The orbit lattice: turning without travelling, and how a driver writes it

Magnons, phonons, light, the standing stroke and the tide on one picture; deterministic control by pulses and other drivers

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Fundamental Density Theory

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Abstract

In the animation every dot turns on its own fixed circle and no dot travels; the waves that cross the screen are the phase law between neighbouring dots. The blank between the dots stands for the omnium, which is never empty: no term of the central invariant vanishes, so no site, no wave and no interval between sites is ever at rest. This manuscript measures the animation, writes its kinematics exactly, and reads it on the locked core [1]. The orbit lattice is the quintuple identity (I-2) drawn. One turning, read between neighbours (I-3a), is at once a packing wave, a twist wave and a tide: the scalar grade, the bivector grade and the shear of one deviation dyad. The rate of every orbit wave is the cost of a unit of circulation, $\omega = dE/dL$, so each quantum carries $\hbar$. The phonon is that turning held by the inertia of the lattice's sites; the magnon is the same turning of moments held in a twist; light is the stroke turning at its own run; the gluon is the turning standing in a closure; gravity's ripple is read as the turning of free patterns against a reference path. Because every term of the chain adjusts at once, the universal lattice sizes its circles to the density: radius and period rescale together by the clock factor, the speed on the circle stays c, and the growth of the circles between neighbours is the free fall the bookkeeper reads. A driver writes a turning when it carries the wave's own rate, phase law, hand and projection, and the response is then fixed by the driver's time course and the state the lattice is in: pulse arrival sets the start azimuth, the delay of two crossed pulses sets the hand, an equal pulse half a period later cancels the written turning, a sweeping front writes the wave vector. FDT returns the measured record of each, and no divergence follows.

1 The animation

A square lattice of circles of one radius fills the frame, with one dot on each circle. Every dot runs its circle clockwise at one rate. The centres never move. Bands of crowded dots travel up and to the left at a steady speed. Follow any one dot and it goes nowhere: the bands are made of different dots at every instant. What travels is a relation between neighbours, each dot a fixed fraction of a turn behind the next. The screen between the dots is blank only in the drawing: in every reading below it is the omnium, the one medium, and the omnium settles the whole system at every step, from the state it is in (0.1, 0.1a, 0.1b).

Table 1 gives the animation's numbers as measured from its 212 frames (Appendix A), and Fig. 1 redraws it from them.

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quantity	measured	in lattice units
lattice	square, about 20×20 orbits in view	spacing $\mathfrak{d} = 39.83$ px
orbit radius $\mathfrak{r}$	35.54 px, every orbit	$\mathfrak{r}/\mathfrak{d} = 0.892$: neighbouring circles overlap
turning	clockwise, 30.2 frames per turn	2.11 s at the recording's 70 ms per frame
phase step per site	0.448 rad along rows, 0.449 along columns, each ± 0.001	$2\pi/14 = 0.449$: fourteen sites per turn of phase
run of the fronts	the diagonal, 135° from the rows	wavelength $\lambda = 9.90\,\mathfrak{d}$
turn ratio $\mathfrak{h} = \mathfrak{r}k$	0.566	fronts move $1/\mathfrak{h} = 1.77$ times faster than dots
packing, crest and trough	2.31 and 0.638 of the mean	ratio 3.61
first harmonic of the packing	0.267	$J_1(\mathfrak{h}) = 0.272$, eq. (5)

Table 1: The animation, measured. The turning rate is the same for every dot to a part in ten thousand, and the phase law fitted on the lattice's sites holds across the frame with coherence 0.987.[A01–A10]

2 The orbit lattice, exactly

2.1 The phase law and the rates

Let $\mathbf{y}$ label a site, the fixed centre of an orbit, and let $\mathbf{e}(\varphi) = \cos\varphi\,\mathbf{e}_1 + \sin\varphi\,\mathbf{e}_2$ with $\mathbf{e}_1, \mathbf{e}_2$ spanning the plane of the orbits and $\mathbf{N} = \mathbf{e}_1 \times \mathbf{e}_2$ its normal. The dot of site $\mathbf{y}$ is at

$$\mathbf{x}(\mathbf{y}, t) = \mathbf{y} + \mathbf{u}(\mathbf{y}, t), \qquad \mathbf{u} = \mathfrak{r}\,\mathbf{e}(\varphi), \qquad \varphi(\mathbf{y}, t) = \mathbf{k}\cdot\mathbf{y} - \omega t + \varphi_0. \tag{1}$$

The whole pattern is the *phase law* φ: one radius $\mathfrak{r}$, one rate ω, one wave vector $\mathbf{k}$, one hand (here clockwise about $\mathbf{N}$; the other hand is $\mathbf{e}_2 \mapsto -\mathbf{e}_2$) and one start azimuth φ_0. In this section the run lies in the orbit plane, $\mathbf{k} = k\,\mathbf{e}_1$, as in the animation; Section 2.5 takes the general case.

Three rates belong to the pattern, and a fourth is absent:

turning rate	$\mathfrak{r}\omega$	the speed of each dot on its circle
phase rate	ω/k	the speed of the fronts
run	$d\omega/dk$	the speed of a modulation of the turning; it needs a law $\omega(\mathbf{k})$, which a kernel supplies (Sections 5, 6)
drift	0	no dot leaves its circle

The *turn ratio* is the turning rate over the phase rate,

$$\mathfrak{h} \equiv \mathfrak{r}k = \frac{\mathfrak{r}\omega}{\omega/k}. \tag{2}$$

2.2 The packing: what a bookkeeper at a fixed place counts

The map $\mathbf{y} \mapsto \mathbf{x}$ has the gradient $\mathsf{F} = 1 + \mathfrak{r}\,\mathbf{e}'(\varphi) \otimes \mathbf{k}$, a rank-one change of the identity, so its determinant is exact:

$$\det \mathsf{F} = 1 - \mathfrak{h}\sin\varphi, \qquad \frac{\mathfrak{n}}{\mathfrak{n}_0} = \frac{1}{1 - \mathfrak{h}\sin\varphi}, \tag{3}$$

with $\mathfrak{n}$ the packing density of the dots and $\mathfrak{n}_0$ that of the sites.[B01] The crest is at $\varphi = \pi/2$, where the dot moves with the fronts; the trough is at $\varphi = -\pi/2$, where it moves against them. The pattern is single-valued exactly on the open interval

$$0 < \mathfrak{h} < 1.$$

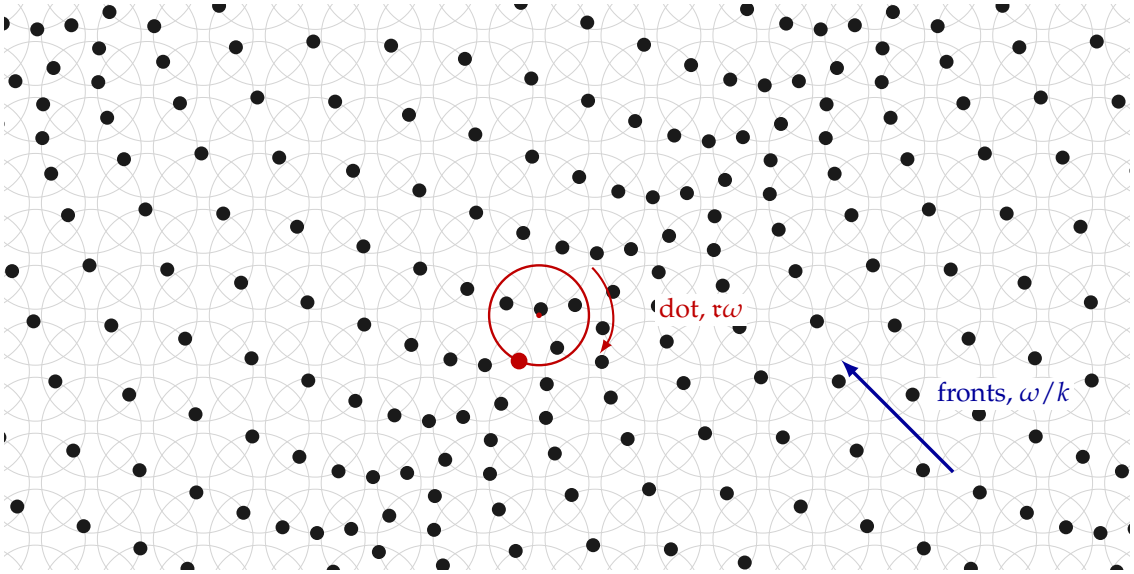


Figure 1: The orbit lattice at the animation's measured values: $\mathfrak{r}/\mathfrak{d} = 0.892$, phase step $2\pi/14$ per site on each axis, clockwise turning. Every dot stays on its own circle (one is marked). The crowded bands are the crests of the packing; they run up and to the left at the phase rate while each dot turns at $\mathfrak{h} = 0.566$ of that rate.

As $\mathfrak{h} \to 1^-$ the dot at the crest keeps pace with the front and the crest packing grows without bound: the caustic, the cusp of a cycloid. The two ends are one-sided limits. The lower end, $\mathfrak{h} \to 0^+$, is the lattice with no turning, and it is never reached: no term of the chain vanishes (0.1), so every wave of a lattice keeps a floor of turning (Section 9.7). On the interval the crest excess and the trough deficit are

$$\frac{\mathfrak{n}_{\text{crest}}}{\mathfrak{n}_0} - 1 = \frac{\mathfrak{h}}{1-\mathfrak{h}}, \qquad 1 - \frac{\mathfrak{n}_{\text{trough}}}{\mathfrak{n}_0} = \frac{\mathfrak{h}}{1+\mathfrak{h}}. \tag{4}$$

The turn ratio is a ratio of two rates, kinematic as the translation share $\mathfrak{s}$ of 0.1d is: it is no application of 0.4 and stands in no force or optical slot, and the density it sets is the packing density of the dots, named as such at every use (0.6).[B10] Whether a packing ratio of the orbit lattice is the odds of application A-3 is D-L2 (Section 11); no A-3 token is used here.

A bookkeeper at a fixed place $\mathbf{x}$ reads the phase $\Xi = kx_1 - \omega t + \varphi_0$, and (1) gives $\Xi = \varphi + \mathfrak{h}\cos\varphi$. This is Kepler's equation, with $\pi/2 - \varphi$ the eccentric anomaly, $\pi/2 - \Xi$ the mean anomaly and the turn ratio in the place of the eccentricity.[B09] Its classical series [3] gives the packing the bookkeeper counts,

$$\frac{\mathfrak{n}}{\mathfrak{n}_0} = 1 + 2\sum_{\ell \geq 1} J_\ell(\ell\mathfrak{h})\,\cos\big[\ell(\Xi - \tfrac{\pi}{2})\big], \tag{5}$$

with J_ℓ the Bessel functions.[B08] The first harmonic of the animation's packing measures 0.267; $J_1(0.566) = 0.272$ (Fig. 2a).

No dot travels: the mean velocity of every dot is zero, and so is the count-weighted flux.[B12] The bookkeeper at a fixed place meets the stretched troughs for longer than the crowded crests and reads a mean velocity $-(\mathfrak{h}^2/2)\,\omega/k$, against the fronts.[B13] The two numbers are the two readings of one motion, the pattern's own and the bookkeeper's (0.1c).

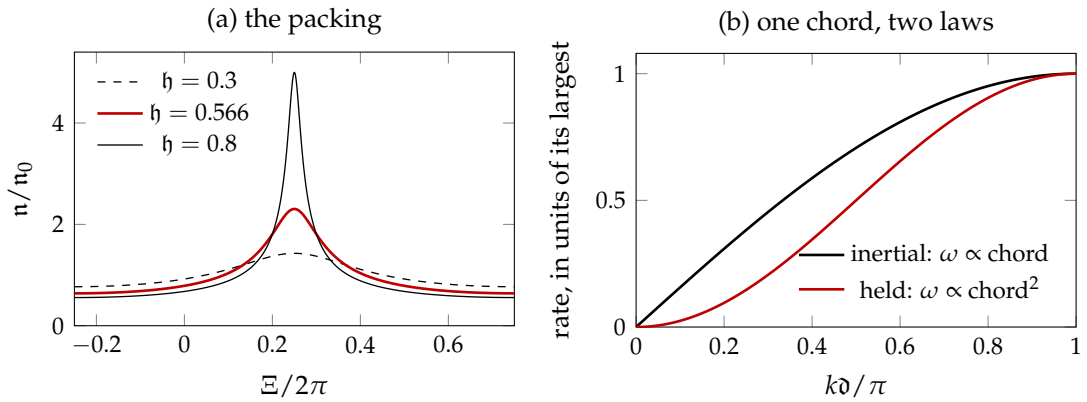


Figure 2: (a) The packing a fixed bookkeeper counts over one wavelength, eq. (3) against $\Xi = \varphi + \mathfrak{h}\cos\varphi$; the red curve is the animation. (b) The rates of the two held turnings on a chain of nearest neighbours against the chord between neighbouring dots, $2\mathfrak{r}\sin(k\mathfrak{d}/2)$: the phonon's rate follows the chord, the magnon's its square (Sections 5, 6).

2.3 Read between neighbours: three projections of one dyad

One dot shows a circle and no wave. The wave is in the comparison of neighbours, and core I-3a names that comparison: the deviation map of a family of patterns, split into a trace (expansion: density), an antisymmetric part (the bivector grade) and a symmetric trace-free part (shear: the tide). For the orbit lattice the map is one dyad,

$$\nabla\mathbf{u} = \mathfrak{r}\,\mathbf{e}'(\varphi)\otimes\mathbf{k}, \qquad \partial u_a/\partial y_b = \mathfrak{r}\,e'_a k_b. \tag{6}$$

Let $\mathbf{\Pi} = \mathbf{e}_1\mathbf{e}_2$ be the unit bivector of the orbit plane, $\mathbf{\Pi}^2 = -1$, and let φ_k be the direction angle of $\mathbf{k}$ in that plane.

Proposition 1 (the three projections)**.** *For the phase law* (1) *with the run in the orbit plane:*

(i) The scalar and bivector grades of the dyad are the two quadratures of one rotor of constant size,

$$\mathfrak{N} \equiv \mathbf{k}\,(\mathfrak{r}\,\mathbf{e}') = \nabla\cdot\mathbf{u} + \nabla\wedge\mathbf{u} = \mathfrak{h}\,\mathbf{\Pi}\,e^{\mathbf{\Pi}(\varphi-\varphi_k)}, \qquad \mathfrak{N}\widetilde{\mathfrak{N}} = \mathfrak{h}^2, \tag{7}$$

that is, $\nabla\cdot\mathbf{u} = -\mathfrak{h}\sin(\varphi-\varphi_k)$ *and* $\nabla\wedge\mathbf{u} = \mathfrak{h}\cos(\varphi-\varphi_k)\,\mathbf{\Pi}$*. The rotor turns once per turn of the dot.*

(ii) The packing is its scalar grade, exactly: $\mathfrak{n}_0/\mathfrak{n} = 1 + \langle\mathfrak{N}\rangle_0$.

(iii) The shear has the constant principal stretch $\mathfrak{h}/2$*, and its stretch axis stands at the angle* $\frac{1}{2}(\varphi + \varphi_k + \frac{\pi}{2})$*: the axis turns at half the dot's rate, and half a turn of the axis restores the shear.*

(iv) The rates of expansion $\mathfrak{x}$ *and of rotation* $\mathfrak{w}$ *of the dots' velocity field are*

$$\mathfrak{x} + i\mathfrak{w} = \frac{\mathfrak{h}\,\omega\,e^{i(\varphi-\varphi_k)}}{1-\mathfrak{h}\sin(\varphi-\varphi_k)}.$$

Proof. With $\mathbf{k} = k\,\mathbf{e}(\varphi_k)$ the geometric product $\mathbf{k}\,\mathbf{e}'(\varphi)$ has scalar part $\mathbf{k}\cdot\mathbf{e}' = -k\sin(\varphi-\varphi_k)$ and bivector part $\mathbf{k}\wedge\mathbf{e}' = k\cos(\varphi-\varphi_k)\mathbf{\Pi}$, which is (i). A rank-one change of the identity has $\det(1+\mathsf{A}) = 1 + \operatorname{tr}\mathsf{A}$, which is (ii). The symmetric trace-free part of $\mathbf{e}'\otimes\mathbf{k}$ has eigenvalues $\pm k/2$ with its positive axis bisecting $\mathbf{e}'$ and $\mathbf{k}$, at $\frac{1}{2}(\varphi + \frac{\pi}{2} + \varphi_k)$, which is (iii). For (iv), $\dot{\mathsf{F}}\mathsf{F}^{-1} = (\mathfrak{r}\omega/\det\mathsf{F})\,\mathbf{e}(\varphi)\otimes\mathbf{k}$ by the Sherman–Morrison inverse, whose trace and antisymmetric part are the two rates. □

A turning that one dot shows as a circle is read between neighbours as a packing wave, a twist wave and a tide at once.[B02–B07] The first two are the scalar and bivector grades of one rotor, exchanged as the phase advances, and neither is constant alone while their quadrature sum is. The third turns twofold. Which of the three a medium prices decides the name the wave is given: Sections 5 to 7.

2.4 Chords and world-lines

Two sites a separation $\boldsymbol{\Delta}$ apart have dots displaced relative to each other by a chord of the orbit circle,

$$\mathbf{u}(\mathbf{y}+\boldsymbol{\Delta}) - \mathbf{u}(\mathbf{y}) = \mathfrak{c}_{\boldsymbol{\Delta}}\, \mathbf{e}'\big(\varphi + \tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\big), \qquad \mathfrak{c}_{\boldsymbol{\Delta}} = 2\mathfrak{r}\sin\big(\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\big), \tag{8}$$

constant in length and turning with the dots.[B14] The discrete turn ratio is $\mathfrak{h}_{\boldsymbol{\Delta}} = |\mathfrak{c}_{\boldsymbol{\Delta}}|/|\boldsymbol{\Delta}|$, at most $\mathfrak{h}$ and equal to it in the long-wave limit along the run. In the plane and time together each dot's world-line is a helix.

Proposition 2 (winding)**.** *The world-lines of two sites wind about each other once per period if* $\mathfrak{h}_{\boldsymbol{\Delta}} > 1$ *and not at all if* $\mathfrak{h}_{\boldsymbol{\Delta}} < 1$*. Passing* $\mathfrak{h}_{\boldsymbol{\Delta}} = 1$ *takes a contact of the two dots.*

Proof. The separation of the two dots is $\boldsymbol{\Delta}$ plus a vector of length $|\mathfrak{c}_{\boldsymbol{\Delta}}|$ turning once per period; it encircles the origin exactly when $|\mathfrak{c}_{\boldsymbol{\Delta}}| > |\boldsymbol{\Delta}|$, and it passes through the origin at equality. □

In the animation $\mathfrak{h}_{\boldsymbol{\Delta}}$ is 0.40 for neighbours on the axes and 0.55 for diagonal neighbours along the run: nothing winds.[B15, B16] An orbit wave below the caustic is an unbraided family of helices. The winding is an integer and it changes only through a contact, never smoothly: the form of the class rule of I-5, shown here on dots, which are closures and not strands.

2.5 The run and the orbit plane

Let χ be the angle between the run and the orbit normal, $\mathbf{k} = k(\sin\chi\,\mathbf{e}_1 + \cos\chi\,\mathbf{N})$. The packing takes the in-plane share of the run,

$$\det \mathsf{F} = 1 - \mathfrak{h}\sin\chi\,\sin\varphi. \tag{9}$$

For $\chi = \pi/2$ this is the animation. For $\chi = 0$ the run is along the normal: a line of sites along the run is, at one instant, a helix of radius $\mathfrak{r}$ and pitch λ standing at $\arctan\mathfrak{h}$ to its axis; the trace of the dyad vanishes, there is no packing wave and no caustic at any $\mathfrak{h}$, and the displacement is its own curl, $\nabla\times\mathbf{u} = -k\,\mathbf{u}$, with the sign of the hand.[B18, B19] The wave is then twist and shear alone.

3 Where the core enters

The medium is never empty. The chain $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$ lets none of its terms vanish (0.1). Three consequences are used throughout. There is no vacuum: between the sites of a lattice, and where no lattice stands, is the omnium, whose response coefficients ε_0 and μ_0 are the kernel of the open stroke (0.1d). No wave is ever at rest: the turning nobody wrote has a floor (Section 9.7). And every interval of this manuscript that ends at no turning, $\mathfrak{h}\to 0^+$ or a cone closing, is open at that end (I-6).

Chart inventory. One chart of the Alpha Lock is used: the exterior branch of application A-1 for a lattice site as a source, $\alpha_{\text{ext}} = \mathscr{M}c^2/(2F_{\max}r_f)$, evaluated at the neighbour distance. No interior chart is entered, and no β. The turn ratio $\mathfrak{h}$, its discrete form $\mathfrak{h}_\Delta$ and the translation share $\mathfrak{s}$ of 0.1d are kinematic ratios and are no application of 0.4. No A-3 token is used (D-L2). Table 2 declares the symbols this manuscript adds.

The seven invariants.

I-1. The force law holds between sites as it holds everywhere. For the nuclei of two neighbouring ^{28}Si sites, rest energy 26.05 GeV each with supports of a few femtometres, at $d = 2.352$ Å: $\alpha_{\text{ext}} = 2.9 \times 10^{-43}$ and $F = F_{\max}\alpha_1\alpha_2 = G\mathscr{M}^2/d^2 = 2.6 \times 10^{-42}$ N on the pair domain, $d \geq R_{b,1} + R_{b,2}$.[F01–F03] The small number is a correct reading. A lattice's rates are reached through a declared response kernel (rate is not force): the kernels Φ_Δ and J_Δ below are taken from the measured laws $\omega(\mathbf{k})$, and no map from a charge or a circulating charge to an I-1 input is written here (O-6). Section 9 uses I-1's rule for a driven observable: a causal convolution of the driver with the lattice's response.

I-2. One carrier. The orbit lattice shows what a carrier is: a phase law of local turnings. No carrier crosses from site to site: the omnium settles the whole system at every step, from the state it is in (0.1a), and the sites are its patterns (0.1b); the five names are five ways the same turning is held and read.

I-3, I-3a. Curvature is read between neighbours, and so is a wave: Proposition 1. Dots on an interface carry one phase each, so the phase law is continuous along the interface and $k_1 \sin\theta_1 = k_2 \sin\theta_2$ at one ω: the Fermat jump law in its exact index form, one refraction for sound, spin waves [20] and light (P-4).

I-4. The picture is the quantum face drawn site by site: each dot is $\psi_j = A\, e^{i\varphi_j}$, the projection of a helix on the plane normal to its axis. An observer who has lost φ_0 sees every circle evenly occupied; that ring is superposition, ignorance of the helical phase. Section 9 is about not losing it.

I-5. A carrier changes no helix count: an orbit wave leaves every closure of the lattice in its class. A stroke standing in a closure has the cost $m_q c^2 = \hbar\omega_q$ (Section 7); no form of the confinement operator D, its domains, boundary data or spectrum is written here (O-9).

I-6. Every bounded quantity here is bounded by its domain: $0 < \mathfrak{h} < 1$ from single-valuedness, and $0 < \mathfrak{m} < 1$ on the exterior branch (Section 8), each with one-sided limits.

I-7. The energy of an orbit wave enters $\mathscr{M}$ once, as E/c^2, and a thermal population of orbit waves is part of that same entry, entered once; the radiation term aT^4V/c^2 of 0.5 is the open strokes' and is not used here. An isotope, thermal or pressure shift of a wave's rate is one density mechanism, written as a sensitivity-weighted sum $\delta\ln\omega = \sum_i S_i\,\delta\ln x_i$: the inertial sensitivity at a fixed kernel is $S_m = -\frac{1}{2}$ (Section 5), the density-channel input $\delta\ln\alpha_{\text{ext}} = \delta\ln\mathscr{M}$ is positive for a heavier isotope at fixed geometry, and its sensitivity for an orbit wave is D-L3.

$\mathfrak{r}$, $\mathfrak{d}$	orbit radius; site spacing	$\Phi_{\mathbf{\Delta}}$	chord stiffness between sites $\mathbf{\Delta}$ apart
$\mathbf{y}$, $\mathbf{x}$, $\mathbf{u}$	site; dot; displacement $\mathbf{x}-\mathbf{y}$	$\mathsf{J}_{\mathbf{\Delta}}$, S	chord coupling of held moments; held length
φ, φ_0, Ξ	dot's phase; start azimuth; phase at a fixed place	$\mathbf{t}_j$, $\mathfrak{g}$	unit vector to a moment's tip; moment per held angular momentum
$\mathfrak{h}$, $\mathfrak{h}_{\mathbf{\Delta}}$	turn ratio $\mathfrak{r}k$; its discrete form	$\mathfrak{u}$, $\mathfrak{w}$	a wave's turning; the driver's component on it
$\mathfrak{n}$, $\mathfrak{n}_0$	packing density of dots; of sites	$\mathfrak{z}$	damping rate of a turning
$\mathfrak{N}$, $\mathbf{\Pi}$	neighbour rotor; orbit-plane bivector	ς	strain amplitude of the ripple
$\mathfrak{c}_{\mathbf{\Delta}}$, χ	chord; angle of run to orbit normal	E_0	rest energy of a site
F, $\mathfrak{x}$	gradient of the map $\mathbf{y}\mapsto\mathbf{x}$; expansion rate	Φ_0, $\mathscr{T}$	holding stiffness of a site; period of a turning
Θ, $\omega_{\mathfrak{z}}$	half-angle of a moment's cone; damped rate	$\mathfrak{r}_{\text{spot}}$, $\mathscr{T}_{\text{spot}}$	a circle's radius and period on the spot

Table 2: Symbols this manuscript adds. Core symbols keep their core meanings: c the strand rate, $\mathscr{M}$ the total gravitating energy, α_{ext} the exterior density coordinate, $\mathfrak{s}$ the translation share, $\mathfrak{m}$ a held moment, B the twist's field, $\mathfrak{m}$ the clock factor.

4 One unit of circulation

Proposition 3 (the rate is the cost of circulation)**.** *Let a lattice hold a family of orbit waves of one phase law, and let its energy be unchanged when every dot is advanced along its own circle by one common angle. Let L be the generator of that advance, the circulation of the turning about the orbit normal, and $E(L)$ the energy along the family. Then*

$$\omega = \frac{dE}{dL}. \tag{10}$$

Where the response is linear, $E=\omega L$: the wave carries $1/\omega$ of circulation per unit energy.

Proof. Advancing every dot along its own circle by one angle is the transformation generated by L, and the phase φ is the coordinate it shifts. Hamilton's equation for that pair is $\dot{\varphi}=\partial E/\partial L$ up to the sign of the hand. If E is homogeneous of degree one in L the derivative is the ratio. □

A kernel that prices the two components of $\mathbf{u}$ differently lacks this symmetry; Section 5 shows that the circle is then not kept.

With Planck's bridge $E_q=\hbar\omega_q$ of 0.1a, one quantum is one unit $\hbar$ of circulation. With the run along the orbit normal ($\chi=0$) the fourth and fifth of the six readings that type the stroke [2], $E\mathscr{T}=h$ and $|S|=\hbar$ about the run, are this one statement; with the run in the orbit plane the same unit lies across the run. The record carries it for light [6], for the lattice, computed in [7] and measured in [8, 9], for the held moments [11] and for the standing stroke [12].

An orbit wave has two radii. Its amplitude $\mathfrak{r}$ says how far each site turns and grows with the number of units. Its phase cylinder $1/k$ is the lever arm at which the wave's $E/(\omega/k)$ carries its E/ω, the same for every orbit wave and fixed per unit.[C11] The turn ratio is the first over the second.

One unit is spread over the whole lattice. For sites of mass m turning by inertia, $Nm\omega\mathfrak{r}_1^2=\hbar$: one unit at 10 THz in 10^{22} sites of ^{28}Si turns each of them on a circle of radius 6.0×10^{-23} m.[C10] The quantum has no place narrower than the lattice: it is the one pattern turning by that much everywhere (0.1b).

5 The phonon: turning held by inertia

The dots are the lattice's sites, closures whose saturated cores P-3 names the pinning sites of a material's Chladni volume, and the load between them is taken by the interstitial omnium. A core is fixed in its saturation, which a load does not compress (P-3); in an orbit wave it turns with its site and loads the line there, as on the loaded line of [2]. The lattice's response is one kernel summed over every separation, an energy $\frac{1}{2}\Phi_{\boldsymbol{\Delta}}|\mathbf{u}(\mathbf{y}+\boldsymbol{\Delta})-\mathbf{u}(\mathbf{y})|^2$ for each; no pair responds alone (0.1a). In an orbit wave each site's turning energy $\frac{1}{2}m(\mathfrak{r}\omega)^2$ and each chord's energy are constant in time, and the equation of motion sets them equal:

$$m\,\omega^2\mathfrak{r}^2=\sum_{\boldsymbol{\Delta}>0}\Phi_{\boldsymbol{\Delta}}\,\mathfrak{c}_{\boldsymbol{\Delta}}^2 \quad\Longleftrightarrow\quad \omega^2=\frac{4}{m}\sum_{\boldsymbol{\Delta}>0}\Phi_{\boldsymbol{\Delta}}\sin^2\!\big(\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\big). \tag{11}$$

The circulation is inertial, $L=Nm\omega\mathfrak{r}^2$, and $E=\omega L$.[C01–C04] On a chain of nearest neighbours $\omega=\omega_{\max}|\sin(k\mathfrak{d}/2)|$: the rate follows the chord (Fig. 2b).[C05] A site that the rest of the pattern also holds in place with the stiffness Φ_0, as the other sublattice of a compound lattice holds it, has the law

$$m\,\omega^2=\Phi_0+4\sum_{\boldsymbol{\Delta}>0}\Phi_{\boldsymbol{\Delta}}\sin^2\!\big(\tfrac{1}{2}\mathbf{k}\cdot\boldsymbol{\Delta}\big), \tag{12}$$

with the rate $\sqrt{\Phi_0/m}$ at $\mathbf{k}=0$.[C12] Without Φ_0 the turning in step has no rate, and a push uniform over the sites moves the lattice bodily.

The lattice's rate on the strand rate. The long-wave phase rate of the chain is $c_s^2=\Phi_1\mathfrak{d}^2/m$. A site's inertia is its rest energy over c^2, closure included (0.1d), so

$$c_s=c\sqrt{\frac{\Phi_1\mathfrak{d}^2}{E_0}}, \tag{13}$$

the strand rate scaled by the square root of the chord energy at unit strain over the site's rest energy. For silicon along [100], $c_s=8.44\ \mathrm{km\,s^{-1}}$ [19] and $E_0(c_s/c)^2=20.6$ eV against $E_0=26.06$ GeV.[F04] On the loaded line of [2] (its regime R4 and eq. 13), clamps of loading μ on a stroke that runs at c between them give $c_s=c/\sqrt{1+\mu/\mathfrak{d}}$, and (13) is its heavy-load form once the line between clamps is given the stiffness $\Phi_1\mathfrak{d}$ and the loading is read as $\mu=E_0/(\Phi_1\mathfrak{d})$, an identification made here.[D06, D07] The open stroke runs at c on the spot, the third reading; c_s is the rate of the phase law over closures, whose circulating share is the sixth reading changed.

The longitudinal branch. The two hands of the turning with the run in its plane are $\mathbf{u}_\pm=\mathfrak{r}(\cos\varphi\,\mathbf{e}_1\pm\sin\varphi\,\mathbf{e}_2)$. Their sum is a displacement along the run, a packing wave with shear and no twist. Their difference lies across the run, twist and shear with no packing.[B17] Each hand carries $\mp1/\omega$ of circulation per unit energy about the orbit normal, so the longitudinal branch carries none, about the normal or about the run. The longitudinal branch is to the in-plane hands what a linearly polarised stroke is to the hands about its run, and its reading between neighbours is the scalar grade of Proposition 1: P-3's change of the interstitial packing, $-\Delta V/V=\mathfrak{h}\sin\varphi$ for one hand. This is the kinematic half of D-05 of [2]. Its rate from the stroke between the clamps is not derived here.

Which turnings a lattice keeps. A kernel that treats the two components of $\mathbf{u}$ alike, as (11) does, prices the three projections alike. A lattice's kernel is a tensor: it prices packing and

shear at two rates and does not price a uniform twist, so with the run in the orbit plane the two quadratures part and a circle opens into an ellipse. The circle is kept with the run along the normal on an axis where the two transverse branches share one rate, and with the run in the plane where the lattice's symmetry makes the turning a mode: the valley points of a hexagonal sheet [8], and a free surface, where the dots of a water wave run circles [4, 5].

The site's clock. Every site of an orbit wave of one hand moves at the one speed $\mathfrak{r}\omega$. Its strand's helix triangle (0.1d) has $v = \mathfrak{r}\omega$, and it keeps time at the rate $\sqrt{1 - \mathfrak{r}^2\omega^2/c^2}$, the same at every site and steady: among sites turned alike nothing is read (0.1c). A linearly polarised wave of equal energy has the same mean and a modulation at 2ω.[F06] For thermal turning this is the temperature shift of a carried clock, the time dilation of its own thermal motion (0.1c), written by Pound and Rebka as $-C_L/(2Mc^2)$ per kelvin, with C_L the lattice's specific heat per site, and measured by them [18]; its classical limit for ^{57}Fe is $-3k_B/(2Mc^2) = -2.44 \times 10^{-15}$ K^{-1}.[F05]

Standing. Two orbit waves of one hand running against each other give $\mathbf{u} = 2\mathfrak{r}\cos(\mathbf{k}\cdot\mathbf{y})\,\mathbf{e}(-\omega t)$: every dot turns in step on a circle of radius $2\mathfrak{r}|\cos\mathbf{k}\cdot\mathbf{y}|$, and on the lines where that radius has its least value this pair of waves writes no turning. Those are the nodal lines of P-2; the floor's turning remains on them as everywhere.

6 The magnon: turning held in a twist

A magnetised lattice is the twist grade of the density field aligned along an axis (P-5). Each site holds a moment of size $\mathfrak{g}\hbar\mathsf{S}$ along a unit vector $\mathbf{t}_j$ and the angular momentum $-\hbar\mathsf{S}\,\mathbf{t}_j$, running against its moment, the measured sign [13, 14]. The animation is the view along the magnetisation, from behind it, which is why its dots turn clockwise: the dots are the tips of the moments, each on a cone of half-angle Θ, with the phase law $\varphi_j = \omega t - \mathbf{k}\cdot\mathbf{y}_j$.

The kernel couples tips through the chord between them, $\mathfrak{c}_{\mathbf{\Delta}} = 2\sin\Theta\,\sin(\frac{1}{2}\mathbf{k}\cdot\mathbf{\Delta})$ on the unit sphere: a pair $\mathbf{\Delta}$ apart stores $\mathsf{J}_{\mathbf{\Delta}}\mathsf{S}^2\mathfrak{c}_{\mathbf{\Delta}}^2$ above alignment. The twist's own field B along the axis adds the term $-\sum \mathsf{m}\cdot B$ of 0.1c. Per site, above the aligned state,

$$\mathcal{E}(\Theta) = \mathsf{S}^2\sum_{\mathbf{\Delta}>0}\mathsf{J}_{\mathbf{\Delta}}\,\mathfrak{c}_{\mathbf{\Delta}}^2 + \mathfrak{g}\hbar\mathsf{S}B\,(1-\cos\Theta), \qquad \mathcal{L}(\Theta) = \hbar\mathsf{S}\,(1-\cos\Theta), \tag{14}$$

with $\mathcal{L}$ the circulation of the turning, directed along the magnetisation. The aligned state is the limit $\Theta \to 0^+$ and is never held: the floor keeps every cone open. Proposition 3 gives the law at every cone angle,

$$\omega = \frac{d\mathcal{E}}{d\mathcal{L}} = \mathfrak{g}B + \frac{8\mathsf{S}}{\hbar}\cos\Theta\sum_{\mathbf{\Delta}>0}\mathsf{J}_{\mathbf{\Delta}}\sin^2\!\big(\tfrac{1}{2}\mathbf{k}\cdot\mathbf{\Delta}\big), \tag{15}$$

and the equation of motion of the held moments, $\dot{\mathbf{t}}_j = -(2\mathsf{S}/\hbar)\sum_{\mathbf{\Delta}\neq 0}\mathsf{J}_{\mathbf{\Delta}}\,\mathbf{t}_j\times\mathbf{t}_{j+\mathbf{\Delta}} - \mathfrak{g}\,\mathbf{t}_j\times\mathbf{B}$, integrated on a ring, returns the same rate and keeps the cone.[C06, C07] For a small cone on a chain of nearest neighbours $\hbar\omega = \hbar\mathfrak{g}B + 4\mathsf{J}\mathsf{S}(1-\cos k\mathfrak{d})$, the law of record [15, 16].[C08]

- *The unit.* $N\mathcal{L} = N_q\hbar$: each unit of the wave is $\hbar$ along the magnetisation, and one unit opens the cone by $1-\cos\Theta = 1/(N\mathsf{S})$.[C09]
- *The hand.* The tips turn right-handed about the magnetisation, the reverse of the sites' held circulation, since the kernel's energy is lowest at alignment. A lattice of one sublattice has

no orbit wave of the other hand. A lattice of two opposed sublattices has both, and its long-wave law has the form $\omega^2 = \omega_0^2 + c_m^2 k^2$ of I-2's closed-pattern phase, phase rate times run equal to c_m^2 at the lattice's rate.[D01]

- *One chord, two laws.* The phonon and the magnon store the same chord. Inertial circulation grows with ω, so $\omega^2 \propto \text{chord}^2$; held circulation is fixed by the cone, so $\omega \propto \text{chord}^2$ (Fig. 2b). The linear law of sound and the quadratic law of the spin wave are one chord read against two ways of holding circulation.
- *The gap.* At $\mathbf{k} = 0$ every tip turns in step and there are no fronts; the rate is $\mathfrak{g}B$, the twist term alone. Where B varies along the lattice the unit's cost $\hbar\mathfrak{g}B$ varies with it and the bookkeeper reads a push on a packet of the wave toward the weaker twist. No clock factor is assigned to the wave: a carrier keeps no time (I-2).
- *The run in the plane.* With $\chi \neq 0$ the tips' packing is the pole density $-\nabla\cdot\mathbf{t}$, the scalar grade. The dipolar part of the kernel prices it: the rate rises with $\sin^2\chi$ and the circle becomes an ellipse [17]. With $\omega \to 0$ the phase law stands still: the screw of a helical magnet for $\chi = 0$, the cycloid for $\chi = \pi/2$.

This section derives, for a declared kernel, what D-06 of [2] lists: the turning of the twist, its unit along the magnetisation, its hand and its law. The kernel J_Δ from the stroke through the helical gradient is not written here (O-6).

7 Light, the standing stroke, the tide

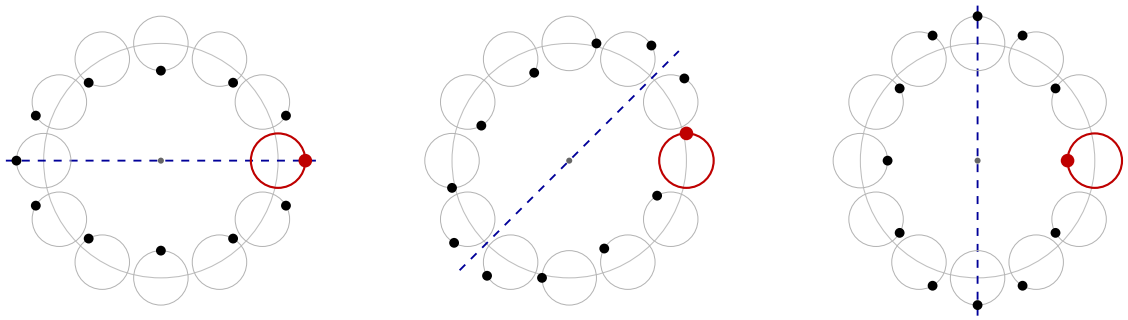
Light. The open stroke is the orbit wave of the omnium itself. No closure loads it and no lattice holds it; its kernel is the medium's own pair of responses, ε_0 and μ_0, whose product sets its rate, $c^2 = 1/(\varepsilon_0\mu_0)$ (0.1, 0.1d). What turns is the omnium, never an empty background: there is no vacuum (0.1d). Its two grades turn about the run ($\chi = 0$) as the two quadratures of one null multivector, $(\mathbf{E} + cI_3\mathbf{B})^2 = 0$ (0.1d), and it has no scalar grade, $\nabla\cdot\mathbf{E} = 0 = \nabla\cdot\mathbf{B}$. Its amplitude radius is its phase cylinder, $\mathfrak{r} = 1/k = \bar{\lambda}$, so its turn ratio is one: the turning rate equals the run, the helix at $\pi/4$ of I-2.[D05] A held turning reaches $\mathfrak{h} \to 1^-$ only at the caustic, with the run in its plane; the stroke stands at $\mathfrak{h} = 1$ with the run along its normal, where its turning has no scalar grade and leaves the omnium's packing as it is.

A closure carrying charge, turned by a stroke of one hand, runs a circle across the run with the momentum $a_0\, mc$, where a_0 is the field amplitude times the charge over $m\omega c$. Its translation share (0.1d) is

$$\mathfrak{s} = \frac{v^2}{c^2} = \frac{a_0^2}{1 + a_0^2}, \tag{16}$$

with $\mathfrak{s}/(1-\mathfrak{s}) = a_0^2$, the squared ratio of the two grades of $\mathbf{P} = pc + mc^2\boldsymbol{\Sigma}$, linear in the stroke's intensity.[D03, D04] The dot's turn ratio is $\sqrt{\mathfrak{s}} < 1$ at every intensity. The share stays kinematic.

The clamped and the standing stroke. A stroke held between clamps has $\omega^2 = \omega_c^2 + c^2k^2$. Its run is $v = c^2k/\omega$ and the share that does not run stands across the clamps at $u = c\,\omega_c/\omega$, with $v^2 + u^2 = c^2$: the helix triangle of 0.1d read on a clamped stroke.[D02] Standing wholly, it turns and does not run, and its cost $\hbar\omega_c$ is the cost of a mode in its support, the form $m_q c^2 = \hbar\omega_q$ of I-5. The gluon is that stroke standing inside a closed pattern's own region (I-2):



(a) $\omega t = 0°$ (b) $\omega t = 90°$ (c) $\omega t = 180°$

Figure 3: The tide on a ring of free patterns about a reference path, eq. (17). Each pattern turns once per period on its own small circle (one is marked). The ring's stretch axis (dashed) stands at $\omega t/2$: after half a period it has made a quarter turn and after a full period half a turn, where the ring's shape is the same again.

two components running against each other, with nodal surfaces where the mode writes no turning, as in Section 5. A standing stroke changes no helix count.

The tide. The graviton is the stroke in gravity's geometry, and the ripple is that geometry adjusting. Let a ripple of one hand and strain amplitude ς run along the normal of a ring of free patterns of radius R, small against the ripple's wavelength, about a reference path. A pattern at the azimuth ψ is displaced by

$$u_1 + iu_2 = \tfrac{1}{2}\varsigma R\, e^{i(\omega t - \psi)} : \tag{17}$$

it turns once per period on a circle of radius $\varsigma R/2$ (Fig. 3).[E01] The radius grows with the distance from the reference path, so nothing is read at one point and the reading is a comparison (I-3a). The deviation map is symmetric and trace-free, the shear projection alone, and its stretch axis stands at $\omega t/2$. In one period each pattern makes a whole turn on its circle and the ring's stretch axis makes half a turn, after which the ring's shape is the same again.[E02] The onefold turn belongs to each pattern. The twofold turn belongs to the map between neighbours, and it is the rank of the tide. Proposition 1(iii) shows the same half-rate shear in every orbit wave.

The standard account quantises the ripple as a second carrier of helicity two. FDT reads one carrier of unit $\hbar$ whose geometry's adjustment is read between neighbours as a rank-two map (I-2, I-3a); the measured tensor pattern [21] is accepted in full. How the stroke's quanta compose the ripple is D-04 of [2] and is not derived here.

8 The universal lattice: circles sized to the density

All the terms of the chain adjust at once (0.1a). The universal lattice, the omnium's own turning, therefore does one thing when the density changes: it resizes its circles, radius and period together, and their ratio stays the strand rate.

The lattice. The universal lattice is the open stroke of Section 7 read as an orbit lattice. No closures mark its sites, which are any points of the omnium. Its turn ratio is one. On the spot each circle has the radius c/ω_{spot} and the period $2\pi/\omega_{\text{spot}}$, and the dot on it runs at c (I-2, 0.1d).

Proposition 4 (circles sized to the density)**.** *Let a steady stroke have the rate ω for the distant bookkeeper, and let it cross the static exterior of a source at the field radius r_f, on the exterior branch of A-1, where the clock factor is $\mathfrak{m} = \sqrt{1-\alpha_{\text{ext}}}$. Then on the spot*

$$\mathfrak{r}_{spot} = \mathfrak{m}\,\frac{c}{\omega}, \qquad \mathscr{T}_{spot} = \mathfrak{m}\,\frac{2\pi}{\omega}, \qquad \frac{2\pi\,\mathfrak{r}_{spot}}{\mathscr{T}_{spot}} = c, \qquad \mathfrak{h} = 1, \tag{18}$$

at every r_f. Drawn in the bookkeeper's areal coordinates the same circle is an ellipse with the semi-axes

$$(1-\alpha_{\text{ext}})\,\frac{c}{\omega}\ \textit{along the radius}, \qquad \sqrt{1-\alpha_{\text{ext}}}\,\frac{c}{\omega}\ \textit{across it}. \tag{19}$$

Proof. A pattern's kept time and thread length rescale together by $\mathfrak{m}$ (0.1c), and the stroke's rate on the spot is the bookkeeper's over the lapse, $\omega_{\text{spot}} = \omega/\mathfrak{m}$; with $\mathfrak{r}_{\text{spot}} = c/\omega_{\text{spot}}$ this is (18). Areal coordinates keep lengths across the radius and shorten lengths along it by the lapse, which turns the circle of radius $\mathfrak{m}c/\omega$ into (19). The same two lengths are what the stroke runs per radian of phase on the exact optical metric of I-3, $d\ell^2_{\text{opt}} = dr^2/(1-\alpha_{\text{ext}})^2 + r^2 d\Omega^2/(1-\alpha_{\text{ext}})$, whose coordinate rates are $c(1-\alpha_{\text{ext}})$ along the radius and $c\sqrt{1-\alpha_{\text{ext}}}$ across it. □

Radius and period carry the one factor $\mathfrak{m}$: the circles are smaller and slower where the omnium is denser, and on the spot nothing has changed, because the clocks and rulers there are closed strokes that resized with them (0.1a, 0.1c). The turn ratio is one at every density: the chain is all ratios and selects no size (0.3a). The clock factor sizes the circle on the spot and bends no stroke. The bending is in (19), where the semi-axis along the radius carries $\mathfrak{m}^2$ and the one across it carries $\mathfrak{m}$: the factor of two of I-3.[H01, H02]

- *The circles never close.* On the exterior branch $0 < \mathfrak{m} < 1$. As $\alpha_{\text{ext}} \to 1^-$ the circles shrink and do not reach a point: no term of the chain vanishes, and the floor $2\ell_P$ bounds every stroke (0.1, 0.1a).
- *Read between neighbours.* One circle carries no reading of its size (0.1c). Neighbours at two heights do (I-3a):

  $$c^2\,\frac{\partial \ln \mathfrak{r}_{\text{spot}}}{\partial r_f} = \frac{G\mathscr{M}}{r_f^2\,(1-\alpha_{\text{ext}})}, \tag{20}$$

  at leading order the free fall $G\mathscr{M}/r_f^2$ that the bookkeeper reads for a slow pattern.[H03] At the Earth's surface the circles grow upward by 1.09×10^{-16} of their size per metre, and over the 22.5 m of Pound and Rebka's tower by 2.46×10^{-15}, the shift read there [37]. At the solar limb they are smaller than far away by 2.12×10^{-6}.[H04, H05]
- *Refraction is the radii.* Across a change of density the phase law is continuous while the circles on the two sides differ, so the fronts tilt by the jump law of I-3 with the exact index. Past the solar limb the tilt adds up to $4G\mathscr{M}/(bc^2)$, the 1.7516″ of I-3.[H05]
- *Every driver is a density driver.* Mass, energy, temperature and pressure enter one density (I-7), so a driver that deposits any of them resizes and re-centres the circles where it lands. In a lattice of closures the deposit acts through the same channel, and the lattice's circles re-centre and resize by its kernel: the moved-centre start of Section 9.3. The size of the resizing for a given deposit is D-L6.

The standard account reads clocks and rulers as readers of a background metric that curves. FDT reads closed patterns that resize with the circles of the universal lattice, and their adjustment is the geometry (0.1a); the measured shifts and deflections are accepted in full.

9 Writing the turning: drivers and deterministic control

An orbit wave is five numbers: its rate, wave vector, hand, radius and start azimuth. A driver controls the lattice's response when it sets them. FDT is deterministic (I-4): the response of the lattice is the settlement of the state it is in under the driver that arrives (0.1a), and no part of it is left open.

9.1 The control law

Write a site's displacement in the orbit plane as $u_j = u_1 + iu_2$ and a driver's push on it as $f_j(t)$, a force on an inertial site or a torque on a held moment. The lattice's turning in the wave $(\mathbf{k}, \text{hand})$ and the driver's component on that wave are

$$\mathfrak{u}(t) = \frac{1}{N}\sum_j e^{-i\mathbf{k}\cdot\mathbf{y}_j}\, u_j(t), \qquad \mathfrak{w}(t) = \frac{1}{N}\sum_j e^{-i\mathbf{k}\cdot\mathbf{y}_j}\, f_j(t). \tag{21}$$

For small turnings, with $\mathfrak{z}$ the damping rate and $\omega_\mathfrak{z} = \sqrt{\omega^2 - \mathfrak{z}^2}$,

$$\text{held:}\quad \dot{\mathfrak{u}} = (i\omega - \mathfrak{z})\,\mathfrak{u} + \mathfrak{w}, \qquad \mathfrak{u}(t) = e^{(i\omega-\mathfrak{z})t}\Big[\mathfrak{u}(0) + \int_0^t e^{-(i\omega-\mathfrak{z})t'}\,\mathfrak{w}(t')\,dt'\Big], \tag{22}$$

$$\text{inertial:}\quad m(\ddot{\mathfrak{u}} + 2\mathfrak{z}\dot{\mathfrak{u}} + \omega^2\mathfrak{u}) = \mathfrak{w}, \quad \mathfrak{u}(t) = \mathfrak{u}_{\text{free}}(t) + \int_0^t \frac{e^{-\mathfrak{z}(t-t')}\sin\omega_\mathfrak{z}(t-t')}{m\,\omega_\mathfrak{z}}\,\mathfrak{w}(t')\,dt'. \tag{23}$$

This is I-1's rule for a driven observable,[G17, G19] a causal convolution of the driver with the lattice's response, which maps the driver to the observable and is not the force law. It says three things. The response is fixed by the driver's time course and by the state $\mathfrak{u}(0)$ the lattice is in when the driver arrives. The lattice keeps the part of the driver that turns with it: after a pulse, $\mathfrak{u} \propto \int e^{-i\omega t'}\mathfrak{w}(t')\,dt'$, the driver's component at the wave's own rate, phase law and hand. And the phase of the response is the phase of that component: the driver writes φ_0.

9.2 Four matchings

A driver writes a wave when its component $\mathfrak{w}$ on that wave is not zero. It must carry four things of the wave's.

1. *The rate.* A tone carries one rate. A pulse shorter than the period carries every rate up to the inverse of its length.
2. *The phase law.* A driver uniform over the sites writes $\mathbf{k} = 0$, every dot in step, on a branch that has a rate there: the held lattice's gap $\mathfrak{g}B$, or an inertial lattice with the holding term Φ_0 of (12). The pulse records of Section 9.3 are of such branches. A driver with the law $\mathbf{q}\cdot\mathbf{y} - \Omega t$ writes $\mathbf{k} = \mathbf{q}$ alone, modulo the reciprocal lattice.[G08] Two crossed strokes write the difference of their wave vectors at the difference of their rates. A front sweeping the sites at the speed v writes the wave whose phase rate is v.[G09] A comb of period $2\pi/k$ pressed on a face writes k.
3. *The hand.* An inertial lattice takes either hand, and takes the driver's.[G01] A held lattice has its own hand and takes only that component of any driver: a linear driver delivers half, and the other hand draws a response smaller by the factor $2\omega/\mathfrak{z}$.[G03]
4. *The projection.* A driver writes the projection it carries. One stroke is transverse and has no scalar grade: its magnetic grade turns held moments by the torque $\mathsf{m}\times\mathbf{B}$, and its electric

grade turns sites that carry charge, across its run. Two strokes together, or one stroke twice, write through their dyad $\mathbf{E} \otimes \mathbf{E}^*$, which splits as the lattice's dyad (6) does:

part of $\mathbf{E} \otimes \mathbf{E}^*$	what sets it	projection it writes
trace $\lvert\mathbf{E}\rvert^2$	the intensity	packing: the stroke's pressure on the packing, and heat (I-7)
antisymmetric $i\,\mathbf{E} \times \mathbf{E}^*$	the hand, with its sign	twist: an axial push along the run on held moments [27]
symmetric trace-free	a linear stroke's axis, twofold	shear [22]; by I-3a the rank of the tide

A stroke of one hand has the antisymmetric part and no shear; a linear stroke has the shear and no antisymmetric part.[G12] A packing wave along a stroke's own run is therefore written by the dyad and never by one stroke alone.

9.3 Pulses: where the dot starts

A pulse much shorter than the period is one step of 0.1a's settlement: it leaves the lattice in a new state and the turning starts from there.

- *An impulse on an inertial site.* An impulse P along an axis at the time t_0 gives the displacement $(P/m\omega)\sin\omega(t-t_0)$ along that axis: a line through the centre, both hands in equal parts, its clock set by the arrival time.
- *A moved centre.* A pulse that shifts the kernel's minimum by $\mathbf{s}$ leaves every dot off its new centre: $\mathbf{u} = -\mathbf{s}\cos\omega(t-t_0)$ about it, a quarter turn from the impulse's start.[G07] The record distinguishes the two starts by that quarter turn [22, 23].
- *A transverse pulse on a held moment* of area $\mathfrak{g}\int B\,dt$ tips it by exactly that angle, toward the azimuth a quarter turn behind the field's axis.[G13] It then turns on its cone in its own hand. One linear pulse writes a full circle: the radius by the pulse's area, the start azimuth by its polarisation axis, the start time by its arrival [27, 28]. An area of π reverses the moment.
- *Pairs.* Small responses add as arrows on the orbit circle. Two equal impulses on one axis a delay τ apart, with $\mathfrak{z}\tau \ll 1$, leave the energy $4\cos^2(\omega\tau/2)$ times one pulse's: fourfold at whole periods, and none of it at half periods, where the second pulse cancels what the first wrote.[G05] Two equal impulses on crossed axes leave the circulation

$$L(\tau) = L_{\max}\,\sin\omega\tau\,: \tag{24}$$

one hand at a quarter period, the other at three quarters, none at a half (Fig. 4).[G06] On an inertial lattice the hand is a clock setting. The record has the building train [24], the cancelling pulse [25, 28] and the written hand [29, 30].

9.4 Sustained drivers

- *Resonance.* With the rate, phase law and hand matched, the driver's push lies along every dot's velocity and each dot runs a quarter turn behind the driver.[G02] The radius grows as $f_0 t/(2m\omega)$ on an inertial lattice and as $f_0 t$ on a held one, until damping holds it at $f_0/(2m\omega\mathfrak{z})$ or $f_0/\mathfrak{z}$.[G04]
- *Locking.* A turning that sustains itself, gain against loss, keeps a free phase. A weak driver holds it: the phase φ_r relative to the driver obeys $\dot{\varphi}_r = \Delta - K\sin\varphi_r$ [34], and for a detuning $|\Delta| < K$ it rests at $\arcsin(\Delta/K)$. The driver's detuning sets the phase.[G10]

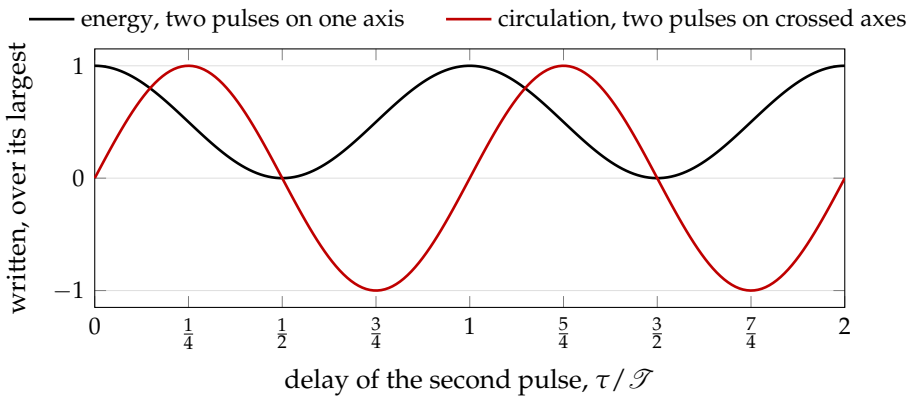


Figure 4: Two equal short pulses on an inertial lattice, eq. (24). On one axis the second pulse doubles the turning at whole periods and cancels it at half periods. On crossed axes it writes one hand at $\mathscr{T}/4$, none at $\mathscr{T}/2$ and the other hand at $3\mathscr{T}/4$.

- *A modulated kernel.* Modulating the chord stiffness, the twist or the held length at 2ω, by a fraction ϵ of ω^2, makes the turning grow at the rate $\epsilon\,\omega/4$ with its phase fixed to the modulation up to a half turn. Which of the two phases is taken is decided by the state the growth starts from: opposite seeds grow into opposite phases, and a seed pulse larger than the turning nobody wrote (Section 9.7) decides it.[G11] Waves grow in pairs $\pm\mathbf{k}$ with the sum of their phases set by the driver, the parallel pumping of spin waves [35]; the gas so pumped is that of [38].
- *Other drivers.* A transducer on a face carries the scalar and shear grades directly and writes the phase law by its comb, with the run in the orbit plane at the surface. A heat pulse writes packing through I-7 and moves centres. A flow of held circulation into a magnet feeds or drains the held turning directly. An antenna writes the twist grade with the phase law of its conductors.

9.5 Passing the unit between the two ways of holding it

A driver that writes a turning of one hand in the sites puts the circulation $Nm\omega\mathfrak{r}^2$ into the lattice. Twist induces magnetism (P-5): the sites' driven circulation is a twist, and held moments answer it as they answer a field along the orbit normal, with the sign of the hand. Reverse the hand of the driver and the answer reverses; a linear driver of equal energy writes no circulation and draws no answer. The record has every leg: driven circular turning of the sites acts on spin waves as a field [29], magnetises a crystal that holds no ordered moments [30] and reverses a magnet in the direction the pulse's hand chooses [31]; in the other direction spin waves in a graded field become lattice waves of one hand [10], and a magnet losing its order hands its circulation to the lattice [9]. A kernel unchanged by a common turn of sites and moments keeps the sum of the two circulations, so a unit leaving one holding enters the other. The size of the answer runs through a kernel that is not written here (O-6).

The standard account reads these as couplings between two kinds of quasi-particle, the phonon and the magnon. FDT reads one turning held two ways, by inertia and by a twist, with one unit passed between them (I-2, P-5); every measured rate and efficiency is accepted in full.

9.6 Large turnings: the order of the pulses

Small turnings add as arrows, and arrows commute. A held moment's large turnings compose as turns of a sphere and do not: two quarter-turn pulses about crossed axes leave the moment in different places according to their order.[G14] That is 0.1a in the laboratory: each pulse settles the state the last one left, and events do not commute. Two consequences govern control at large angles. The law (15) slows the turning as the cone opens, by $\cos\Theta$, so a driver that keeps pushing along the velocity follows the falling rate. And a moment carried over a barrier stays where the pulse leaves it, so the end of the pulse is timed to the turning: the record has the reversal by one shaped pulse [33] and the choice between states by the axis of a linear pulse [32].

9.7 What bounds the control

- *Damping.* A written turning lasts $1/\mathfrak{z}$, and a sequence of pulses fits inside it.
- *The run.* A driver confined in space writes a band of wave vectors; the written pattern moves at $d\omega/dk$ and spreads with the band.
- *The open intervals.* $\mathfrak{h} < 1$ with the run in the orbit plane, and $\Theta < \pi$ for a held moment.
- *Turning nobody wrote.* Before the driver arrives the lattice is already turning in every wave, with phases no one set. Part of it is heat, $\bar{n}$ units in a wave of rate ω. The rest is the floor: no term of the chain vanishes (0.1), so no wave is ever at rest, and the record's least share is half a unit in every wave [36]. Against a written turning of N_q units the phase spread is

$$\delta\varphi \simeq \sqrt{\frac{\bar{n}+\frac{1}{2}}{2N_q}}, \qquad \bar{n} = \frac{1}{e^{\hbar\omega/k_BT}-1}, \qquad N_q \gg \bar{n}+\tfrac{1}{2}. \tag{25}$$

  At 5 GHz and 300 K, $\bar{n} = 1.25\times10^3$. As the lattice is cooled $\bar{n}\to 0^+$ and the spread tends to $1/(2\sqrt{N_q})$, never to zero.[G15, G16, G18]

The standard account reads the half unit as the fluctuation of an empty vacuum and the unwritten phase as random. FDT reads a medium that is never empty and the helical phase of a state the experimenter did not write (0.1, I-4): the spread (25) is the experimenter's ignorance of $\mathfrak{u}(0)$ in the control law, and it shrinks as the written state outweighs the unwritten one, or as $\mathfrak{u}(0)$ is read first and the driver timed to it. The measured statistics of unseeded starts and of the floor are accepted in full.

9.8 The dials, and six protocols

Each protocol below is a comparison with a stated zero, the setting at which the written quantity cancels by the control law; the floor's turning remains in every case.

1. *Hand by delay.* Two crossed linear pulses at the delay τ; the circulation read through the twist it induces follows $\sin\omega\tau$. Zero: $\tau = \mathscr{T}/2$.
2. *Cancel on command.* A second pulse on the same axis at a half period, with its area matched to the first after damping, $e^{-\mathfrak{z}\mathscr{T}/2}$ of it. Zero: the written turning after the pair.
3. *A swept wave vector.* A front swept across the face at the speed v writes the wave with $\omega(k)/k = v$; changing v reads the law $\omega(k)$ off the lattice. Zero: the response at a sweep speed that no branch has.

dial of the wave	driver setting that writes it	check	record
rate ω	a tone at ω, or a pulse shorter than the period	G17	[22]
wave vector $\mathbf{k}$	crossed strokes; a front swept at the phase rate; a comb	G08, G09	[26]
hand	the driver's hand (inertial); the delay $\mathscr{T}/4$ or $3\mathscr{T}/4$ of two crossed pulses; the lattice's own (held)	G01, G03, G06	[8, 30]
start azimuth φ_0	arrival time; polarisation axis; impulse or moved centre	G07, G13	[23, 27]
radius $\mathfrak{r}$, cone Θ	pulse area; time on resonance	G04, G13	[28]
cancel	an equal pulse half a period later	G05	[25, 28]
projection	one stroke: twist; the dyad's trace, antisymmetric and trace-free parts: packing, twist by hand, shear	G12	[27, 22]
phase of a sustained turning	detuning inside the locking range	G10	[34]
one of two phases	a seed before the kernel is modulated	G11	[35]

Table 3: What a driver sets and how. Checks refer to the receipt (Appendix B).

4. *Handing over the unit.* A turning of one hand written in the sites, and the held moments' cone read against the driver's hand. Zero: a linear driver of equal energy.
5. *Seeded growth.* A kernel modulated at 2ω after a seed pulse of chosen axis; the grown phase follows the seed and reverses with it. Zero: the difference between runs with equal seeds.
6. *Read, then write.* A weak probe reads $\mathfrak{u}(0)$ of the wave to be controlled, and the pulse is timed to it; the spread (25) falls to the probe's own. Zero: the residual phase spread at the probe's limit.

10 Convergence statement

Every identity of Section 2 is kinematic and holds for any dots on circles. With the kernels taken from the measured laws, FDT returns the consensus record by the following maps.

- *Lattice waves.* Equation (11) is the measured law of lattice vibrations for the fitted Φ_{Δ}; $E = \omega L$ returns the angular momentum of circular lattice modes [7, 8, 9]; (13) is $c_s^2 = \Phi_1 \mathfrak{d}^2/m$ with $m = E_0/c^2$; the site clock returns the thermal shift of [18].
- *Spin waves.* Equation (15) at small cone is the law of record [15, 16]; the unit along the magnetisation returns the spin flip of neutron scattering [11]; the scalar grade returns the dipolar law [17].
- *Light.* One unit per quantum returns the measured torque [6]; (16) is the orbit of a charge in a wave of one hand.
- *The tide.* Equation (17) is the transverse trace-free pattern of general relativity for the same ripple (I-3 containment), the measured one [21].
- *The universal lattice.* Equation (18) between two heights returns the shift read on the tower of [37], and (19) is the exact optical metric, which returns the deflection of I-3.
- *Control.* Equations (22) and (23) are the linear response of the two lattices, and Section 9 returns the records of pulse pairs, written hands, impulsive and displacive starts, locking

and seeded growth cited there; (25) at $\bar{n} \to 0^+$ is the measured phase spread of a written wave at low temperature.

No divergence follows from any identity of this manuscript: it supplies no additional source, no constitutive input and no changed observable dictionary, and its kernels are the measured ones.

Two standard readings are declined. The standard account reads a phonon or a magnon as a quasi-particle that travels through the lattice carrying $\hbar\omega$ and $\hbar$ from place to place. FDT reads the lattice's one phase-ordered turning, each unit $\hbar$ of circulation of the whole pattern, and what runs is the modulation of the turning (0.1a, 0.1b, I-2); the measured energies, wave vectors and angular momenta are accepted in full. The standard account reads the interaction of two sites as an exchange of such carriers. FDT reads one carrier and no mediation, interactions being curvature and geodesic motion (I-2).

11 Status ledger

result	rests on	status
The animation's numbers (Table 1)	Appendix A	measured
Packing, the Kepler form, the open interval, the crest excess (3)–(5)	kinematics	derived, exact
The three projections (Proposition 1)	I-3a	derived, exact
Winding of world-lines (Proposition 2)	kinematics	derived, exact
$\omega = dE/dL$; $\hbar$ per quantum (Proposition 3)	0.1a	derived
The inertial law (11), with the holding term (12), and $E = \omega L$	P-3; Φ_0, Φ_Δ from data	derived, given the kernel
The lattice's rate on the strand rate (13)	0.1d	derived
The longitudinal branch as the sum of the in-plane hands	I-3a, P-3	derived (kinematics); its rate from the stroke between clamps: to be derived (D-05 of [2])
The held law (15), its unit, its hand	P-5; J_Δ from data	derived, given the kernel; the kernel from the stroke through the gradient: to be derived (D-06 of [2])
The charge's share (16); the clamped stroke's triangle	0.1d	derived
Circles sized to the density (Proposition 4), their growth (20)	0.1a, 0.1c, I-3	derived, exact on the exterior branch
The tide: onefold circles, a twofold map (17)	I-3, I-3a	derived; the ripple composed of the stroke's quanta: to be derived (D-04 of [2])
The control law (22)–(23), the four matchings, pulse pairs (24), locking, seeded growth	I-1 (driven observable), 0.1a	derived for small turnings; exact pulse areas for a held moment
The phase spread (25)	0.1, I-4	derived for $N_q \gg \bar{n} + \frac{1}{2}$; the half unit is the record's

not written here	binds at	status
Maps from a charge or a circulating charge to an I-1 input: the kernels Φ_Δ, J_Δ and the size of the answer of Section 9.5	I-1, P-5	to be derived (O-6)
The confinement operator D and the standing stroke's spectrum	I-5	to be derived (O-9)
A geometric odds for application A-3	0.4, I-1a	to be derived (O-2)
D-L1. How a lattice's tensor kernel prices the three projections, from the stroke between clamps	I-3a, P-3	to be derived
D-L2. Whether an odds of the orbit lattice is A-3's: the crest ratio $\mathfrak{n}_{\mathrm{crest}}/\mathfrak{n}_0 = 1/(1-\mathfrak{h})$ reaches 2 at $\mathfrak{h} = \frac{1}{2}$, and the crest excess $\mathfrak{h}/(1-\mathfrak{h})$ reaches 2 at $\mathfrak{h} = \frac{2}{3}$	0.4, I-1a (A-3)	to be derived; no threshold is used
D-L3. The density-channel sensitivity of an orbit wave's rate	I-7	to be derived
D-L4. The control law beyond small turnings on an inertial lattice, where the chord kernel is no longer quadratic	0.1a	to be derived
D-L5. The half unit of a wave's floor, from the floor of the chain	0.1, 0.1a	to be derived
D-L6. The resizing of a lattice's circles for a given deposit of a driver	I-7, 0.1c	to be derived

A Measuring the animation

The recording has 212 frames of 788×800 pixels at 70 ms per frame. In each frame the dots were separated from the thin circles by eroding the thresholded image and their centroids taken; 152 dots were followed through every frame. A circle fitted to each track gives the centre, the radius (35.54 px, radial residual 0.24 px) and the angle against time (-0.20822 rad per frame, clockwise). The centres lie on a square lattice of spacing 39.83 px, to 0.5 px. The phase law was fitted on the lattice indices of the sites by maximising the coherence $|\sum_j e^{i(\varphi_j - \mathbf{k}\cdot\mathbf{y}_j)}|/N$: the steps are -0.448 and $+0.449$ rad per site along rows and columns, each ± 0.001 by resampling the dots, with coherence 0.987. A wave vector on a lattice is defined modulo the reciprocal lattice, and this is its shortest representative, the one the eye reads. Fitted on the centres' pixel positions, the same phases follow a wave vector in a higher zone, of wavelength 0.71 spacings, with coherence 0.998: the phases follow the centres' positions, which sit 0.5 px off the perfect lattice, more closely than they follow the indices. The direction of the fronts was confirmed from the drift of the phase of $\sum_j e^{-i\mathbf{k}\cdot\mathbf{x}_j(t)}$, whose size, 0.267 per dot, is the first harmonic of the packing. The script `measure_animation.py` reproduces these numbers from the recording.

B Receipt

The script `receipt_orbit_lattice.py` recomputes every identity and every printed number of this manuscript: the animation's arithmetic (A01–A10), the kinematics by finite differences and quadrature (B01–B19), the two held turnings by normal modes and by integration of a ring of sites and a ring of moments (C01–C11), the rates, the charge's orbit and the loaded

line (D01–D07), the tide (E01–E02), the printed numbers (F01–F07) the control results by integration of the driven equations (G01–G19) and the universal lattice (H01–H05). Its output is `receipt_orbit_lattice.out.txt`. Result: **82/82 PASS**. The grey tags in the text name the checks.

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