

One Substance, One Carrier, One Geometry, Myriad Forms

Why the books of the universe balance themselves:
conservation as identity in Fundamental Density Theory

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Fundamental Density Theory

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Abstract

Fundamental Density Theory has one substance, the omnium (0.1); one carrier, the spin-1 stroke of the density field (I-2); one geometry, the density cone that is at once the light cone, the Einstein geometry and the stroke's helical geodesic, on an atlas with the shape of the three-sphere split at the Clifford torus (P-4, I-3, I-4, 0.3a); and myriad forms, every one of them a configuration of the stroke—unbound, open strokes whose grades turn on open helices, read as light, gravity's ripple, sound and spin waves; bound, the closed braids read as matter (I-2, I-5). This configuration is the only one in which the books of the universe balance as identities: a connected ledger balances automatically, in the sense of Definition 5, if and only if it has one substance, one carrier, one geometry and no fixed background, with discrete labels, a chain of ratios and bounded readings (Theorem 5). Every book is a reading of one state. Energy, mass, frequency, the period, which a closed pattern keeps as its time, and the length its thread runs in that period are tied by bridges fixed by the one chain $E/m = c^2 = 1/(\epsilon_0\mu_0) = (d/t)^2$; the terms adjust collectively and none ever changes independently of the others (0.1a), so no transaction posts to one reading alone (Proposition 1) and every exchange cycle closes (Theorem 1). The four-momentum book is the contracted Bianchi identity of the one geometry applied to the one total source, read at the configuration's boundary as $\mathcal{M}$ and R_s (Theorem 2). The spacetime book is the decompressed phase of the same substance: on the support its share and the matter phase's sum to one (Proposition 3), and every increment of the spacetime phase is posted against a release from the matter phase (Theorem 2(b), 0.7). The books of charge, handedness and fermion class are integers of framed closures, held by $Lk = Tw + Wr$ through every deformation (Theorem 3). Because the chain is built of ratios, one ledger holds at every size—nucleus, star, galaxy, domain—bounded below by the floor $2\ell_P, 2t_P$: the universe is fractal because its central invariant requires it (Theorem 4). Every other configuration needs an entry posted by hand: a second substance needs a cross-rate for each process; a second carrier speed lets an internal relabelling move an isolated configuration's centre of inertia; a second geometry leaves vector constraints on its metrics that hold on solutions but not identically; a fixed background lets a kilogram lowered and returned as light post $mg\Delta h$ from nothing; a size-selecting chain makes the books depend on grouping; a vanishing term or a singular interior leaves an account unpriced or unaudited (Lemmas 2–8). Theorem 5 states both directions. The record shows no hand-posted entry: one mass–energy rate to 4×10^{-7} , one weight for every energy channel to 10^{-15} , one stroke speed for gravity and light to 10^{-15} , and kept times adjusting across a single millimetre (Table 3). FDT recovers each of these measurements exactly (§9).

1 Conventions, charts and sources

This document is self-contained on the locked FDT core v6.3 (`CORE-FDT-INVARIANTS.md`, author lock 2026-10-26) [1], which it quotes by section: 0.1, 0.1a, 0.1b, 0.1c, 0.1d, 0.2, 0.3, 0.3a, 0.4, 0.5, 0.6, 0.7, 0.8, I-1 to I-7, P-3 to P-5. The rate c is the strand rate derived in *What c Is* [2]; the pattern's reading and the bookkeeper's are those of *The Extended Equivalence Principle* [3]; the one carrier is that of *The Quintuple Identity* [4]. Every statement below is of one of three kinds: a quotation of the core; a derivation from the core and displayed mathematics; or an item marked to be derived (D-01 to D-05) and collected in §8. The receipt script of Appendix A checks the displayed identities and numbers; check labels appear in brackets, [OS04f]. The consensus record enters through measured values only: the mass–energy relation [13], the universality of free fall [14], the speed of gravitational strokes [15], the gravitational shift of the kept time [16, 17], the solar and Jovian nominal values [18] and the masses of the closures [19].

Notation. $\kappa = 8\pi G/c^4$, $F_{\max} = c^4/4G$, $\kappa F_{\max} = 2\pi$ exactly (0.2); $\hbar = h/2\pi$. $\mathcal{M}$ is the total gravitating energy of a configuration, every channel entering once (0.5), and $R_s = 2GM/c^2$. A *pattern* is a configuration of the omnium (0.1b); a *stroke* is the carrier of I-2, open, running straight at c and keeping no time, its grades turning about the run on a helix of radius c/ω (I-2, 0.1d); a *closure* is a bound form of I-5, n strands of the stroke closed into a braid, which keeps time by circulating (0.1d). $\mathcal{D}$ is the set of admissible configurations: those that keep the chain (0.1), the floor (0.1a) and the open interval (I-6).

Chart inventory. Every α in this document is the A-1 compactness coordinate of 0.4, branch-tagged: $\alpha_{\text{ext}}(r_f) = R_s/r_f$ on the exterior branch, $R_s < r_f < \infty$, and $\alpha_{\text{int}}(r_f) = r_f/R_s$ on the interior branch, $r_c \leq r_f < R_s$; $\alpha_\sigma = (R_s/r_f)^\sigma$ (0.3). The I-1 inputs $\alpha_i(d) = R_{s,i}/d$ are exterior cross-centre evaluations on the I-1 domain. On the A-1 chart the seam $\alpha_\sigma = \frac{1}{2}$ is the Clifford torus of the atlas—at $r_f = 2R_s$ on the exterior branch and $r_f = R_s/2$ on the interior branch—splitting the support S^3 into the general-relativistic solid torus $\alpha_\sigma < \frac{1}{2}$, the spacetime phase, and the quantum-mechanical solid torus $\alpha_\sigma > \frac{1}{2}$, the matter phase; the share is $|z_1|^2 = \alpha_\sigma$, $|z_2|^2 = 1 - \alpha_\sigma$, and the jacket flip $\mathfrak{J} : \alpha_\sigma \mapsto 1 - \alpha_\sigma$ exchanges the two moduli at fixed phases and fixes the seam pointwise (0.3a). The branch exchange $\mathfrak{i}$ does not enter. No other application of the bounded map enters, and no computational-chart token appears. A bound configuration is carried by its two declared charts—the exterior atlas of its source and its own interior chart—with no transfer map between them (I-3); the two charts read one geometry.

2 Where the seven invariants enter

The force (I-1). $F = F_{\max} \alpha_1 \alpha_2$ at every scale, $0 < \alpha_i < 1$, $F < F_{\max}$. On the I-1 domain the pair energy is the open line integral $U(d) = -\int_d^\infty F ds = -F_{\max} \alpha_1 \alpha_2 d$; it is the entry a composite's read-out carries for its parts (Theorem 4). The ceiling $F_{\max}$ keeps every force entry finite (Proposition 4).

The carrier (I-2). One medium and one speed give one carrier: photon = graviton = gluon = phonon = magnon, one spin-1 stroke read in different density and geometry regimes. Every unbound form is this stroke; every bound form is closed from it (I-5). One carrier is one invariant speed and one mass–energy rate (Lemma 3); the speed is the strand rate of the omnium, derived (0.1d) [2].

The geometry (I-3). $G_{\mu\nu} = \kappa T_{\mu\nu}$, unchanged, with $T_{\mu\nu}$ the one total source. Its contracted Bianchi identity is the four-momentum book (Theorem 2). The I-3 optical index is the adjustment of the kept time written for a distant bookkeeper (0.1a); the gravitational shift of strokes is that

adjustment, and the books require it (Lemma 5). Moving at constant speed, on a pattern's own clocks and rulers, through a density gradient is gravity: the distant bookkeeper reads the pattern accelerating, slowing down and bending (0.1c) [3].

The faces (I-4). Newton = GR = QM = FDT, each the same statement read in a different limit. The composite read-out of Theorem 4 is evaluated at the leading weak-field, slow-motion order (the containment layer L2), with the order stated. Quantum mechanics is helical geodesics on truncated density cones; the helical phase of every stroke is a configuration variable, and the dynamics is deterministic.

The braids (I-5, I-5a, I-5b). Matter is braided photon helices: closures of n strands of the I-2 stroke, mass a confinement eigenvalue, $m_q c^2 = \hbar \omega_q$. The books of charge, handedness and fermion class are integers of framed closures (Theorem 3); every reclosure balances every exact book (I-5b), and the strand count is a per-closure label, never a book.

The bounded map (I-6). $0 < \alpha_\sigma < 1$ strictly. The branch-aware kinematics $\dot{\alpha}_\sigma / \alpha_\sigma = \sigma(\dot{\mathcal{M}}/\mathcal{M} - \dot{r}_f/r_f)$ means that internal relabelling at fixed $\mathcal{M}$ and r_f cannot change α_σ ; phase exchange obeys $T_\Omega^{\mu\nu} = \sum_a T_a^{\mu\nu}$, $\nabla_\mu T_a^{\mu\nu} = Q_a^\nu$, $\sum_a Q_a^\nu = 0$. These are the read-out and the phase book of Theorem 2.

Density equivalence (I-7, 0.5). $\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \dots$, every channel entering once: mass $\equiv$ energy $\equiv$ temperature $\equiv$ pressure $\rightarrow$ density. Every form of energy posts to one account, $\mathcal{M}$, and one read-out, R_s ; this is the single numeraire of Lemma 1.

Definitions used. One substance and the chain (0.1); the chain as one state read four ways, the floor, general relativity as the chain kept state by state, and every step a system-wide settlement (0.1a); only omnium and patterns (0.1b); the pattern's reading, the bookkeeper's reading and the clock factor (0.1c); c the strand rate (0.1d); the atlas, its seam, its two phases and the repetition of its form (0.3, 0.3a); the dilute centre and the densest skin (0.6); dark matter and dark energy as states of the one substrate (0.7); no singularity, universal domains and the flow from the matter phase to the spacetime phase (0.8); clamps (P-3), density cones as light cones (P-4), twist as magnetism (P-5).

3 The inventory: one substance, one carrier, one geometry, myriad forms

3.1 One substance

FDT has one substance, the omnium: matter = energy = spacetime (0.1). The chain

$$E/m = c^2 = 1/(\varepsilon_0 \mu_0) = (d/t)^2 \quad (1)$$

is one identity read by quantum mechanics (E/m), electromagnetism ($1/\varepsilon_0 \mu_0$) and general relativity ($(d/t)^2$); ε_0 and μ_0 are response coefficients of the omnium, so c^2 is a property of the medium. There are no separate particles and no boxes: there is only omnium, and omnium patterns (0.1b). The omnium carries two phases on one support. The matter phase is compressed omnium, closed braids, on the quantum-mechanical solid torus $\alpha_\sigma > \frac{1}{2}$ (A-1, either branch); the spacetime phase is decompressed omnium on the general-relativistic solid torus $\alpha_\sigma < \frac{1}{2}$; the seam between them is the Clifford torus $\alpha_\sigma = \frac{1}{2}$ (0.3a). The Heegaard splitting splits the support, never the omnium.

3.2 One carrier

A carrier is a disturbance of a medium at that medium's speed. The chain gives one speed to every reading—the mass–energy exchange rate (gravity's ripple), the electromagnetic propagation

constant (light) and the medium's own stroke (sound)—and 0.1b leaves one medium for them to be disturbances of. Hence one carrier (I-2):

$$\text{photon} = \text{graviton} = \text{gluon} = \text{phonon} = \text{magnon}, \quad (2)$$

the same spin-1 stroke read in different density and geometry regimes [4]. Its propagation through structured density—sound through a lattice of clamps (P-3), the stroke through a helical density gradient (P-5), light through a dielectric—is the one stroke read in a regime. The effective aperture speed $c_{\text{eff}} = c\sqrt{1 - \alpha_\sigma}$ is chart-dependent (P-4); the invariant c of local null-cone kinematics is one; and every energy the stroke carries enters $\mathcal{M}$ as E/c^2 (0.5).

3.3 One geometry

The truncated density cone at a point is the local light cone (P-4). Its curvature is the Einstein geometry, $G_{\mu\nu} = \kappa T_{\mu\nu}$ (I-3); generalised Snell's law is the Fermat jump law of its exact optical metric (I-3); the stroke's paths on it are helical geodesics, and $\psi = Ae^{i\varphi}$ is their projection on the plane normal to the cone axis (I-4). Its atlas has the shape of the three-sphere, with α_σ the share, the Clifford torus the seam and the two Hopf core circles the two ends of the open interval (0.3a). One geometry carries gravity, optics, the quantum phase and the two regimes.

3.4 Myriad forms

Every form is a configuration of the one stroke on the one geometry (Table 1, Figure 1). Unbound, the stroke runs straight at c and keeps no time, its grades turning about the run on an open helix of radius c/ω standing at $\pi/4$ (I-2, 0.1d); on a density cone it is read as light, as gravity's ripple, as sound and as the spin wave of a helical gradient, and standing inside a closed pattern's own region it is read as the gluon (I-2). Bound, n strands of it close into braids (I-5): the $n = 2$ closure is read as the neutrino in the free room and as the neutron in the nuclear room, the $n = 3$ closure as the electron and as the proton, and the rungs $n = 5, 7, 9$ stay closed only at higher density, with footing in the skin terrain of a bound configuration (I-5, I-3). Matter and antimatter are opposite handedness of one configuration (P-4). A magnetic field is a helical density gradient (P-5); dark matter is the compressed omnium around matter, the mass shadow, and dark energy the stored potential of compressed omnium released as matter unwinds (0.7); the spacetime phase is decompressed omnium (0.3a); the cosmic microwave background is the substrate's cumulative standing-wave state (0.8). The count of forms is open. The count of substances, carriers and geometries is one.

3.5 Three facts that carry the argument

(F1) The terms adjust collectively. The four terms of the chain adjust simultaneously: the omnium settles matter, energy and spacetime by one global optimisation, never one term at a time (0.1a). None ever changes independently of the others. If the left side steps, the right side steps with it: a pattern quantised in energy is quantised in mass, in the time it keeps and in the length its thread runs, by the identical stroke (0.1a). Every step is a system-wide settlement: the optimisation is solved at every step from the state the system is in, responses are many-to-many, events do not commute, and a process run backward is never the mirror of the forward event (0.1a).

(F2) There is no fixed background. Clocks and rulers are closed patterns of the omnium; their kept time and thread length move together so that d/t measured on the spot stays c . Their adjustment is the geometry: the metric's response to $T_{\mu\nu}$ is the chain's simultaneity at the scale

Table 1: Myriad forms, one stroke. Every row is a configuration of the I-2 carrier on the one geometry.

configuration of the stroke	state	read as	core
open stroke on a density cone, its grades on an open helix	unbound	light; gravity's ripple; sound through a lattice of clamps; the spin wave of a helical gradient	I-2, I-4, P-3, P-5
the stroke standing inside a closed pattern's own region	standing	gluon	I-2
$n = 2$ closure	bound	neutrino (free room), neutron (nuclear room)	I-5
$n = 3$ closure	bound	electron (free room), proton (nuclear room)	I-5
$n = 5, 7, 9$ closures	bound at higher density	the $s, c; b; t$ rungs, footing in the skin terrain	I-5, I-3
one configuration, opposite handedness	bound	matter and antimatter	P-4
helical density gradient	gradient	magnetic field	P-5
compressed omnium around matter	compressed	dark matter, the mass shadow	0.7
stored potential released by unwinding	released	dark energy, Λ_{eff}	0.7
decompressed omnium	phase	spacetime	0.3a
cumulative standing-wave state of the substrate	standing	cosmic microwave background	0.8

of bodies, and there is no background metric that clocks and rulers merely read (0.1a). On a pattern's own clocks and rulers the physics is unchanged, and the distant bookkeeper reads the common rescaling (0.1c).

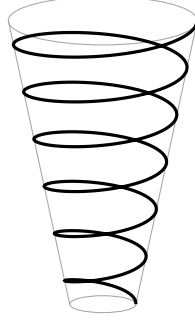
(F3) The chain is built of ratios, so it holds at every scale. $E/m, c^2, 1/(\varepsilon_0\mu_0)$ and $(d/t)^2$ fix relations and select no size, and α_σ inherits the invariance under $(R_s, r_f) \mapsto (\lambda R_s, \lambda r_f)$ (0.3). The support, its seam and its two phases have the same form at every size—nucleus, compact object, galaxy, domain, one construction read at successive sizes. The universe is fractal, as a central invariant built on ratios requires, and the repetition is bounded below: $F_{\text{max}} d_{\text{min}} t_{\text{min}} = \hbar$ fixes $d_{\text{min}} = 2\ell_P$, $t_{\text{min}} = 2t_P$, a minimum and not a staircase (0.1a, 0.3a).

Section 4 turns (F1) into the statement that every book is a reading of one state; §5 shows that the books of such a state balance as identities; §6 shows that a configuration giving up one substance, one carrier, one geometry, (F1), (F2) or (F3) keeps a book that balances only by an entry posted by hand.

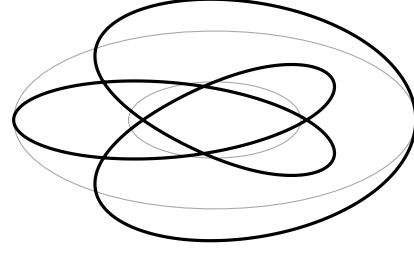
4 The books are readings of one state

Definition 1 (reading). A *reading* is a positive function on $\mathcal{D}$ carrying a unit. The readings of a pattern are its energy E ; its mass m , the gravitating mass of its energy, $m = E/c^2$, positive for every pattern, open or closed; its frequency ω ; its period T , which a closed pattern keeps as its time and an open stroke does not keep (0.1d); and the length ℓ its thread runs in one period, for an open stroke its advance per cycle (0.1a). The readings of a configuration are its total gravitating energy $\mathcal{M}$ (0.5), its fold radius R_s and its density coordinate $\alpha_\sigma(r_f)$ at a declared field radius (0.3).

Definition 2 (bridge). A *bridge* is an identity between two readings that holds on all of $\mathcal{D}$ with a



unbound: an open stroke's helix on a truncated
density cone



bound: strands closed into a braid

Figure 1: Myriad forms of one stroke. Left: the unbound form on a truncated density cone, the stroke running at c along the axis with its grades turning on the open helix drawn (I-2, I-4). Right: a bound form, strands of the same stroke closed into a braid closure, drawn as a $(3, 2)$ torus closure on a guide torus (I-5). Schematic: no values, no identification of a knot type with a named closure.

constant of the medium (0.2), or, for α_σ , with the declared field radius:

$$m = \frac{E}{c^2}, \quad \omega = \frac{E}{\hbar}, \quad T = \frac{2\pi}{\omega} = \frac{h}{E}, \quad \ell = cT, \quad R_s = \frac{2G\mathcal{M}}{c^2}, \quad \alpha_\sigma(r_f) = \left(\frac{R_s}{r_f}\right)^\sigma. \quad (3)$$

For a single pattern read alone, $\mathcal{M} = m$. The constants are the omnium's: $c^2 = 1/(\varepsilon_0\mu_0)$ (0.1), and $G, \hbar$ enter through $F_{\max}$ and the floor (0.2, 0.1a).

Definition 3 (posting). A change of configuration $z \rightarrow z'$ in $\mathcal{D}$ posts $\Delta r = r(z') - r(z)$ to every reading r . A posting is *single-entry* if it changes a reading without the change that a bridge requires of another reading.

Proposition 1 (no single-entry posting). *Every change of configuration posts to every reading in the proportions fixed by (3):*

$$\begin{aligned} \Delta m = \frac{\Delta E}{c^2}, \quad \Delta \ln \omega = \Delta \ln E, \quad \Delta \ln T = -\Delta \ln E, \quad \Delta \ln \ell = \Delta \ln T, \\ \Delta R_s = \frac{2G}{c^2} \Delta \mathcal{M}, \quad \Delta \ln \alpha_\sigma = \sigma (\Delta \ln \mathcal{M} - \Delta \ln r_f). \end{aligned} \quad (4)$$

No single-entry posting exists.

Proof. Each bridge holds at z and at z' ; subtract, or take logarithms and subtract. The last relation is the finite form of the I-6 kinematics $\dot{\alpha}_\sigma/\alpha_\sigma = \sigma(\dot{\mathcal{M}}/\mathcal{M} - \dot{r}_f/r_f)$. $\square$

Proposition 1 is (F1) in ledger form. A book that can be posted alone is a coordinate; a book that cannot is a reading. In FDT every book is a reading: the universe keeps one state and reads it many ways, and a transaction is a change of that state, posted to every reading at once (Figure 2).

Proposition 2 (the two invariants of a stroke). *For every unbound stroke $ET = h$ and $\ell/T = c$, with T its period, read by any bookkeeper. A change of bookkeeper that multiplies the stroke's period by $1 + z$ multiplies E, m and ω by $(1 + z)^{-1}$ and T and ℓ by $1 + z$, and leaves both invariants unchanged.*

Proof. $ET = h$ and $\ell = cT$ are bridges of (3); the factor $1 + z$ enters E and T with opposite exponents and ℓ and T with equal ones. [OS02a] The I-3 optical index is this adjustment written for a distant bookkeeper (0.1a). $\square$

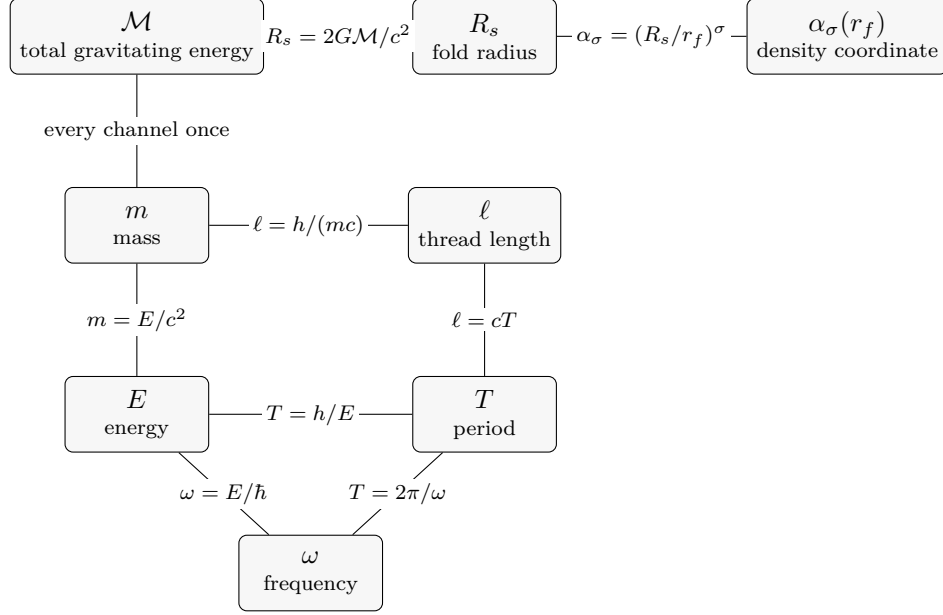


Figure 2: One state, eight readings. Every edge is an identity on the admissible configurations: the bridges of Eq. (3); $\ell = h/(mc)$, composed from them; $\mathcal{M} = m$ for a pattern read alone (I-7, 0.5); and $\alpha_\sigma = (R_s/r_f)^\sigma$ at the declared r_f . Every cycle closes (Theorem 1), and every change of configuration posts to every node at once (Proposition 1).

Table 2 reads one stroke of advance 500 nm per cycle by two bookkeepers whose kept times differ by a factor two. [OS02b, OS02c] Every reading moves; the two invariants do not. A gravitational shift (Lemma 5) and a stroke crossing a decompressing domain (§5.3) are instances.

Table 2: One stroke, two bookkeepers (h , c exact; CODATA 2022 [19]).

$1+z$	E (eV)	m (kg)	ω (s ⁻¹)	T (s)	ℓ (nm)	ET (Js)	ℓ/T (m s ⁻¹)
1	2.47968	4.42044×10^{-36}	3.76730×10^{15}	1.66782×10^{-15}	500	6.62607×10^{-34}	299 792 458
2	1.23984	2.21022×10^{-36}	1.88365×10^{15}	3.33564×10^{-15}	1000	6.62607×10^{-34}	299 792 458

5 Why the books balance

5.1 Exchange cycles close

Definition 4 (exchange graph). The *exchange graph* has the readings as vertices and the bridges as edges. With $x_r = \ln(r/[r])$ for a reading r of unit $[r]$, each bridge is an affine relation $x_b = s_{ab}x_a + w_{ab}$ with $s_{ab} \in \{+1, -1\}$ and w_{ab} the logarithm of a constant of the medium, or of the declared radius r_f , in the chosen units. The *holonomy* of a closed cycle is the composite of the affine relations along it.

Lemma 1 (one numeraire). *On a connected exchange graph the following are equivalent: (i) every cycle has trivial holonomy; (ii) there is a reading r_o , with $x_{r_o}(\mathcal{D})$ containing more than one point, such that every reading is $r = k_r r_o^{e_r}$ with $k_r > 0$ a constant and $e_r \in \{+1, -1\}$.*

Proof. (ii) $\Rightarrow$ (i): every reading is a fixed affine function of x_{r_o} , so composing the relations around a cycle returns each x_r as the same function of x_{r_o} ; the composite fixes every value x_r takes, which

are more than one, and an affine map of slope ± 1 fixing two points is the identity. (i) $\Rightarrow$ (ii): fix r_o ; for a reading r choose a path from r_o and let A_r be the composite affine relation along it, so that $x_r = A_r(x_{r_o})$. Two paths differ by a cycle, whose holonomy is trivial, so A_r is independent of the path. A_r has slope ± 1 , hence $r = k_r r_o^{\pm 1}$. $\square$

Theorem 1 (exchange cycles close). *Every cycle of the exchange graph of (3) has trivial holonomy, and every reading of a pattern read alone is a constant multiple of $E^{\pm 1}$:*

$$m = c^{-2}E, \quad \omega = \hbar^{-1}E, \quad T = hE^{-1}, \quad \ell = hcE^{-1}, \quad \mathcal{M} = c^{-2}E, \quad R_s = 2Gc^{-4}E. \quad (5)$$

Proof. The readings are functions of one state and each bridge is an identity on $\mathcal{D}$. Following a cycle from a reading a returns $x_a(z)$ for every $z \in \mathcal{D}$, so the composite affine relation fixes every point of $x_a(\mathcal{D})$. That set contains an interval: unbound strokes carry a continuum of energies, and no lattice of allowed intervals exists (0.1a). An affine map of the line that fixes two points is the identity. [OS01d] Equation (5) follows from (3) by substitution; it is Lemma 1(ii) with $r_o = E$. $\square$

The cycle $E \rightarrow m \rightarrow \ell \rightarrow T \rightarrow E$ reads $m = E/c^2$, $\ell = h/(mc) = hc/E$, $T = \ell/c = h/E$, and returns $h/T = E$. [OS01a–c] It closes because the four readings are one state. The closing is a statement about readings, not about events: composing identities of one state returns every reading, and no process is run backward; the events themselves do not commute (0.1a). The constants c , h and G are displayed once, in (3), and never fitted. The single reading r_o of Lemma 1 is the measure of one substance in one unit—the numeraire—and density equivalence (I-7) says the same of every channel: mass, energy, temperature and pressure all post to one $\mathcal{M}$ and one R_s .

5.2 The four-momentum book

Theorem 2 (four-momentum is an identity). *Let $G_{\mu\nu} = \kappa T_{\mu\nu}$ with $T_{\mu\nu}$ the one total source $T_{\mu\nu}^{(\Omega)}$, every channel entering once (I-3, 0.5). Then:*

- (a) $\nabla_\mu T^{\mu\nu} = 0$ follows from the identity $\nabla_\mu G^{\mu\nu} \equiv 0$, with no condition beyond the field equation;
- (b) the phase book closes identically: with $T^{(\Omega)\mu\nu} = \sum_a T_a^{\mu\nu}$ and $\nabla_\mu T_a^{\mu\nu} = Q_a^\nu$, $\sum_a Q_a^\nu = 0$ (I-6);
- (c) a configuration's total $\mathcal{M}$, the mass aspect of its exterior solution read on its exterior atlas, is constant while nothing is radiated to the far exterior and decreases by exactly the energy radiated outward; internal relabelling at fixed $\mathcal{M}$ and r_f cannot change α_σ , and every change of α_σ is posted to $\mathcal{M}$ or to the declared radius by $\dot{\alpha}_\sigma/\alpha_\sigma = \sigma(\dot{\mathcal{M}}/\mathcal{M} - \dot{r}_f/r_f)$ (I-6).

Proof. (a) The contracted Bianchi identity holds for every metric; with $\kappa \neq 0$ the field equation gives $\kappa \nabla_\mu T^{\mu\nu} = \nabla_\mu G^{\mu\nu} = 0$. (b) $\sum_a Q_a^\nu = \nabla_\mu \sum_a T_a^{\mu\nu} = \nabla_\mu T^{(\Omega)\mu\nu} = 0$ by (a). (c) The mass aspect of the exterior solution is constant at spatial infinity [5] and, at null infinity, decreases by the radiated flux [6]; the kinematics are the logarithmic derivative of $\alpha_\sigma = (R_s/r_f)^\sigma$ with $R_s = 2G\mathcal{M}/c^2$. $\square$

Remark. The contracted Bianchi identity is the geometric statement that the boundary of a boundary is empty [7, ch. 15]: the conservation of the source is a property of the geometry, not a law laid on it. FDT makes it a property of the one substance. The source is the omnium's one stress tensor; the geometry is the adjustment of the omnium's own patterns (F2); and the read-out $\mathcal{M}$ is the one account every channel posts to (0.5, I-7). The phases are two phases of one substance, so the sum of their exchange rates vanishes identically, (b). In integral form the same statement is the superpotential identity: the source, together with the contribution read from the solved geometry,

is the divergence $\partial_\nu H^{\mu\nu}$ of an antisymmetric superpotential, and $\partial_\mu \partial_\nu H^{\mu\nu} \equiv 0$ —the boundary of a boundary, written in coordinates [7, ch. 20]. The angular-momentum read-out of an isolated configuration is likewise a surface quantity of its exterior solution.

5.3 The spacetime book

Proposition 3 (the support balances its shares). *On the support $S^3 = \{(z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^2 = 1\}$:*

- (a) *at every point the share $\alpha_\sigma = |z_1|^2$ and its complement $1 - \alpha_\sigma = |z_2|^2$ sum to one;*
- (b) *the jacket flip $\mathfrak{J} : (\sqrt{\alpha_\sigma} e^{i\varphi_1}, \sqrt{1 - \alpha_\sigma} e^{i\varphi_2}) \mapsto (\sqrt{1 - \alpha_\sigma} e^{i\varphi_1}, \sqrt{\alpha_\sigma} e^{i\varphi_2})$, the exchange of the two moduli at fixed phases, carries α_σ to $1 - \alpha_\sigma$, exchanges the two solid tori, carries Hopf fibres to Hopf fibres and fixes the Clifford torus pointwise; it is not an isometry of S^3 . The coordinate swap $(z_1, z_2) \mapsto (z_2, z_1)$, $\mathfrak{J}$ composed with the exchange of the two phases, is an isometry that also carries α_σ to $1 - \alpha_\sigma$ and fixes on the Clifford torus only the great circle $z_1 = z_2$;*
- (c) *the round measure of S^3 is uniform in the share: $\text{vol}\{a < |z_1|^2 < b\} = 2\pi^2(b - a)$ for $0 < a < b < 1$, so each solid torus carries π^2 , half of the support's $2\pi^2$, and $\mathfrak{J}$ and the swap both preserve the measure.*

Proof. In toroidal coordinates $z_1 = \cos \eta e^{i\xi_1}$, $z_2 = \sin \eta e^{i\xi_2}$, $0 < \eta < \pi/2$, the round metric is $d\eta^2 + \cos^2 \eta d\xi_1^2 + \sin^2 \eta d\xi_2^2$ and the volume element is $\sin \eta \cos \eta d\eta d\xi_1 d\xi_2$. (a) is the defining equation. [OS04a] (b) $\mathfrak{J}$ is $\eta \mapsto \pi/2 - \eta$ at fixed ξ_1, ξ_2 : it sends $\cos^2 \eta$ to $\sin^2 \eta$, fixes $\eta = \pi/4$ point by point, and sends the Hopf fibre $\{(e^{it} z_1, e^{it} z_2)\}$ through a point to the Hopf fibre through its image. It carries the metric to $d\eta^2 + \sin^2 \eta d\xi_1^2 + \cos^2 \eta d\xi_2^2$, so it is no isometry. [OS04d–f] The swap is unitary on $\mathbb{C}^2$, hence an isometry of S^3 ; it exchanges $|z_1|^2$ and $|z_2|^2$, and on the Clifford torus it fixes exactly the points with $z_1 = z_2$. [OS04g] (c) With $u = \cos^2 \eta = |z_1|^2$, $|du| = 2 \sin \eta \cos \eta d\eta$, so $dV = \frac{1}{2} du d\xi_1 d\xi_2$: uniform in u , with total $\frac{1}{2}(2\pi)^2 = 2\pi^2$. [OS04b, OS04c] $\mathfrak{J}$ keeps $\sin \eta \cos \eta$ and reverses $d\eta$, so it preserves the measure; so does every isometry. $\square$

“Flipping a jacket inside out is the whole of it: the lining becomes the outside, nothing is added and nothing is taken away, and the seam is the one locus that reads the same garment either way” (0.3a). Proposition 3 states this on the support: the two phases share one support whose measure is uniform in the share; the seam divides it into equal halves; the jacket flip carries one half onto the other, preserves the measure and leaves the seam in place point by point (Figure 3). The three-sphere is the shape of the atlas, never a container the omnium occupies (0.1b): the measure is the atlas’s, and the balance it states is between two readings of one substance.

The spacetime phase is decompressed omnium. Expansion is the flow of omnium from the matter phase to the spacetime phase (0.8), which is passage across the seam (0.3a), and the effective cosmological term is the released stored potential accumulated over the domain’s history (0.7):

$$\frac{d\Lambda_{\text{eff}}}{dt} = \frac{8\pi G}{c^2} \frac{1}{V_\Omega} \int_\Omega \delta \dot{m} dV, \quad \Lambda_{\text{eff}} = \frac{8\pi G}{c^4} u_{\text{rel}}. \quad (6)$$

Every increment of the spacetime phase is posted against a debit of the matter phase; nothing is posted from nothing. The release of a stored potential is not a new channel and enters $\mathcal{M}$ once (0.7); Λ_{eff} is a running total that grows with cumulative unwinding. Compression around matter, the mass shadow, is the same substrate moving the other way along the compression axis (0.7). Spacetime is conserved in the one sense one substance allows: the total source is conserved,

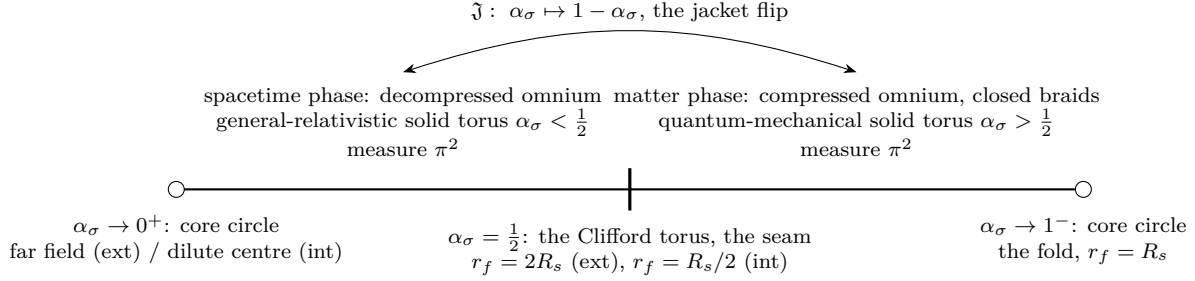


Figure 3: The support on the A-1 chart, read on either branch. The share $|z_1|^2 = \alpha_\sigma$ and its complement $|z_2|^2 = 1 - \alpha_\sigma$ sum to one at every point; the round measure is uniform in the share, so the seam divides the support into halves (Proposition 3). Both ends are circles, never points, and the seam is never the fold; $\mathfrak{J}$ exchanges the two halves and fixes the seam pointwise. Schematic: no trajectory; the only values are the A-1 seam, the fold and the measures of Proposition 3(c).

$\nabla_\mu T^{\mu\nu} = 0$ (Theorem 2), and its spacetime phase changes only by the phase flow. The phase-exchange rates Q_a^ν across the seam and the release function $\dot{m}(x)$, $\delta(x)$ —which bodies unwind, and at what rate—are to be derived (D-02). A stroke crossing a decompressing domain is read with T and ℓ grown and E lowered by one factor (Proposition 2); the domain-level ledger that receives the difference, with what a merger of domains conserves, is to be derived (D-03).

5.4 The helix books

Theorem 3 (integers of framed closures). *Let K be a closed strand of a bound form with declared framing Γ , the push-off $K + \epsilon U$ along a unit normal field U , with ϵ small enough that the ribbon is embedded (I-5). Then*

$$\text{Lk}(K, \Gamma) = \text{Tw}(K, U) + \text{Wr}(K), \quad (7)$$

$$\text{Tw} = \frac{1}{2\pi} \oint (\mathbf{t} \times U) \cdot \frac{dU}{ds} ds, \quad \text{Wr} = \frac{1}{4\pi} \oint \oint \frac{(X(s) - X(s')) \cdot (\mathbf{t}(s) \times \mathbf{t}(s'))}{|X(s) - X(s')|^3} ds ds',$$

with $\text{Lk} \in \mathbb{Z}$, $X(s)$ the strand and $\mathbf{t}$ its unit tangent [8, 9, 10]. Along every evolution of the framed closure that passes no strand through a strand, its ribbon or the axis A , Lk is constant while Tw and Wr exchange continuously, $\Delta \text{Tw} = -\Delta \text{Wr}$. The books of I-5 built on such linking numbers—the charge sign $\text{sgn}(\nu_o \nu_i)$ with $\nu_o = \text{Lk}(A, K)$ about the closure’s axis A and $\nu_i = \text{Lk}(K, \Gamma)$, and the helicity sign $\text{sgn} \nu_i$ —and the charge magnitude $n \bmod 2$ are constant along every such evolution, and change only at a reclosure, where the admissible final states are those that preserve every exact book: four-momentum, charge, fermion parity and angular momentum (I-5b).

Proof. Equation (7) is the Călugăreanu–White–Fuller theorem for a closed curve with a unit normal field. The linking number of the two-component link (K, Γ) , and of (A, K) , is an isotopy invariant with integer values; an integer that varies continuously is constant. The strand count n of a closure changes only when strands are cut and rejoined, which is a reclosure. On framed (2, 3) and (3, 2) torus closures under deformation the Gauss integrals return Lk as an integer, fixed while Tw and Wr trade. [OS05a, OS05b] $\square$

This is the helix’s own double-entry book. A closure coils, precesses, supercoils and deforms through myriad shapes; its geometry trades twist for writhe, and the integer never moves. No law is consulted: the balance is (7). A reclosure is the one event at which an integer changes, and there every exact book balances across the event: decay is loop unwinding through the one channel left open, $m_i c^2 = \sum_k E_{f,k}$ in the rest frame (I-5b). The unbound stroke carries helicity ± 1 as the

spin-1 carrier. The map from a closure's framing integers to the helicities and angular momenta of the strokes it opens into, and the fermion class $\nu_F = B_s \bmod 2$ on the exact framed closures, are to be derived (D-01).

5.5 Nested ledgers: one book at every size

Theorem 4 (nested read-outs).

- (a) *A composite configuration's total is read from its own exterior solution, every channel entering once and binding never added as a local term (0.5). For two parts on the I-1 domain, at the leading weak-field, slow-motion order (I-4, L2), with I-1 inputs $\alpha_i = R_{s,i}/d$ and kinetic content K ,*

$$R_{s,C} = R_{s,1} + R_{s,2} - \frac{1}{2} \alpha_1 \alpha_2 d + \frac{2GK}{c^4}, \quad (8)$$

up to terms of third order in the α_i ; for a bound Kepler orbit of semi-major axis d , $K + U = U(d)/2$ and $R_{s,C} = R_{s,1} + R_{s,2} - \frac{1}{4} \alpha_1 \alpha_2 d$.

- (b) *No bridge of (3) refers to the size of a configuration containing the pattern: a pattern's local readings and the bridges between them are the same read alone or as a part of any composite.*
- (c) *The ledger has one form at every size: every configuration's book is its exterior read-out, its parts' books are theirs, and the parent's read-out differs from the sum of the parts' by the binding read from the solved geometry—at the order of (a), the pair entries—and by what has crossed its boundary.*

Proof. (a) On the I-1 domain $U(d) = -\int_d^\infty F_{\max} R_{s,1} R_{s,2} s^{-2} ds = -F_{\max} \alpha_1 \alpha_2 d$. At leading order the mass aspect of the composite's exterior solution is $\mathcal{M}_C = \mathcal{M}_1 + \mathcal{M}_2 + (K + U)/c^2$; multiply by $2G/c^2$ and use $2GF_{\max}/c^4 = \frac{1}{2}$. [OS08a, OS08b] On a bound Kepler orbit of semi-major axis d the total $K + U$ is $U(d)/2$. [OS08c, OS08d] (b) The bridges contain c , $\hbar$, G and the pattern's own readings and no other length or time (0.1b); α_σ is invariant under $(R_s, r_f) \mapsto (\lambda R_s, \lambda r_f)$ (0.3). (c) follows from (a) and (b) by induction on the depth of nesting, starting from a single pattern read alone. The least closed pattern approaches the floor $d_{\min} = 2\ell_P$, $t_{\min} = 2t_P$ and never occupies it (0.1a), so the nesting is bounded below and never repeats downward without end. $\square$

The pair entry of (8) is built from the product $\alpha_1 \alpha_2$ that I-1 uses for the force: the force and the pair entry are one geometry read twice. For the Sun and Jupiter, with the nominal $(GM)_\odot$ and $(GM)_J$ [18] and the semi-major axis $d = 7.785 \times 10^{11}$ m (on any Kepler orbit $K + U = U(d)/2$), $R_{s,\odot} = 2953.25$ m, $R_{s,J} = 2.8192$ m, and the exterior I-1 inputs are $\alpha_\odot(d) = 3.79 \times 10^{-9}$ and $\alpha_J(d) = 3.62 \times 10^{-12}$; the pair entry of the orbiting system is $-\frac{1}{4} \alpha_\odot(d) \alpha_J(d) d = -2.67$ nm of fold radius. [OS08e] At nuclear size the read-out difference is measured directly (§7.5); the I-1 inputs that return it at nuclear compactness are to be derived (D-05). The rule is one at every size. That is (F3) written as a ledger.

5.6 Every account priced and finite

Proposition 4 (bounded accounts).

- (a) *Every bridge of (3) is defined at every configuration of $\mathcal{D}$: no term of the chain vanishes (0.1), so no ratio of the chain has a vanishing denominator.*

- (b) *Every reading is finite: forces are capped by $F_{\max}$ (0.2, I-1), strokes are bounded below in length and time by $d_{\min} = 2\ell_P$, $t_{\min} = 2t_P$ (0.1a), and α_σ lies on the open interval $(0,1)$ of its branch (I-6).*
- (c) *No admissible region has a singular interior (0.8). The integral balance over an admissible region is therefore a statement about that region's own boundary, with no excised internal boundary: for a stationary configuration through the Killing current $J^\mu = -T^\mu_\nu \xi^\nu$, and for every configuration through the superpotential form of the field equation, in which the source together with the contribution read from the solved geometry is $\partial_\nu H^{\mu\nu}$ with $H^{\mu\nu}$ antisymmetric [7, ch. 20].*

Proof. (a) and (b) are the cited sections. (c) is 0.8 with the divergence theorem applied to J^μ , whose divergence vanishes along a Killing direction, or to $\partial_\nu H^{\mu\nu}$, whose divergence vanishes identically, over a region whose interior is regular. $\square$

The floor is demanded by the chain: $d_{\min} = 2\ell_P = 3.2325 \times 10^{-35}$ m and $t_{\min} = 2t_P = 1.0782 \times 10^{-43}$ s. [OS11a] A ratio cannot be evaluated where a term vanishes, so a chain built of ratios admits no empty state; an empty state would not be a cheap account but an unpriced one. The lowest state of every region is a pattern of positive, finite energy that enters $\mathcal{M}$ once (0.1, 0.5). Of the three boundaries that exclude a singular interior in 0.8, the regular core chart is to be derived (D-04).

6 Why nothing else balances

Definition 5 (automatic balance). A ledger is *connected* when every reading converts into every other through some sequence of processes. A connected ledger *balances automatically* when

- (B1) every rate between readings is a constant of the structure, and every cycle of conversions returns each reading to its starting value;
- (B2) the conservation of the total source is an identity of the geometry: once matter obeys its own equations, the Bianchi identity constrains no geometric variable;
- (B3) every structure that enters a balance is posted by the configuration;
- (B4) every conserved label—every conserved quantity other than the readings of Definition 1—is constant under every continuous deformation of the configuration, with no conservation law of its own;
- (B5) no entry depends on how patterns are grouped into configurations;
- (B6) every reading is defined, positive and bounded on every admissible configuration, and the integral balance over every admissible region is a statement about its own boundary.

The universe's ledger is connected: light is absorbed by matter and emitted by it, closures open into strokes and strokes close into closures, configurations lose mass as strokes and as closures, and one stroke is read in every regime (I-2, I-5, §7). The lemmas below show what each condition of Definition 5 demands.

Lemma 2 (a second substance). *Let the readings fall into two classes A and B, each closed under bridges with constants of its own and each with its own base reading, with no constant of either class relating them. In a connected ledger every conversion process P from A to B supplies a relation $x_B = s_P x_A + w_P$ between the base readings, and every process Q from B to A a relation*

$x_A = s_Q x_B + w_Q$, each offset a datum of its process. Condition (B1) holds if and only if $s_P s_Q = 1$ and $s_Q w_P + w_Q = 0$ for every such pair; when it holds, every process from A to B supplies one relation, a constant joining the two base readings, and by Lemma 1 the two classes are readings of one substance.

Proof. The cycle that converts by P and then by Q has holonomy $x_A \mapsto s_Q(s_P x_A + w_P) + w_Q$, which is the identity on an interval only if $s_P s_Q = 1$ and $s_Q w_P + w_Q = 0$. [OS03a] Two processes P and P' from A to B closed by one Q then have $s_P = s_{P'} = s_Q$ and $w_P = w_{P'} = -s_Q w_Q$. Each process is its own settlement, and the cycle runs two forward processes and reverses neither (0.1a). When every such holonomy is trivial the union graph satisfies Lemma 1(i), hence (ii), with one base reading for both classes. $\square$

Two substances with one universal exchange rate are one substance with two names. Two substances that stay two need a rate for each process, and an arbitrage cycle wherever two processes disagree. The record shows one rate: $1 - \Delta m c^2 / E = (-1.4 \pm 4.4) \times 10^{-7}$ for nuclear gamma strokes against Penning-trap mass differences [13]; and the weight of matter per unit total energy is the same for titanium and platinum, whose mass is divided among rest-mass, nuclear and electromagnetic channels in different proportions, to $\eta = [-1.5 \pm 2.3(\text{stat}) \pm 1.5(\text{syst})] \times 10^{-15}$ [14]. Both are I-7: every channel posts to one $\mathcal{M}$.

Lemma 3 (a second carrier speed). *Let two carriers have invariant speeds $c_1 \neq c_2$, and let a configuration's content convert between them.*

- (a) *Energy E carried by carrier i has inertia E/c_i^2 .*
- (b) *In an isolated configuration at rest with total inertia M , a conversion of content of energy E from carrier 1 to carrier 2 at a point displaced by s from the centre of inertia moves the centre of inertia by $sE(c_2^{-2} - c_1^{-2})/M'$, with $M' = M + E(c_2^{-2} - c_1^{-2})$, and no external impulse.*
- (c) *With weight equal to inertia in a uniform field g , the cycle that lowers the content through Δh in one form and raises it in the other delivers the net work $Eg\Delta h |c_2^{-2} - c_1^{-2}|$ per cycle.*

Conditions (B1) and (B2), through the energy and momentum books, therefore require $c_1 = c_2$: one invariant speed. With one medium (0.1b), one speed is one carrier (I-2), and the speed is the strand rate c (0.1d).

Proof. (a) A body of inertia M and length L emits from one end a pulse of energy E carried at c_i with momentum E/c_i and absorbs it at the other end after L/c_i . The body recoils at $E/(Mc_i)$ for that time and is displaced by $EL/(Mc_i^2)$; the centre of inertia of the isolated system does not move, so the pulse carried inertia E/c_i^2 through L [11]. (b) The first moment changes by $sE(c_2^{-2} - c_1^{-2})$ while the total inertia becomes M' . (c) The work out on lowering in form 2 less the work in on raising in form 1 is $Eg\Delta h(c_2^{-2} - c_1^{-2})$; the cycle run with the two forms exchanged gives the opposite sign. [OS06a–c] $\square$

Effective propagation speeds are not invariant speeds: sound through a lattice of clamps and light through a dielectric are the one stroke read in a regime, and the energy they carry weighs E/c^2 (§3, 0.5). The record shows one stroke speed: gravitational and electromagnetic strokes from one source arrived 1.74 s apart after 1.3×10^8 years of travel, so the fractional difference of their speeds lies between -3×10^{-15} and $+7 \times 10^{-16}$ [15]—gravity's ripple and light are one stroke (I-2).

Lemma 4 (a second geometry). *Let N metric structures $g^{(1)}, \dots, g^{(N)}$ over the same events each obey a field equation of its own, $G_{\mu\nu}^{(i)} = \kappa_i T_{\mu\nu}^{(i)}$, derived from an action invariant under one relabelling*

of events, the source of each containing its share of the interaction. The N contracted Bianchi identities give N divergence conditions $\nabla_\mu^{(i)} T^{(i)\mu\nu} = 0$ on solutions; invariance of the action under the one relabelling supplies one identity among them; $N - 1$ remain as vector constraints on the metrics, satisfied on solutions but not identically. Condition (B2) holds if and only if $N = 1$.

Proof. Write $S = \sum_i S_i[g^{(i)}] + S_{\text{int}}[g^{(1)}, \dots, g^{(N)}] + S_m[g^{(1)}, \psi]$ and $V^{(i)\mu\nu} = (2/\sqrt{-g^{(i)}}) \delta S_{\text{int}}/\delta g_{\mu\nu}^{(i)}$. Matter on its equations obeys $\nabla_\mu^{(1)} T_m^{\mu\nu} = 0$, so the Bianchi identities require $\nabla_\mu^{(i)} V^{(i)\mu\nu} = 0$ for every i . Under a relabelling generated by ξ^λ , $\delta g_{\mu\nu}^{(i)} = \nabla_\mu^{(i)} \xi_\nu^{(i)} + \nabla_\nu^{(i)} \xi_\mu^{(i)}$ with $\xi_\nu^{(i)} = g_{\nu\lambda}^{(i)} \xi^\lambda$; invariance of S_{int} for every ξ gives, after integration by parts,

$$\sum_{i=1}^N \sqrt{-g^{(i)}} g_{\nu\lambda}^{(i)} \nabla_\mu^{(i)} V^{(i)\mu\nu} = 0,$$

one vector identity. For $N = 1$ it is the single condition, satisfied identically, and the Bianchi identity constrains no geometric variable. For $N \geq 2$ it relates the N conditions and leaves $N - 1$ of them as vector constraints that restrict the metrics. $\square$

Two declared charts of one bound configuration (I-3) are two readings of one geometry with one field equation; they add no condition. A second invariant speed is a second cone structure (P-4), so Lemma 3 is the kinematic face of Lemma 4.

Lemma 5 (a fixed background).

- (a) *On a geometry $\bar{g}$ that is not varied, the source obeys $\bar{\nabla}_\mu T^{\mu\nu} = 0$, and the energy current $J^\mu = -T^\mu{}_\nu \xi^\nu$ obeys $\bar{\nabla}_\mu J^\mu = -T^{\mu\nu} \bar{\nabla}_\mu \xi_\nu$: the energy book closes only along a Killing direction of the background, and where the background has none it supplies or absorbs energy with no account of its own. On the solved geometry the same exchange is posted to the mass aspect read on the exterior (Theorem 2(c)); on an unvaried background it is posted nowhere.*
- (b) *Lower a pattern of mass m through Δh in a field g , extracting $mg\Delta h$; open it into strokes of total energy mc^2 ; send them up. If the patterns at the top kept the same time as the patterns at the bottom—a background whose intervals are untied from every pattern—the strokes would be read at the top with energy mc^2 , and closing them into a pattern of mass m would complete a cycle that posts $mg\Delta h$ from nothing. The cycle closes if and only if the strokes are read at the top with energy $mc^2(1 - g\Delta h/c^2)$ to first order in $g\Delta h/c^2$: the kept time of patterns adjusts with height by $\Delta T/T = g\Delta h/c^2$.*

Condition (B3) therefore excludes a fixed background and requires (F2).

Proof. (a) $\bar{\nabla}_\mu (T^\mu{}_\nu \xi^\nu) = (\bar{\nabla}_\mu T^\mu{}_\nu) \xi^\nu + T^{\mu\nu} \bar{\nabla}_\mu \xi_\nu$, and $T^{\mu\nu}$ is symmetric. (b) In a static field the number of crests a stroke carries between two background instants is the same at both heights, so a background whose intervals are untied from the patterns reads the same frequency, hence the same energy, at both. The work out over the cycle is $mg\Delta h$; the pattern closed at the top has energy equal to the strokes' reading there; returning to the start requires that reading to be $mc^2 - mg\Delta h$. By Proposition 2 the strokes' energy reading scales inversely with their period, and the period is read against the time the patterns keep, which fixes $\Delta T/T$ [12]. $\square$

The record shows the adjustment. Over the 74-ft (22.56 m) tower the two-way shift of strokes is $-(5.13 \pm 0.51) \times 10^{-15}$ against the predicted 4.92×10^{-15} , the ratio 1.05 ± 0.10 [16]. [OS07d] Across a single millimetre-scale sample of atoms the kept time runs at $-9.8(2.3) \times 10^{-20}$ per millimetre against the expected -10.9×10^{-20} per millimetre [17]. The kept time of patterns adjusts across a millimetre; no interval is held fixed beneath them.

Lemma 6 (labels). *A conserved label that is not fixed by the configuration needs a conservation postulate of its own. Condition (B4) holds if and only if every conserved label is locally constant on the space of configurations—a discrete invariant—and such a label needs no law between the events that change the configuration’s topology; for closures these are the integers of framed closures (Theorem 3).*

Proof. A label not fixed by the configuration takes values along a given evolution that the evolution does not determine; only a postulate fixes them. A label constant under every continuous deformation is constant on each connected component of the space of configurations, which is what a discrete invariant is; conversely, an isotopy invariant is fixed along every isotopy with no law consulted. $\square$

Lemma 7 (a size-selecting chain). *If a bridge rate depends on the size L of the configuration in which a reading is taken, $k = k(L)$, then regrouping a pattern into a composite of another size changes its entry by $(k(L_2) - k(L_1))r$ with no transaction. Condition (B5) holds if and only if every rate is free of size: the chain is built of ratios that select no size, and the ledger has one form at every size.*

Proof. The entry of a reading r through the bridge is $k(L)r$; regrouping changes L and nothing else. Conversely, size-free rates give Theorem 4(b). A floor is a bound, not a rate: it fixes no spacing above it and changes no bridge (0.1a). $\square$

(F3) is therefore not an extra assumption: it is what a balanced ledger requires of its chain. Built of ratios, the chain selects no size, and the universe repeats its form at every size: it is fractal. The bound below is supplied by (B6), through Lemma 8: bounded readings give strokes a floor.

Lemma 8 (vanishing terms, divergent readings, singular interiors).

- (a) *At a configuration where a term of the chain vanishes, the ratio with that term as denominator is undefined: the account is unpriced, and no cycle through it can be evaluated.*
- (b) *A reading allowed to diverge absorbs every finite entry without change: its balance is void.*
- (c) *An admissible region containing a singular point acquires, in the integral balance of Proposition 4(c), an excised boundary whose flux no reading fixes.*

Condition (B6) holds if and only if no term of the chain vanishes, every reading is bounded, and no admissible region has a singular interior. Bounded readings give strokes a floor: an energy bounded above per stroke bounds its period below through $T = h/E$, and its advance per cycle through $\ell = cT$.

Proof. (a) and (b) are immediate. (c) The integral balance over a region with a point removed carries the flux through a small surface about the point; at a singular point that flux has no limit fixed by the readings of the region. The floor follows from (3). $\square$

Theorem 5 (parsimony). *A connected ledger balances automatically if and only if*

- (i) *it has one substance: every reading is a constant multiple of a power ± 1 of one numeraire (Lemma 1), so no reading changes independently of the others (F1);*
- (ii) *it has one carrier: one invariant speed, the strand rate, and one mass–energy rate;*
- (iii) *it has one geometry, obeying $G_{\mu\nu} = \kappa T_{\mu\nu}$ with the one total source;*
- (iv) *it has no fixed background: the kept time and thread length of every closed pattern adjust with the configuration (F2);*

Table 3: Each multiplicity and the entry it would post by hand, against the record.

multiplicity	entry posted by hand	lemma	the record
second substance	a cross-rate for each conversion process	2	$1 - \Delta m c^2 / E = (-1.4 \pm 4.4) \times 10^{-7}$ [13]; $\eta(\text{Ti, Pt}) = [-1.5 \pm 2.3 \pm 1.5] \times 10^{-15}$ [14]
second carrier speed	a centre-of-inertia jump; a work cycle	3	$-3 \times 10^{-15} \leq \Delta v / c \leq +7 \times 10^{-16}$ for gravitational against electromagnetic strokes [15]
second geometry	$N - 1$ vector constraints on the metrics	4	one cone structure for both strokes, to the same bound [15]
fixed background	$mg\Delta h$ per cycle; energy only along Killing directions	5	two-way shift ratio 1.05 ± 0.10 over the 22.56 m tower [16]; $-9.8(2.3) \times 10^{-20}$ per mm against -10.9×10^{-20} [17]
labels not fixed by configuration	a postulate for each label	6	structural: integers of framed closures (Theorem 3)
size-selecting chain	entries that depend on grouping	7	one c^2 from nuclear transitions [13] through laboratory matter [14]; one stroke speed across 40 Mpc [15]
vanishing term, singular interior	unpriced or unaudited accounts	8	structural: the floor the chain demands (0.1, 0.1a)

(v) *every conserved label is a discrete invariant of the configuration—for closures, an integer of a framed closure;*

(vi) *its chain is built of ratios: one ledger at every size;*

(vii) *no term of its chain vanishes and every reading is bounded, so strokes have a floor, and no admissible region has a singular interior.*

FDT is this configuration: (i) 0.1, 0.1a, 0.1b; (ii) I-2, 0.1d; (iii) I-3, P-4, 0.3, 0.3a; (iv) 0.1a, 0.1c; (v) I-5, I-5a, I-5b; (vi) 0.3, 0.3a; (vii) 0.1, 0.1a, 0.2, I-6, 0.8.

Proof. Necessity. (B1) gives (i) by Lemma 2; (B1) and (B2) give (ii) by Lemma 3; (B2) gives (iii) by Lemma 4; (B3) gives (iv) by Lemma 5; (B4) gives (v) by Lemma 6; (B5) gives (vi) by Lemma 7; (B6) gives (vii) by Lemma 8. Sufficiency. (i) and (ii) make every rate a constant of the structure and close every cycle of bridges (Theorem 1); (iii) gives (B2) by Theorem 2(a); with (iv), every structure that enters a balance is posted by the configuration, which is (B3), and every cycle of conversions closes by Theorem 2 and Lemma 5(b), which completes (B1); (v) gives (B4) by Theorem 3; (vi) gives (B5) by Theorem 4(b); (vii) gives (B6) by Proposition 4. $\square$

The weight of the theorem sits in Lemmas 2–5: they show that one substance, one carrier, one geometry and the absence of a fixed background are forced by the books themselves. Lemmas 6–8 state what the remaining conditions of Definition 5 mean for the chain and its forms.

7 Worked ledgers

7.1 The Sun’s stroke entry

The Sun’s photon luminosity read far from it is $L_{\odot} = 3.828 \times 10^{26}$ W and $(GM)_{\odot} = 1.3271244 \times 10^{20} \text{ m}^3 \text{ s}^{-2}$ [18]; with G from [19], $\mathcal{M}_{\odot} = 1.98841 \times 10^{30}$ kg. Table 4 posts the stroke entry to every reading of Proposition 1. [OS09a–c] The exterior reading at the photosphere, $\alpha_{\text{ext}}(R_{\odot}) = 4.245 \times 10^{-6}$,

lies in the general-relativistic regime of the A-1 chart; the exterior seam $r_f = 2R_s$ sits far inside the photosphere, where the exterior reading is not taken. [OS09d]

Table 4: The Sun’s stroke entry, posted to every reading (Julian year).

reading	per second	per year
$\Delta\mathcal{M} = -L_\odot/c^2$	$-4.2592 \times 10^9 \text{ kg}$	$-1.3441 \times 10^{17} \text{ kg}$
$\Delta R_s = -2GL_\odot/c^4$ (of $R_{s,\odot} = 2953.25 \text{ m}$)	$-6.3259 \times 10^{-18} \text{ m}$	$-1.9963 \times 10^{-10} \text{ m}$
$\Delta \ln \alpha_{\text{ext}}$ at every fixed r_f (exterior branch)	-2.1420×10^{-21}	-6.7597×10^{-14}

Static patterns at the photosphere read the same strokes with $L_{\text{loc}} = L_\odot/(1 - \alpha_{\text{ext}}(R_\odot)) = L_\odot + 1.625 \times 10^{21} \text{ W}$. The difference is not lost and is not a binding account: it is the collective adjustment of 0.1a written for two bookkeepers. Each stroke’s energy reading carries one factor $(1 - \alpha_{\text{ext}}(R_\odot))^{1/2} = 1 - 2.1225 \times 10^{-6}$, and the rate at which strokes are counted carries the other, the kept time of the emitting patterns. [OS09e] The far reading of the Sun’s account changes by $L_\odot/c^2$ per second, and that is the entry. The $n = 2$ closures that leave in the free room and the $n = 3$ closures of the outflow post to the same $\mathcal{M}$ through the same bridges.

7.2 A kilogram lowered and returned as light

The cycle of Lemma 5(b) with $m = 1 \text{ kg}$, $g = 9.80 \text{ m s}^{-2}$ and $\Delta h = 22.56 \text{ m}$, the 74-ft tower of [16]: lowering extracts $mg\Delta h = 221.09 \text{ J}$; the pattern opens into strokes of $mc^2 = 8.987552 \times 10^{16} \text{ J}$; read at the top they carry $mc^2 - 221.09 \text{ J}$, the fraction $g\Delta h/c^2 = 2.4599 \times 10^{-15}$; closed at the top they make a pattern of $1 \text{ kg} - 2.4599 \times 10^{-15} \text{ kg}$; and the cycle posts zero. [OS07a, OS07b] On a background whose intervals were untied from the patterns, the same cycle would post $+221.09 \text{ J}$ every time it ran. The two-way fraction $2g\Delta h/c^2 = 4.92 \times 10^{-15}$ is the prediction against which [16] measured $-(5.13 \pm 0.51) \times 10^{-15}$; at Earth’s surface the kept time adjusts by $g/c^2 = 1.09 \times 10^{-19}$ per millimetre, measured within one atomic sample [17]. [OS07c]

7.3 Two closures open into light

An electron and a positron at rest are one $n = 3$ configuration in opposite handedness (P-4). With total angular momentum $J = 0$ they open into two strokes of 510.999 keV , back to back. The books: four-momentum, $2m_e c^2$ into two strokes of equal energy and opposite momentum; charge, $(+1) + (-1) = 0$; fermion class as measured, $1 + 1 \equiv 0 \pmod{2}$, so a purely bosonic final state is admissible (I-5b); angular momentum, two strokes of equal helicity, with zero projection on their common axis. With $J = 1$ the same pair opens into three strokes. Each stroke advances $\ell = h/(m_e c) = 2.42631 \text{ pm}$ per cycle, in its period $T = 8.0933 \times 10^{-21} \text{ s}$: the bridge $\ell_q = cT_q = h/(m_q c)$ of 0.1a, carried from the closed reading, the length the thread runs in the time the closure keeps, to the open one, which keeps no time (0.1d)—the length a thread runs, never the support of the closure (I-5). [OS10a]

7.4 One closure recloses into three

The $n = 2$ closure read as the neutron recloses as $2 \rightarrow 3 + 3 + 2$: the $n = 3$ closures read as the proton and the electron and the $n = 2$ closure read as the antineutrino (I-5b). With the CODATA masses [19] the read-out difference is $Q = (m_n - m_p - m_e)c^2 = 0.78233 \text{ MeV}$ [OS10b], shared among three products, which is why the free $n = 3$ closure leaves with a continuous range of energies. The books: four-momentum, $m_n c^2 = m_p c^2 + m_e c^2 + E_{\bar{\nu}}$ kinetic; charge, $0 = (+1) + (-1) + 0$; fermion

class as measured, $1 \equiv 1 + 1 + 1 \pmod{2}$. [OS10e] The strand count goes from 2 to 8 and is not a book: n is a per-closure label and never an additive charge (I-5b). The third entry is what makes the books balance; the $n = 2$ closure is that entry.

7.5 Two closures reclose into one pattern and a stroke

In $n + p \rightarrow d + \gamma$ two closures reclose into one pattern and one stroke. The read-outs give $Q = (m_p + m_n - m_d)c^2 = 2.224566 \text{ MeV}$ [19]. The stroke carries 2.223249 MeV and the pattern takes the recoil, 1.318 keV , which the momentum book requires. The composite's rest read-out is less than the sum of the two closures' by $Q/c^2 = 3.9656 \times 10^{-30} \text{ kg}$, 0.118% : [OS10c, OS10d] the stroke carries its share away, the recoil stays with the pattern as kinetic content, and no binding account is added (0.5). It is the rule of Theorem 4(c) at nuclear size, the rule that gives the Sun–Jupiter system its 2.67 nm : one ledger, two sizes.

7.6 Unwinding into the spacetime phase

A heavy closure that unwinds by fission (I-1a) posts its read-out difference to the strokes and the kinetic content of the fragments, and the stored potential of its compression to the spacetime phase (0.7). The matter phase posts the debit and the spacetime phase the credit; Λ_{eff} is the running total of such credits, Eq. (6), whose two sides carry the units of a curvature per unit time and of a curvature. [OS11b]

8 The record and the tests

Table 3 sets each multiplicity against the record, and the record shows no entry posted by hand. Each test below is the converse of a derived result, the measurement that would end it:

- T-1 *One rate.* A conversion of matter into strokes at a rate other than c^2 , a measured $1 - \Delta m c^2/E \neq 0$, ends the one numeraire of Theorem 1 and Lemma 2.
- T-2 *One weight.* A composition-dependent free fall of bodies whose energy is divided among rest-mass, nuclear and electromagnetic channels in different proportions ends the one account of I-7 that Lemma 2 requires.
- T-3 *One stroke speed.* A measured difference between the speeds of the gravitational and the electromagnetic strokes of one source ends Lemma 3 and Lemma 4.
- T-4 *No fixed background.* A gravitational shift of the kept time other than $g\Delta h/c^2$ at first order ends Lemma 5(b).
- T-5 *Integers.* A charge or a helicity sign that changes along a deformation with no reclosure ends Theorem 3.

Five pieces sit outside the derivation and leave every result in it unchanged:

- D-01 *Framing to strokes.* The map from a closure's framing integers (ν_o, ν_i, f) to the helicities and angular momenta of the strokes it opens into, and the fermion class $\nu_F = B_s \pmod{2}$ on the exact framed closures, are to be derived (core O-11, O-12).
- D-02 *The phase book's rates.* The phase-exchange rates Q_a^ν across the seam, and the release function $\dot{m}(x)$, $\delta(x)$ that feeds Λ_{eff} , are to be derived (core O-12, O-14).

- D-03 *The domain ledger.* Each universal domain's $\mathcal{M}$ read on its own boundary, what a merger of domains conserves, and the ledger that receives the energy strokes give up crossing a decompressing domain are to be derived (core O-15).
- D-04 *The regular core.* The regular core chart, with the finite-core radius r_c , used by Proposition 4(c), is to be derived (core O-1).
- D-05 *Nuclear inputs.* The I-1 inputs at nuclear size, the maps from a seat to an I-1 input, that return the measured read-out differences of reclosures such as $n + p \rightarrow d + \gamma$ are to be derived (core O-6).

9 Declined readings and convergence

The standard account reads an interaction as the exchange of virtual quanta that borrow energy for a time of order $\hbar/\Delta E$, the books reopened and closed inside each exchange. FDT reads every interaction as curvature plus geodesic motion of one carrier with no mediation (I-2), and every configuration as keeping the chain at every instant with its terms adjusting together (0.1a): the books are never open.

The standard account reads a reaction and its reverse through one matrix element, detailed balance, and a response as one-to-one between particles. FDT reads every step as a system-wide settlement solved from the state the system is in, many-to-many, with events that do not commute (0.1a): the closing of an exchange cycle (Theorem 1) is an identity among the readings of one state, never a forward event mirrored by its reverse.

The standard account sets the energy of the empty state to zero by subtracting a zero-point sum that diverges without a cutoff. FDT has no empty state and no divergent sum: no term of the chain vanishes (0.1), the floor bounds every stroke (0.1a), and the lowest state of every region is a pattern of positive, finite energy that enters $\mathcal{M}$ once (0.5); there is nothing to subtract.

The standard account carries dark energy as a cosmological constant, a vacuum energy fixed for all time. FDT reads the same acceleration as the released stored potential of compressed omnium, with no Λ term and Λ_{eff} the running total of releases (0.7): every increment of the spacetime phase is posted against a debit of the matter phase.

The standard account carries charge and the fermion property as intrinsic labels, each conserved by a symmetry postulated for it. FDT reads them as integers of framed closures of the one carrier (I-5), held by $L_k = Tw + Wr$ through every deformation and changed only at reclosures where every exact book balances (I-5b).

The standard account computes fields on a fixed spacetime background whose energy book closes only along the background's symmetries. FDT has no background: clocks and rulers are patterns whose adjustment is the geometry (0.1a), and the geometry posts every change through $G_{\mu\nu} = \kappa T_{\mu\nu}$ (I-3).

The standard account admits interiors where curvature and density diverge, regions whose content leaves every book. FDT has none: no term of the chain vanishes, force is capped by F_{max} , and the density coordinate lives on the open interval with a regular core (0.8); a compact object is a hollow, double-sided density boundary, dilute at the centre and densest at the skin (0.6).

Convergence. For the same source and data FDT and general relativity solve the same equation and return the same curvature (I-3). The local identity $\nabla_\mu T^{\mu\nu} = 0$, the mass aspect and its loss by radiated flux, the gravitational shift of strokes and kept times, the composite read-out of Eq. (8) and every read-out difference of §7 are the consensus results, recovered exactly at the orders

stated. The measurements used here are accepted in full: $1 - \Delta m c^2/E = (-1.4 \pm 4.4) \times 10^{-7}$ [13]; $\eta(\text{Ti, Pt}) = [-1.5 \pm 2.3(\text{stat}) \pm 1.5(\text{syst})] \times 10^{-15}$ [14]; $-3 \times 10^{-15} \leq \Delta v/c \leq +7 \times 10^{-16}$ [15]; the two-way shift ratio 1.05 ± 0.10 over the 22.56 m tower [16]; $-9.8(2.3) \times 10^{-20}$ per millimetre [17]; the solar and Jovian nominal values [18]; and the CODATA masses [19]. Each is a reading of the configuration of Theorem 5, and each shows the absence of an entry that Lemmas 2–5 would require. What FDT declines—borrowed energy, mirrored events, a subtracted empty state, a constant Λ , intrinsic labels, a fixed background, singular interiors—are readings, not observables. FDT supplies an additional source in one book: the spacetime phase receives the released stored potential of unwinding matter, and Λ_{eff} is the running total of those releases (0.7), with its release function to be derived (D-02).

10 One substance, one ledger

The universe keeps one state and reads it many ways. Energy, mass, frequency, the period and the length a thread runs in it are tied by the bridges of one chain, so no transaction posts to one reading alone and every cycle of readings closes. The four-momentum book is the boundary of a boundary of the one geometry; the spacetime book is the decompressed phase of the same substance, credited only by a debit of the matter phase; the books of charge, helicity and fermion class are integers of framed closures that no deformation moves. Because the chain is built of ratios, the ledger has one form at every size, from nucleus to domain, bounded below by the floor. Every other configuration needs an entry posted by hand, and the record shows none: one rate, one weight, one stroke speed, and kept times that adjust across a millimetre.

11 Status register

result	rests on	status
No single-entry posting (Proposition 1)	0.1, 0.1a, I-6	derived
The two invariants of a stroke (Proposition 2)	0.1a, 0.1d	derived
One numeraire; exchange cycles close (Lemma 1, Theorem 1)	0.1, 0.1a	derived
Four-momentum and the phase book as identities (Theorem 2)	I-3, I-6, 0.5	derived
The support's shares and measure; the jacket flip and the swap (Proposition 3)	0.3a	derived
Integers of framed closures (Theorem 3)	I-5, I-5b	derived
Nested read-outs (Theorem 4)	0.3, 0.5, I-1, I-4	derived, at the order stated
Bounded accounts (Proposition 4)	0.1, 0.1a, 0.2, 0.8, I-6	derived; the regular core D-04
Parsimony (Lemmas 2–8, Theorem 5)	Definition 5	derived
The worked ledgers (§7)	I-5b, 0.5, 0.7	derived from the record
Framing integers to stroke helicities; ν_F on exact closures	I-5, I-5b	to be derived (D-01; core O-11, O-12)
Phase-exchange rates; the release function	I-6, 0.7, 0.8	to be derived (D-02; core O-12, O-14)
The domain ledger and merger kinematics	0.8	to be derived (D-03; core O-15)

result	rests on	status
The regular core chart	0.3, I-6, 0.8	to be derived (D-04; core O-1)
I-1 inputs at nuclear size	I-1, I-5a	to be derived (D-05; core O-6)

invariant	where it enters
I-1	the pair energy $U(d) = -F_{\max}\alpha_1\alpha_2d$, the entry a composite carries for its parts (Theorem 4)
I-2	one carrier: one invariant speed and one mass–energy rate (Lemma 3); the open stroke keeps no time (Proposition 2)
I-3	the contracted Bianchi identity as the four-momentum book (Theorem 2); the optical index as the kept time’s adjustment (Lemma 5)
I-4	the composite read-out at the containment order L2 (Theorem 4); the deterministic helical phase
I-5	closures, their framing integers and their reclosures (Theorem 3, §7)
I-6	the branch-aware kinematics and the phase book (Proposition 1, Theorem 2)
I-7	every channel posts to one $\mathcal{M}$: the single numeraire (Lemma 1, Lemma 2)

A Receipt

The script `receipt_one_substance.py`, in the folder of this paper, checks the displayed identities symbolically with SymPy and the displayed numbers numerically with the CODATA 2022 values and the IAU 2015 nominal values, with Gauss integrals for the linking, twist and writhe of framed torus closures. Run it with `python3 receipt_one_substance.py`; its output is `receipt_one_substance.out.txt`. Result: **41/41 PASS**.

check	content
OS01a–d	the bridges and the numeraire (5); the cycle $E \rightarrow m \rightarrow \ell \rightarrow T \rightarrow E$; an affine map of slope ± 1 fixing two points is the identity
OS02a–c	the two invariants under $1 + z$; the two rows of Table 2
OS03a	the holonomy of two substances: $s_P s_Q = 1$, $s_Q w_P + w_Q = 0$
OS04a–g	the shares; the volume element; $2\pi^2(b - a)$, and π^2 for each solid torus; $\mathfrak{J}$ carries α_σ to $1 - \alpha_\sigma$ and keeps the measure; $\mathfrak{J}$ is no isometry; $\mathfrak{J}$ fixes the Clifford torus pointwise and keeps the Hopf fibres; the swap is an isometry fixing there only $z_1 = z_2$
OS05a–b	$Lk = Tw + Wr$ with Lk an integer on framed $(2, 3)$ and $(3, 2)$ torus closures; Lk constant along a deformation while Tw and Wr trade
OS06a–c	the inertia E/c_i^2 of a carried pulse; the shift of the centre of inertia; the work per cycle, zero only for $c_1 = c_2$
OS07a–d	the kilogram cycle: 221.09 J , 2.4599×10^{-15} ; 4.92×10^{-15} and 1.09×10^{-19} per mm; the Pound–Rebka ratio
OS08a–e	$U(d) = -F_{\max}\alpha_1\alpha_2d$; the composite read-out (8); $K + U = U(d)/2$ on a Kepler orbit, checked at perihelion of an ellipse; the Sun–Jupiter entry -2.67 nm
OS09a–e	$\mathcal{M}_\odot$; the Sun’s stroke entry per second and per Julian year; $\alpha_{\text{ext}}(R_\odot)$ and the seam; $L_{\text{loc}} - L_\odot$ and $(1 - \alpha_{\text{ext}})^{1/2}$
OS10a–e	annihilation: 510.999 keV , 2.42631 pm , $8.0933 \times 10^{-21} \text{ s}$; beta: 0.78233 MeV ; capture: Q , the stroke, the recoil, Q/c^2 and 0.118% ; the charge and fermion-class books
OS11a–b	the floor; the units of the released-potential law (6)

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