad (36)$$

The share is $\alpha_\sigma = \cos^2 \chi$, so the seam is $\chi = \pi/4$; $\mathfrak{J}$ sends (χ, ϕ_1, ϕ_2) to $(\pi/2 - \chi, \phi_1, \phi_2)$ and J sends it to $(\pi/2 - \chi, \phi_2, \phi_1)$. The reference metric is

$$g_{\text{round}} = R^2 (d\chi^2 + \cos^2 \chi d\phi_1^2 + \sin^2 \chi d\phi_2^2). \quad (37)$$

It is smooth across the Clifford interface. At $\chi = 0$ or $\pi/2$, one angular coordinate degenerates in the usual polar-coordinate sense; the corresponding core is a circle, not a singular point of S_R^3 . These two core circles are the ends of the open interval, $\alpha_\sigma \rightarrow 1^-$ at $\chi = 0$ and $\alpha_\sigma \rightarrow 0^+$ at $\chi = \pi/2$, which no configuration occupies (I-6, 0.3a). The parametrization fixes a round reference geometry. The S^3 describes the three-dimensional phase organization; physical spacetime evolution includes the temporal and Lorentzian metric structure.

The two truncated density cones of I-3 read the same organization on one support. Their narrow ends meet apex circle to apex circle at the fold, where both branches approach 1^- and neither reaches it: both ends of the interval are Hopf core circles, so the cones have no point at which to meet (0.3a, I-3). The Clifford torus is not this meeting; it is the seam $\alpha_\sigma = 1/2$ inside each cone, at $r_f = 2R_s$ on the exterior branch and $r_f = R_s/2$ on the interior branch. The density cone at a point is the local light cone, its causal aperture set by the density through the I-3 reading [1, P-4]; the constitutive derivation of that aperture is to be derived (D-4).

The topology fixes the interface geometry. Field regularity is controlled by the evolution estimates on the stated dynamical domain.

5.1.1 A smooth exchange and an unbounded field

The distinction can be checked directly. For the round reference metric the swap J in (35) is an isometry. For any vector field v ,

$$|(J_*v)(Jx)|_{g_{\text{round}}} = |v(x)|_{g_{\text{round}}}, \quad \|J_*v\|_{L^\infty(S_R^3)} = \|v\|_{L^\infty(S_R^3)}. \quad (38)$$

Indeed J exchanges the two angular metric terms and reverses the χ coordinate, leaving their squared lengths unchanged. The seam $\mathfrak{J}$ exchanges the moduli at fixed phases and is not an isometry of the round metric off $\mathcal{T}_{\text{Cl}}$: $\mathfrak{J}^*g_{\text{round}} - g_{\text{round}} = -R^2 \cos 2\chi (d\phi_1^2 - d\phi_2^2)$, which vanishes on $\mathcal{T}_{\text{Cl}}$ and nowhere else. As a passive change of chart, with the metric carried along with the vector, it preserves the physical norm all the same, as every passive coordinate change does. Thus a smooth inside-outside atlas removes a coordinate obstruction while preserving the field's invariant norm, including any unboundedness. Section 8.1 couples the passage across the seam to a physical pattern response.

5.2 The force ceiling and the length and time floors

The core gives the force ceiling [1, sections 0.1a–0.2 and I-1]

$$F_{\text{max}} = \frac{c^4}{4G}. \quad (39)$$

Its minimal one-action stroke relation and the central speed relation are

$$F_{\text{max}} d_{\text{min}} t_{\text{min}} = \hbar, \quad d_{\text{min}} = ct_{\text{min}}. \quad (40)$$

The first equality is the core's one-action stroke (0.1a); the following length and time formulas are its algebraic consequences together with (39). The action in (40) is $\hbar$; the full-period action h appearing in (32) describes the separate mode-period reading.

Proposition 5.1 (Physical stroke infima). *Under (39)–(40), the characteristic lower bounds are*

$$d_{\min} = 2\ell_P, \quad t_{\min} = 2t_P, \quad \ell_P = \sqrt{\frac{\hbar G}{c^3}}, \quad t_P = \frac{\ell_P}{c}. \quad (41)$$

For a one-action physical stroke with length d_{st} , duration t_{st} , and mean force $\bar{F} = b_F F_{\max}$ with $0 < b_F < 1$,

$$\bar{F} d_{\text{st}} t_{\text{st}} = \hbar, \quad d_{\text{st}} = c t_{\text{st}} \implies d_{\text{st}} = \frac{d_{\min}}{\sqrt{b_F}} > d_{\min}, \quad t_{\text{st}} = \frac{t_{\min}}{\sqrt{b_F}} > t_{\min}. \quad (42)$$

Proof. Eliminating $t_{\min}$ from (40) gives

$$d_{\min}^2 = \frac{\hbar c}{F_{\max}} = \frac{4\hbar G}{c^3} = 4\ell_P^2. \quad (43)$$

Taking the positive root and dividing by c proves (41). For the sub-ceiling stroke, $d_{\text{st}}^2 = \hbar c / (b_F F_{\max}) = d_{\min}^2 / b_F$; positivity and $b_F < 1$ give (42). The dimensional products are $[Fdt] = \text{J s}$ and $[\hbar c / F] = \text{m}^2$. $\square$

The generic mean-force fraction b_F is not identified with a spherical pair's product of compactness inputs. It specifies the stroke's mean force under the universal ceiling. The lower bounds are approached but unoccupied: they fix no lattice or spacing above the floor. Mathematical coordinate differences are distinguished from the physical strokes to which these relations apply.

The central ratio, the one-action stroke and the ceiling operate together. The ratio $d/t = c$ by itself permits d and t to approach zero together; equations (40)–(42) exclude that continuation for the stated physical strokes. A physical trajectory must satisfy the chain, force ceiling, and stroke bounds at the same time. Its resolved fluid velocity is obtained from the same motion by the reconstruction below.

5.3 From the same state to resolved fluid motion

Velocity, pressure, stress, and viscosity describe motions and responses of this same medium. There is no second substance to which fluid dynamics belongs. Different equations specify different representations and constitutive conditions of omnium's motion. The comparison equation in this paper is viscous, incompressible Navier–Stokes flow on a fixed flat domain. It is a resolved approximation; the coupled relativistic description has the additional variables and equations of Section 4.3.

Let v_{hel} denote the velocity of a strand of the omnium's closed patterns, and let u_{loc} denote resolved transport in an observer rest space. A closed pattern's strand runs at c on a helix, translating at v and circulating at u with $v^2 + u^2 = c^2$, and keeps time by circulating (0.1d); an open stroke runs straight at c , its grades turning on a helix that stands at $\pi/4$ (I-2). In a local orthonormal measurement frame the strand obeys

$$|v_{\text{hel}}| = c = 299\,792\,458 \text{ m s}^{-1}. \quad (44)$$

A closed pattern separates internal circulation from net transport: symmetric closed circulation has zero mean velocity while every segment of the strand moves at c . These are two readouts of the same motion. The rate c belongs to the total tangent velocity along the complete nested path; nested components are resolved as one path, not added as independent velocities of magnitude c .

5.4 Observed orbital nesting and its FDT interpretation

Planetary orbital motion and the Solar System's motion through the Galaxy are observed phenomena [12, 13]. In the two-body approximation, a planet's bound orbit is an ellipse with the Sun at a focus; actual ephemerides include mutual perturbations and specify their reference frames [14]. In a common nonrotating spatial basis at a common coordinate time, the planet's position can be written

$$\mathbf{x}_p(t) = \mathbf{R}_\odot(t) + \mathbf{r}_{p/\odot}(t). \quad (45)$$

This decomposes one trajectory into the position of the Sun and the planet's relative position. For the illustrative case of a circular orbit traversed at constant angular rate and uniform translation perpendicular to the orbital plane,

$$\mathbf{x}_p(t) = \mathbf{X}_0 + R_{\text{orb}} \cos(\omega_{\text{orb}} t) \mathbf{e}_1 + R_{\text{orb}} \sin(\omega_{\text{orb}} t) \mathbf{e}_2 + V_d t \mathbf{e}_3. \quad (46)$$

Here the $\mathbf{e}_i$ form a fixed orthonormal basis, $R_{\text{orb}} > 0$ is the orbital radius, $\omega_{\text{orb}} > 0$ its angular rate, and $V_d \neq 0$ the signed drift speed. This is a circular helix of signed pitch $2\pi V_d / \omega_{\text{orb}}$. Ellipticity, oblique translation, nonuniform angular motion, and curvature of the Sun's galactic path produce more general winding trajectories. Nested systems admit another relative-position term, for example

$$\mathbf{x}_m(t) = \mathbf{R}_\odot(t) + \mathbf{r}_{p/\odot}(t) + \mathbf{r}_{m/p}(t) \quad (47)$$

for a moon. This is a kinematic decomposition, not a decomposition into independent substances.

The visible motions provide a macroscopic example of orbital circulation combined with transport. Their spatial appearance depends on the chosen frame: subtracting $V_d t \mathbf{e}_3$ from (46) recovers the circular orbit. Naked-eye records reveal apparent angular positions and their changes; complete spatial trajectories additionally use distance measurements, reference frames, and dynamical reconstruction.

FDT reads this nesting as a macroscopic instance of circulation and transport within one medium. The observations establish the orbital motion; the microscopic reconstruction is specified by the helical average and the I-5 confinement eigenproblem below.

In the density-cone reading the Sun is the source of its exterior cone (I-3). The density cone at an event is the local light cone, its aperture set by the density through the I-3 reading; the flat cone of special relativity is its uniform-density idealisation [1, P-4]. A planet's locally measured motion is timelike, inside that cone [15]; the ellipse describes the spatial orbit, the cone the causal directions.

5.5 The microscopic transport reconstruction

Definition 5.2 (Local resolved omnium transport). At each resolved location and time (x, t) , let ξ label local helical directions of the omnium configuration. Let $d\mu_{x,t}(\xi)$ be a nonnegative normalized averaging measure:

$$\int d\mu_{x,t} = 1, \quad v_{\text{hel}}(x, t, \xi) = c \hat{\tau}(x, t, \xi), \quad |\hat{\tau}| = 1 \quad \mu_{x,t}\text{-almost everywhere.}$$

Define

$$u_{\text{loc}}(x, t) = \int v_{\text{hel}}(x, t, \xi) d\mu_{x,t}(\xi). \quad (48)$$

The measure gives deterministic positive weights for the stated internal directional coarse-graining. These weights describe resolved transport; the quantum phase projection is the separate I-4 readout. All vectors in the average are measurable and expressed in the same local orthonormal frame; transporting vectors from different points requires that frame identification as part of the reconstruction.

Proposition 5.3 (The helical average gives a local speed bound). *The resolved transport defined in (48) satisfies*

$$|u_{\text{loc}}(x, t)| \leq c. \quad (49)$$

Proof. The triangle inequality and normalization give

$$|u_{\text{loc}}| = \left| \int c \hat{\tau} d\mu \right| \leq c \int |\hat{\tau}| d\mu = c.$$

□

Where the reconstruction is maintained, it carries this local bound throughout the evolution. Timelike massive transport requires strict inequality; the equality case of this average is the aligned null limit. The fixed-coordinate PDE velocity u is a different variable. Its identification with u_{loc} requires the maintained flat-frame calibration stated in Section 3.1. Differing orientations give resolved transport below c ; symmetric closed circulation gives zero net transport. The underlying stroke still obeys $d/t = c$. The invariant local speed and the chart-dependent optical aperture have the distinct roles specified below.

Proposition 5.4 (Directional concentration near the speed ceiling). *Under Definition 5.2, at each fixed (x, t) ,*

$$\int |v_{\text{hel}} - u_{\text{loc}}|^2 d\mu = c^2 - |u_{\text{loc}}|^2. \quad (50)$$

Consequently, $|u_{\text{loc}}| \rightarrow c$ forces this directional variance to vanish. At $|u_{\text{loc}}| = c$, all contributing velocities equal u_{loc} almost everywhere with respect to μ .

Proof. Expanding the square and using normalization and the definition of u_{loc} ,

$$\begin{aligned} \int |v_{\text{hel}} - u_{\text{loc}}|^2 d\mu &= \int |v_{\text{hel}}|^2 d\mu - 2u_{\text{loc}} \cdot \int v_{\text{hel}} d\mu + |u_{\text{loc}}|^2 \int d\mu \\ &= c^2 - 2|u_{\text{loc}}|^2 + |u_{\text{loc}}|^2 = c^2 - |u_{\text{loc}}|^2. \end{aligned}$$

At equality the integral of a nonnegative function is zero, giving the almost-everywhere statement. □

This identity gives the internal directional dispersion at each resolved point and constrains reorganization near c . For a single closed helix with μ uniform over one turn, (50) is the helix triangle of 0.1d, $v^2 + u^2 = c^2$, with $v = |u_{\text{loc}}|$ the translation and u the circulation; for a closed pattern $u > 0$, so $|u_{\text{loc}}| < c$, the open interval read on the strand.

The combined constraints exclude physical continuations that require a vanishing stroke or violation of the force or local reconstructed-speed bounds. On the fixed-domain PDE, the maintained Euclidean velocity bound controls the transport term and gives the derivative estimate of Theorem 3.1. The coupled geometric and phase constructions have their own evolution variables and boundary data.

5.6 Containment and the theorem's domain

FDT is a true extension of general relativity and quantum mechanics: it contains both as descriptions of the same omnium. In the GR reading, source and geometry obey the tensor field equation. In the QM reading, helical phase and confinement modes determine the state's quantum readout. The relativistic fluid reading resolves the same motion into observer-relative velocity, stress, and conserved currents.

Theorem 3.1 establishes continuation for the fixed flat momentum equation (4), its stated initial-data class, and a uniform Euclidean velocity bound throughout the maximal solution. Its proof uses those hypotheses directly. The coupled Einstein–matter construction is a separate evolution problem.

6 The seven invariants and their roles

Notation. n_h is helix count; n_{iso} , n_{cone} , and n_{dens} are indices. ν is kinematic viscosity; κ is Einstein coupling; a is radiation energy-density constant. ξ labels averaged directions; q labels confinement modes. d_{st} and t_{st} are a physical stroke's length and duration; b_F is its mean force as a fraction of F_{max} ; s is the Sobolev regularity index in the fluid theorem. u_{loc} is the local orthonormal transport average, u the Euclidean PDE velocity, and C_E its maintained coordinate bound. The styled symbols $\mathbf{N}, \mathbf{b}, \mathbf{h}, \mathbf{U}, \mathbf{W}$ retain the lapse, shift, spatial metric, and observer-velocity roles of Section 4.2; h is distinct from Planck's h . The driving exchange $\mathcal{Q}_{\text{drv}}^\nu$ is distinct from the internal phase exchange Q^ν below. Ω is the fluid domain and Ω_p a pattern support. Greek letters μ, ν, λ as tensor indices are spacetime indices; ν standing alone is the viscosity. h_{min} is the lower eigenvalue bound of the spatial metric and $\mathbf{m}$ the medium's clock factor of 0.1c; ς_o, ς_i are the linking signs of the charge rule. These definitions follow the FDT core [1].

6.1 I-1 and I-4: force, its evaluations, and its continuum reading

The force law introduced in Section 4.1 and Einstein coupling are

$$F = \frac{c^4}{4G} \alpha_1 \alpha_2 = F_{\text{max}} \alpha_1 \alpha_2, \quad F_{\text{max}} = \frac{c^4}{4G}, \quad \kappa = \frac{8\pi G}{c^4}, \quad \kappa F_{\text{max}} = 2\pi. \quad (51)$$

In this pair expression the inputs are exterior, cross-centre compactness values: $\alpha_i = \alpha_{i,\text{ext}}(d_{12}) = R_{s,i}/d_{12}$. The explicit spherical monopole evaluation has nonoverlapping supports, $R_{b,i} \geq R_{s,i}$ and $d_{12} \geq R_{b,1} + R_{b,2}$, with $R_{s,i} = 2GM_i/c^2$. On this declared domain,

$$F = \frac{c^4}{4G} \frac{2GM_1}{c^2 d_{12}} \frac{2GM_2}{c^2 d_{12}} = \frac{GM_1 M_2}{d_{12}^2} = G \lambda_1 \lambda_2 = \frac{2\pi}{\kappa} \alpha_1 \alpha_2, \quad \lambda_i = \mathcal{M}_i/d_{12}. \quad (52)$$

The universal open coordinate interval gives $F < F_{\text{max}}$ wherever the force inputs occupy that interval. The sharper pair-domain bound is $F \leq F_{\text{max}}/4$, since $R_{s,1}R_{s,2}/d_{12}^2 \leq R_{s,1}R_{s,2}/(R_{s,1} + R_{s,2})^2 \leq 1/4$; equality needs $R_{s,1} = R_{s,2}$, saturated supports $R_{b,i} = R_{s,i}$, and contact $d_{12} = R_{b,1} + R_{b,2}$ [1, I-1]. This evaluation domain qualifies the closed-form calculation, not the universality of the invariant.

I-4 keeps three layers distinct: the pair algebra above is exact in its declared model (L1); general relativity reduces to $GM_1 M_2/d_{12}^2$ in the weak-field, slow-motion limit, with first corrections of relative order v^2/c^2 and $GM/(c^2 d_{12})$ (L2); the FDT source programme and each claimed divergence are the third layer, the core's register (L3). In the fluid representation, internal interaction enters through stress and the associated momentum balance.

The quantum face of I-4 reads quantum motion as helical geodesics on density cones: $\psi = Ae^{i\phi}$ records a projected helix, and phase ignorance gives the superposition reading. These are states of the same deterministic omnium. The mode energy in I-5 belongs to this same helical dynamics.

6.2 I-2 and I-5: the carrier and the closed patterns of the fluid

The one-medium chain gives the FDT carrier identity

$$\text{photon} = \text{graviton} = \text{gluon} = \text{phonon} = \text{magnon}. \quad (53)$$

One medium and the same propagation stroke underlie these names: the mass-energy, electromagnetic, and mechanical readings are disturbances of omnium. The carrier is the spin-1 density-field excitation. Fluid stress and viscous response describe reconfiguration of that medium.

Matter modes are closed patterns of the excitation; they are not additional entries in (53). I-5 distinguishes the integer helix count $n_h \in \{2, 3, \dots\}$, which fixes a braid class, from the confinement coordinate within that class. A change of confinement preserves the class; a change of helix count requires re-braiding. For a pattern with its own support Ω_p , boundary data $\mathcal{B}_p$, orientation χ_p , and confinement coordinate ζ_p , the core's spectral form is

$$\mathcal{D}(\Omega_p, \mathcal{B}_p; n_h, \chi_p, \zeta_p) \varphi_q = \omega_q \varphi_q, \quad m_q c^2 = \hbar \omega_q. \quad (54)$$

The support is fixed by the pattern's geometry, independently of the mass to be calculated; it is not identified with $\hbar/(m_q c)$. The inertial and energy channels of the fluid refer to the same medium and its patterns, with total energy counted as in I-7.

6.3 I-3: geometry, the Alpha Lock, and the optical roles

For total gravitating mass $\mathcal{M} > 0$, let $R_s = 2G\mathcal{M}/c^2$. The field equation and branch-tagged atlas are

$$G_{\mu\nu} = \kappa T_{\mu\nu}^{(\text{om})}, \quad (55)$$

$$\alpha_{\text{ext}}(r_f) = R_s/r_f \quad (r_f > R_s), \quad \alpha_{\text{int}}(r_f) = r_f/R_s \quad (r_c \leq r_f < R_s), \quad (56)$$

where $r_c > 0$ is the core-chart boundary. Both branches have values in $(0, 1)$; their fold values are approached as one-sided limits. The atlas is downstream of the source and the solved geometry, not a replacement for the tensor equation. Given the same source and initial and boundary data, the FDT and GR readings give the same curvature.

For the spherical exterior, the exact optical metric in areal coordinates is

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\Omega_2^2}{1 - \alpha_{\text{ext}}}. \quad (57)$$

Generalized Snell refraction is the Fermat jump law of this metric, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, with the exact index appropriate to the declared optical chart. Three index roles remain distinct:

$n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2)$	isotropic weak-field reading,
$n_{\text{cone}} = (1 - \alpha_{\sigma})^{-1/2}$	causal-aperture assignment,
$n_{\text{dens}} = (1 - \beta_{\sigma})^{-1/2}$	density-chart diagnostic, excluded from optics.

Here $\sigma \in \{\text{ext}, \text{int}\}$ names a branch and β_{σ} is defined in the chart inventory below. A standalone n_{cone} is not the complete lensing geometry. In particular, $c_{\text{eff}} = c\sqrt{1 - \alpha_{\sigma}}$ is a chart-dependent aperture, not the invariant c in the local bound (49).

For an internal bound configuration, the core additionally carries its own interior chart $\alpha_{\text{int}}^{(\text{cfg})}$, with the skin at the fold and the measured skin radius R_{skin} specified as configuration data. The boundary-data source records the measured radius and probe weighting. That chart is not obtained by rescaling a laboratory radius into the exterior gravitational-radius atlas. Its centre is the dilute region, its skin the densest region (0.6); a measured interior quantity is a profile-weighted reading, not a point coordinate. The local nuclear profile is a constitutive quantity. The configuration shell, measured radius, and exterior gravitational fold retain their distinct geometric roles.

6.3.1 Schwarzschild density and finite support

The spherical Schwarzschild density scale of a fixed $\mathcal{M} > 0$ is [1, section 0.6]

$$\rho_s(\mathcal{M}) = \frac{\mathcal{M}}{(4\pi/3)R_s^3} = \frac{3c^6}{32\pi G^3 \mathcal{M}^2} < \infty. \quad (58)$$

For a whole configuration with spherical support radius $R_b > R_s$,

$$\bar{\rho}(R_b) = \frac{\mathcal{M}}{(4\pi/3)R_b^3} = \rho_s \alpha_{\text{ext}}(R_b)^3 < \rho_s. \quad (59)$$

This configuration cannot collapse its fixed mass into zero support while retaining $R_b > R_s(\mathcal{M}) > 0$. The density statement and the physical stroke floors belong to the same geometric account, with distinct roles: ρ_s sets a fixed-mass comparison scale, while d_{min} and t_{min} constrain physical strokes.

The core distinguishes this mean-density scale from the actual local profile. The interior comparison proxy obeys $\bar{\rho}(r_f)/\rho_s = \alpha_{\text{int}}(r_f)^{-3}$; the local profile is determined by its constitutive and core data. ρ_s separates the two branches; it is not a density cap (0.6). Also, $\rho_s(\mathcal{M})$ has no uniform upper bound as $\mathcal{M} \rightarrow 0^+$. The flat-domain fluid derivative estimate is supplied by the maintained Euclidean bound and viscosity, with the density and stroke conditions retained as simultaneous physical constraints.

6.4 I-6: chart inventory and bounded updates

Application	Coordinate	Role here
A-1	$\alpha_{\text{ext}}, \alpha_{\text{int}}$	Geometric compactness atlas; force inputs use declared exterior evaluations.
Config.	$\alpha_{\text{int}}^{(\text{cfg})}$	The pattern's own skin chart, with measured skin radius as boundary data; no transfer map from the gravitational atlas is used.
A-2	$\beta_\sigma = \frac{\alpha_\sigma}{1+\alpha_\sigma}$	Computational compactification; no force, optical, or density-law substitution.
A-3	α_{ovl}	Overlap/contrast token; no numerical threshold or continuum stress is inferred from it here.
A-4	α_{conf}	Confinement-position token; no numerical mass address is assigned.
A-5	α_{mode}	Naming token for manifestation regimes; no ordering or trajectory is assigned.
Share	$\alpha_\sigma = z_1 ^2/R^2$	The A-1 compactness read as the share of S^3 (0.3a): the seam $\alpha_\sigma = 1/2$, at $r_f = 2R_s$ (exterior) and $r_f = R_s/2$ (interior), and its exchange $\mathfrak{J} : \alpha_\sigma \mapsto 1 - \alpha_\sigma$; matching across the seam in Section 8.1.

The A-1 odds are

$$X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \quad X_{\text{int}} = \frac{r_f}{R_s - r_f}, \quad \alpha_\sigma = \frac{X_\sigma}{1 + X_\sigma}. \quad (60)$$

An A-1 odds-enhancing update $X_\sigma = X_{\sigma,0}(1+x)$, with dimensionless $x \geq 0$, gives

$$\alpha_\sigma(x) = \frac{\alpha_{\sigma,0}(1+x)}{1 + \alpha_{\sigma,0}x}, \quad 1 - \alpha_\sigma(x) = \frac{1 - \alpha_{\sigma,0}}{1 + \alpha_{\sigma,0}x} > 0. \quad (61)$$

No signed negative drive is placed in this input; polarity has its own orientation sign when required. An interior odds-enhancing update raises α_{int} and reduces the comparison density $q_{\text{int}} = \alpha_{\text{int}}^{-3}$; the exterior comparison density is $q_{\text{ext}} = \alpha_{\text{ext}}^3$. These comparison densities belong to the gravitational atlas; the configuration chart carries the distinct local-profile reading stated above. No sign is transferred between chart applications without an explicit map.

The physical endpoints remain excluded (I-6); the finite core, regular core chart or outward phase transfer at the endpoints is the core's register item O-1.

6.5 I-7: one energy account for the fluid

The total gravitating-energy account is

$$\mathcal{M} = M + \frac{E_{\text{other}}}{c^2} + \frac{PV}{c^2} + \frac{aT^4V}{c^2} + \dots, \quad a = \frac{4\sigma_{\text{SB}}}{c}, \quad R_s = \frac{2GM}{c^2}. \quad (62)$$

All non-rest channels enter the single omnium stress tensor exactly once. E_{other} excludes contributions separately listed in the chosen account. The radiation term uses an energy-density constant; gravitational binding is already present in the solved mass aspect and is not an added local stress

channel. Kinetic motion, pressure work, and thermal changes are responses of the same omnium, not independent substances or unrelated density mechanisms.

The thermodynamic pressure P in (62) has its physical reference and state relation. The pressure multiplier π in the incompressible equation is defined modulo a function of time. Its relation to thermodynamic pressure is part of the specified stress representation, not an identification of an arbitrary pressure gauge with absolute gravitating energy. The pressure and the enthalpy per count are also the clock factor of the medium's own patterns: the pressure force in (1) is the bookkeeper's reading of their common rescaling, (2), and the entries E and PV enter $\mathcal{M}$ once (0.1c, I-7).

7 Coupled phase evolution of the same omnium state

The central invariant is applied to the complete state. The omnium settles matter, energy and spacetime by one global optimisation, with all readings adjusting together, and every step is a system-wide settlement solved from the state the system is in; responses are many-to-many, never particle by particle [1, 0.1a]. A drive that would violate a constraint of the chain induces a joint reorganisation that prevents the violation: closed matter patterns pass into the spacetime phase across the seam, and the spacetime phase closes into matter the other way (0.3a, 0.8). The invariant remains satisfied throughout the allowed evolution. Energy belongs to both phases; transfer does not remove it from the total account.

7.1 Simultaneous adjustment as constrained evolution

Use an evolution parameter s_{ev} distinct from the time a pattern keeps. For a pattern's matched state reading let $X(s_{\text{ev}}) = (E, m, \varepsilon_0, \mu_0, \ell, T)$ with positive entries, where T is the time the pattern keeps and ℓ the length its thread runs in that time (0.1a). At constant local c the chain and its bridge read, without division,

$$C(X) = \begin{pmatrix} E - mc^2 \\ c^2\varepsilon_0\mu_0 - 1 \\ \ell - cT \\ ET - h \end{pmatrix} = 0. \quad (63)$$

A differentiable allowed evolution satisfies the tangency condition

$$DC(X) X' = 0, \quad E' = c^2 m', \quad \mu_0 \varepsilon'_0 + \varepsilon_0 \mu'_0 = 0, \quad \ell' = cT', \quad \frac{E'}{E} = -\frac{T'}{T} = -\frac{\ell'}{\ell}, \quad (64)$$

where primes denote derivatives with respect to s_{ev} : a step in energy is a step in mass, in the time kept and in the length run, by the identical stroke. For a one-action stroke the bridge is $\bar{F} d_{\text{st}} t_{\text{st}} = \hbar$ of Proposition 5.1 in place of $ET = h$. These relations tie changes in the same local mode energy to its period and thread length. Coordinate time, proper time, and an observer's kinetic-energy reading retain the distinct roles established in Section 4.2. The mode identity uses the local mode energy and period. At a discrete reorganization, both allowed endpoint configurations satisfy their state constraints and the transition obeys the conserved accounts.

The joint state also includes density, geometry, phase organization, and momentum. Equations (63)–(64) specify the admissible state variations. The core names no objective for the optimisation (0.1a); this analysis writes one, as its own reading, as a *matter action* $\mathcal{S}_{\text{mat}}[\mathcal{X}]$, where $\mathcal{X}$ denotes the complete coupled configuration or history, including its geometry and phase data. Its constrained stationarity condition is

$$\delta \mathcal{S}_{\text{mat}}[\mathcal{X}; \delta \mathcal{X}] = 0 \quad \text{for admissible variations } \delta \mathcal{X}. \quad (65)$$

The expected behaviour of the evolving system is not necessarily the optimum of $\mathcal{S}_{\text{mat}}$ (0.1a). The action density, boundary data, selection law and estimates for the coupled evolution are to be derived (D-5). Tangency specifies the allowed variations throughout.

“Global” describes optimization of the coupled state; physical signals retain the characteristic speed of the medium. Classical Navier–Stokes already couples a whole fluid through its nonlinear velocity field, pressure, and viscosity. The coupled geometric description evolves the physical metric together with the source, apparatus, and phase state. It introduces dynamical variables beyond a material-coordinate pullback and collective pressure coupling.

7.2 Recurrent organization and the range of scale transformations

The universe is fractal because the chain is all ratios: E/m , c^2 , $1/(\varepsilon_0\mu_0)$ and $(d/t)^2$ fix relations and select no size, and α_σ is invariant under $(R_s, r_f) \mapsto (\lambda R_s, \lambda r_f)$, so the support, its seam and its two phases have the same form at every size, matter and spacetime manifesting at every scale (0.3, 0.3a). The repetition is bounded below by the floor $2\ell_P$, $2t_P$ (0.1a). The scaling relations below show which maps keep which readings.

For example, the following transformation of a *mode* family, with $\lambda > 0$, preserves the central ratios and quantum bridges:

$$\begin{aligned}(\ell_{q,\lambda}, T_{q,\lambda}) &= \lambda(\ell_q, T_q), \\ (E_{q,\lambda}, m_{q,\lambda}, \omega_{q,\lambda}) &= \lambda^{-1}(E_q, m_q, \omega_q).\end{aligned}\tag{66}$$

Indeed $E_{q,\lambda}/m_{q,\lambda} = c^2$, $\ell_{q,\lambda}/T_{q,\lambda} = c$, $E_{q,\lambda}T_{q,\lambda} = h$, and $E_{q,\lambda} = \hbar\omega_{q,\lambda}$. These are exact scaling identities for the displayed mode relations. Admissible confinement modes also satisfy their eigenproblem.

For a *one-action stroke*, considered separately, preserving $\bar{F}d_{\text{st}}t_{\text{st}} = \hbar$ while scaling $(d_{\text{st}}, t_{\text{st}})$ by λ requires

$$\bar{F}_\lambda = \lambda^{-2}\bar{F}, \quad \bar{F}_\lambda < F_{\text{max}} \implies \lambda > \sqrt{\frac{\bar{F}}{F_{\text{max}}}}.\tag{67}$$

The full-period action h and the one-action stroke $\hbar$ retain their distinct roles. This stroke scaling cannot be continued to arbitrarily small physical lengths at fixed F_{max} .

The geometry repeats with it: under $(\mathcal{M}, r_f) \mapsto (\lambda\mathcal{M}, \lambda r_f)$, so that $R_s \mapsto \lambda R_s$,

$$\alpha_{\text{ext},\lambda}(\lambda r_f) = \alpha_{\text{ext}}(r_f), \quad \alpha_{\text{int},\lambda}(\lambda r_f) = \alpha_{\text{int}}(r_f),\tag{68}$$

so the seam, the fold and the two phases have the same form at every size, and the stroke scaling (67) bounds the repetition below at fixed F_{max} . The mode family (66) is a different map: it scales mass inversely with size, $\mathcal{M} \mapsto \mathcal{M}/\lambda$, and moves α_{ext} by λ^{-2} ; it is a statement about one pattern’s bridges, not the recurrence of the support. The copies at successive sizes are distinct patterns, each with its own confinement eigenvalues (I-5), and every stroke of every copy respects the floor.

7.3 Bidirectional phase exchange

For a closed system including the channels involved in the transfer, a two-phase stress-energy account can be written

$$T_{\text{om}}^{\mu\nu} = T_{\text{mat}}^{\mu\nu} + T_{\text{sp}}^{\mu\nu},\tag{69}$$

$$\nabla_\mu T_{\text{mat}}^{\mu\nu} = -Q^\nu, \quad \nabla_\mu T_{\text{sp}}^{\mu\nu} = Q^\nu,\tag{70}$$

$$\nabla_\mu T_{\text{om}}^{\mu\nu} = 0.\tag{71}$$

The labels identify phases of the one medium. The exchange four-vector Q^ν accounts for energy and momentum transferred between them. For an externally driven subsystem, the external four-force density is added to the total balance; including the driving apparatus restores a closed total account. Every channel enters the I-7 source once.

In this construction internal phase transfer cancels identically: $-Q^\nu + Q^\nu = 0$, whatever its direction. Summing the two phase balances therefore gives the total covariant conservation

law. Conversion within one omnium therefore preserves the combined account while each phase exchanges energy. Matter, energy, and spacetime name the common substance; $E = mc^2$ gives its dimensionally correct energy–mass relation. The displayed balance equations specify total conservation.

The energy-transfer component of Q^ν , read in a declared local measurement frame, has either sign: one sign denotes matter-to-spacetime transfer and the opposite sign denotes the reverse. The four-vector itself has no scalar ordering. Both directions of the phase response sit in one account. The seam $\mathfrak{J}$ of (35) is the interchange of the phase regions. The paired E, m in (63) belong to the same selected state or mode. A global conserved energy additionally uses the geometric structure required to define that charge on the spacetime.

This is the conservation architecture of I-6 [1, I-6 and O-12]. The constitutive phase law determines Q^ν , including the onset and rates of unwinding and the matter-phase to spacetime-phase flow law, which are to be derived (D-7). Admissible transitions preserve the exact charge, angular-momentum, and fermion-parity accounts of I-5b. These selection rules determine which outgoing configurations are available.

7.4 Acceleration, energy, and phase response

The mode relation $E_q = m_q c^2$ describes a specified confinement-energy and mass reading. An observer’s total energy of a moving pattern is a different quantity. For a body whose nonzero rest mass m_0 is held fixed, the relativistic relation is

$$E_{\text{obs}} = \gamma m_0 c^2, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \quad K_{\text{obs}} = (\gamma - 1)m_0 c^2. \quad (72)$$

Thus $K_{\text{obs}} \rightarrow \infty$ as the local relative speed $v \uparrow c$; the body cannot reach c by finite-energy acceleration while retaining that fixed massive state [16]. FDT derives the same inertia from the strand: a stationary closed pattern, its closure stress included, has inertia E/c^2 (von Laue), and its energy in uniform motion is γE_0 [1, 0.1d]. This relation uses local relativistic kinematics. Accelerated proton and ion beams remain identifiable material beams near c before collisions [17]. The beam record constrains any phase-response model at high local speed.

The FDT phase reading represents pattern loading through confinement and phase response subject to (70). A speed-dependent trigger uses declared local frames or covariant relative-motion variables. Equation (50) constrains directional organization; Q^ν belongs to the constitutive phase law. The observer energy E_{obs} remains distinct from confinement energy at fixed rest mass. Energy leaving T_{mat} enters the receiving phase and total source. The central state thus has one coupled account.

7.5 Energy injection, compensation, and the compactness readings

An energy injection into one part of a closed omnium system is supplied by another part of the complete account. This includes the driving apparatus and its fields when the fluid is externally forced. The response of α depends on the region’s mass and geometry: α is a branch-dependent ratio, and its transfer response carries geometric weights.

For an explicit exterior-chart comparison, take two source regions with fixed support radii $R_{b,1}, R_{b,2}$, both read on their declared exterior branches with positive masses. In a quasistatic source-energy bookkeeping approximation where transferred energy ΔE changes their gravitating masses by $\Delta \mathcal{M}_1 = \Delta E/c^2$ and $\Delta \mathcal{M}_2 = -\Delta E/c^2$, one obtains

$$\Delta \alpha_{1,\text{ext}} = \frac{2G\Delta E}{c^4 R_{b,1}}, \quad \Delta \alpha_{2,\text{ext}} = -\frac{2G\Delta E}{c^4 R_{b,2}}, \quad R_{b,1}\Delta \alpha_{1,\text{ext}} + R_{b,2}\Delta \alpha_{2,\text{ext}} = 0. \quad (73)$$

Equal and opposite changes of α follow in this example only when the radii agree. With changing geometry, the infinitesimal exterior variation is

$$\delta \alpha_{\text{ext}} = \frac{2G}{c^2} \left(\frac{\delta \mathcal{M}}{r_f} - \frac{\mathcal{M} \delta r_f}{r_f^2} \right). \quad (74)$$

This fixed-geometry bookkeeping example uses the stated source-energy identification and neglects changes in gravitational binding. Interior charts have their own variation. The complete account includes geometry and field-energy changes; α responds through its chart definition.

For the forced fluid representation with a fixed inertial coefficient ρ_0 , the exact resolved kinetic-energy balance is

$$\frac{d}{dt} \left(\frac{\rho_0}{2} \|u\|_2^2 \right) + \rho_0 \nu \|\nabla u\|_2^2 = \rho_0 \int_{\Omega} f \cdot u \, dx. \quad (75)$$

The source term measures work on the fluid and may have either sign. Its compensation can lie in the apparatus outside the resolved fluid domain. The viscous term is likewise part of the accounting for energy leaving resolved kinetic motion. Those channels complete the physical energy account around the resolved forced-fluid equation.

7.6 Jets and the physical meaning of phase exchange

FDT reads black-hole jets as the passage of the matter phase into the spacetime phase across the seam at its most extreme (0.3a, 0.8), which ties the phase reading to observed outflow. The GMVA/ALMA/GLT observations of M87 image its emission ring and jet base together, while EHT polarization constrains the magnetic field, the helical density gradient of P-5, in the radiating plasma near the black hole [22, 23]. Those observations establish near-hole outflow and constrain its environment. Jet power, matter content, polarization, and launch geometry give the observable content of a physical phase account.

In the standard relativistic account, observable jets launch from the region outside the event horizon; they are not escaping trajectories from inside a true horizon [24]. In FDT the phase interface is the seam, $r_f = 2R_s$ on the exterior branch, outside the fold $r_f = R_s$; for a rotating source the fold sits at the horizon's areal radius $R_H \leq R_s$, and the band $R_H < r_f < R_s$ is read through the multipole remainder of the core's register item O-5 [1, 0.3a, 0.4].

The quantitative jet account, its power, matter content, polarization and launch geometry from the phase-exchange rates with every incoming, outgoing and stored contribution of energy and momentum included, is to be derived (D-7). The fluid continuation proof uses its fixed-domain equation and maintained velocity bound.

7.7 Transfer forcing in the fixed-domain estimate

The momentum equation is a specified fixed-domain comparison with constant kinematic viscosity and incompressible Euclidean transport. A physical relativistic description couples momentum, constitutive, apparatus, and metric equations. Phase transfer changes the resolved momentum balance through its stress, inertial coefficient, or source terms. If it reduces to the same equation with an extra acceleration f_{tr} , and

$$f_{ext} + f_{tr} \in L^2(0, T; L^2(\Omega)) \quad \text{for every finite } T, \quad (76)$$

then the forced continuation estimate applies under the same maintained Euclidean velocity bound. This reduction assumes a fixed flat domain and the same differential operator; evolving geometry brings the additional terms exemplified by (28). Conservation (71) supplies the total balance; (76) is the force-integrability hypothesis of this reduction. A different resolved momentum operator has its own regularity problem.

8 Seam continuation, force balance, and stable patterns

8.1 Complementary readings at the seam and mode-selective continuation

Crossing the seam exchanges the two regimes, $\mathfrak{J} : \alpha_{\sigma} \mapsto 1 - \alpha_{\sigma}$ (0.3a): the inside becomes the outside and the outside the inside, as a jacket turned inside out. Patterns compatible with the seam keep their pattern and reverse their normal motion label; other patterns undergo a joint

reconfiguration. The following formulation gives the analytic chart identity and the operator and interface conditions for this transmission rule.

An analytic complement and its orientation. On a collar of an interface Σ of the support, write α for the A-1 share $\alpha_\sigma = |z_1|^2/R^2$ of the declared branch (0.3a) and $\alpha^\mathfrak{J} = 1 - \alpha$ for the reading $\mathfrak{J}$ carries it to. With α_Σ the share on Σ ,

$$\alpha^\mathfrak{J} = 1 - \alpha, \quad \alpha|_\Sigma = \alpha_\Sigma, \quad \alpha^\mathfrak{J}|_\Sigma = 1 - \alpha_\Sigma. \quad (77)$$

Thus the centered normal coordinates obey

$$q_{\text{out}} = \alpha - \alpha_\Sigma, \quad q_{\text{in}} = \alpha^\mathfrak{J} - (1 - \alpha_\Sigma) = -q_{\text{out}}. \quad (78)$$

The transition has nonzero derivative -1 , so it is an analytic, invertible change of chart. If a scalar profile F is analytic on the collar, its two representations satisfy

$$F_{\text{in}}(\alpha) = F_{\text{out}}(1 - \alpha), \quad F_{\text{in}}^{(j)}(\alpha) = (-1)^j F_{\text{out}}^{(j)}(1 - \alpha). \quad (79)$$

For an analytic field on the collar, this identity matches its two representations. Physical continuation uses a solution of the field equation with the required regularity across the collar; the change of chart preserves the physical norm as in (38).

For example, with $L > 0$ a physical length scale, a local mode transforms as

$$e^{ikLq_{\text{out}} - i\omega t} = e^{-ikLq_{\text{in}} - i\omega t}, \quad \dot{q}_{\text{in}} = -\dot{q}_{\text{out}}. \quad (80)$$

The normal wave-number label reverses while the represented wave is the same. Reversal of the *entire physical motion*, including tangential winding, is a separate dynamical operation: $\mathfrak{J}$ keeps the phases ϕ_1, ϕ_2 , and with them every tangential winding. Geometric inside–outside exchange and reversal of physical time evolution are distinct maps.

The two readings of an interface coincide only where $\alpha_\Sigma = 1 - \alpha_\Sigma$, at $\alpha_\Sigma = 1/2$: the seam, the one locus that reads the same either way (0.3a). An interface at any other share has the two readings α_Σ and $1 - \alpha_\Sigma$, exchanged by $\mathfrak{J}$. The seam names its application, A-1, and its branch; it is not the fold, where both branches approach 1^- (I-3), and it carries no threshold to or from the $2/3$ of I-1a.

What it means for the seam to select eigenmodes. Let $\mathcal{A}_{\text{out}}(\alpha_\Sigma)$ and $\mathcal{A}_{\text{in}}(1 - \alpha_\Sigma)$ be self-adjoint operators on the two sides of the interface, on Hilbert spaces $\mathcal{H}_{\text{out}}$ and $\mathcal{H}_{\text{in}}$, with their geometry, domains and boundary conditions included in their definition. Take their eigenvalues to have units of squared frequency. Let $U : \mathcal{H}_{\text{out}} \rightarrow \mathcal{H}_{\text{in}}$ be the unitary map that implements the specified spatial orientation identification. The exact spectral-matching condition, with P the orthogonal projector onto a candidate transparent subspace, is

$$\mathcal{A}_{\text{in}}(1 - \alpha_\Sigma)UP = U\mathcal{A}_{\text{out}}(\alpha_\Sigma)P, \quad [P, \mathcal{A}_{\text{out}}(\alpha_\Sigma)] = 0, \quad (81)$$

on the relevant operator domain. Consequently, whenever $P\phi_n = \phi_n$ and $\mathcal{A}_{\text{out}}\phi_n = \lambda_n\phi_n$,

$$\mathcal{A}_{\text{in}}(U\phi_n) = \lambda_n U\phi_n. \quad (82)$$

The interface preserves that eigenvalue under the orientation exchange. Transmission also satisfies the interface conditions, whose reflection coefficient is calculated below.

The statement that the spectrum is determined by the share α means that α selects a *specified operator family*, including its physical length scale and boundary data. For example, the explicitly chosen scalar operator $-c^2\Delta_\Sigma$ on a flat product torus of circle radii R_1, R_2 has

$$\lambda_{nm} = c^2 \left(\frac{n^2}{R_1^2} + \frac{m^2}{R_2^2} \right), \quad n, m \in \mathbb{Z}. \quad (83)$$

On the round reference geometry of Section 5.1, the torus at share α has $R_1 = R\sqrt{\alpha}$ and $R_2 = R\sqrt{1-\alpha}$, so this particular operator gives

$$\begin{aligned}\lambda_{nm}(\alpha) &= \frac{c^2}{R^2} \left(\frac{n^2}{\alpha} + \frac{m^2}{1-\alpha} \right), \\ \lambda_{nm}(1/2) &= \frac{2c^2}{R^2} (n^2 + m^2).\end{aligned}\tag{84}$$

$\mathfrak{J}$ carries the torus at α onto the torus at $1-\alpha$ with the phases kept, so $\lambda_{nm}(1-\alpha) = \lambda_{mn}(\alpha)$, and at the seam it fixes every mode. The torus modes $\phi_{nm} = e^{i(n\phi_1 + m\phi_2)}$ exhibit the orientation distinction explicitly: the swap J exchanges their indices, $(n, m) \mapsto (m, n)$. The additional angle reversal $(\phi_1, \phi_2) \mapsto (-\phi_2, -\phi_1)$ gives $(n, m) \mapsto (-m, -n)$ and preserves the Clifford eigenvalue as well. This is the spatial mode matching and scale dependence of the chosen scalar torus operator. Physical nuclear spectra and charge-preserving transmission belong to the confinement and transition constructions, which are to be derived (D-5, D-7).

A local transmission calculation. Consider one diagonal channel across the interface, with a common physical normal x increasing from the half-space $x < 0$, read by α_Σ , to the half-space $x > 0$, read by $1 - \alpha_\Sigma$: the outside and the inside of the jacket $\mathfrak{J}$ turns, labelled out and in. These are readings of the seam's two sides, not the exterior and interior branches of the atlas, which the branch exchange $\mathfrak{i}$ relates at fixed α_σ (0.3a). As an explicit local model, let a displacement amplitude ψ_j obey

$$\rho_j \partial_t^2 \psi_j - K_j \partial_x^2 \psi_j + \rho_j \lambda_j \psi_j = 0, \quad c_j^2 = K_j / \rho_j, \quad j \in \{\text{out}, \text{in}\},\tag{85}$$

where $\rho_j > 0$ is mass density, $K_j > 0$ is a modulus and $\lambda_j \geq 0$ is the channel's eigenvalue on the interface. At fixed frequency $\omega > 0$, assume $\omega^2 > \lambda_j$, and define

$$k_j = \frac{\sqrt{\omega^2 - \lambda_j}}{c_j}, \quad Z_j = \frac{K_j k_j}{\omega} > 0.\tag{86}$$

Write the incident, reflected and transmitted fields, suppressing the common factor $e^{-i\omega t}$, as $\psi_{\text{out}} = A(e^{ik_{\text{out}}x} + r e^{-ik_{\text{out}}x})$ and $\psi_{\text{in}} = A t e^{ik_{\text{in}}x}$. Continuity of displacement and normal traction gives

$$1 + r = t, \quad Z_{\text{out}}(1 - r) = Z_{\text{in}} t, \quad r = \frac{Z_{\text{out}} - Z_{\text{in}}}{Z_{\text{out}} + Z_{\text{in}}}, \quad t = \frac{2Z_{\text{out}}}{Z_{\text{out}} + Z_{\text{in}}}.\tag{87}$$

Therefore transmission with no reflected wave requires $Z_{\text{out}} = Z_{\text{in}}$, in which case $r = 0$ and $t = 1$. The energy-flux check is

$$\mathcal{R} = |r|^2, \quad \mathcal{T} = \frac{Z_{\text{in}}}{Z_{\text{out}}} |t|^2, \quad \mathcal{R} + \mathcal{T} = 1.\tag{88}$$

For $c_{\text{out}} = c_{\text{in}} = c$, transparency still has the impedance condition above. At $\omega^2 < \lambda_j$ that channel is evanescent; at equality it has zero normal group velocity. A propagating transparent channel satisfies both the frequency and impedance conditions.

The joint reconfiguration law. Let $\mathcal{S}$ act on complete incoming and outgoing configurations, including the interface, apparatus and other omnium channels. With ι_{in} and ι_{tr} the channel inclusions, the direction-only transmission rule becomes

$$\mathcal{S} \iota_{\text{in}} P = \iota_{\text{tr}} U P.\tag{89}$$

Here U must contain every asserted spatial winding reversal; the normal chart reversal alone supplies only Eq. (80). Direction-only passage preserves electric charge when $\hat{Q}_{\text{in}} U = U \hat{Q}_{\text{out}}$ on the matched

sector. It is not a charge-conjugation rule. Reconfiguration of two electrons, for example, must retain their total charge in the full outgoing state. For a complete closed set of flux-normalized channels, conservation is represented by $\mathcal{S}^\dagger \mathcal{S} = I$ together with the conserved generator relations on their domains. The stronger rule that every input in the complementary sector $P^\perp = I - P$ changes its pattern requires a nonzero coupling into the corresponding reconfigured stable-pattern channels. Spectral mismatch by itself also permits ordinary reflection or evanescence, so that coupling is a property of the complete constrained dynamics, to be derived (D-5). Equations (81) and (87) identify the operator and interface properties that the dynamics must produce. The smooth chart exchange preserves invariant norms as in (38); these interface calculations specify the model's additional dynamics.

8.2 The equal boundary value and the Clifford torus

The two sides of an interface have the same reading, so that their opposing forces can balance, only at the fixed value of the complement:

$$\alpha_\Sigma = 1 - \alpha_\Sigma, \quad \alpha_\Sigma = \frac{1}{2}. \quad (90)$$

The S^3 geometry realizes this statement explicitly:

$$\alpha(z_1, z_2) = \frac{|z_1|^2}{R^2} = \cos^2 \chi, \quad \alpha^\mathfrak{J} = \frac{|z_2|^2}{R^2} = 1 - \alpha. \quad (91)$$

These are the two complementary squared-radius fractions of the round reference geometry. Their equality holds exactly on $\mathcal{T}_{\text{Cl}}$, where $\chi = \pi/4$. The value $1/2$ here is the A-1 seam; it does not move the 2/3 attraction–repulsion threshold of the overlap chart (I-1a, A-3).

The regimes read on the share are (0.3a)

$$\begin{aligned} 0 < \alpha < 1/2 & : \text{ general-relativistic regime, spacetime phase, in } V_{\text{sp}}^\circ, \\ \alpha = 1/2 & : \text{ the transition, the seam } \mathcal{T}_{\text{Cl}}, \\ 1/2 < \alpha < 1 & : \text{ quantum-mechanical regime, matter phase, in } V_{\text{mat}}^\circ. \end{aligned} \quad (92)$$

Here the superscript denotes the interior of each solid torus; the coordinate collar avoids its core circles, the ends $\alpha \rightarrow 0^+$ and $\alpha \rightarrow 1^-$ of the open interval (I-6). The seam $\mathfrak{J}$ sends α to $1 - \alpha$, exchanges the two open regimes, and fixes the transition surface pointwise; the swap J does the same to the readings and fixes there only the great circle $z_1 = z_2$. A passive change from one reading to its complement describes the same state; a physical phase exchange is a step of the system-wide settlement, with rates that are the core's register item O-12. All seven invariants govern both phase regions. The respective dynamics are specified by the field equation and the confinement eigenproblem.

8.2.1 Balanced tractions and an extremum of the interface energy

Equal readings give equal opposing forces only if the two sides have the corresponding constitutive symmetry. Let $\mathbf{n}$ point from the matter region into the spacetime region, and let $\boldsymbol{\sigma}_{\text{mat}}, \boldsymbol{\sigma}_{\text{sp}}$ be their stress representations. With $\mathbf{f}_\Sigma$ the force per area due to any surface stress, a stationary interface obeys the vector balance

$$(\boldsymbol{\sigma}_{\text{sp}} - \boldsymbol{\sigma}_{\text{mat}})\mathbf{n} + \mathbf{f}_\Sigma = 0. \quad (93)$$

The opposite normals fix the signs. The constitutive tensor stresses include the swirling and anisotropic response of each pattern.

Let $\mathcal{W}(\alpha)$ be the constrained energy of a declared conservative model of the interface along a specified family of complete configurations. $\mathcal{W}$ is a model energy of this analysis, not the objective of the global optimisation, which the core leaves unnamed (0.1a). Suppose exchanging the two phase regions is a symmetry of this family, including all its response variables, so that

$$\mathcal{W}(\alpha) = \mathcal{W}(1 - \alpha). \quad (94)$$

Differentiation gives $\mathcal{W}'(\alpha) = -\mathcal{W}'(1 - \alpha)$ and hence $\mathcal{W}'(1/2) = 0$. This is a derived force balance under the stated exchange symmetry, which applies to the complete state, including both phase responses.

Use proper reference normal distance $s = R(\chi - \pi/4)$, so

$$\alpha(s) = \frac{1 - \sin(2s/R)}{2}, \quad \alpha'(0) = -\frac{1}{R}. \quad (95)$$

If $\mathcal{W}$ is smooth, its normal restoring force is

$$F_s = -\frac{d\mathcal{W}}{ds} = -\frac{\mathcal{W}''(1/2)}{R^2}s + O(s^2). \quad (96)$$

Thus $\mathcal{W}''(1/2) > 0$ gives a restoring force along this admissible mode; a negative value gives an instability. If it vanishes, higher orders or other coupled variables decide the result.

The distinction can be demonstrated within the same geometry. A constant- χ torus has area

$$\mathcal{A}(\chi) = 2\pi^2 R^2 \sin(2\chi), \quad \mathcal{A}(s) = 2\pi^2 R^2 \cos(2s/R). \quad (97)$$

Its first derivative vanishes on the Clifford torus, but $\mathcal{A}''(0) = -8\pi^2$. A positive constant surface-tension energy $\gamma_\Sigma \mathcal{A}$ therefore has negative second variation for this uniform normal displacement, if that displacement is allowed. For this surface-tension functional the interface is balanced and unstable along the allowed displacement. A model's constrained energy has its own admissible displacements and restoring contributions.

8.2.2 The stability operator uses all adjustable terms

For an equilibrium energy $\mathcal{E}$ with complete conserved constraints C_b , the relevant quadratic form is the Hessian of $\mathcal{E} - \sum_b \Lambda_b C_b$ restricted to the admissible tangent space $\delta C_b = 0$. Denote its operator by K_{adm} . Local energetic stability requires a nonnegative quadratic form. In a finite reduction to physical, nonsymmetry coordinates, a positive-definite constrained Hessian gives a strict local energy minimum; an operator formulation requires coercivity in the chosen perturbation norm. For finite normalized coordinates $\mathbf{q}$ with positive inertia M , the corresponding conservative linear modes satisfy

$$K_{\text{adm}} \mathbf{v} = \omega^2 M \mathbf{v}. \quad (98)$$

These are the small oscillations of the declared interface model (Lagrange–Dirichlet). The stability of a closed pattern is the ledger form of I-5b: it is stable exactly when every final state that keeps every exact book has zero transition amplitude, and its masses are the confinement eigenvalues of I-5, whose operator is the core's register item O-9.

In particular, write a finite admissible energy Hessian in blocks for a displacement $\delta\alpha$ of the share and other adjusting medium variables ξ . If the latter have positive-definite block $K_{\xi\xi}$, stationary elimination gives

$$K_{\text{eff}} = K_{\alpha\alpha} - K_{\alpha\xi} K_{\xi\xi}^{-1} K_{\xi\alpha}. \quad (99)$$

This is the stiffness after the other terms have adjusted together. Checking $K_{\alpha\alpha}$ alone would freeze part of the system and can give the wrong stability verdict. The equal boundary value and the Clifford surface establish the symmetric interface; the sign of this constrained response determines local energetic stability of the model. The transition rates and decay channels are the core's register item O-12.

8.3 Cooper pairing as a charge-preserving joint pattern

Compatible joint patterns are selected through the surrounding medium: an allowed joint pattern persists, while an incompatible one reconfigures, the whole settling as one system (0.1a). Cooper pairing makes the charge and binding requirements of this account concrete. In conventional BCS

pairing, the paired electron states have opposite momentum and spin [19]. The standard account reads the pairing attraction as the exchange of virtual phonons. FDT reads it as two opposite-helicity electron cones meeting tip to tip through the medium's deformation, with one carrier and no mediation (P-4, I-2); the measured pairing is accepted in full, and its induced interaction is computed below. Each electron, an $n = 3$ braid, reads charge $-e$, $e > 0$, a topological invariant of its closure that pairing leaves unchanged (I-5). Neither opposite momentum nor opposite spin makes it the positively charged positron [20].

8.3.1 The nesting operation must preserve the electron sector

I-5 fixes the charge sign of an odd braid by the nested-helix rule, $\text{sgn } Q = \text{sgn}(\nu_o \nu_i)$, with ν_o the linking of the outer cable about the pattern's axis and ν_i that of the inner cable about the outer, in a declared framing and with $\nu_o \nu_i \neq 0$. With $\varsigma_o = \text{sgn } \nu_o$ and $\varsigma_i = \text{sgn } \nu_i$, the charge sign on the charged class reads

$$q_{\text{el}} = q_* \varsigma_o \varsigma_i, \quad q_* > 0. \quad (100)$$

The label algebra distinguishes

$$\begin{aligned} \mathcal{G} : (\varsigma_o, \varsigma_i) &\mapsto (-\varsigma_o, -\varsigma_i), & q_{\text{el}} &\mapsto q_{\text{el}}, \\ \mathcal{C} : (\varsigma_o, \varsigma_i) &\mapsto (-\varsigma_o, \varsigma_i), & q_{\text{el}} &\mapsto -q_{\text{el}}. \end{aligned} \quad (101)$$

The first operation reverses both linking signs: the charge stays and the helicity sign $\text{sgn } H = \text{sgn } \nu_i$ flips (I-5), which is the opposite-helicity partner a Cooper pair meshes with (P-4). The second reverses one linking sign and is charge conjugation in this label representation. They cannot be identified as the same physical operation, and neither is a reversal of the pattern's external velocity: reversing the whole moving pattern, its axis included, reverses the orientation of all three curves and leaves both linking numbers, and the charge, unchanged (P). The normal reversal at the seam alone selects neither.

The distinction is also exact at the many-body level. In a reference fermion description let

$$P_{\mathbf{k}}^\dagger = c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger, \quad [Q, P_{\mathbf{k}}^\dagger] = -2e P_{\mathbf{k}}^\dagger. \quad (102)$$

Here $c^\dagger$ creates an electron state and $[Q, c^\dagger] = -ec^\dagger$. The displayed commutator follows by the product rule. The elementary pair channel has zero total momentum; the conventional singlet is the corresponding antisymmetric spin combination with an even orbital amplitude. The spin and exchange map from $(\varsigma_o, \varsigma_i)$ to this singlet is to be derived (D-6).

For example a number-conserving pair-scattering term is

$$H_{\text{pair}} = \sum_{\mathbf{k}, \mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} P_{\mathbf{k}}^\dagger P_{\mathbf{k}'}, \quad [Q, H_{\text{pair}}] = 0, \quad (103)$$

with $V_{\mathbf{k}\mathbf{k}'} = V_{\mathbf{k}'\mathbf{k}}^*$ for self-adjointness. It reconfigures the joint pair states while preserving the charge sector: a complete-pattern to complete-pattern reconfiguration, many to many (0.1a, I-5b).

8.3.2 Why a two-electron pattern does not unwind into light alone

Let $\mathcal{S}_{\text{om}}$ be the transition operator on complete configurations, including the medium and apparatus. Charge is an exact book of every transition (I-5b):

$$[Q_{\text{tot}}, \mathcal{S}_{\text{om}}] = 0. \quad (104)$$

For initial and final charge eigenconfigurations it gives

$$(Q_F - Q_I) \langle F | \mathcal{S}_{\text{om}} | I \rangle = 0. \quad (105)$$

If the surrounding configuration has unchanged charge, two electrons carry $-2e$, whereas an outgoing state of photons alone carries zero. The transition matrix element therefore vanishes. For an

electron–positron pair the charge difference vanishes and the charge-neutral channel is open; its matrix element is fixed by the transition operator. If charge is transferred to the surroundings, those changed surroundings must appear in F and the same total selection rule applies. The selection rule acts on the complete configuration. Pair binding depends on the complete energy, including the medium interaction calculated in the next subsection.

8.3.3 An induced pair interaction from a stable medium response

A concrete route to pair binding is available once the medium’s response is specified. In a stationary, adiabatic approximation, take real medium deformations ξ with quadratic energy

$$\mathcal{E}[\Psi, \xi] = \mathcal{E}_0[\Psi] + \frac{1}{2} \xi^T K \xi + \xi^T j[\Psi], \quad K = K^T > 0. \quad (106)$$

Here K is the stiffness of the stable medium and j is the coupling produced by the complete pattern Ψ . With ξ a displacement, K has units of force per length and j of force. Both are specified inputs to this stationary calculation; their constitutive origin is to be derived (D-5). Minimizing over the common medium yields

$$\begin{aligned} \xi_* &= -K^{-1}j, \\ \mathcal{E}_{\text{eff}}[\Psi] &= \mathcal{E}_0[\Psi] - \frac{1}{2} j^T K^{-1}j. \end{aligned} \quad (107)$$

Writing $j = j_1 + j_2$, the induced interaction relative to the two self-response terms is

$$V_{12}^{\text{ind}} = -j_1^T K^{-1}j_2. \quad (108)$$

It is negative for equal nonzero coupling vectors; the bilinear form determines its sign for general vectors. The force uses the separation dependence of those vectors. A paired state is energetically favored when this interaction, together with kinetic and direct interaction terms in $\mathcal{E}_0$, lowers the complete paired-state energy. The derivation expresses the medium mechanism through the stiffness K and coupling j ; the resulting pair kernel enters the complete binding calculation. The approximation is stationary and adiabatic; the dynamic response retains the characteristic speed and coupled invariant.

The Cooper-pair comparison identifies the charge-preserving many-body channel required by compatible nesting. In this calculation the medium operator describes the material response. Connecting it to an individual nuclear shell is a further material map, with the pairing spectrum determined by the complete operator and its boundary data.

9 Combined constraints and the physical model class

The central invariant, force ceiling, and physical stroke floors constrain the same omnium state. A physical continuation requiring a vanishing stroke violates Proposition 5.1; one requiring an unbounded locally measured reconstructed velocity violates Proposition 5.3. These physical statements concern proper quantities. An unbounded Euclidean PDE velocity is first a statement about the fixed-background mathematical model and its observation map.

Theorem 3.1 establishes global regularity for the declared class of maximal Navier–Stokes trajectories. Its maintained bound is a hypothesis on the evolution. The source’s solution has $\limsup_{t \uparrow 1} \|u(t)\|_\infty = \infty$, so it lies outside the hypothesis of Theorem 3.1 and of its forced extension: the two theorems are consistent, and the source’s unbounded velocity is the necessary condition that Fefferman’s description records for any finite blowup time [8, pp. 2–3].

The relativistic physical extension uses conserved total source, causal constitutive evolution, apparatus response, and a dynamical metric. Equation (28) identifies the geometric terms in its continuation calculation. Section 10.1 records the tasks belonging to that extension.

9.1 Comparison with the OpenAI blowup construction

The source's theorem concerns the fixed Euclidean equation introduced in Section 1. It claims bounded integrated kinetic energy and divergent velocity supremum for one smooth compactly supported forcing [18, Theorem 1.1]. The nonlinear momentum fluxes in its construction redistribute momentum inside the same classical system, with the forcing work accounted for by its energy identity. Its prescribed background fixes the limit of the physical extrapolation examined here.

9.1.1 The vortex core and the truncated density-cone tip

The source's geometry supports a two-sided helical comparison. It needs a precise orientation convention. Sections 2.1 and 3.1 of the source describe a leading axisymmetric vortex with radial inflow and oppositely directed axial outflow. With its positive similarity scale q_{NS} and exponent $A_{\text{NS}} = 1/2 + h_{\text{NS}}$, where h_{NS} is the source's exponent h , $0 < h_{\text{NS}} < 1/100$, and η_{NS} its axial similarity variable, the leading azimuthal velocity is

$$u_{\theta}^{(0)} = q_{\text{NS}}^{-A_{\text{NS}}} E_{\text{NS}}(X_{\text{NS}}, \eta_{\text{NS}}) > 0 \quad (X_{\text{NS}} > 0, |\eta_{\text{NS}}| \leq 1). \quad (109)$$

The profile positivity on both sides is stated in Theorem 4.6(i), page 33. Thus the leading swirl keeps the same sign in a fixed cylindrical frame. Wherever $u_z^{(0)} \neq 0$, the projection of a streamline on the cylinder of radius r_{NS} has pitch

$$\frac{d\theta}{dz} = \frac{u_{\theta}^{(0)}}{r_{\text{NS}} u_z^{(0)}}. \quad (110)$$

Opposite axial directions reverse this pitch without reversing the angular velocity. Looking from opposite ends also reverses an apparent clockwise label. Section 2.1, page 5, deliberately introduces a slight axial asymmetry; the two branches are not exact reflected copies.

The source's equation (3.2) gives a further geometric comparison. In its normalization, a surface of fixed $X_{\text{NS}} = X_c > 0$ satisfies

$$\begin{aligned} q_{\text{NS}} - z^2 q_{\text{NS}}^{2h_{\text{NS}}} &= \tau_{\text{NS}}, & r_{\text{NS}} &= \sqrt{2X_c q_{\text{NS}}}, \\ r_{\text{NS}}(0, \tau_{\text{NS}}) &= \sqrt{2X_c \tau_{\text{NS}}} > 0, & r_{\text{NS}}(z, 0) &= \sqrt{2X_c} |z|^{1/(1-2h_{\text{NS}})}. \end{aligned} \quad (111)$$

The last expression is the terminal extension away from the origin: a two-sided power-law horn. The actual inner core has an additional axial similarity cutoff and contracts entirely to the origin. These formulas describe a pinched two-branch geometry. The incompressible source uses constant density; its map onto the density chart is to be derived (D-10).

The FDT description begins with the two phases and their seam, the Clifford torus, in Section 5.1; the paired truncated density cones read the same organization on one support and meet circle to circle at the fold. The seam is a smooth torus. A quantitative comparison assigns the source to the atlas in (56) using its total gravitating mass $\mathcal{M}$, field radius r_f , and exterior or interior branch. Cylindrical vortex radius, spherical field radius, and constant inertial density ρ_0 retain their defined meanings. On the exterior chart the excluded endpoint is approached through $r_f \rightarrow R_s^+$ and $\alpha_{\text{ext}} \rightarrow 1^-$. The interior branch uses its own domain and regular-core data. These chart assignments specify the endpoint to which a physical vortex correspondence refers.

The OpenAI field is smooth on its axis for every $t < 1$, with azimuthal velocity zero on the axis. Its Theorem 3.1(iv), Eq. (3.6), instead gives unbounded velocity along a path whose positive radius shrinks to zero as $t \uparrow 1$. Writing $\tau_{\text{NS}} = 1 - t$ in that paper's normalization, with h_{NS} its small exponent, distinct from Planck's constant, the path has

$$r_{\text{NS}}(\tau_{\text{NS}}) = \sqrt{2X_{\text{in}} \tau_{\text{NS}}}, \quad u_{\theta}|_{\text{path}} = \tau_{\text{NS}}^{-1/2-h_{\text{NS}}} \left(e_0 + O(\tau_{\text{NS}}^{2h_{\text{NS}}}) \right), \quad (112)$$

where $X_{\text{in}} > 0$, $e_0 > 0$, and $0 < h_{\text{NS}} < 1/100$. The limit is an unbounded Euclidean-coordinate velocity; the axis is smooth at every earlier time. Identifying that entire trajectory with the

normalized physical average in one maintained flat frame contradicts the bound in Proposition 5.3. The source's concentrating Newtonian energy integral and the fixed-rest-mass relativistic energy in (72) refer to different states and energy readings.

FDT's physical objection is that the source continues the slow limit (3) of the medium's free fall beyond its domain: as the fluid concentrates, it holds the Euclidean ruler, Newtonian time, and differential operator fixed while $|u|$ passes c and π passes c^2 . The source establishes a claim about that prescribed equation. FDT declines the fixed-background reading of its pressure (0.1c) and is under no obligation to reproduce an unbounded velocity reading (0.1d). A physical extrapolation with evolving geometry uses the coupled fluid, apparatus, and metric equations. Their solution determines local speed, proper intervals, curvature, and phase response. The theorem established in Section 3 excludes blowup in its maintained-bound class; the coupled physical extension has the status recorded below.

10 Invariant structure and convergence to the measured reading

Invariant	Role in the fluid representation
I-1	The force ceiling and the core's one-action relation give the physical stroke floors; the declared pair evaluation constrains the source and stress account.
I-2	The carrier identity identifies fluid response as motion of the one medium; observed orbital nesting is distinguished from the microscopic transport reconstruction.
I-3	The conserved total source drives the metric; proper ruler/clock intervals and observer-relative speed use that metric. The reciprocal atlas and optical coordinates remain downstream of its solution. The medium window runs on the clock factor (2).
I-4	Exact pair algebra, weak-field/slow-motion containment, and the source calculation are kept distinct; the quantum face interprets the same medium.
I-5	Material configurations and their physical strokes belong to patterns; the quantum bridges connect mode energy, mass, kept time, and thread length.
I-6	Named compactness inputs and nonnegative odds updates retain their open domains together with the state and physical-stroke constraints.
I-7	Fluid, driving apparatus, medium response, radiation, and other channels enter one total tensor once; exchange terms cancel in its covariant balance. Gravitational binding is read from the solved geometry. The pressure and the enthalpy per count are the medium's clock factor (0.1c).

FDT is a true extension of general relativity and quantum mechanics; it reads both as descriptions of the same medium and recovers their measured record. All seven invariants govern the joint state throughout its evolution. The domains declared here specify individual calculations and observation maps. For the same complete source, initial and boundary data, and gauge, (25) returns the same curvature as GR. A new compactness label or cone exchange changes no invariant observable.

The limiting map by which FDT recovers the measured record of laboratory flows is the medium window (2), completed by the first-order viscous stress $\Pi^{\mu\nu}$ of a causal relativistic fluid [4] and the drive's exchange $\mathcal{Q}_{\text{drv}}^\nu$:

$$\nabla_U U^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathfrak{h}_N + \frac{\mathfrak{P}^\mu{}_\nu}{c^2} \frac{\mathcal{Q}_{\text{drv}}^\nu - \nabla_\lambda \Pi^{\nu\lambda}}{n\mathfrak{h}_N/c^2} \xrightarrow[\alpha_{\text{ext}} \rightarrow 0^+]{|u|^2/c^2 \rightarrow 0^+, \pi/c^2 \rightarrow 0^+} \partial_t u + (u \cdot \nabla) u = -\nabla \pi + \nu \Delta u + f, \quad (113)$$

with $\Pi^{ij} \rightarrow -\rho_0 \nu (\partial_i u_j + \partial_j u_i)$, $\mathcal{Q}_{\text{drv}}^i \rightarrow \rho_0 f^i$ and $n\mathfrak{h}_N/c^2 \rightarrow \rho_0$ on the flat laboratory chart ($\mathbf{N} = 1$, $\mathbf{b} = 0$, $\mathbf{h} = \delta$). The limits are those of the measured record: bulk speeds small against c , pressures small against $\rho_0 c^2$, a gravitational window α_{ext} small for the laboratory pattern, metric and frame changes small over the measurement interval, and a controlled incompressible, constant-viscosity

reduction of the material. With the same reduced equation, viscosity, forcing, initial and boundary data, and pressure convention, the same strong solution is recovered on its existence interval. The status ledger names the source, constitutive and observation inputs that additional predictions need.

The central chain, the strict force ceiling and the one-action stroke give $d_{\min} = 2\ell_P$ and $t_{\min} = 2t_P$ as unattained infima of physical strokes. The local helical reconstruction gives $|u_{\text{loc}}| \leq c$. The independently stated flat-domain result is

$$\boxed{
\begin{aligned}
&\text{fixed flat incompressible equation, } u_0 \in H^s, \ s \geq 3, \ \nu > 0, \\
&\|u(t)\|_\infty \leq C_E \quad (0 \leq t < T_*) \\
&\implies \int_0^T \|u(t)\|_\infty^2 dt \leq C_E^2 T < \infty \quad (T < T_*) \\
&\implies T_* = \infty.
\end{aligned}
} \tag{114}$$

Equation (7) provides the uniform H^1 estimate toward any finite candidate endpoint. The calibrated $C_E = c$ corollary uses the flat-frame identification stated in its corollary.

10.1 Status ledger: derived here, and to be derived

Derived here on the stated domains: Theorem 3.1, with the weighted estimate (10) and uniqueness, for the fixed flat incompressible equation with strong data, positive viscosity and the maintained Euclidean bound, and its forced extension under the finite-interval $L_t^2 L_x^2$ condition on the acceleration; the corollary $C_E = c$ for the helical average identified with the Euclidean velocity throughout the maximal interval in one maintained flat frame, whose persistence for arbitrary classical data is D-11; the medium window (2), its slow limit (3) and the limiting map (113); the pair evaluation and its response (14)–(17), exact on the spherical nonoverlap domain, with the Newtonian vector closure in its weak-field, slow-motion reduction; the clock relation and coordinate bound (21)–(22); the moving-metric identity (28); the singular Gross–Pitaevskii norm and the concentration family; the seam and its two exchanges (35) with the norm identity (38); the stroke infima of Proposition 5.1; Propositions 5.3 and 5.4; the scaling identities; the seam transmission, traction balance and constrained Hessian on their declared model operators and functionals; the charge-selection condition (105) and the induced interaction (108).

To be derived.

id	item	binds
D-1	the finite-core cutoff, regular core chart or outward phase-transfer law of the deep interior, and the local constitutive density profile with its boundary, surface and core data	0.3, 0.6, I-6; the core's O-1, O-4
D-2	a geometrically constructed overlap-contrast odds and a canonical confinement odds	0.4, I-1a, I-5; the core's O-2, O-3
D-3	the multipole and tidal remainders of the pair evaluation, the band $R_H < r_f < R_s$ outside a rotating source's horizon, the overlap complex, how the settlement signs a pair it does not join, and the maps from a charge, a circulating charge or a colour seat to an I-1 input	I-1, P-4; the core's O-5, O-6
D-4	the constitutive derivation of the causal-aperture speed and cone index, and their reconciliation with lensing	I-3, P-4; the core's O-8
D-5	the density of the matter action this analysis writes, its boundary data and selection law; the confinement operator with its domains, boundary data and spectra; the seam transmission couplings and the pairing strengths	0.1a, I-5; the core's O-9
D-6	the seat-lattice premises and probe embedding, the framed-closure parity and absorption calculation, a nontrivial spinorial class, and the spin and exchange map from $(\varsigma_o, \varsigma_i)$ to the BCS singlet	I-5a, I-5b; the core's O-10, O-11
D-7	the phase-exchange rates and the complete transition kernel, spectrum, chirality and rates, the matter-phase to spacetime-phase flow law, and the quantitative jet account	I-5b, I-6, 0.8; the core's O-12, O-15
D-8	the full junction data and stresses at an interface	P; the core's O-13
D-9	a causal matter–apparatus model with its initial and boundary data and the estimates for the coupled metric and fluid evolution: the coupled continuation theorem	I-3, I-7
D-10	the map from the source's vortex geometry to the density chart, and the quantitative phase-conversion and particle correspondence	I-3, 0.8
D-11	the persistence of the maintained flat-frame identification of the helical average with the Euclidean velocity for arbitrary classical data	0.1d, I-3

11 Conclusions

The classical continuation criterion, proved here in full, excludes finite-time breakdown for the declared fixed-domain incompressible flow. A maintained Euclidean speed bound and positive viscosity give the gradient estimate (7), which closes strong continuation. FDT's maintained flat-frame helical average supplies the specialization $C_E = c$. The force ceiling and the one-action stroke fix the physical stroke infima.

The OpenAI construction holds Euclidean space and Newtonian time fixed and reads its pressure as a force across that background. FDT reads the same pressure gradient as the medium's clock factor, of which the source's equation is the slow limit (3); the source's blowup continues that limit past $|u| \sim c$. FDT places the fluid, apparatus, rulers, clocks, and spacetime in one coupled state. The identity $F = (c^4/4G)\alpha_1\alpha_2 = F_{\max}\alpha_1\alpha_2$ makes the declared pair inputs functions of source energy and geometry. The conserved total stress sources the metric, and proper intervals describe the resulting ruler and clock comparisons.

The singular Gross–Pitaevskii branch separates finite integrated norm from bounded local density and ties stability to the energy domain and medium response. The two phases and their seam specify the reference geometry; the seam calculations derive transmission conditions and traction balance, and the constrained Hessian tests energetic stability. The Cooper calculation derives the charge-selection condition and the induced interaction for the specified stable medium

response.

The additional vortex studies resolve recoil, energy-lowering knot fusion and fission, active rotor pairing, collective rearrangement, and turbulent coarsening. Pairs, knots, and chains identify local patterns within the evolving medium, drive, and surroundings (Section 4.6). In FDT, injected work is taken up by that coupled omnium state: it rescales the medium’s clocks and rulers together through its enthalpy per count, whose gradient the bookkeeper reads as the pressure force (0.1c), and the material reconfiguration keeps the central invariant at every step (0.1a).

These comparisons identify the physical source, geometric response and constitutive information the construction holds fixed or leaves unspecified. Section 10.1 records what is derived here and what is to be derived. The theorem holds on its stated domain with the classical estimate.

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