

The omnium vortex

Frame drag as the omnium's flow, and the magnetic field as its kept vorticity: two windows of one substance, read against the record of fifty-one bodies

The omnium around a spinning body flows, and the frame drag a gyroscope reads is that flow's local rotation; the magnetic field a body carries is not the reading of that flow but the kept vorticity of the body's own medium, and the record of fifty-one bodies shows which is which. Read in the time its patterns keep, the exterior of a static source is a pressureless Euler flow with zero Bernoulli constant, and the field equation on its flat slices is the second invariant of the flow's strain rate: outside a source the convergence and the shear balance, and inside, their difference is the density. A rotating source adds a swirl with the form of the Stokes rotlet. It entrains the omnium at the skin by the fraction $k_I \alpha_{\text{ext}}(R_b)$ of the body's spin, and at first order in J it reaches no slip at the fold. It is the unique least-shear configuration with its skin value, and half its curl is the gyroscope's precession. The inflow lines wind by a finite angle on the whole exterior branch. The swirl turns once in $P/(k_I \alpha_{\text{ext}}(R_b))$: 49 seconds at a magnetar, 5.9 million years at the Earth. The free-falling patterns of the flow are themselves irrotational, so the field a body carries belongs to a second window, the medium's. There the vorticity 2-form of the medium's flow obeys Maxwell's source-free pair as an identity, the medium's free fall is the frozen-in law with a baroclinic source, and P-5's two grades are one 2-form whose circulation Kelvin keeps whole. The record decides between the two windows. If magnetism were the present swirl read through I-1, every body would carry Blackett number $B_k = 1$. The measured values span ten decades, differ by four decades between the Earth and Mars at the same spin, and change sign and axis from body to body: the field is the medium's kept vortex, carried by Kelvin through every change of support while the spin is not kept.

The argument runs in nine propositions and a reading of the record. Propositions 1–5 are the gravitational window: the inflow, the swirl, the winding of the inflow in three azimuths, the swirl's own clock, and the two vorticities kept apart. Propositions 6–9 are the medium window: Maxwell's source-free pair, the frozen-in law with its baroclinic source, P-5's single 2-form with its three limits, and the I-1 reading of Coulomb's law that gives the swirl a magnetic reading. Every identity is checked in two scripts, 85 checks in all, and every number of the record is computed from a sourced table of fifty-one bodies (appendix). No divergence from general relativity or from the measured record is claimed; what would separate the omnium reading from the standard one is listed as D-01 to D-07.

The chart inventory and the domain

The inventory is declared once and holds throughout. A-1 is the only application. Its atlas is the branch-tagged compactness of 0.3,

$$\alpha_{\text{ext}}(r_f) = \frac{R_s}{r_f}, \quad \sigma = +1, \quad R_s < r_f < \infty, \quad R_s = \frac{2GM}{c^2} = \frac{Mc^2}{2F_{\text{max}}},$$

the bounded output of the gap odds $X_{\text{ext}} = R_s/(r_f - R_s)$, with $\alpha_{\text{ext}} = X_{\text{ext}}/(1 + X_{\text{ext}})$ and M the total gravitating energy of 0.5. It is evaluated at field radii r_f and at a body's support radius R_b (the configuration evaluation). The seam $\alpha_{\text{ext}} = 1/2$ of A-1 ($r_f = 2R_s$) and the fold $\alpha_{\text{ext}} \rightarrow 1^-$ ($r_f \rightarrow R_s^+$) enter only as places on the exterior branch. The interior branch carries no load-bearing equation: each body's interior enters through its measured M , R_b , moment of inertia I with $k_I \equiv I/(MR_b^2)$, spin $\Omega = 2\pi/P$ and dipole moment μ . No A-2, A-3, A-4 or A-5 token appears, and no map between applications is displayed or used. The computational β_σ and the saturation diagnostic $\alpha_{\text{core}} \rightarrow 1^-$ do not enter.

The clock factor m of 0.1c enters as the medium's enthalpy per count $\mathfrak{h} = (E + \mathcal{P}V)/N$, under 0.1c's premise of a perfect form with conserved count N and fixed entropy per count; where the entropy per count s varies, the temperature T and s enter through the term $T ds$ of Proposition 7. Neither $\mathfrak{h}$ nor T stands in an α , force, optical or

density slot. An enclosed-mass ratio $2GM(< r_f)/(c^2 r_f)$ appears once, for the Galaxy, as a flow speed squared; it is not an α and carries no branch tag.

Evaluation domain. Stationary, axisymmetric sources. The swirl is evaluated at first order in J and exactly in α_{ext} at that order: the slow-rotation exterior of Hartle [8]. Kerr enters at the fold, at that order, and its exact horizon is placed against the atlas in Proposition 2. Propositions 6–9 hold for a medium of perfect form; the departure from fixed entropy per count is the term $T ds$, and departures from perfect form are D-03. The Galaxy’s entries in Proposition 4 and in the last section lie outside this domain and are labelled estimates where they stand.

The fluid is the omnium itself (0.1b). The flat slices on which the flow is drawn are the bookkeeper’s reading of that flow (0.1c), never a background the omnium moves across.

invariant	where it enters
I-1	the universal law below; the I-1 reading of Coulomb’s law that normalises the Blackett number (Proposition 9); orbits as free fall through the inflow (last section)
I-2	one carrier: the strokes along a kept vortex’s lines and the strokes of its magnetic grade are the one spin-1 stroke; the magnon is the stroke through a helical density gradient (P-5)
I-3	containment: the inflow, the swirl and the gyroscope’s reading are general relativity’s exterior for the same source and data; the field equation unchanged with α downstream; I-3a’s three projections read on the bookkeeper’s flow (Propositions 1, 2, 5)
I-4	the Newtonian face of the orbits, $v_{\text{orb}} = v/\sqrt{2}$; the helix whose phase closes on a loop, which quantises circulation (Proposition 8)
I-5	charge per count is a braid reading (I-5a); the condensate’s inertia per pair is a confinement-position mass whose operator is O-9’s; helicity counts the linking of vortex lines
I-6	every α_{ext} lies in $(0, 1)$ on the declared branch; $k_I \alpha_{\text{ext}}(R_b) < 1$ at every skin; the fold is approached as $\alpha_{\text{ext}} \rightarrow 1^-$; the inflow’s winding is finite on the whole branch
I-7	the energies of the swirl and of the field are channels of the one $T_{\mu\nu}$ and enter $\mathcal{M}$ once; the medium’s clock factor is its enthalpy per count

The invariants as used. I-1 holds always and at every scale. With the per-body exterior inputs $\alpha_i = R_{s,i}/d$ on the pair domain of nonoverlapping spherical supports at monopole order, it reduces exactly to Newton’s law:

$$F = \frac{c^4}{4G} \alpha_1 \alpha_2, \quad 0 < \alpha_i < 1 \Rightarrow F < F_{\text{max}}, \quad F = \frac{c^4}{4G} \frac{2GM_1}{c^2 d} \frac{2GM_2}{c^2 d} = \frac{GM_1 M_2}{d^2}.$$

I-3 keeps $G_{\mu\nu} = \kappa T_{\mu\nu}$, $\kappa = 8\pi G/c^4$, unchanged, with α downstream of the solution; its exact optical face is $d\ell_{\text{opt}}^2 = dr_f^2/(1 - \alpha_{\text{ext}})^2 + r_f^2 d\Omega^2/(1 - \alpha_{\text{ext}})$. I-7 enters every channel once,

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{\mathcal{P}V}{c^2} + \frac{aT^4 V}{c^2} + \dots, \quad a = \frac{4\sigma_{\text{SB}}}{c} = 7.566 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4},$$

the rotational and field energies among them. The bridges of 0.1a are constants of the medium,

$$E_q = m_q c^2 = \hbar \omega_q, \quad T_q = \frac{2\pi}{\omega_q} = \frac{h}{E_q}, \quad \ell_q = c T_q = \frac{h}{m_q c}, \quad \frac{d}{t} = c = 299\,792\,458 \text{ m s}^{-1},$$

and the quantum of action h that quantises circulation in Proposition 8 is the one they carry. Mass in a closed pattern is a confinement eigenvalue, $m_q c^2 = \hbar \omega_q$, whose operator is O-9’s; no mass formula, anchor radius or density address is printed for any pattern.

Notation guard. One symbol carries one meaning. The renames below keep the reserved tokens free.

source or usual symbol	written here	reason
pulsar literature's $\kappa^{(\text{src})} = (r_g/R)(I/MR^2)$	$k_I\alpha_{\text{ext}}(R_b)$	κ is the Einstein coupling (0.2)
Blackett's coefficient $\beta^{(\text{src})}$	B_k	β_σ is the A-2 chart
shear of the strain rate σ_{ij}	Σ_{ij}	σ is the branch tag
expansion θ	Θ	θ is the I-3 deflection
flow 1-form π	ϖ	π is the number
energy per count Φ	$\mathcal{E}$	Φ is the magnetic flux
Maxwell 2-form F	$\mathcal{F} = dA$	F is the I-1 force
charge q	Q ; per count q	q labels modes and density order
winding integer n	n	n is the strand count (I-5)
electron number density n_e	N_e	n is the strand count
pressure P in the I-7 ledger	$\mathcal{P}$	P is the spin period
Kerr parameter a , orbit radius a	$a_\star = cJ/(GM^2)$; r_{GPB}	a is the radiation constant
four-acceleration a^μ	$\nabla_{\mathcal{U}}\mathcal{U}^\mu$	a is the radiation constant
unit radial vector $\hat{n}$; body radius R	$\hat{e}_r$; R_b	n is reserved; R_b is the support token (0.3)
scale factor λ	$\mathfrak{s}$	λ_i is the I-4 three-body input

The bookkeeper's flow is $\mathbf{v} = \mathbf{v}_{\text{in}} + \mathbf{v}_{\text{sw}}$, the inflow and the swirl; $v = c\sqrt{\alpha_{\text{ext}}}$ is the inflow speed of 0.1c and $\omega_{\text{sw}}(r_f)$ the swirl's angular speed. $\mathcal{U}$ is the four-velocity of the omnium's patterns, $g(\mathcal{U}, \mathcal{U}) = -c^2$, $\mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu$, and t_{own} the time they keep. S_{ij} is the symmetric part of $\nabla\mathbf{v}$, $\Theta = \text{tr } S$ and $\Sigma_{ij} = S_{ij} - \Theta\delta_{ij}/3$.

The gravitational window

The inflow

0.1c reads the exterior of a static source as a flow: patterns falling from rest far away move inward at $v = c\sqrt{\alpha_{\text{ext}}}$ in the time they keep, each slice of that time is flat, and the whole curvature sits in the flow. The same form is the Painlevé–Gullstrand exterior, which the river model of black holes reads as space flowing through a flat background [9]; FDT reads the flow as the omnium's own.

Proposition 1 (the inflow is an Euler flow, and the field equation is its strain invariant). On the exterior branch the bookkeeper's inflow $\mathbf{v}_{\text{in}} = -c\sqrt{\alpha_{\text{ext}}}\hat{e}_r$ satisfies

$$(\mathbf{v}_{\text{in}} \cdot \nabla)\mathbf{v}_{\text{in}} = -\frac{GM}{r_f^2}\hat{e}_r, \quad \frac{1}{2}|\mathbf{v}_{\text{in}}|^2 - \frac{GM}{r_f} = 0, \quad \mathbf{v}_{\text{in}} = \nabla Y, \quad Y = -2\sqrt{2GM}r_f, \quad \Theta = -\frac{3}{2}\frac{c\sqrt{\alpha_{\text{ext}}}}{r_f},$$

and the flow form $ds^2 = -c^2 dt_{\text{own}}^2 + (dr_f + v dt_{\text{own}})^2 + r_f^2 d\Omega^2$ solves $G_{\mu\nu} = 0$. On its flat slices

$$\Theta^2 - S:S = \frac{2}{3}\Theta^2 - \Sigma:\Sigma = \begin{cases} 0 & \text{outside a source,} \\ 16\pi G\rho & \text{inside a source carried on the same slices,} \end{cases}$$

with ρ the mass density the observers normal to the slices measure.

Proof. With $v = \sqrt{2GM/r_f}$: $v dv/dr_f = -GM/r_f^2$, $v^2/2 = GM/r_f$, $dY/dr_f = -v$, and $\Theta = r_f^{-2}d(r_f^2 v_r)/dr_f = \frac{3}{2}v_r/r_f$ (receipt A1–A5). The strain components are $S_{rr} = \frac{1}{2}c\sqrt{\alpha_{\text{ext}}}/r_f$ and $S_{\theta\theta} = S_{\varphi\varphi} = -c\sqrt{\alpha_{\text{ext}}}/r_f$, so $S:S = \frac{9}{4}c^2\alpha_{\text{ext}}/r_f^2 = \Theta^2$; the identity $\Theta^2 - S:S = \frac{2}{3}\Theta^2 - \Sigma:\Sigma$ holds for every symmetric S (A6, A9). The Einstein tensor of the flow form vanishes identically (A7). For a flow form with an arbitrary enclosed $M(r_f)$ and unit lapse, $G_{\mu\nu}n^\mu n^\nu = 2M'/r_f^2$ in geometric units and $\Theta^2 - S:S = 4M'/r_f^2$, which is $16\pi G\rho$ with $M' = 4\pi r_f^2 \rho$ (A8). $\square$

This is general relativity's Hamiltonian constraint, written as the second invariant of the flow's strain rate. Outside a source the omnium's convergence and its shear balance exactly; inside, the excess of $\frac{2}{3}\Theta^2$ over $\Sigma:\Sigma$ is the density. I-3a names the trace of the deviation map “expansion: density, by continuity”; on the flow's flat slices the field equation fixes how trace and shear together carry the density, and the trace alone is nonzero where the density vanishes.

The swirl

A rotating source adds a swirl to the flow (0.1c). At first order in J the flow form becomes $ds^2 = -c^2 dt_{\text{own}}^2 + (dr_f + v dt_{\text{own}})^2 + r_f^2 d\vartheta^2 + r_f^2 \sin^2 \vartheta (d\varphi - \omega_{\text{sw}} dt_{\text{own}})^2$, the Hartle exterior carried onto flat slices (Proposition 3).

Proposition 2 (the swirl). For a stationary source of angular momentum J , at first order in J and exactly in α_{ext} at that order,

$$\mathbf{v}_{\text{sw}} = \frac{2G}{c^2} \frac{\mathbf{J} \times \mathbf{x}}{r_f^3} = \frac{2G}{c^2} \nabla \times \frac{\mathbf{J}}{r_f}, \quad \omega_{\text{sw}}(r_f) = \frac{2GJ}{c^2 r_f^3},$$

and:

- (a) $\nabla \cdot \mathbf{v}_{\text{sw}} = 0$ and $\nabla^2 \mathbf{v}_{\text{sw}} = 0$ for $r_f > R_b$: the operator of creeping flow at uniform pressure. Inside the support, at leading order in the source's compactness, the same vector Laplacian is driven by the source's momentum density $\rho \mathbf{v}_b$, with $\mathbf{v}_b$ the velocity of the source's matter, $\nabla^2 \mathbf{v}_{\text{sw}} = -(16\pi G/c^2) \rho \mathbf{v}_b$: the momentum constraint of general relativity on the flat slices.
- (b) At the skin the swirl is a rigid rotation at the fraction $\omega_{\text{sw}}(R_b)/\Omega = 2GI/(c^2 R_b^3) = k_I \alpha_{\text{ext}}(R_b)$ of the body's spin.
- (c) Among all divergence-free swirls $v_\varphi = \omega(r_f) r_f \sin \vartheta$ with the same skin value that vanish far out, the total shear $\int S:S dV = \frac{4\pi}{3} \int r_f^4 (d\omega/dr_f)^2 dr_f$ is least for $\omega \propto r_f^{-3}$ and only for it. The Euler–Lagrange operator $(r_f^4 \omega')'$ is the operator of Hartle's exterior frame-drag equation at every compactness. By Helmholtz's minimum-dissipation theorem for Stokes flow [11, 12] the property holds against every divergence-free field with the same skin values.
- (d) $\nabla \times \mathbf{v}_{\text{sw}} = (2G/c^2) [3(\mathbf{J} \cdot \hat{\mathbf{e}}_r) \hat{\mathbf{e}}_r - \mathbf{J}]/r_f^3$, curl-free and divergence-free outside the support: a dipole field. Half of it is the precession a gyroscope reads.

Proof. (a) and (d) are direct (B1–B5); the interior statement is solved for a uniform sphere turning rigidly, where matching at the skin returns the exterior coefficient $2G/c^2$ with $J = \frac{2}{5} \mathcal{M} R_b^2 \Omega$ (review receipt V4). (b) is $\omega_{\text{sw}}(R_b) = 2GI\Omega/(c^2 R_b^3)$ with $\alpha_{\text{ext}}(R_b) = 2GM/(c^2 R_b)$ (B10). For (c), $S_{r\varphi} = \frac{1}{2} r_f \sin \vartheta \omega'$ is the only strain component, so $S:S = \frac{1}{2} r_f^2 \sin^2 \vartheta \omega'^2$ and the angular integral gives $\frac{4\pi}{3}$. The Euler–Lagrange equation $(r_f^4 \omega')' = 0$ has the decaying solution $\omega \propto r_f^{-3}$. Its cross term with any perturbation vanishing at the skin integrates to zero and the second variation is positive (B11, B12); Hartle's exterior equation for $\bar{\omega} = \Omega - \omega$ is $(r_f^4 \bar{\omega}')' = 0$ (B13). The slow-rotation exterior with $\omega = 2GJ/(c^2 r_f^3)$ satisfies the vacuum field equation at first order in J (B9). For (d), at the pole half the curl is $2GJ/(c^2 R_b^3) = \omega_{\text{sw}}(R_b)$ (B6), and half the curl is the Lense–Thirring precession of a gyroscope at rest against the distant stars. $\square$

The entrainment. The omnium at the skin is entrained by the fraction $k_I \alpha_{\text{ext}}(R_b)$ of the body's spin and slips past it by the rest: 4.6×10^{-10} at the Earth's surface, where $\alpha_{\text{ext}}(R_b) = 1.39 \times 10^{-9}$; 0.123 at the skin of a $1.4 M_\odot$, 12 km neutron star (receipt F1, F3). Partial slip $k_I \alpha_{\text{ext}}(R_b) < 1$ is every skin's state.

An identity of operators, not a stress. Part (a) gives the swirl the form of the Stokes rotlet; it does not give the gravitational window a viscosity. A rotlet in a fluid of viscosity η exerts the torque $8\pi\eta R_b^3$ times the angular slip on its sphere. The exterior of general relativity exerts none, and J is kept (containment, I-3). The medium window of Propositions 6–9 runs in perfect form, and its departures from perfect form are D-03.

Least shear. Part (c) is Dirichlet’s principle for the momentum constraint: the swirl general relativity returns is the unique least-shear configuration with its skin datum, and the skin datum $\omega_{\text{sw}}(R_b) = 2GJ/(c^2 R_b^3)$ is fixed by J through the source. Read through 0.1a, the omnium settles the exterior by one global optimisation. The core names no objective for it (0.1a); this manuscript’s reading is that (c) names what the optimisation leaves least around a spinning body. Uniqueness and minimality are proved; how the omnium relaxes to the minimum is a statement about its dissipation, which is D-03.

The fold, at first order and beyond. At first order in J the fold $r_f \rightarrow R_s^+$ is the horizon, and the swirl there turns with it: $\omega_{\text{sw}}(R_s) = \Omega_H = a_\star c^3/(4GM)$ (review receipt K4). No slip is the fold’s limit, approached as $\alpha_{\text{ext}} \rightarrow 1^-$ and never reached (I-6). Beyond first order the exact Kerr horizon of a source of total gravitating energy $\mathcal{M}$ lies below that source’s R_s . With $r_+ = (GM/c^2)(1 + \sqrt{1 - a_\star^2})$, its areal radius is $R_H = \sqrt{r_+^2 + a_\star^2 (GM/c^2)^2} = \sqrt{2GM r_+/c^2}$, and

$$\frac{R_s}{R_H} = \sqrt{\frac{2GM}{c^2 r_+}} = 1 + \frac{a_\star^2}{8} + O(a_\star^4),$$

which is 1.035 at $a_\star = 0.5$, 1.180 at 0.9 and 1.371 at 0.998 (K1–K3). The exact identity $\omega(r_+) = \Omega_H$ of Kerr (B7) therefore sits below A-1’s monopole fold $r_f = R_s$ of the source’s $\mathcal{M}$. A-1 is the monopole chart: the rotating source’s fold is its horizon, at the areal radius R_H , and the band $R_H < r_f < R_s$ outside it is exterior, read through the multipole remainder of O-5 and never on either branch of A-1 (0.4; D-07).

The gyroscope’s reading. Gravity Probe B carried four gyroscopes through the Earth’s swirl in a polar orbit of semi-major axis $r_{\text{GPB}} = 7027.4$ km. The orbit-averaged precession $GJ_\oplus/(2c^2 r_{\text{GPB}}^3) = 40.80$ mas/yr, projected on the axis to the guide star IM Pegasi at declination $+16^\circ 50' 28''$, is 39.05 mas/yr (F2). The general-relativistic prediction is -39.2 mas/yr and the measurement -37.2 ± 7.2 mas/yr, in the experiment’s sign convention [14]. The node drag of LAGEOS and LAGEOS 2 returned $99 \pm 5\%$ of the general-relativistic value ($\pm 10\%$ allowing for unknown errors) [15]; LARES with LAGEOS returned 0.9910 ± 0.02 [16]; and LARES-2 with LAGEOS and the GRACE gravity field reaches a relative uncertainty at the one-part-in-a-thousand level [17].

Newton’s vortex. Principia Book II, Proposition LII, sets a sphere turning in an infinite fluid and concludes that “the periodic times of the parts of the fluid will be as the squares of the distances from the centre of the sphere” [13]. The viscous solution gives the cubes, $\omega \propto r_f^{-3}$, and the cube law is the frame-drag law $\omega_{\text{sw}} = 2GJ/(c^2 r_f^3)$: one operator, the vector Laplacian, governs both. Newton’s vortex around a turning sphere and the Lense–Thirring frame drag share one radial law.

The whirlpool: one inflow, three azimuths

Proposition 3 (the winding of the inflow). A pattern falling from rest far away with zero angular momentum keeps the inflow’s radial motion and turns with the swirl. At first order in J :

- (i) in the azimuth φ of the flow form its line winds by

$$\Delta\varphi = \frac{a_\star}{3} \alpha_{\text{ext}}^{3/2}, \quad a_\star = \frac{cJ}{GM^2},$$

finite on the whole exterior branch and $a_\star/3$ as $\alpha_{\text{ext}} \rightarrow 1^-$;

- (ii) in the static azimuth φ_S of the Hartle exterior, read in the static time t_S , the same line winds by

$$\Delta\varphi_S = a_\star \left[\operatorname{artanh} \sqrt{\alpha_{\text{ext}}} - \sqrt{\alpha_{\text{ext}}} \right] \longrightarrow \frac{a_\star}{3} \alpha_{\text{ext}}^{3/2} \quad \text{far out,}$$

which grows without bound as $\alpha_{\text{ext}} \rightarrow 1^-$;

- (iii) in the azimuth $\varphi_R = \varphi - \int (\omega_{\text{sw}}/v) dr_f$ the shift has no azimuthal part, and the frame drag sits in the twist of the flowing frame: the river model’s inflow without swirl [9, 10].

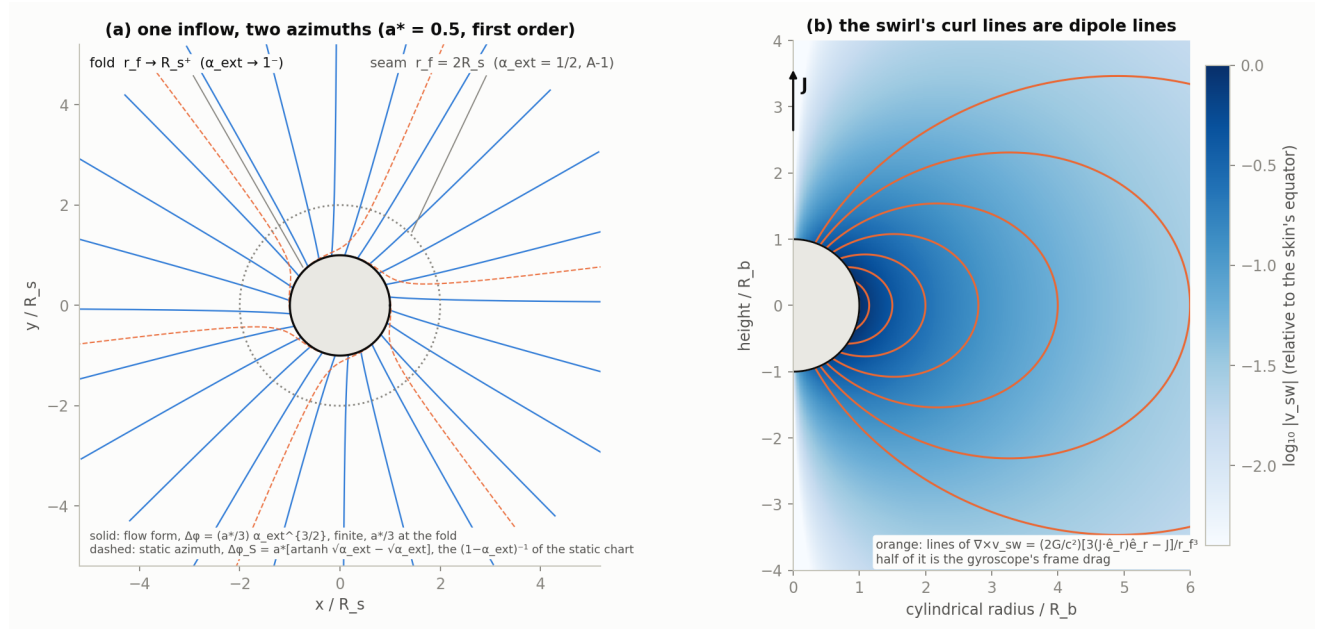


Figure 1: (a) Inflow lines at $a_\star = 0.5$ (first order), in units of R_s . Solid: the flow form's azimuth, $\Delta\varphi = (a_\star/3)\alpha_{\text{ext}}^{3/2}$, finite and $a_\star/3$ at the fold. Dashed: the static azimuth, $\Delta\varphi_S = a_\star[\text{artanh } \sqrt{\alpha_{\text{ext}}} - \sqrt{\alpha_{\text{ext}}}]$, whose unbounded wrap is the static chart's factor $(1 - \alpha_{\text{ext}})^{-1}$. The dotted circle is the A-1 seam $\alpha_{\text{ext}} = 1/2$; the solid circle is the fold, approached as $\alpha_{\text{ext}} \rightarrow 1^-$. (b) The swirl's magnitude in the meridional plane and its curl lines, $\nabla \times \mathbf{v}_{sw} = (2G/c^2)[3(\mathbf{J} \cdot \hat{\mathbf{e}}_r)\hat{\mathbf{e}}_r - \mathbf{J}]/r_f^3$: dipole lines. Half the curl is the gyroscope's frame drag.

Proof. In the flow form $dr_f/dt_{\text{own}} = -v$ and $d\varphi/dt_{\text{own}} = \omega_{sw}$, so $d\varphi/dr_f = -\omega_{sw}/v$. With $\omega_{sw} = 2GJ/(c^2 r_f^3)$ and $v = c\sqrt{R_s/r_f}$ the integral in from rest is $(4GJ/3c^3)R_s^{-1/2}r_f^{-3/2} = (a_\star/3)\alpha_{\text{ext}}^{3/2}$ (review receipt W1, W2). The charts are related by

$$t_S = t_{\text{own}} - \int \frac{v dr_f}{c^2(1 - \alpha_{\text{ext}})}, \quad \varphi_S = \varphi - \int \frac{\omega_{sw} v dr_f}{c^2(1 - \alpha_{\text{ext}})},$$

which carries the Hartle exterior onto the flow form at first order in J (W4). Along the line $d\varphi_S/dr_f = -(\omega_{sw}/v)/(1 - \alpha_{\text{ext}})$ (W5), whose integral is (ii) (W3; receipt B14, B15). In (iii) the shift's azimuthal component in φ_R is $-\omega_{sw} + (\omega_{sw}/v)v = 0$ (W6). $\square$

The three azimuths are three bookkeepings of one geometry, and what all three agree on is the gyroscope's precession of Proposition 2(d). In the flow form the whirlpool is gentle. At the skin of a millisecond pulsar, with $a_\star = 0.33$, the inflow line has turned by 0.023 rad; at a magnetar spinning in 6 s, with $a_\star = 8.7 \times 10^{-5}$, by 5.9×10^{-6} rad (W7). The static azimuth's unbounded wrap is the distant bookkeeper's static time running without bound toward the fold: read in that time, an inflowing pattern turns with the fold at Ω_H and never crosses it. No azimuth continues the flow across the fold into the interior chart (0.1c, I-3).

One whirlpool at every size. Under $(\mathcal{M}, R_b, r_f, \Omega) \mapsto (\mathfrak{s}\mathcal{M}, \mathfrak{s}R_b, \mathfrak{s}r_f, \Omega/\mathfrak{s})$ the swirl $\mathbf{v}_{sw}$ and α_{ext} are unchanged (B16). The planet, the star and the neutron star carry the same exterior form, read at successive sizes (0.3a), and differ only in $k_I \alpha_{\text{ext}}(R_b)$ and Ω .

The swirl's own clock

The stationary metric records the omnium's velocity at each place, the same at every instant. The flow is what each element of the omnium does in time: how often it turns and what it carries.

Proposition 4 (the swirl’s clock). At the skin the swirl completes one turn in

$$\tau_s = \frac{2\pi}{\omega_{\text{sw}}(R_b)} = \frac{P}{k_I \alpha_{\text{ext}}(R_b)},$$

and a line carried by the swirl between two field radii winds linearly in time, $\Delta\varphi_{12}(t) = (2GJ/c^2)(r_{f,1}^{-3} - r_{f,2}^{-3})t$.

Proof. Proposition 2(b) and $\omega_{\text{sw}} \propto r_f^{-3}$ (receipt S1, S2). $\square$

Rheology has a number for the ratio of a material’s own time to the time of observation, the Deborah number $\text{De} = \tau/T_{\text{obs}}$, named for Deborah’s song, “the mountains flowed before the Lord” [19]. With $T_{\text{obs}} = 100$ yr the swirl’s ladder reads:

body	$\omega_{\text{sw}}(R_b)/\Omega$	spin P	swirl τ_s	De (100 yr)
millisecond pulsar B1937+21	0.15	1.6 ms	10 ms	3×10^{-12}
Crab pulsar	0.15	33 ms	0.22 s	7×10^{-11}
magnetar SGR 1806–20	0.15	7.5 s	51 s	2×10^{-8}
white dwarf RE J0317–853	1.6×10^{-4}	725 s	51 d	1.4×10^{-3}
Jupiter	1.0×10^{-8}	9.9 h	1.1×10^5 yr	1.1×10^3
the Sun	3.0×10^{-7}	25.4 d	2.3×10^5 yr	2.3×10^3
the Earth	4.6×10^{-10}	23.9 h	5.9×10^6 yr	5.9×10^4
the Galaxy at the Sun’s circle (estimate)	6×10^{-7}	2.2×10^8 yr	3.6×10^{14} yr	3.6×10^{12}

Neutron stars are on the catalogue convention ($R_b = 10$ km, $I = 10^{38}$ kg m², $1.4 M_\odot$), for which $k_I \alpha_{\text{ext}}(R_b) = 0.149$. The Galaxy’s row is an estimate outside the evaluation domain: the exterior formula with the enclosed $J(< r_f) = v_{\text{circ}}^3 r_f^2 / (2G)$ of a flat curve, $v_{\text{circ}} = 233$ km s^{−1} at 8.2 kpc [20], gives $\omega_{\text{sw}} = (v_{\text{circ}}/c)^2 v_{\text{circ}}/r_f$. The Sun’s circle lies inside the Galaxy’s mass, and for co-rotating spherical shells with a flat curve the mass outside r_f drags twice what the mass inside does, $\omega_{\text{sw}} = 3(v_{\text{circ}}/c)^2 v_{\text{circ}}/r_f$ and $\tau_s = 1.2 \times 10^{14}$ yr (review receipt G1, G2). The order, $\text{De} \sim 10^{12}$, is unchanged.

What the ladder sorts. The ladder spans twenty-four decades. The swirl is stationary, so each turn repeats the last, and a record of any length reads it as a rate. The rate observables read it at every Deborah number: Gravity Probe B and LARES-2 at the Earth’s $\text{De} = 5.9 \times 10^4$, the precessing inner flows of X-ray binaries and the polar caps of pulsars at $\text{De} \ll 1$. Around accreting black holes and neutron stars the low-frequency quasi-periodic oscillations of the X-ray flux are attributed to the Lense–Thirring precession of the inner flow [21, 22]. In the black-hole binary H1743–322 the iron line’s energy “varies systematically over the ~ 4 s QPO cycle (3.70 sigma significance), as predicted” [23]: the swirl read as a precession, cycle by cycle. What the ladder counts is the number of turns of the omnium at the skin that a century holds, 3.0×10^{11} above B1937+21 and one in six million years at the Earth. The swirl’s own change is the change of J , on the spin-down clock $P/\dot{P}$. A flow with a history to read is the medium window’s kept vortex, whose clocks are its own (Proposition 8 and the record).

Two vorticities

Proposition 5 (the free-falling patterns are irrotational). In the flow form at first order in J , a pattern falling from rest far away with zero angular momentum has $\mathcal{U}^b = -c^2 dt_{\text{own}}$. Its congruence is normal to the flat slices and carries no vorticity 2-form, $d\mathcal{U}^b = 0$. The swirl’s curl $\nabla \times \mathbf{v}_{\text{sw}}$ is the nonzero dipole of Proposition 2(d), and half of it is the rotation of a gyroscope carried by those patterns against the bookkeeper’s grid.

Proof. With $\mathcal{U}^\mu = (1, -v, 0, \omega_{\text{sw}})$ the metric of the flow form gives $\mathcal{U}_t = -c^2$ and $\mathcal{U}_r = \mathcal{U}_\theta = \mathcal{U}_\varphi = 0$ at first order in J (review receipt V1, V2). The curl is computed in V3. $\square$

Two objects carry the name vorticity, and they are kept apart. The bookkeeper’s is the antisymmetric part of the gradient of $\mathbf{v}$ on the slices of constant t_{own} , flat at the Lense–Thirring order; its axial vector, half the curl of $\mathbf{v}$, is the frame drag (0.1c). The patterns’ own is the 2-form $\mathcal{W} = d\varpi$ of their flow 1-form, the object of Propositions 6–9.

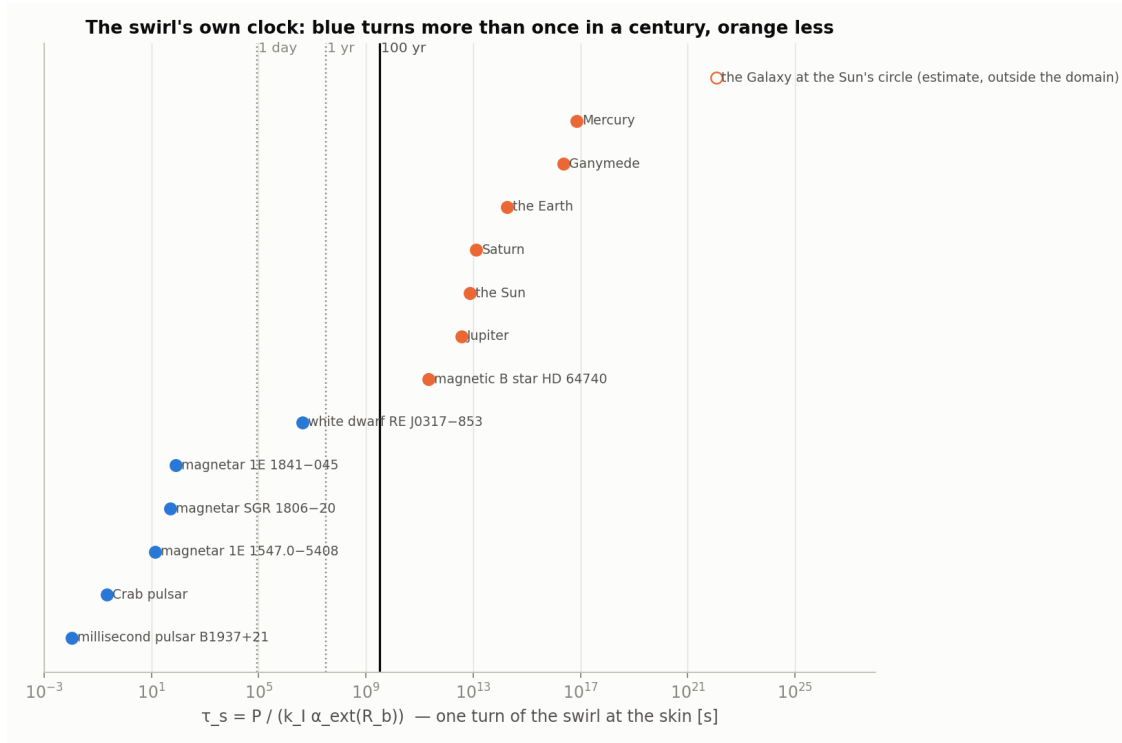


Figure 2: The swirl's own clock $\tau_s = P / (k_I \alpha_{\text{ext}}(R_b))$ against a century of watching. Blue: more than one turn per century; orange: less. The open marker is the Galaxy's estimate, which lies outside the evaluation domain.

For the gravitational flow the second vanishes identically, so the gravitational window carries no field of the kind Proposition 8 describes. The two windows meet in one equation (I-3a): the medium's patterns fall freely on the metric whose shift is the swirl, and the polar cap of a pulsar is where the meeting is read (the engine, below).

The medium window

The vorticity 2-form

Let $\mathcal{U}$ be the four-velocity of a medium's patterns and $\mathfrak{h}$ their enthalpy per count, the clock factor of 0.1c under its premise. The flow 1-form and its vorticity 2-form are

$$\varpi = \frac{\mathfrak{h}}{c^2} \mathcal{U}^\flat, \quad \mathcal{W} = d\varpi.$$

In a 3+1 split $\varpi = -\mathcal{E} dt + \mathbf{p} \cdot d\mathbf{x}$, with $\mathcal{E}$ an energy and $\mathbf{p}$ a momentum per count, and $\mathcal{W}_E = -\nabla \mathcal{E} - \partial_t \mathbf{p}$, $\mathcal{W}_B = \nabla \times \mathbf{p}$ are its electric and magnetic parts.

Proposition 6 (Maxwell's source-free pair is an identity). For every flow 1-form, $d\mathcal{W} = d d\varpi = 0$; in 3+1, $\nabla \cdot \mathcal{W}_B = 0$ and $\partial_t \mathcal{W}_B + \nabla \times \mathcal{W}_E = 0$.

Proof. $d^2 = 0$ for arbitrary ϖ_μ , and the 3+1 components follow (receipt C1, C2). $\square$

The two source-free Maxwell equations are not laws laid on the omnium: they state that the vorticity of a flow is the exterior derivative of something.

Proposition 7 (the medium's free fall is the frozen-in law with a baroclinic source). For a medium of perfect form with conserved count,

$$(i_{\mathcal{U}}\mathcal{W})_{\mu} = \frac{1}{c^2} \mathcal{U}^{\nu} \nabla_{\nu} (\mathfrak{h} \mathcal{U}_{\mu}) + \partial_{\mu} \mathfrak{h} = \frac{\mathfrak{h}}{c^2} \left[\nabla_{\mathcal{U}} \mathcal{U}_{\mu} + \mathfrak{P}_{\mu}^{\nu} \partial_{\nu} \ln \mathfrak{h} \right].$$

0.1c's medium equation, $\nabla_{\mathcal{U}} \mathcal{U}^{\mu} = -\mathfrak{P}^{\mu\nu} \partial_{\nu} \ln \mathfrak{m}$ with $\mathfrak{m} = \mathfrak{h}$ at fixed entropy per count, is therefore $i_{\mathcal{U}}\mathcal{W} = 0$, the homentropic case of the Carter–Lichnerowicz equation [24, 25]

$$i_{\mathcal{U}}\mathcal{W} = T ds.$$

In 3+1, with the medium's Euler equation,

$$\mathcal{W}_E + \mathbf{v} \times \mathcal{W}_B = -T \nabla s, \quad \partial_t \mathcal{W}_B = \nabla \times (\mathbf{v} \times \mathcal{W}_B) + \nabla T \times \nabla s.$$

Its consequences:

Kelvin. $\frac{d}{dt} \oint_C \varpi = \oint_C T ds$ for a loop C carried by the flow: at fixed entropy per count the circulation of every carried loop is kept.

Helmholtz. $\mathcal{L}_{\mathcal{U}}\mathcal{W} = d(i_{\mathcal{U}}\mathcal{W}) = dT \wedge ds$: at fixed entropy per count the vortex lines are carried and the vortex flux through every carried surface is kept.

Newcomb. The lines of $\mathcal{W}_B$ are carried by some flow $\mathbf{w}$ if and only if $\mathcal{W}_E \cdot \mathcal{W}_B = -\mathcal{W}_B \cdot \nabla \psi$ for a single-valued ψ ; then $\mathcal{W}_E + \mathbf{w} \times \mathcal{W}_B = -\nabla \psi$ with $\mathbf{w}_{\perp} = (\mathcal{W}_E + \nabla \psi) \times \mathcal{W}_B / |\mathcal{W}_B|^2$ [26]. Perfect form, $\mathcal{W}_E \cdot \mathcal{W}_B = 0$, is the case $\psi = 0$. On closed lines with $\oint \mathcal{W}_E \cdot d\mathbf{l} \neq 0$ no single-valued ψ exists, no flow carries the lines, and they reconnect.

Helicity. $\partial_t (\mathbf{p} \cdot \mathcal{W}_B) + \nabla \cdot (\mathcal{E} \mathcal{W}_B + \mathcal{W}_E \times \mathbf{p}) = -2 \mathcal{W}_E \cdot \mathcal{W}_B$: the linking of vortex lines [27] is kept wherever $\mathcal{W}_E \cdot \mathcal{W}_B = 0$.

The baroclinic source. $\nabla \times (T \nabla s) = \nabla T \times \nabla s$: vorticity is born where the temperature gradient crosses the entropy gradient, and in perfect form nowhere else. For the electron patterns of a plasma, inertia negligible, this is the Biermann battery $\partial_t \mathbf{B} = (k_B / (e N_e)) \nabla T \times \nabla N_e$ [28].

Proof. The first display is $\mathcal{U}^{\nu} (\nabla_{\nu} \varpi_{\mu} - \nabla_{\mu} \varpi_{\nu})$ with $\mathcal{U}^{\nu} \nabla_{\mu} \mathcal{U}_{\nu} = 0$ and $\mathcal{U}^{\nu} \mathcal{U}_{\nu} = -c^2$; the identity is checked at random points of a nontrivial flow (receipt C5). With the Euler equation per count, $m(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v}) = -\nabla \mathfrak{h} + T \nabla s$, and $\mathcal{W}_E = -\nabla (\mathfrak{h} + \frac{1}{2} m v^2) - m \partial_t \mathbf{v}$, $\mathcal{W}_B = m \nabla \times \mathbf{v}$, the 3+1 law follows, and its curl with Proposition 6 gives the induction law (C13, C14). Kelvin and Helmholtz are Cartan's formula for $\mathcal{L}_{\mathcal{U}}$ on ϖ and on $\mathcal{W}$ (C3, C4). For Newcomb, $(\mathbf{w} \times \mathcal{W}_B) \cdot \mathcal{W}_B = 0$, so $\mathcal{W}_E + \mathbf{w} \times \mathcal{W}_B = -\nabla \psi$ requires $\mathcal{W}_E \cdot \mathcal{W}_B = -\mathcal{W}_B \cdot \nabla \psi$; conversely the stated $\mathbf{w}_{\perp}$ solves it (C10). A uniform $\mathcal{W}_E \parallel \mathcal{W}_B$ of magnitude $\mathcal{W}_0$ is carried by $\mathbf{w} = 0$ with $\psi = -\mathcal{W}_0 z$ although $\mathcal{W}_E \cdot \mathcal{W}_B \neq 0$, and closed lines with $\oint \mathcal{W}_E \cdot d\mathbf{l} = 2\pi \mathcal{W}_0$ need $\psi = -\mathcal{W}_0 \varphi$, multi-valued (review receipt N1, N2). Helicity and the baroclinic source are direct (C6–C8). $\square$

The first 3+1 line in perfect form, $\mathcal{W}_E + \mathbf{v} \times \mathcal{W}_B = 0$, is the frozen-in law that magnetohydrodynamics writes for its electric and magnetic fields and derives from an infinite conductivity. Here it is the free fall of the medium's patterns (0.1c). The term $T ds$ is the departure from fixed entropy per count, inside perfect form. Viscous stress, heat flux and resistive terms are departures from perfect form, and they are D-03.

Magnetism is the omnium's vorticity

Proposition 8 (one 2-form per kind of pattern). For patterns of one kind with charge reading q per count (I-5a), the canonical flow 1-form and its vorticity are

$$\varpi = \frac{\hbar}{c^2} \mathcal{U}^b + qA, \quad \mathcal{W} = d\left(\frac{\hbar}{c^2} \mathcal{U}^b\right) + q\mathcal{F}, \quad \mathcal{F} = dA,$$

a mechanical grade and a magnetic grade of one 2-form. Propositions 6 and 7 hold for it, and Kelvin keeps the circulation of the whole ϖ along that kind's flow, never a grade alone. Three limits:

- (i) *Neutral patterns*, $q = 0$: $\mathcal{W}$ is mechanical vorticity, and Proposition 7 is the whirlpool and the cyclone.
- (ii) *Inertialess carriers*, $\hbar/(c^2 q) \rightarrow 0$ per unit charge (the electrons of a magnetosphere): $\mathcal{W} \approx q\mathcal{F}$, and Proposition 7 is ideal magnetohydrodynamics, with flux freezing, line motion, magnetic helicity and the Biermann battery.
- (iii) *The condensate*, $\mathcal{W} = 0$ in one state for every pattern: the two grades cancel. For a body turning rigidly at Ω , $\nabla \times (\Omega \times \mathbf{x}) = 2\Omega$, so

$$\mathbf{B} = -\frac{2m^*}{q} \Omega,$$

the London moment. For Cooper pairs ($m^* = 2m_e$, $q = -2e$) it is $\mathbf{B} = (2m_e/e)\Omega = 1.137 \times 10^{-11}$ T per rad s⁻¹, parallel to the spin [29].

Around any loop threading a condensate the circulation is quantised, $\oint \varpi = nh$ with n an integer and h the quantum of 0.1a. The flux tube of a proton condensate is $h/(2e) = 2.0678 \times 10^{-15}$ Wb, and the vortex of a neutron condensate carries $h/(2m_n) = 1.978 \times 10^{-7}$ m² s⁻¹.

Proof. The 2-form statements are Propositions 6 and 7 applied to ϖ . (iii) is $\mathcal{W}_B = m^* \nabla \times \mathbf{v} + q\mathbf{B} = 0$ with $\nabla \times (\Omega \times \mathbf{x}) = 2\Omega$ (receipt C9). The quantisation is the phase of a pattern, the turning of its helix (I-4), returning to itself around a closed loop: its advance is a whole number of turns, and each turn carries one h (C11). $\square$

P-5 states that mechanical vorticity and magnetic moments are different grade projections of one density-gradient topography. Proposition 8 gives that statement its exact 2-form setting. Each kind of pattern has one flow 1-form ϖ and one vorticity $\mathcal{W} = d\varpi$; A is the grade of ϖ that a charge probe reads, $\mathcal{F}$ is its vorticity, and $\mathcal{F}$ is the grade every kind shares. The identification of $\mathcal{F}$ with a grade of the omnium's flow is P-5's; the identities above are derived from it. The measured fields are accepted in full.

The London limit is P-5's grade-projection sentence as a measurement. Gravity Probe B read its rotors through it: “the spinning rotor generated a London moment equivalent at 80 Hz to a uniform 5×10^{-5} G field aligned with the spin axis” [14], and $(2m_e/e) 2\pi \cdot 80 \text{ s}^{-1}$ is 5.7×10^{-5} G (review receipt R5). The same law weighs the Cooper pair: $m^*/(2m_e) = 1.000084 \pm 0.000021$ in niobium against the theoretical 0.999992 [30], an excess of 92 ppm at 4.4 standard deviations of the stated 21 ppm accuracy, in the one precision reading of the twist-to-field kernel (D-04). A 12 km star at spin Ω carries $N_v = 2\pi R_b^2 \Omega / (h/2m_n) = 4.6 \times 10^{15}$ ($\Omega/1 \text{ rad s}^{-1}$) spin vortices, and 1.5×10^{31} flux tubes run through a 10 km core at 10^{12} G (C12). The star is a bundle of omnium vortex lines of both grades, each one a circulation quantum, and P-5's “vortex lines are bivector topological defects” is that bundle.

The I-1 reading of Coulomb's law, and the swirl's magnetic reading

Proposition 9 (the monopole reading). Coulomb's law written in the I-1 form, $F_{\max} \alpha_{Q,1} \alpha_{Q,2} = Q_1 Q_2 / (4\pi\epsilon_0 d^2)$ with $\alpha_Q = 2GM_Q/(c^2 d)$, fixes the product $M_{Q,1} M_{Q,2} = Q_1 Q_2 / (4\pi\epsilon_0 G)$. For a per-body input linear in the charge it fixes the magnitude

$$M_Q = \frac{|Q|}{\sqrt{4\pi\epsilon_0 G}}, \quad \sqrt{4\pi\epsilon_0 G} = 8.6175 \times 10^{-11} \text{ C kg}^{-1}.$$

Read the other way, a kilogram reads in the charge window as $\sqrt{4\pi\epsilon_0 G}$ coulombs, and the swirl's magnetic reading is the field of that charge carried with the mass:

$$\mathbf{B}_{I1} = \frac{\nabla \times \mathbf{v}_{sw}}{4\sqrt{4\pi\epsilon_0 G}}, \quad B_{I1,pole} = \frac{k_I \alpha_{\text{ext}}(R_b) \Omega}{2\sqrt{4\pi\epsilon_0 G}}, \quad \mu_{I1} = \frac{\sqrt{4\pi\epsilon_0 G} J}{2},$$

and $B_{I1,pole} = \alpha_{\text{ext}}(R_b) \Omega / (5\sqrt{4\pi\epsilon_0 G})$ for a uniform sphere: compactness times spin. A body's measured dipole is compared with its own swirl's reading by the Blackett number

$$\text{Bk} = \frac{2\mu}{\sqrt{4\pi\epsilon_0 G} J} = \frac{B_{\text{pole}}}{B_{I1,pole}}.$$

Proof. $F_{\max}(2G/c^2 d)^2 M_{Q,1} M_{Q,2} = GM_{Q,1} M_{Q,2} / d^2$, and a product $f(Q_1)f(Q_2) = Q_1 Q_2 / (4\pi\epsilon_0 G)$ for all pairs with f linear fixes $|f(Q)| = |Q| / \sqrt{4\pi\epsilon_0 G}$ (receipt D1, D5). A charge density $\sqrt{4\pi\epsilon_0 G} \rho$ moving with the mass has the dipole $\sqrt{4\pi\epsilon_0 G} J / 2$, whose exterior potential is $\mathbf{v}_{sw} / (4\sqrt{4\pi\epsilon_0 G})$ (review receipt M1); the polar and uniform-sphere forms follow (D2–D4). $\square$

The sign of the pair force is P-4's selection rule: same flow direction repels, opposite attracts. The map from a charge to an I-1 input, derived from the charge readings of I-5a and the cone geometry, is O-6's. Proposition 9 is the magnitude any such map returns, and it is used here to normalise Bk. $\text{Bk} = 1$ is the field a body would carry if its magnetism were its present swirl read through I-1, the dipole of the Schuster–Wilson hypothesis [1, 2]; Bk is Blackett's coefficient [3] written in this normalisation. The record's verdict does not rest on the normalisation. A change of the constant moves every body's Bk by the same factor, and the measured values span ten decades (Section), so no choice of the constant gathers the bodies at one value. Only the reading of $\text{Bk} = 1$ as the Schuster–Wilson dipole uses the magnitude.

The record

The snapshot law against the kept vortex

The snapshot law. Magnetism is the reading of the present swirl, a body's rotation read through I-1 [1, 2, 3]. Then every body carries one Blackett number, $\text{Bk} \approx 1$; the field is parallel to the spin and never reverses; every spinning body carries one; and inside a body the field changes with depth as the rotating matter above the point is removed.

The kept vortex. Magnetism is the kept vortex of the body's own medium (Propositions 7 and 8). It is born where the medium's flow is helical and where the temperature and entropy gradients cross; it is kept by Kelvin when the drive stops; it is carried with its flux through every change of support; and it answers to the body's later spin only through the slow exchange of D-03.

Fifty-one bodies, every entry sourced and computed by script (appendix; Figure 3):

bodies	n	Bk	what else the record shows
active vortex: Mercury, the Earth, Ganymede, Jupiter, Saturn, Uranus, Neptune, the Sun	8	0.015 – 0.94	the Earth and Ganymede carry their dipoles against their spins (Ganymede at 176°), Jupiter and Saturn with theirs; Uranus at 58.6°, Neptune at 46.9°; the Sun reverses every ~11 yr
active vortex: the T Tauri star BP Tau	1	6.9	a 1.2 kG dipole and a 1.6 kG octupole [33]
no active vortex: Venus, Mars, the Moon	3	$< 1.1 \times 10^{-3}$, $< 2.4 \times 10^{-5}$, $< 1.3 \times 10^{-4}$	Mars turns in 24.6 h, the Earth in 23.9 h; Mars and the Moon keep remanent fields in their crusts
kept vortex: magnetic O and B stars	10	1.4 – 500; NGC 1624-2 $> 7.2 \times 10^2$	flux $1 \times 10^{26} - 3 \times 10^{28}$ G cm ²
kept vortex: magnetic white dwarfs	2	3.5 (RE J0317–853); Grw+70°8247 $> 2.9 \times 10^7$	flux $5 \times 10^{25} - 3 \times 10^{26}$ G cm ²
kept vortex: magnetars	20	2.0 – 560	flux $4 \times 10^{25} - 1.3 \times 10^{28}$ G cm ²
young radio pulsars	5	0.005 – 2.3	flux $2 \times 10^{25} - 3 \times 10^{26}$ G cm ²
millisecond pulsars	2	$2 \times 10^{-8} - 6 \times 10^{-8}$	flux $2 \times 10^{21} - 3 \times 10^{21}$ G cm ²

Sources: the NASA planetary fact sheets [31]; Kivelson et al. for Ganymede [32]; Phillips & Russell, Connerney et al. and Runcorn for the limits [34, 35, 36]; Petit et al. for the O and B stars [37]; Ferrario, de Martino & Gänsicke with Nauenberg's radii for the white dwarfs [38, 39]; Olausen & Kaspi for the magnetars [40]; the ATNF catalogue v2.9.0 for the pulsars [41]. Neutron stars are on the catalogue convention, on which the dipole field is the spin-down estimate $B_{\text{eq}} = 3.2 \times 10^{19} (P\dot{P})^{1/2}$ G; the flux is $\Phi = \pi R_b^2 B_{\text{pole}}$.

Reading the record. The snapshot law predicts one value, and the record refutes it.

Ten decades. The measured values span 10.4 decades, from 2.4×10^{-8} (B1937+21) to 5.6×10^2 (SGR 1806–20). The lower limit on Grw+70°8247, $Bk > 2.9 \times 10^7$, carries the span to 15.1 decades; the upper limits on Venus, Mars and the Moon lie inside the measured range (review receipt R1).

Same spin, different vortex. The Earth and Mars turn in 23.9 h and 24.6 h. The Earth's Bk is 0.32; Mars carries no global dipole, $Bk < 2.4 \times 10^{-5}$, more than four decades lower. SGR 0418+5729 and SGR 1806–20 turn in 9.1 s and 7.5 s, and their Bk are 2.0 and 560, a factor of 270. The present spin cannot tell these pairs apart. Mars has no helical flow in its core now, and its crust keeps the vortex of an old one; the two magnetars carry different fluxes.

Sign and axis. The snapshot law fixes the sign of μ against J . The Earth and Ganymede carry their dipoles against their spins, Jupiter and Saturn with theirs; Uranus and Neptune hold theirs at 58.6° and 46.9°; the Sun reverses its field every eleven years and the Earth on geological time. A helical flow sets its own axis and its own sign.

Where the source sits. Measured down five mines, the change of the field with depth favours a source in the core [4], and Blackett's own test of a massive body at rest in the laboratory, turning with the Earth, found no field of the kind the modified hypothesis predicts [5].

Spin and its history, together. On the spin-down convention $Bk \propto P^{3/2} \dot{P}^{1/2} I^{-1/2}$, and the radius cancels (review receipt B1). Across the 27 neutron stars $\log Bk$ rises with $\log P$ with least-squares slope 2.4, and $\log P$ alone carries 91 % of the variance of $\log Bk$; the Spearman coefficient is 0.72 for all 27 and 0.35 inside the magnetar class (B2; Figure 3b). Among neutron stars Bk sorts with the spin and its spin-down history together, and the same-spin pairs above are where the history shows alone. On $I = 1.43 \times 10^{38}$ kg m² (a 12 km star) every catalogue Bk is multiplied by 0.84 (B3).

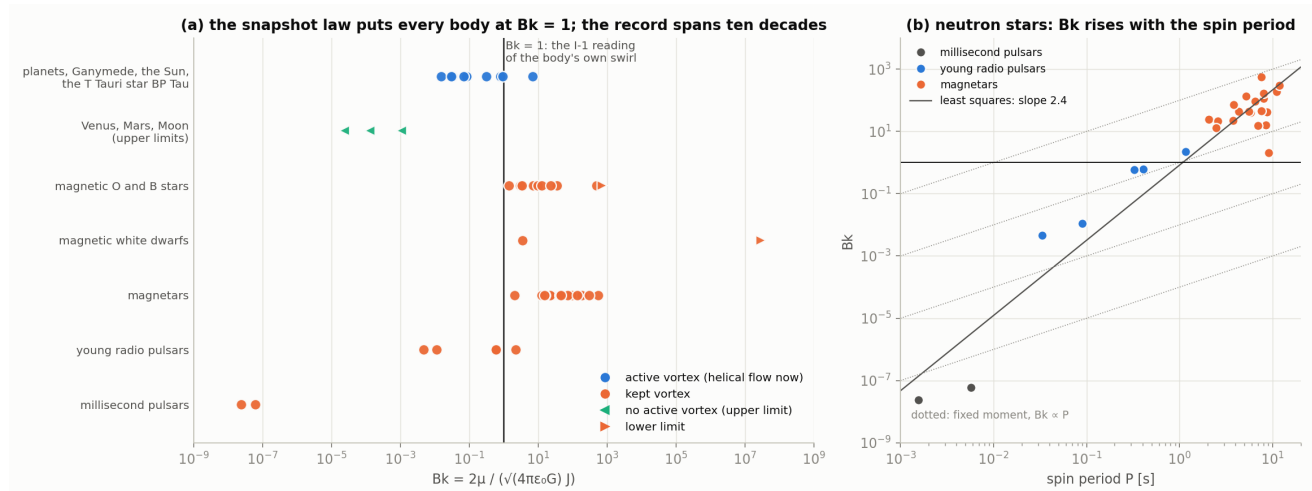


Figure 3: (a) The Blackett number $Bk = 2\mu / (\sqrt{4\pi\epsilon_0 G} J)$ by class. The snapshot law puts every body at $Bk = 1$; the record spans ten decades measured and fifteen with limits. (b) Neutron stars on the catalogue convention: Bk against the spin period, with the least-squares line (slope 2.4) and lines of fixed dipole moment, $Bk \propto P$ (dotted).

One point misleads. Sirag read the accreting pulsar Her X-1, 5×10^8 T at 5 rad s^{-1} , as support for the snapshot law [6]. On the catalogue convention its Bk is 0.12; the neutron stars of the same table span 10^{-8} to 560.

Memory (Kelvin), generation (the baroclinic source and the stretching of carried lines) and transport of flux through a change of support are the properties of a fluid's kept vortex, and they are the record's: the snapshot law sorts nothing. The narrow band of the active vortices, 0.015 to 6.9, is the old “magnetic Bode law”; Cain et al. trace most of that correlation to the common radius scaling of μ and J [7], and D-01 must reproduce the band against that null. The same record is the content the standard account carries as dynamo and fossil-field theory for conducting plasma, so the record selects the kept vortex over the snapshot law, and the convergence statement recovers it on both readings.

The flux is kept, and the spin is not

The flux. The magnetars carry $\Phi = \pi R_b^2 B_{\text{pole}}$ from 4×10^{25} to $1.3 \times 10^{28} \text{ G cm}^2$, and the magnetic O and B stars from 1×10^{26} to $3 \times 10^{28} \text{ G cm}^2$: the same range. The magnetic white dwarfs carry 5×10^{25} to $3 \times 10^{26} \text{ G cm}^2$ and the young radio pulsars 2×10^{25} to $3 \times 10^{26} \text{ G cm}^2$, the flux of a $10 R_\odot$ star at 14 to 220 G. Kelvin keeps a vortex's flux through every change of a configuration's support, in either direction: $\pi R_b^2 B$ is one number at both ends. A 4 kG field at $2.5 R_\odot$ and $8.4 \times 10^{13} \text{ G}$ at 12 km are one flux, the factor between them $(R_b/12 \text{ km})^2 = 2.1 \times 10^{10}$ (review receipt E3), and NGC 1624-2's flux, above $2.9 \times 10^{28} \text{ G cm}^2$, is more than $6 \times 10^{15} \text{ G}$ at 12 km.

The spin. Angular momentum kept from a star of $2.5 R_\odot$ turning in 5.47 d down to 12 km gives $23 \mu\text{s}$ at equal k_I , or $115 \mu\text{s}$ with k_I from 0.07 to 0.357, against a break-up period of 0.61 ms. Kept from a 6 s, 12 km star out to $2.5 R_\odot$, it gives $4.0 \times 10^3 \text{ yr}$ at equal k_I , or $7.8 \times 10^2 \text{ yr}$ with k_I from 0.357 to 0.07 (E1, E2). The magnetic O and B stars of the table turn in 1.2 d to 537 d, and young neutron stars in tens of milliseconds to seconds. The spins of the two populations are linked by kept angular momentum in neither direction, and their fluxes are linked. At kept flux and support, Bk grows in proportion to the period as the spin changes, $Bk \propto P$ (receipt E1; Figure 3b, dotted).

The direction of the change. The standard account forms a neutron star by the collapse of a massive star's core and reads a magnetar's field as the progenitor's fossil flux [42, 43] or as a dynamo in a fast-spinning newborn star [44]. FDT reads compact objects as inherited dense seeds that unwind by decompressive fission (0.8, I-1a), each with its kept vortex carried by Kelvin. The shared flux range is what a kept vortex carried through a change of support requires, and the record of fluxes does not fix the change's direction; FDT's direction is I-1a's. Which

bodies unwind, and at what rate, is the release function of O-14, and what a configuration inherits is O-15's (D-02). The remnants around three magnetars carry explosion energies close to the canonical 10^{51} erg, suggesting initial spin periods above about 5 ms, and give no evidence that magnetars form from rapidly rotating proto-neutron stars [45]. A kept vortex needs no fast-spin phase. Had the vortex been generated by an active flow at $Bk \leq 1$, where the active vortices of the solar system sit, the period at generation was at most P/Bk : 14 ms for SGR 1806–20, 39 ms for 1E 1841–045 and SGR 1900+14, and 4.4 s for SGR 0418+5729 (review receipt R2).

The magnetar

For a $1.4 M_\odot$ neutron star of radius 12 km, with k_I from the fit of Lattimer & Schutz [18], $\alpha_{\text{ext}}(R_b) = 0.345$, $k_I = 0.357$ and $I = 1.43 \times 10^{38} \text{ kg m}^2$, and the omnium at the skin is entrained by $k_I \alpha_{\text{ext}}(R_b) = 0.123$ of the spin. At a spin period of 6 s the swirl turns once in 49 s. The catalogued magnetars on the catalogue convention ($R_b = 10 \text{ km}$, $I = 10^{38} \text{ kg m}^2$, the equatorial field from spin-down [40]):

magnetar	P [s]	B_{eq} [G]	Bk	τ_s [s]
SGR 0418+5729	9.078	6.1×10^{12}	2.0	61
Swift J1834.9–0846	2.482	1.4×10^{14}	12.8	17
1E 2259+586	6.979	5.9×10^{13}	15.2	47
Swift J1822.3–1606	8.438	5.1×10^{13}	15.9	57
SGR 1627–41	2.595	2.2×10^{14}	21.1	17
SGR J1745–2900	3.764	1.6×10^{14}	22.2	25
1E 1547.0–5408	2.072	3.2×10^{14}	24.5	14
SGR 0501+4516	5.762	1.9×10^{14}	40.4	39
4U 0142+61	8.688	1.3×10^{14}	41.7	58
XTE J1810–197	5.540	2.1×10^{14}	43.0	37
PSR J1622–4950	4.326	2.7×10^{14}	43.1	29
SGR 1833–0832	7.565	1.6×10^{14}	44.7	51
CXOU J171405.7–381031	3.825	5.0×10^{14}	70.6	26
1E 1048.1–5937	6.458	3.9×10^{14}	93.0	43
CXOU J010043.1–721134	8.020	3.9×10^{14}	115.5	54
SGR 1900+14	5.200	7.0×10^{14}	134.4	35
SGR 0526–66	8.054	5.6×10^{14}	166.6	54
1RXS J170849.0–400910	11.003	4.6×10^{14}	187.0	74
1E 1841–045	11.783	6.9×10^{14}	300.3	79
SGR 1806–20	7.548	2.0×10^{15}	557.6	51

The magnetic reading of a magnetar's present swirl is 5×10^{12} to 3×10^{13} G at the pole, and the catalogue fields are 2 to 560 times that: the vortex a magnetar carries is not the reading of the swirl it has now.

The engine. The field lines of a neutron star corotate with it at Ω , held by the star's own patterns. The omnium swirls past them at $\omega_{\text{sw}}(r_f) = k_I \alpha_{\text{ext}}(R_b) \Omega (R_b/r_f)^3$ (Proposition 2). The charge density the corotating lines require, the Goldreich–Julian density [46], scales with the slip $\Omega - \omega_{\text{sw}}(r_f)$, and the slip changes with height (Figure 4). Muslimov & Tsygan computed the electric potential drop this sets above the polar cap [47], through the parameter the pulsar literature writes $\kappa^{(\text{src})} = (r_g/R)(I/MR^2)$ [48], which is $k_I \alpha_{\text{ext}}(R_b)$, the omnium's entrainment at the skin: 0.149 on the catalogue convention and 0.123 for the 12 km star. For low-obliquity pulsars, kinetic simulations “in flat spacetime show weak particle acceleration and no pair production near the poles”; with frame dragging the current along the field exceeds the local Goldreich–Julian value, “which leads to unscreened parallel electric fields and the ignition of a pair cascade”, and “general relativistic effects are essential for the existence of pulsar mechanism in low obliquity rotators” [49]. At the skin of the 12 km star the ratio of the current to its Goldreich–Julian value is $1/(1 - 0.123) = 1.14$. Along the open lines $\mathcal{W}_E \cdot \mathcal{W}_B \neq 0$ in the inertialess limit, and the medium leaves perfect form there. In flat spacetime, with no swirl, a low-obliquity pulsar is dark at its poles; the omnium's swirl, slipping past the field lines it does not carry, lights it.

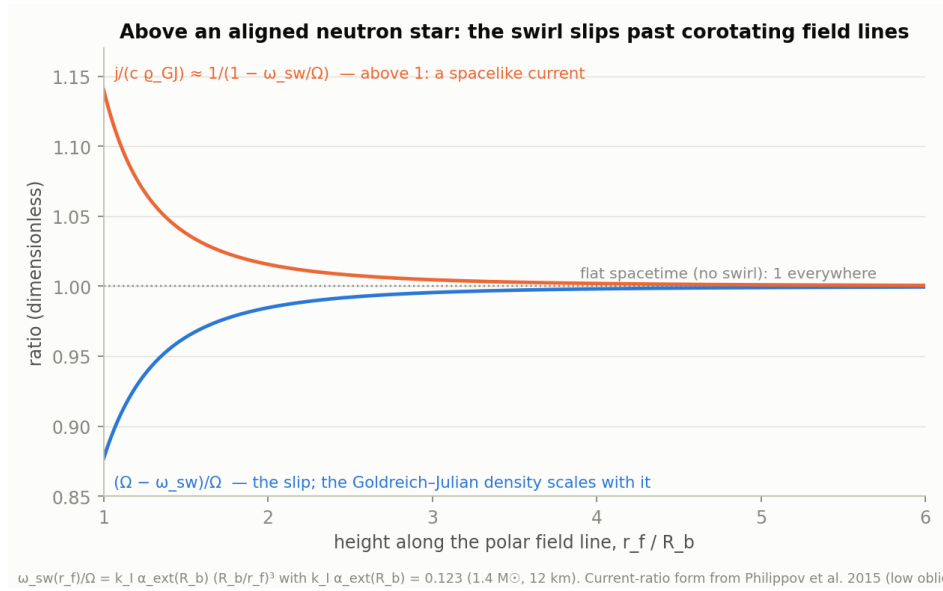


Figure 4: Above an aligned neutron star of $1.4 M_{\odot}$ and 12 km, $k_I \alpha_{\text{ext}}(R_b) = 0.123$. Blue: the slip $(\Omega - \omega_{\text{sw}})/\Omega$ along the polar field line, with which the Goldreich–Julian density scales. Orange: the current ratio $j/(c q_{\text{GJ}}) \approx 1/(1 - \omega_{\text{sw}}/\Omega)$, above 1, a spacelike current [49]. Flat spacetime has 1 everywhere.

The twist. A magnetar’s magnetosphere is twisted: currents run along its field lines, and its outbursts and giant flares are the twist released [50]. In the vortex reading the twist is the helicity of the medium’s vortex (Proposition 7), kept wherever $\mathcal{W}_E \cdot \mathcal{W}_B = 0$ and released where lines reconnect. The laboratory shows the same in water: knotted vortex rings untie by reconnection [51], and their helicity is carried across the reconnection from linking into twist [52]. The kept vortex is more than its dipole grade. SGR 0418+5729 has a catalogue dipole of 6.1×10^{12} G at the equator, and a phase-variable absorption feature, read as a proton cyclotron line, gives “a magnetic field ranging from 2×10^{14} gauss to more than 10^{15} gauss” [53].

Four clocks. The spin, $P = 2$ to 12 s, $\text{De} \sim 10^{-9}$ against a century. The swirl’s turn, $\tau_s \approx 7P$, 14 to 79 s across the catalogue, $\text{De} \sim 10^{-8}$, read through its rate at the polar cap. The twist, released in outbursts that rise within days and fade over months to years. The spin-down clock, $P/(2\dot{P})$ from 2.3×10^2 yr for SGR 1806–20 (from the catalogue’s P and B ; review receipt R4) upward, on which J , and with it the swirl, changes; $\text{De} \gtrsim 1$. The kept flux changes on its own decay clock, which is D-03’s. A galaxy shows the omnium moving on none of these clocks inside a century; a magnetar shows its spin and its twist moving, and its swirl read as a rate.

Systems and galaxies

Every stationary rotating configuration carries the inflow of Proposition 1, the swirl of Proposition 2 and, where its medium’s flow has been helical, the kept vortex of Proposition 7. The swirl each one carries is the unique least-shear configuration its skin allows (Proposition 2(c)).

A star system. The orbits of a system are free fall through the flow (0.1c). A circular orbit at r_f runs at $c\sqrt{\alpha_{\text{ext}}/2}$ and the inflow at the same radius at $c\sqrt{\alpha_{\text{ext}}}$: the orbital speed is the inflow speed divided by $\sqrt{2}$, at every radius, in the Newtonian face (I-4, the L2 layer). The planets are not carried by the swirl, which at the Earth’s orbit is 4.3×10^{-13} of the orbital speed (receipt S4); they fall through the inflow. Newton closed Book II by rejecting Descartes’ vortices, because a vortex carrying the planets cannot give the sesquuplicate ratio of their periodic times [13]. The objection binds a vortex that carries the planets. The omnium’s flow carries none: the orbits are free fall, the swirl is the rotlet of Proposition 2, and the 3/2 law is the free fall’s.

A galaxy, read as an estimate. A galaxy lies outside the evaluation domain: its rotation is read inside its own mass. The interior form of Proposition 1 with the enclosed $M(< r_f)$ gives the inflow speed $\sqrt{2GM(< r_f)/r_f}$. For a flat rotation curve this is $\sqrt{2} v_{\text{circ}} = 330 \text{ km s}^{-1}$ at the Sun's circle, the same at every radius, and its square over c^2 , the enclosed-mass ratio $2v_{\text{circ}}^2/c^2 = 1.2 \times 10^{-6}$, is constant across the disk. What holds that ratio constant against the visible matter is the mass shadow of 0.7, whose response function is O-14's (D-06). The disk turns once in $2.2 \times 10^8 \text{ yr}$ and its swirl once in 10^{14} yr (Proposition 4). The spiral arms read as patterns, not as material: a material line in a disk with a flat curve winds once more for every turn of the inner disk against the outer, and the observed arms are not wound. P-2 reads the arms as the nodal pattern of the disk's standing waves, with the stars as the sand. The galaxy's field of a few microgauss is a kept vortex of the disk's medium, and its growth and decay run on clocks of the disk's turn and longer, far beyond a century.

What remains to derive

D-01	The grade split of an active vortex: the fraction of the medium's vorticity the charge probe reads, from the helical flow of the interior, reproducing the active band $B_k = 0.015$ to 6.9 against the radius-scaling null of Cain et al.
D-02	Inheritance and generation of a kept vortex: the share carried from the configuration a dense seed was inherited with (0.8) and the share generated by an active flow (the baroclinic source and the stretching of carried lines), with its saturation; the kinematics bind to O-14 and O-15
D-03	The medium's departures from perfect form: viscous stress, heat flux and resistive terms, which set the decay clock of a kept vortex, its exchange with the spin, and the relaxation of the swirl to its least-shear minimum; O-12 binds here
D-04	The London kernel at the 21 ppm accuracy of Tate et al.: the condensate's inertia per pair against the 92 ppm excess, through I-5's confinement operator (O-9)
D-05	The polar-cap slip in omnium variables: the Goldreich–Julian density as the charge reading of the slip $\Omega - \omega_{\text{sw}}(r_f)$, and the accelerating potential from $k_I \alpha_{\text{ext}}(R_b)$, reducing to Muslimov & Tsygan and to Philippov et al.
D-06	The galactic vortex: the mass shadow that holds the enclosed-mass ratio constant across a disk (O-14), and the growth of the disk's field in the medium window
D-07	The rotating exterior beyond first order in J : the band $R_H < r_f < R_s$ outside the horizon, exterior and read through the multipole remainder of O-5 (0.4)

Convergence

FDT is a true extension of general relativity and quantum mechanics: for the same source and data it returns the measured record, and it reads that record as one substance seen through two windows.

The gravitational window. The flow form is the Schwarzschild exterior (receipt A7); the swirl is Hartle's slow-rotation frame drag (B9, B13), and Kerr's at the fold at first order in J (B7, B8). The gyroscope's reading is the Lense–Thirring precession: 39 mas/yr for Gravity Probe B, against -39.2 mas/yr predicted and $-37.2 \pm 7.2 \text{ mas/yr}$ measured [14]. The node drag of LAGEOS, LARES and LARES-2 is returned with it [15, 16, 17], and so is the iron-line modulation over the QPO cycle of H1743–322 [23]. The pair cascade of a low-obliquity pulsar with frame dragging is the slip of Proposition 2 at the polar cap [49].

The medium window. Maxwell's source-free pair is an identity; flux freezing, magnetic helicity, the Biermann battery, the London moment, flux quantisation and the neutron vortex are the medium window's Propositions 6–8, and the London readout of Gravity Probe B and the Cooper-pair mass of Tate et al. are accepted as measured [14, 30]. Every measured field, moment, spin and period of the record is accepted as measured; the catalogue fields of neutron stars are the spin-down estimates of their stated convention.

The record. The Schuster–Wilson–Blackett hypothesis is refuted by the record, as the mine measurements and Blackett’s negative experiment found [4, 5]. The kept-vortex reading recovers the same record as the dynamo and fossil-field accounts [42, 43, 44].

Declined readings. The standard account reads spacetime flow and frame drag as strong-field effects of compact objects. FDT reads them as the flow of the geometry at every size, with the frame drag the local rotation of the bookkeeper’s flow, half the curl of its swirl (0.1c); Gravity Probe B and LARES-2 read it at the Earth, where $\alpha_{\text{ext}}(R_b) = 1.39 \times 10^{-9}$.

The standard account reads the magnetic field as an independent entity. FDT reads it as a helical density gradient of the omnium, the magnetic grade of a medium’s vorticity 2-form (P-5), with the measured fields accepted in full.

The standard account reads a plasma as a fluid pushed by pressure forces across a fixed spacetime background. FDT reads its patterns as falling freely on clocks and rulers rescaled together by their enthalpy per count, the pressure force being the bookkeeper’s reading (0.1c, I-7); the Carter–Lichnerowicz equation is that free fall written as a vorticity law (Proposition 7).

The standard account reads fluid flows that reproduce a metric as analogue models. FDT reads the identity directly, since the flow is the omnium’s own (0.1c).

The standard account forms neutron stars by the collapse of massive stellar cores. FDT reads compact objects as inherited dense seeds unwinding by decompressive fission (0.8, I-1a), with their kept vortices carried by Kelvin; the record of fluxes and spins is accepted as measured.

No divergence is claimed from α_{ext} , a chart or a branch tag. The observables that separate the omnium reading from the standard one are the targets D-01 to D-07: the grade split of an active vortex, the inheritance of a kept vortex, its dissipation, the London kernel at the ppm level, the polar-cap slip in omnium variables, the galactic vortex, and the rotating exterior near the fold.

Status ledger

The core's owed register where it binds, the core clauses used as posited, and the items this analysis enters, each named once.

Owed by the core (v6.0), in its words, and where it binds

O-5	multipole and tidal remainders of the closed-form pair expression, and the reading they give of the band $R_H < r_f < R_s$ outside a rotating source's horizon; the overlap complex; how P-4's many-cone settlement signs a pair it does not join. Binds the rotating exterior beyond first order in J , where the exact horizon lies below R_s (D-07)
O-6	the maps from a charge or a colour seat to an I-1 input. Binds Proposition 9, whose magnitude any such map returns; the map itself is owed
O-9	the confinement operator D , its domains, boundary data and spectrum; every mass ratio. Binds the condensate's inertia per pair m^* (D-04)
O-12	the phase-exchange rates and the decay kernel, spectrum, chirality and rates. Binds the dissipation of D-03
O-13	full junction data $[K_{ab}]$ at any interface. Binds the skin, where the swirl meets the body
O-14	the mass-shadow response function and its closure; the release function of 0.7. Binds the galactic ratio (D-06) and which bodies unwind (D-02)
O-15	the domain-merger kinematics. Binds what a dense seed inherits (D-02)

Core clauses used as posited (author rulings)

0.1, 0.1a, 0.1b	one substance; the chain read four ways, settled system-wide by one global optimisation at every step; only omnium and patterns
0.1c	the extended equivalence principle and the flow reading, whose identities are derived; the medium's clock factor, conditional on perfect form
0.8, I-1a	inherited structure, and the downhill direction of unwinding
P-4, P-5	the flow-direction selection rule (the sign of Proposition 9); magnetism as the grade projection of one density-gradient topography (Proposition 8)

Derived here

Props. 1–9	receipts A1–A9, B1–B16, C1–C14, D1–D5, S1–S2 and review receipts W1–W7, K1–K4, V1–V4, N1–N2, M1, B1
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Read from the record: in-sample readings, never predictions

R1	the snapshot law refuted; Bk not a function of the present spin; among neutron stars, spin and spin-down history together (review receipt B2)
R2	the flux kept and the spin not kept through a change of support, in either direction (E1–E3)
R3	the polar-cap engine of a low-obliquity pulsar as the swirl's slip; $\kappa^{(\text{src})} = k_I \alpha_{\text{ext}}(R_b)$ (Philippov et al.)
R4	the magnetar's four clocks

Estimates outside the evaluation domain

the Galaxy	$\tau_s = 3.6 \times 10^{14}$ yr (exterior formula) and 1.2×10^{14} yr (co-rotating shells); the enclosed-mass ratio 1.2×10^{-6}
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Entered by this analysis, to be derived

D-01 to D-07	as listed in the preceding section
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No prediction block is entered: the analysis claims no divergence, and its observables are in-sample readings of the record. No PROVISIONAL token appears.

Appendix: the computations

Two scripts check every identity and every derived number, and a third computes the record from the sourced table.

script and block	content (checks)
01_receipt.py, A	the inflow: speed, Euler, Bernoulli, potential, expansion; the flow form's Einstein tensor; the Hamiltonian constraint with an arbitrary enclosed mass; the strain identity (9)
B	the swirl: the rotlet and its curl, divergence and Laplacian; the polar and equatorial readings; Kerr's horizon rate and the first-order rate; the slow-rotation exterior at first order; the entrainment; the least-shear Euler–Lagrange equation, the cross term and second variation; Hartle's operator; the static winding and its change of variables; scale invariance (16)
C	the vorticity 2-form: $d^2 = 0$ and its 3+1 pair; Cartan's formula on ϖ and on $\mathcal{W}$; the Euler identity at random points; helicity; the baroclinic source and Biermann; London; Newcomb's identity; the 3+1 frozen-in and induction laws; the circulation quanta and counts (14)
D, E, F, S	the monopole reading, the swirl's magnetic reading, B_k , the uniform sphere, $\sqrt{4\pi\epsilon_0 G}$; homologous contraction; the Earth, Gravity Probe B, the neutron star; spin and flux kept from $2.5 R_\odot$; the clock, the carried winding, the Deborah ladder, the swirl at 1 AU (15)
04_review_checks.py, W, K, V	the winding in three azimuths and the change of charts; the exact Kerr horizon against R_s and the first-order fold; the irrotational free fall, the curl, and the interior vector Laplacian matched at the skin (15)
N, M, B, G, E, R	Newcomb's condition with a counterexample and a multi-valued case; the co-moving charge; $B_k \propto P^{3/2} \dot{P}^{1/2} I^{-1/2}$ and its regression across 27 neutron stars; the Galaxy's shells; spin and flux in both directions; the ranges, bounds and ages as printed (16)
02_bodies.py	J , μ , B_k , $\alpha_{\text{ext}}(R_b)$, $k_I \alpha_{\text{ext}}(R_b)$, τ_s , De , the polar field and the flux for the 51 bodies of data/bodies.csv, every row sourced in data/bodies_sources.md
05_verify_tex.py	every number this document prints that a script computes, against the scripts' outputs at the digits printed

The receipts are out/receipt.json (54 of 54) and out/review_receipt.json (31 of 31). The symbolic checks run in exact arithmetic with sympy, the Euler identity at forty-digit precision at random points, and the numerical checks at the digits printed.

Source, dimensions, and verification

The physical framework is **CORE-FDT-INVARIANTS.md, v6.0**: 0.1–0.1c, 0.2, 0.3, 0.3a, 0.4, 0.5, 0.6, 0.7, 0.8, I-1–I-7 with I-1a, I-3a and I-5a, P-2, P-4 and P-5, and the register of §O, used as locked. The author lock of the core is dated 17 October 2026. Its verified SHA256 is

a1877568f1505f0f6bbd01270d0871019b97c47dc 2d124dc392e0fec9e87fdab.

Dimensional checks give dimensionless α_{ext} , $k_I \alpha_{\text{ext}}(R_b)$, $a_\star$, $\Delta\varphi$, B_k and De ; inverse time for Θ , S_{ij} , Σ_{ij} , ω_{sw} , Ω and Ω_H ; inverse time squared for $16\pi G\rho$; velocity for $\mathbf{v}_{\text{in}}$, $\mathbf{v}_{\text{sw}}$ and v ; momentum per count for $\mathbf{p}$ and ϖ ; energy per count for $\mathcal{E}$ and $\mathfrak{h}$; force per count for $\mathcal{W}_E$ and $T\nabla s$; tesla for $\mathbf{B}_{\text{II}}$ and $(2m_e/e)\Omega$; webers for $h/(2e)$; $\text{m}^2 \text{s}^{-1}$ for $h/(2m_n)$; and coulombs per kilogram for $\sqrt{4\pi\epsilon_0 G}$.

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