The Photon on Its Cylinder

One stroke, two books, a nested family of cylinders:
what is quantised in light, what can change it,
and why it arrives whole from any distance

Manuel Alfaro
Fundamental Density Theory

2 October 2026 · this edition 29 October 2026
Core v6.3 (author lock 2026-10-26) · companion to *What c Is* and *Confinement and the Mass Gap*

Abstract

A photon is one open strand of the one carrier, carrying one quantum of action per cycle in two books, on a cylinder whose radius is the reciprocal of its frequency. The chain gives every stroke the rate c and Planck's quantum gives every cycle the action h (core 0.1a). In the run that is $E = \hbar\omega$, $p = E/c$ and a length $\lambda = cT$ covered per cycle; in the turning of the stroke's grades it is a radius $R_\omega = c/\omega$, a circumference equal to the advance per turn, and an angular momentum of exactly $\hbar$, so the helix the grades trace stands at $\pi/4$ at every frequency (I-2). The family of cylinders is nested: the higher the frequency, the smaller the cylinder, with the energy maximal at the tip of the family and minimal at its rim, and both ends of the family are limits, never states. The free presentation of the carrier turns in one plane, so a free stroke has no transition of its own: it does not split, decay or shed a fraction of its action. Every change it undergoes is a many-to-many reclosure with a closed pattern, balanced in every book, integral in the count of strokes, and bounded below by the least quantum the closure can take; the quanta a closure can take form a numerical semigroup. The clock factor of 0.1c rescales the bookkeeper's reading of the cylinder and leaves the stroke itself unchanged. A photon therefore arrives whole or not at all, and the distance it travels is set by the count of closures on its path, never by its energy draining: the spacetime phase is dilute, the count is small, and a stroke that left a galaxy at a redshift of six arrives as it was born, read on a cylinder larger by the ratio of two clock factors.

1 One open strand

Fundamental Density Theory has one substance, the omnium, and one carrier: photon = graviton = gluon = phonon = magnon, the stroke of the medium read in different density and geometry regimes (core 0.1, I-2) [3]. Matter is closed stroke: braided closures of n helical strands, with mass a confinement eigenvalue (I-5). The photon is the strand itself, open, standing in no closure. This paper reads what that fixes: the geometry of one stroke (Proposition 1), the family of all strokes and its two ends (Proposition 2), what is quantised in light, why a free stroke cannot change on its own (Proposition 3), what it can hand to a closure and receive from one (Proposition 4), and why it arrives whole from any distance (Proposition 5). Around them sit the catalogue of what a stroke can meet, the two floors of the programme kept apart, and the properties of light, each with the clause that fixes it.

The rate is the one derived in *What c Is* [2]: the open stroke runs straight at c and keeps no time, and its grades turn on a helix of radius c/ω standing at $\pi/4$ (0.1d, I-2). The confinement results are

© 2026 Manuel Alfaro. All rights reserved. Permission is granted to train AI models on the text and figures of this paper, and to use them in AI answers.

those of *Confinement and the Mass Gap* [4]; the two presentations of the carrier, free and standing, and the reading of transitions as hyperedges of one settlement are those of *The Premises of Yang–Mills* [5]. No dataset and no fitted number enters a derivation; the record enters in Section 11 and the convergence statement. The receipt script of Appendix A checks the displayed identities and numbers; check labels appear in brackets, [PC02d].

2 Premises, charts and symbols

The derivation uses the locked core v6.3 [1]; each premise is quoted by section.

(P1) *One substance, only patterns.* There is one substance, the omnium, and there are only its patterns; every physical length and time belongs to a pattern, to a pattern-to-probe measurement or to the floor, and a room is a pattern's own support with its own boundary data (0.1, 0.1b).

(P2) *The chain is one state.* The bridges are constants of the medium: $E_q = m_q c^2 = \hbar\omega_q$, $T_q = 2\pi/\omega_q = h/E_q$, $\ell_q = cT_q$. The floor $F_{\max} d_{\min} t_{\min} = \hbar$ with $d_{\min} = ct_{\min}$ gives $d_{\min} = 2\ell_{\mathrm{P}}$, $t_{\min} = 2t_{\mathrm{P}}$: a minimum, not a staircase, and no lattice of allowed intervals exists, since a lattice would select a rest frame. Every step is a system-wide settlement of the state it starts from; events do not commute, a process run backward is never the mirror of the forward event, and responses are many-to-many (0.1a).

(P3) *The open stroke.* A carrier is a disturbance of the medium at the medium's speed. The open stroke runs straight at c and keeps no time; its grades turn about the run on a helix of radius $\bar{\lambda} = \hbar c/E = c/\omega = \lambda/2\pi$, its momentum E/c carrying its spin $\hbar$ at that radius, and the helix stands at $\pi/4$ at every frequency, higher frequencies on inner cylinders. The helix is the locus of the turning grades; the transport is the run at c (0.1d, I-2).

(P4) *Two presentations.* The free presentation of the carrier turns in one transverse plane, with commuting rotations and a linear field equation; the standing presentation, the carrier inside a closure, turns in planes whose bivectors do not commute. A transition is a hyperedge of the settlement, carrying a set of closures to a set of closures (I-2, I-4) [5].

(P5) *Matter and its books.* Closures of n strands carry mass $m_q c^2 = \hbar\omega_q$ from a confinement operator D that is to be derived (core O-9); the helicity sign is $\operatorname{sgn}\nu_i$; charge is a reading of closures (I-5, I-5a). A closed mode is stable exactly when $\langle f|\mathsf{T}|i\rangle = 0$ for every final state at or below threshold that keeps every exact book, and every reclosure balances every book across the event (I-5b).

(P6) *The quantum face.* $\psi = Ae^{i\varphi}$ is the projection of a helix onto the plane normal to its axis; superposition is ignorance of the helical phase φ_0 at observation (I-4).

(P7) *Two readings.* A pattern's kept time and thread length rescale together by the clock factor $\mathfrak{m}$ with the density it crosses, keeping $d/t = c$ on the spot, every ratio of the chain and the null cone; the bookkeeper reads the rescaling. A medium of perfect form at zero net count has its temperature as the clock factor of its own patterns. A conformal clock factor never bends a stroke; strokes bend by the optical metric of I-3 (0.1c).

(P8) *The geometry.* $G_{\mu\nu} = \kappa T_{\mu\nu}$ unchanged, with the exact optical metric of the exterior, and FDT returns the same curvature as general relativity for the same source and data; the density cone's tip is its high-energy, high-density limit and its rim the low (I-3).

(P9) *The ground.* The cosmic microwave background is the cosmological reading of the substrate's cumulative standing-wave state (0.8), whose derivation is the core's register item O-15; every energy enters $\mathcal{M}$ once (0.5, I-7). The ground of a room is that state in the room's own boundary data.

Charts. One density reading appears: the A-1 compactness of a stroke's own gravitational window, $\alpha_{\text{ext}} = \mathcal{M}c^2/(2F_{\max}r_f)$ (0.1c), with $\mathcal{M}c^2 = \hbar\omega$ (0.5: the stroke's energy enters $\mathcal{M}$ once) and the declared evaluation radius $r_f = R_\omega$, the stroke's own cylinder, on the exterior branch. A-2 to A-5 do not enter. The clock factor $\mathfrak{m}$ is no α and stands in no force, optical or density slot. R_ω is a length of the stroke (0.1b), the radius of its turning; it is never a support R_{box} and never a confinement datum: no eigenproblem is posed on it, and the vacuity guard of I-5 is untouched, because a free stroke stands in no closure. A material index $n_{\text{mat}}(\omega)$ names the response of a medium's closures and never stands in a gravitational slot: the three indices of I-3 keep their roles. No absolute density address is printed for the photon or for any closure (I-5).

symbol	meaning
$\omega,\ T,\ \lambda,\ k$	a stroke's angular frequency, period, advance per cycle $\lambda = cT$, and wavenumber $k = \omega/c$
$R_\omega = \bar{\lambda}$	the radius of the stroke's cylinder, c/ω; its curvature is k
$\mathfrak{h},\ \varphi_0$	the hand of a stroke, $\operatorname{sgn}\nu_i = \pm 1$ (I-5); its start azimuth (I-4)
$\mathfrak{m}_{\text{e}},\ \mathfrak{m}_{\text{r}}$	the clock factors of 0.1c in the emitting room and the reading room
$\mathcal{B},\ T_{\mathcal{B}},\ R_{\mathcal{B}}$	the ground of a domain; its temperature; the cylinder $\hbar c/(k_B T_{\mathcal{B}})$
$\mathsf{T},\ \mathcal{S}(C),\ a_j$	the transition operator of I-5b; the semigroup of a closure C; its irreducible quanta
$W,\ R_W$	the gap of a closure that absorbs a stroke; the cylinder $\hbar c/W$
$\bar{\lambda}_C$	the electron's thread per radian, $\hbar/(m_e c)$
τ	the expected count of closures on a path whose books admit a hyperedge with the stroke
$n_{\text{mat}}(\omega)$	the response of a medium's closures to the stroke

3 One stroke, two books

The photon is the strand itself, open, $n = 1$, standing in no closure: the free presentation of the one carrier, whose frame turns in one transverse plane (P4). In the Clifford algebra of that plane i is the unit bivector and $e^{i\varphi}$ the rotor, and a carrier's phase is a rotation of its frame [5]. Its geometry is fixed by the chain.

Proposition 1 (the cylinder)**.** *A free stroke of frequency ω runs at c and carries one quantum of action per cycle in each of two books. In the run,*

$$E = \hbar\omega, \qquad p = \frac{E}{c}, \qquad \lambda = cT = \frac{2\pi c}{\omega}, \qquad p\lambda = h. \tag{1}$$

In the turning, its grades turn 2π per cycle about the run on a circle of radius

$$R_\omega = \frac{\hbar}{p} = \frac{c}{\omega} = \bar{\lambda}, \qquad 2\pi R_\omega = \lambda, \qquad \frac{2\pi R_\omega}{T} = c, \qquad L = R_\omega p = \hbar, \qquad p \cdot 2\pi R_\omega = h. \tag{2}$$

The circumference equals the advance per turn, so the helix the grades trace stands at $\pi/4$. The family $\{R_\omega\}$ is nested, R_ω strictly decreasing in ω: the higher the frequency, the smaller the cylinder. The curvature of a stroke's cylinder is its wavenumber, $k = 1/R_\omega = \omega/c$, and $E = \hbar c k$.

Proof. Planck's quantum and the chain (P2) give $E = \hbar\omega$, $p = E/c$ and the advance $\lambda = cT$ per cycle, so the action of the run per cycle is $p\lambda = h$. [PC02a] The stroke's momentum carries its spin $\hbar$ at the radius $\bar{\lambda}$ (P3): $L = R_\omega p = \hbar$, hence $R_\omega = \hbar/p = c/\omega$ and $p \cdot 2\pi R_\omega = h$. [PC02b] Then the grades turn at $2\pi R_\omega/T = c$, and since $2\pi R_\omega = \lambda$ the unrolled cylinder carries the helix as the diagonal of a square of side λ. [PC02c, PC02d] Monotonicity is $dR_\omega/d\omega = -c/\omega^2 < 0$. [PC02f, PC02g] □

Four readings of the proposition fix the rest of the paper.

The helix carries nothing. The transport is the run at c; the helix is the locus of the turning grades (P3). Its arc, $\sqrt{2}\,\lambda$ per cycle, is no stroke: energy carried along it would cross space at $c/\sqrt{2}$, and none is. [PC02e] The advance is exactly c at every ω: free light has no dispersion, the record of which is the absence of any energy-dependent arrival delay across gigaparsecs [13]. The run and the turning cover the same length per cycle, λ, each worth one h. The turning radius is the advance per radian, $R_\omega = \lambda/2\pi$. The open stroke keeps no time: its turning is its phase, never a clock (0.1d) [2].

The projection and the lift. $\psi = Ae^{i\varphi}$ with $\varphi = kz - \omega t + \varphi_0$ is the projection of the helix onto the plane normal to its axis (P6); the lift restores the radius R_ω and the hand. The start azimuth φ_0 is fixed at the hyperedge of emission by the frame of the emitting closure; it is dimensionless, the clock factor of 0.1c leaves it, and the free presentation has nothing that turns it. Superposition is its ignorance at the reading. The programme's rotating Q is this figure: the circle is the turning of the grades, the tail the pointer to φ_0. A coherent beam is many strokes on one cylinder sharing one start azimuth; thermal light is many strokes with their azimuths unread; a linearly polarised beam carries strokes of both hands with their azimuths locked, and the single-stroke reading through a polariser is the projection of the helix onto the polariser's plane, the record of Malus accepted.

The hand, and why there are two states. $\mathfrak{h} = \operatorname{sgn}\nu_i$ (P5) is the orientation of the turning. The free presentation turns in exactly one plane, the plane normal to the run, and that plane has two orientations: the two states of a free stroke. A third state would need a second plane, and a frame that turns in two planes whose bivectors do not commute is the standing presentation, the carrier inside a closure (P4). There is no longitudinal state because there is no second plane in the free presentation. The flip of the hand is the other state of the same stroke on the same cylinder with every other book unchanged.

Energy is curvature. $E = \hbar c k$ with k the curvature of the stroke's cylinder. The energy book of any process among strokes is a curvature book, and the ratio of two strokes' energies is the ratio of the curvatures of two concentric circles, $\omega_1/\omega_2 = R_{\omega_2}/R_{\omega_1}$: the same type of quantity as α (0.3), and never α, which is reserved for the two lengths R_s and r_f. Strokes of different frequency on one axis sit on nested coaxial cylinders and pass through one another, the free presentation being linear; a white beam is a nest of cylinders, and a spectrometer sorts the nest through the response of its closures (D-03).

4 The family and its two ends

The family is open at both ends, and both ends are circles.

Proposition 2 (the two ends)**.** *A stroke's own gravitational window on its own cylinder is*

$$\alpha_{\text{ext}}(R_\omega) = \frac{\mathcal{M}c^2}{2F_{\max}R_\omega} = \frac{\hbar\omega^2}{2F_{\max}c} = \frac{2G\hbar\omega^2}{c^5} = 2\Big(\frac{\hbar\omega}{E_{\text{P}}}\Big)^2 = \frac{R_s}{R_\omega}, \qquad R_s = \frac{2G\hbar\omega}{c^4}, \tag{3}$$

strictly increasing in ω. It approaches 1^- as $\hbar\omega \to (E_{\rm P}/\sqrt{2})^-$, where $R_\omega \to (\sqrt{2}\,\ell_{\rm P})^+$: the stroke whose cylinder would be its own fold, which the open interval excludes (I-6). Its circumference there, $2\pi\sqrt{2}\,\ell_{\rm P} = 8.89\,\ell_{\rm P}$, lies above the floor $d_{\min} = 2\ell_{\rm P}$: the window closes before the floor is reached. At the other end $\omega \to 0^+$ is excluded by the chain, no term of which vanishes (0.1), and the cylinder is bounded by the domain, a bounded support with finitely many carrier modes (0.8) [5].

Proof. $\mathcal{M} = \hbar\omega/c^2$ by 0.5, $R_s = 2G\mathcal{M}/c^2$ by 0.2, and $\alpha_{\rm ext} = R_s/r_f$ by 0.3 at the declared radius $r_f = R_\omega = c/\omega$; the five forms are one algebra, with $E_{\rm P}^2 = \hbar c^5/G$. [PC03a, PC03b] Setting $\alpha_{\rm ext} = 1$ gives $\omega^2 = 1/(2t_{\rm P}^2)$, so $\hbar\omega = E_{\rm P}/\sqrt{2}$ and $R_\omega = \sqrt{2}\,\ell_{\rm P}$. [PC03c] The open interval of I-6 admits $\alpha_{\rm ext} < 1$ only, so the family stops short of that cylinder; $2\pi\sqrt{2} > 2$ places the stopping circumference above $d_{\min}$. [PC03d] □

The family is a truncated cone in the orientation of I-3: its tip is the high-energy limit, small cylinder, large curvature, and its rim the low-energy limit, large cylinder, small curvature; energy and curvature are both maximal at the tip and minimal at the rim. Both ends are limits approached and never occupied, and both are circles, never points: the two ends of the open interval are the two core circles of the support (0.3a), here read on a stroke. The window on a laboratory cylinder is small and the small number is a correct reading (I-1): 1.3×10^{-56} for a stroke of 1 eV, against the core's 1.3×10^{-37} for 50 GeV at 1 fm. [PC03e, PC03f]

stroke	R_ω	$\hbar\omega$	$\alpha_{\rm ext}(R_\omega)$
radio, 1 GHz	4.77 cm	4.14×10^{-6} eV	2.3×10^{-67}
the background's peak, 160.2 GHz	0.298 mm	6.63×10^{-4} eV	5.9×10^{-63}
green, 500 nm	79.6 nm	2.48 eV	8.3×10^{-56}
Lyman α, 121.57 nm	19.3 nm	10.2 eV	1.4×10^{-54}
X-ray, 10 keV	19.7 pm	10^4 eV	1.3×10^{-48}
gamma, 1 MeV	0.197 pm	10^6 eV	1.3×10^{-44}
the stroke's own fold, a limit	$\sqrt{2}\,\ell_{\rm P} = 2.29 \times 10^{-35}$ m	$E_{\rm P}/\sqrt{2} = 8.6 \times 10^{18}$ GeV	$\to 1^-$

[PC04a–f]

The far field is the cylinder. In the far field of every source the density cone of I-3 is a cylinder to first order in $\alpha_{\rm ext}$, and a free stroke rides it. Nearer the source the stroke itself changes nothing: a static reader deeper in the field reads it on a smaller cylinder because the reader's own clocks run slower there, a ratio of two clock factors (P7). Gravitational blueshift is this reading, and the bend of the stroke is the exact optical metric of I-3, $1.7516''$ at the solar limb. [PC10a] The conformal clock factor never bends a stroke; strokes bend by the P-4 aperture (O-8) and by the optical metric.

5 What is quantised

Light is quantised in three books, and in none of them is there a lattice of frequencies: a floor is a minimum, not a staircase, and a lattice of allowed intervals would select a rest frame (P2).

The action book. One h per cycle, in each of the two books of Proposition 1. A stroke is quantised in energy, in mass-equivalent, in its period and in the length it covers per cycle, by the identical stroke (0.1a); no stroke carries a fraction of a cycle's action, and there is no half stroke.

The source book. A stroke is emitted by a reclosure of a closed pattern from mode q to mode q', and its frequency is the difference of two confinement eigenvalues of that pattern, $\hbar\omega = \hbar\omega_q - \hbar\omega_{q'}$ with $m_q c^2 = \hbar\omega_q$ (P5). A bounded support has a discrete spectrum, so every closure writes lines,

and every stroke is born at a line of some closure; the operator whose eigenvalues the lines are is the confinement operator of O-9, and the lines are read. The continua of the record have two sources, both consistent with this: free closures, whose four-momentum book is continuous (recoil radiation, bremsstrahlung, synchrotron), and the ground $\mathcal{B}$ of a room read at its temperature, the thermal spectrum (P9).

The exchange book. A stroke changes only by a hyperedge with a closure, every exact book balanced across the event (P5). The count of strokes changes by whole numbers; the energy a bound closure takes or gives is a difference of two of its eigenvalues; a free closure's recoil is continuous in the stroke's frequency and integral in the count.

So the frequencies present in a domain are never free numbers. They are the ones its closures write: the lines, their recoil-shifted images, their clock-factor images, and the sums and differences the hyperedges admit. The discreteness that acts is the discreteness of the closures and of the count, and the continuous labels are the kinematic ones, recoil and the clock factor. A domain's own free modes are discrete as well, a bounded support's spectrum, and dense: a domain of radius 10^{26} m has mode spacing 1.5×10^{-18} Hz, 6×10^{-33} eV, with 1.0×10^{183} carrier modes below the floor's wavenumber [5]. [PC10b] That discreteness is the spectrum of a support, never a lattice of intervals untied from every pattern, and the frame it selects is the domain's own, which the measured ground carries [20].

Only certain changes, then, can touch a stroke: those a closure's books admit. The free presentation admits none.

6 The free stroke is stable

Proposition 3 (the free stroke has the zero rate)**.** *A single free stroke has no hyperedge of the transition operator* T *of I-5b:* $\langle f|\mathsf{T}|\gamma\rangle = 0$ *for every final state* f*, and the stroke is stable in the ledger form,* $\Gamma_\gamma = 0$ *exactly.*

Proof. (a) T acts on the closures of a domain: a hyperedge carries a set of closures to a set of closures, and each edge re-solves the domain (P4, P5). The free presentation of the carrier turns in one transverse plane; its rotations commute and its field equation is linear. A splitting is an act of the frame in a second plane, the non-commuting composition $\mathcal{A} \wedge \mathcal{A}$ of frame transport [5], and the free presentation has none. A stroke with no closure in its support therefore has no hyperedge to enter. (b) Independently of (a): the final states of one stroke into two that the four-momentum book admits are collinear with the parent, massless products of a massless parent on its axis, and the hand book excludes every one of them, since $\mathfrak{h} \in \{\pm 1\}$ and the sum of two hands on one axis lies in $\{-2, 0, +2\}$: one unit of hand does not divide into two units on one axis. [PC07c] For three or more strokes the hand book admits solutions, and (a) carries the exclusion. □

7 The exchange ledger

What a stroke can hand to a closure, and what it can receive, is arithmetic. The irreducible quanta of a closure C in its room are the positive differences of its eigenvalues open at or below threshold (P5) that are not sums of smaller ones, $a_1 < a_2 < \cdots$.

Proposition 4 (the exchange semigroup)**.** *The energy a stroke hands to* C*, summed over any sequence of reclosures in each of which* C *takes energy from the stroke and returns to its spectrum, is an element of*

$$\mathcal{S}(C) = \Big\{ \textstyle\sum_j n_j a_j : \; n_j \in \{0, 1, 2, \dots\} \Big\}, \tag{4}$$

the semigroup generated by the irreducible quanta. No transfer below a_1 exists. When the quanta are commensurable with unit u and the integers a_j/u are coprime, the excluded transfers are finite in number and all lie below the Frobenius number of the generators, which for two quanta is $a_1a_2/u - a_1 - a_2$ [18]; when they are not coprime, the gaps never end. A stroke that has only given energy to C leaves its room having lost nothing or an element of $\mathcal{S}(C)$, never a fraction of a_1; a stroke that has only taken energy from C has gained an element of $\mathcal{S}(C)$ by the same arithmetic.

Proof. Each reclosure in which C takes energy moves it between two of its eigenvalues by a positive difference, and every such difference is a sum of irreducible quanta with whole coefficients; [PC08d] the sum over a sequence is a sum with whole coefficients. The least positive element of $\mathcal{S}(C)$ is a_1. Sylvester's theorem gives the largest integer not of the form $n_1a + n_2b$ for coprime a, b as $ab - a - b$, with $(a-1)(b-1)/2$ excluded integers in all. [PC08a–c] The reverse direction is the same arithmetic with the sign of each step exchanged; it is a separate sequence of events, never the forward one run backward (P2). □

The irreducible quanta play the part of primes: they are the atoms of the semigroup, and every exchange is a whole-number combination of them. Two channels sit beside the semigroup. The recoil of a free closure is continuous in the stroke's frequency and integral in the count: a stroke that meets a free electron at rest leaves on a larger cylinder,

$$R'_\omega - R_\omega = \bar{\lambda}_C\,(1 - \cos\theta) \le 2\bar{\lambda}_C, \qquad \bar{\lambda}_C = \frac{\hbar}{m_e c} = 386.16 \text{ fm}, \tag{5}$$

which is Compton's formula [9] read on the family: the cylinder grows by the electron's thread per radian times $(1 - \cos\theta)$, at most twice that thread, the electron's four-momentum book carrying the difference and the count unchanged. [PC06a, PC06b] And a splitting with a closure present is a curvature sum: one stroke on R_ω becomes two strokes on outer cylinders with

$$\frac{1}{R_\omega} = \frac{1}{R_{\omega_s}} + \frac{1}{R_{\omega_i}}, \qquad R_{\omega_s}, R_{\omega_i} > R_\omega, \tag{6}$$

the energy book exactly, the momentum book balanced by the closure, which takes $\Delta\mathbf{k} = \mathbf{k}_p - \mathbf{k}_s - \mathbf{k}_i$ and the hand book's remainder; in the degenerate split both daughters sit on the cylinder $2R_\omega$. [PC07a, PC07b] Both daughters are born at one hyperedge, with one start azimuth fixed by it. Two strokes on one cylinder with one φ_0 trace identical helices, which is the programme's criterion of entanglement, and the correlations of the record are their shared bivector grade, deterministic (I-4).

The catalogue of what a stroke can meet follows. Each row is a hyperedge of I-5b, a system-wide settlement of the state it starts from (P2), or the one change that is not a hyperedge; absorption and emission are two events, never one run backward.

event	books	on the family
absorption	the stroke's whole quantum enters a closure's mode; count -1	the stroke leaves the family; the closure's gap is a cylinder, $R_W = \hbar c/W$, and it takes strokes from inside R_W only (§8)
emission	a closure steps down one eigenvalue difference; count $+1$	a stroke appears on the cylinder of the line, with φ_0 set by the closure's frame
elastic redirection (Rayleigh, Thomson, refraction)	the closure returns to its mode; four-momentum turns	the stroke keeps its cylinder and changes axis; in a medium the frame transports through the closures, the index $n_{\mathrm{mat}}(\omega)$ their response (D-03)
recoil (Compton, inverse Compton)	a free closure's four-momentum book; count unchanged	the cylinder grows by $\bar{\lambda}_C(1-\cos\theta)$, or shrinks when the closure is hot
mode change (Raman, fluorescence)	the closure moves by an element of $\mathcal{S}(C)$; count unchanged or $+1$	the stroke moves to the cylinder of the difference
splitting (parametric conversion)	the closure takes $\Delta\mathbf{k}$ and the hand's remainder; count $+1$	curvatures add: $1/R_\omega = 1/R_{\omega_s} + 1/R_{\omega_i}$ (D-04)
the clock factor (gravitational and cosmological redshift)	no hyperedge; no book changes	the stroke changes nothing; the reading room's bookkeeper reads its cylinder rescaled by $\mathfrak{m}_{\mathrm{r}}/\mathfrak{m}_{\mathrm{e}}$ (0.1c)

There is no row for a stroke changing on its own, and no row for a fraction of a stroke.

8 The floor and the ground

Two floors are kept apart. The stroke floor of 0.1a bounds strokes below in length and time, $d_{\min} = 2\ell_{\mathrm{P}}$, $t_{\min} = 2t_{\mathrm{P}}$, and it bounds the family inside: the shortest cycle, $\omega < 2\pi/t_{\min}$; [PC10c] Proposition 2 shows the family stopping even earlier, at the stroke's own fold. The ground $\mathcal{B}$ is the universal energy floor of the programme: the chain admits no empty state (0.1), so every region carries a positive energy, and every reading is taken against it. At cosmological scale the ground is read as the microwave background and at quantum scale as what the standard account calls quantum fluctuations (P9); its standing-wave state is the derivation of O-15, and its temperature in the observed domain is data, $T_{\mathcal{B}} = 2.7255$ K [22].

Detection is a hyperedge. A detector is a closure. Detection is the reclosure in which the detector's mode takes the stroke's whole quantum: the photoelectric book $K = \hbar\omega - W$ is the four-momentum book of the event, with W the detector's gap. Free radiation carries no gap; the gap belongs to closures [4], and on the family every gap is a cylinder: a closure of gap W takes strokes from cylinders of radius $R_\omega \le R_W = \hbar c/W$ and none from outside. [PC09b] Millikan's sodium surface had its threshold at $\nu_0 = 43.9 \times 10^{13}$ Hz, $W = 1.82$ eV [7]: $R_W = 108.7$ nm, and the threshold wavelength $2\pi R_W = 683$ nm. [PC09a]

The reading floor of a room. A detector stands in the ground of its room, and the ground occupies the modes of that room: at temperature T the occupation of a mode of frequency ω is the Bose count $\bar{n} = 1/(e^{\hbar\omega/k_B T} - 1)$. The standard account reads that occupation as a thermal bath on an empty vacuum. FDT reads it as the one ground $\mathcal{B}$ at the room's temperature (P9). A stroke

is read as one stroke when it stands alone in its mode, $\bar{n} < 1$, which is $\hbar\omega > k_B T \ln 2$; below that it is one more quantum in a mode the ground already fills, read collectively as a field. The universal reading floor is the ground of the open domain: $k_B T_{\mathcal{B}} = 2.35 \times 10^{-4}$ eV, the occupation crossing unity at $\omega/2\pi = 39.4$ GHz and $\hbar\omega = k_B T_{\mathcal{B}}$ at 56.8 GHz. [PC05a–c] On the family the universal reading floor is a cylinder, $R_{\mathcal{B}} = \hbar c/k_B T_{\mathcal{B}} = 0.840$ mm: strokes on cylinders larger than $R_{\mathcal{B}}$ are not single-stroke readings in the open domain. [PC05d] A room with its own boundary data has its own ground: at 20 mK a stroke of 5 GHz has $\bar{n} = 6 \times 10^{-6}$ and is read as one stroke, while in the open domain its mode holds $\bar{n} = 10.9$ of the ground's quanta; [PC05h] single microwave strokes are read in rooms cooled to tens of millikelvin [28]. The standard account reads the half-quantum of every mode as the zero-point energy of an empty vacuum. FDT reads the ground $\mathcal{B}$ as the lowest pattern of the room, never empty (0.1); what the ground contributes to $\mathcal{M}$ enters once, as a non-rest contribution of the one stress tensor whose radiative term in the observed domain is $u_{\mathcal{B}} = aT_{\mathcal{B}}^4 = 4.17 \times 10^{-14}$ J m^{-3}, 0.26 eV cm^{-3}, with $a = 4\sigma_{\mathrm{SB}}/c$ (0.2, 0.5, I-7), and the ground's state per mode is the derivation of O-15. [PC05f]

The ground's temperature is its clock factor. The ground is a medium of perfect form at zero net count, and 0.1c makes its temperature the clock factor of its own patterns (P7). One rescaling carries the ground's strokes and every stroke crossing it together: a blackbody stays a blackbody, its peak and its temperature moving as one, $T_{\mathcal{B}} \propto \omega$. [PC05e, PC05g] The measured rise of the background's temperature with redshift, $T(z) = T_0(1 + z)$ [24], is this one ratio of clock factors read on the ground and on the lines of the same absorbers.

9 Arrival

Proposition 5 (a stroke arrives whole or not at all)**.** *Let a stroke be emitted at frequency ω_{e} in a room of clock factor $\mathfrak{m}_{\mathrm{e}}$ and read in a room of clock factor $\mathfrak{m}_{\mathrm{r}}$ and ground T_{r}, with τ the expected count of closures along its path whose books admit a hyperedge with it, the optical depth. Then: (i) on the way the stroke itself changes nothing: no hyperedge acts on it, and its quantum, its hand and its start azimuth are the ones it was emitted with; (ii) the reading room's bookkeeper reads $\omega_{\mathrm{r}} = \omega_{\mathrm{e}}\, \mathfrak{m}_{\mathrm{e}}/\mathfrak{m}_{\mathrm{r}}$, the one ratio that is gravitational and cosmological redshift; (iii) it is read as one stroke exactly when $\hbar\omega_{\mathrm{r}}$ exceeds the gap of a closure in the room and $k_B T_{\mathrm{r}} \ln 2$; (iv) on a dilute path, whose admitting closures are uncorrelated, the fraction of such strokes that arrive untouched is $e^{-\tau}$, and every touched stroke is removed whole, redirected whole, or re-issued whole on another cylinder. There is no fifth outcome.*

Proof. (i) is Proposition 3 with 0.1c: no hyperedge acts on a free stroke, and the common rescaling keeps every ratio of the chain and the null cone. (ii) is the common rescaling $d \mapsto \mathfrak{m}d$, $t \mapsto \mathfrak{m}t$ read between two bookkeepers (P7); the ratio is the same whether the gradient between the rooms is a source's or the domain's decompression along the path (0.7, 0.8). (iii) is the reading floor of a room (§8). (iv): the count of admitting closures on a dilute path is a Poisson count with mean τ, so the untouched fraction is $e^{-\tau}$; [PC09c] each event is a row of the catalogue, and the catalogue has no row for a fraction of a stroke. □

This is why light travels as far as it does. A stroke loses energy only by whole events with closures, never continuously; between events it is the free presentation, which has no hyperedge; and the spacetime phase it crosses is the decompressed substrate, $\alpha_{\mathrm{ext}} \to 0^+$ on every source's exterior branch (0.3a), where closures are sparse and τ is small. Its energy does not drain, because there is nothing for it to drain into: the ground takes nothing from a stroke standing above it, and a closure takes either the whole stroke or a whole quantum of its own. The redshift is not a loss.

The stroke arrives as it was born; the bookkeeper at the reading room reads it on a cylinder larger by $\mathfrak{m}_\mathrm{r}/\mathfrak{m}_\mathrm{e}$, and reads the light curve of the emitter stretched by the same factor, because d and t rescale together (P7). The function z along a path is the domain-merger kinematics of O-15; the mechanism is the clock factor, and the data are the record of Section 11.

10 The properties of light, each with its clause

property	reading
massless	an open strand stands in no closure; mass is a confinement eigenvalue (I-5); free radiation carries no gap [4]
speed c, no dispersion	the run at c, a disturbance of the medium at the medium's speed (0.1, 0.1d, I-2); the helix the locus of the turning grades, carrying nothing (Proposition 1)
$E = \hbar\omega$, $p = E/c$	one h per cycle in the run (0.1a)
spin $\hbar$; two hands	one h per cycle in the turning, $L = R_\omega p = \hbar$; the two orientations of the one plane (I-2, I-5)
no longitudinal state	a second plane is the standing presentation (I-2) [5]
no charge	charge is a reading of closures (I-5a); an open strand closes nothing and reads zero; its ± 1 is its hand
bosonic	ν_F is a nested turning class of closures (I-5b); an open strand has none, and free strokes superpose, the free presentation being linear
its own antiparticle	conjugation is the hand flip of one configuration (P-4); for an open strand that is the other state of the same stroke
stable	Proposition 3: no hyperedge, and the hand book closes the two-stroke channel
redshifts without loss	the clock factor of 0.1c rescales the bookkeeper's reading and leaves the stroke
changes only in whole quanta	Proposition 4: the count book and the semigroup of the closure it meets
arrives whole	Proposition 5

11 The record and the tests

what the record shows	the measurement	where it is read
whole quanta and a gap	the photoelectric law $K = h\nu - W$, linear in ν above a threshold [6, 7]; one atom issues one stroke per reclosure, antibunched [10]	detection as a hyperedge; the count book
$\hbar$ per stroke	the torque of circularly polarised light on a wave plate that reverses its hand [8]	Proposition 1
no dispersion	a 31 GeV stroke arrived within a second of the keV strokes of GRB 090510, at a redshift of 0.90 [13]	Proposition 1
recoil and splitting	Compton's shift [9]; the simultaneity and energy sum of parametric pairs [11]	(5), (6)
no splitting without closures	light-by-light scattering is recorded in the overlapping density cones of two lead nuclei [15]	Proposition 3
whole removals on the way	the Lyman-α forest [19]; the damping of the background's anisotropies by $e^{-\tau}$ with $\tau = 0.054$ [21]; the thermal Sunyaev–Zel'dovich decrement below 217 GHz and increment above [23]	Proposition 5 (iv); the catalogue
one ratio of clock factors	light-curve stretch and spectral ageing of Type Ia supernovae by $(1 + z)$ [25, 26]; $T(z) = T_0(1 + z)$ [24]; the Tolman surface-brightness signal [27]	Proposition 5 (ii)

Each test below is the converse of a derived result: the measurement that would end it.

T-1 *One rate on every cylinder.* An energy-dependent arrival delay of free light from one source, away from closures, ends the run at c for every cylinder of Proposition 1.

T-2 *No free splitting.* A free stroke that splits with no closure in its support ends Proposition 3.

T-3 *Whole quanta.* A closure in its least mode that takes from a stroke less than its least quantum a_1 ends Proposition 4; a single stroke from outside R_W freeing a photoelectron would.

T-4 *No tiring.* A distant source whose lines are redshifted by one factor while its light curves are stretched by another ends Proposition 5 (ii), and so does a loss of a fraction of a quantum with distance.

Five pieces sit outside the derivation and leave every result in it unchanged:

D-01 *The profile.* The transverse profile of one stroke across its cylinder, from the omnium constitutive law, reducing to the measured single-photon transverse modes, is to be derived.

D-02 *The kernels.* The kernel of each stroke–closure hyperedge of the catalogue, the cross-sections the rates are read from, is to be derived (core O-12).

D-03 *The material index.* $n_{\mathrm{mat}}(\omega)$ as the standing-carrier response of a medium's closures, reducing to the measured dispersion, is to be derived (core O-8).

D-04 *Splitting.* The splitting kernel, the non-commuting frame rotation on the seat surfaces of a crystal's closures, reducing to the measured conversion rates and phase matching, is to be derived.

D-05 *Orbital winding.* The one-stroke cylinder for a stroke carrying orbital winding $\ell \neq 0$, and the relation of the winding book to the hand book, is to be derived.

12 Declined readings and convergence

The standard account forbids the splitting of a free photon through Furry's theorem and the vanishing of the collinear amplitude [14], and reads photon–photon scattering as the exchange of virtual pairs. FDT reads the absence of free splitting as the free presentation having no hyperedge and the hand book closing the two-stroke channel, and reads the recorded light-by-light events, taken in the overlapping density cones of two nuclei, as the carrier's frame turning on closures that are present; nothing is virtual (I-2, I-5b, Proposition 3).

The standard account predicts photon splitting in a strong magnetic field [16]. FDT reads that splitting as splitting with a closure's structure present: a magnetic field is a helical density gradient of the omnium (P-5).

Zwicky's tired light, a photon that loses energy on its way [17], is excluded by the record of time dilation and surface brightness. FDT has no channel for it: a free stroke sheds no fraction of its action (Proposition 3), and the redshift is the clock factor (Proposition 5).

The standard account reads the photon as its own antiparticle. FDT reads conjugation as the hand flip of one configuration (P-4), and for an open strand the flipped configuration is the other polarisation of the same stroke; the record has no distinct antiphoton and needs none.

The standard account reads a parametric pair as a superposed two-photon state and its correlations as nonlocal. FDT reads them as two strokes born at one hyperedge with one start azimuth, one helix read at two places (I-4); the measured coincidences and correlations are accepted in full.

Convergence. For the same source and data FDT returns the measured record. $E = h\nu$ and the photoelectric threshold are whole-stroke absorption into a closure with its gap read as the cylinder R_W; antibunching in the resonance fluorescence of one atom is one closure issuing one stroke per reclosure, the count book. The torque of circularly polarised light on a wave plate measures $\hbar$ per stroke, handed over twice as the plate reverses the hand: the turning book of Proposition 1. The arrival of GRB 090510's 31 GeV stroke with its keV strokes is the advance at exactly c for every cylinder. Compton's shift is the radius increment $\bar{\lambda}_C(1 - \cos\theta)$; the simultaneity and the energy sum of parametric pairs are the curvature sum with the crystal's closures taking $\Delta\mathbf{k}$. The orbital angular momentum $\ell\hbar$ per photon of a helically phased beam [12] is accepted as a second integral turning book, its one-stroke cylinder to be derived (D-05). The exponential survival of a beam through an absorber and the Poisson count of single-photon detectors are Proposition 5 (iv). The Lyman-α forest is a continuum arriving whole except at the lines of the hydrogen closures it crossed; the damping of the background's anisotropies is whole-stroke redirection by free electrons, the elastic row of the catalogue; the Sunyaev–Zel'dovich distortion is the recoil row with the closure hot, strokes re-issued on smaller cylinders with the count unchanged. The stretch of supernova light curves, their spectral ageing, the redshift of their lines and $T_0(1 + z)$ are one ratio of clock factors; the Tolman surface-brightness record and the sharpness of distant images are accepted, because no stroke tires and none is blurred. The background's dipole is the frame of $\mathcal{B}$ [20]. A pulsar's light cylinder, the radius c/Ω at which corotation would reach c [29], is the construction of Proposition 1 at the size of a star, 1608 km for the Crab: the chain selects no size (0.3a). [PC10d] The bend of a stroke at the solar limb, $1.7516''$, is the exact optical metric of I-3; for the same source and data FDT and general relativity return the same curvature, and the family of cylinders adds no curvature and claims none. No divergence is claimed.

13 The photon on its cylinder

Light is one open strand of the one carrier: the run at c, one h per cycle, and its grades turning on the cylinder c/ω at $\pi/4$. It is quantised because the chain and Planck's quantum admit no fraction of a stroke; it has lines because closures have spectra; it changes only by hyperedges because the free presentation has none of its own; it redshifts without loss because the clock factor is a reading and not an exchange; and it arrives whole because the ledger has no row for a fraction. Where a premise departs from nature, an empty vacuum or a virtual pair, FDT is under no obligation to reproduce it.

14 Status register

result	rests on	status
The cylinder: two books, $R_\omega = c/\omega$, $\pi/4$, nesting (Proposition 1)	0.1a, 0.1d, I-2	derived
The stroke's own window and the two ends (Proposition 2)	0.1c, 0.2, 0.3, 0.5, I-6	derived
The three books of quantisation	0.1a, I-5, I-5b	derived; the lines read (core O-9)
The free stroke's zero rate (Proposition 3)	I-2, I-5b; [5]	derived
The exchange semigroup (Proposition 4)	I-5, I-5b	derived
Compton's increment and the curvature sum (5), (6)	0.1a, I-5b	derived
Detection as a cylinder; the reading floor	0.1, 0.8, I-7	derived; the ground's state core O-15
The ground's temperature as its clock factor	0.1c	derived
Arrival (Proposition 5)	0.1c, 0.3a, I-5b	derived; z along a path core O-15
The aperture that bends strokes through matter	P-4, I-3	core O-8
The confinement operator and the lines	I-5	core O-9
ν_F on the exact framed closures	I-5b	core O-11
The transverse profile of one stroke	I-2	to be derived (D-01)
The hyperedge kernels and rates	I-5b	to be derived (D-02; core O-12)
The material index $n_{\mathrm{mat}}(\omega)$	I-2, P-4	to be derived (D-03; core O-8)
The splitting kernel	I-2, I-5a	to be derived (D-04)
The orbital winding book	I-2, I-4	to be derived (D-05)

invariant	where it enters
I-1	the stroke's energy is an I-1 input through $\mathcal{M}$ (0.5): light gravitates and is gravitated; its own window on its cylinder is $2(\hbar\omega/E_{\mathrm{P}})^2$ (§4)
I-2	one carrier, two presentations; the photon the free presentation, turning in one plane; the helix at $\pi/4$ (§§3, 6)
I-3	the orientation of the family, energy maximal at the tip; the bend of a stroke by the exact optical metric; the far field as the cylinder (§4)
I-4	the helix, $\psi = Ae^{i\varphi}$ as its projection, the start azimuth, superposition as phase ignorance; the shared bivector grade (§§3, 7)
I-5	the stroke as the open strand; the spectra of closures as the lines; the ledger form of stability; charge as a reading of closures (§§5–7)
I-6	the open interval closes the family at both ends (§4)
I-7	the stroke's energy and the ground's aT^4V/c^2 enter $\mathcal{M}$ once; the ground's temperature is its clock factor (§8)

A Receipt

The script `receipt_photon_cylinder.py`, in the folder of this paper, checks the displayed identities and numbers with the standard library and NumPy: the cylinder on 250 frequencies from 10^3 to 10^{28} rad s^{-1}, Compton's increment in exact rational arithmetic, the semigroup results by exhaustive enumeration, and the attenuation law by sampling 10^6 strokes per optical depth. Run it with `python3 receipt_photon_cylinder.py`; its output is `receipt_photon_cylinder.out.txt`. Result: **46/46 PASS**.

check	content
PC01a–c	$\kappa F_{\max} = 2\pi$; $d_{\min} = 2\ell_{\mathrm{P}}$, $t_{\min} = 2t_{\mathrm{P}}$ from $F_{\max} d\,t = \hbar$; $E_{\mathrm{floor}} = E_{\mathrm{P}}/2$, $E_{\mathrm{floor}} t_{\min} = \hbar$
PC02a–g	on 250 frequencies: $p\lambda = h$; $L = R_\omega p = \hbar$ and $p \cdot 2\pi R_\omega = h$; $2\pi R_\omega = \lambda$ and the turning at c; $\pi/4$; the arc $\sqrt{2}\lambda$ and $c/\sqrt{2}$; $k = \omega/c$, $E = \hbar ck$; nesting
PC03a–f	the five forms of (3); monotone and inside $(0, 1)$ below the fold; the fold at $E_{\mathrm{P}}/\sqrt{2}$ and $\sqrt{2}\ell_{\mathrm{P}}$; the circumference above $d_{\min}$; 1 eV; the core's 50 GeV example
PC04a–f	the six rows of the table of cylinders
PC05a–h	$k_B T_{\mathcal{B}}$; the crossovers at 39.4 and 56.8 GHz; the occupation monotone and below 1 above 39.4 GHz; $R_{\mathcal{B}}$; the peak; a and $u_{\mathcal{B}}$; $T \propto \omega$ under one rescaling; 5 GHz at $T_{\mathcal{B}}$ and at 20 mK
PC06a–b	$\bar{\lambda}_C = 386.16$ fm; Compton's increment exactly, in rationals, on 50 frequencies and 37 angles, bounded by $2\bar{\lambda}_C$
PC07a–c	curvature sums on 300 random splits with both daughters outside; the degenerate cylinder $2R_\omega$; the hand book
PC08a–d	Sylvester's number and count on eight coprime pairs; $(6, 9, 20) \to 43$; unbounded gaps for $(4, 6)$; on 200 model spectra every positive eigenvalue difference a whole-number sum of the atoms
PC09a–c	Millikan's threshold: W, R_W and $2\pi R_W$; strokes taken from inside R_W only, read as single strokes inside $R_{\mathcal{B}}$ only; the Poisson count and $e^{-\tau}$
PC10a–d	the solar-limb bend; the mode count and spacing of a domain of radius 10^{26} m; the shortest cycle; the Crab's light cylinder

References

[1] Fundamental Density Theory, *Core FDT Invariants*, v6.3 (author lock 2026-10-26).

[2] M. Alfaro, "What c is: a geometric derivation of the space–time and matter–energy conversion," Fundamental Density Theory (18 October 2026), densitytheory.com.

[3] M. Alfaro, "The quintuple identity: photon = graviton = gluon = phonon = magnon," Fundamental Density Theory (24 October 2026), densitytheory.com.

[4] M. Alfaro, "Confinement and the mass gap: the common carrier, self-supported omnium patterns, and a background-dependent neutrino minimum," Fundamental Density Theory (22 September 2026; edition of 29 October 2026), densitytheory.com.

[5] M. Alfaro, "The premises of Yang–Mills: frozen geometry, an expansion about the empty configuration and a chosen group, read against one global settlement and the gauge structure of the support and the seats," Fundamental Density Theory (25 September 2026; edition of 27 October 2026), densitytheory.com.

[6] A. Einstein, "Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt," *Annalen der Physik* **17** (1905) 132–148.

[7] R. A. Millikan, "A direct photoelectric determination of Planck's h," *Physical Review* **7** (1916) 355–388.

[8] R. A. Beth, "Mechanical detection and measurement of the angular momentum of light," *Physical Review* **50** (1936) 115–125.

[9] A. H. Compton, "A quantum theory of the scattering of X-rays by light elements," *Physical Review* **21** (1923) 483–502.

[10] H. J. Kimble, M. Dagenais and L. Mandel, "Photon antibunching in resonance fluorescence," *Physical Review Letters* **39** (1977) 691–695.

[11] D. C. Burnham and D. L. Weinberg, "Observation of simultaneity in parametric production of optical photon pairs," *Physical Review Letters* **25** (1970) 84–87.

[12] L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre–Gaussian laser modes," *Physical Review A* **45** (1992) 8185–8189.

[13] A. A. Abdo *et al.* (Fermi LAT and GBM Collaborations), "A limit on the variation of the speed of light arising from quantum gravity effects," *Nature* **462** (2009) 331–334.

[14] W. H. Furry, "A symmetry theorem in the positron theory," *Physical Review* **51** (1937) 125–129.

[15] ATLAS Collaboration, "Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC," *Nature Physics* **13** (2017) 852–858.

[16] S. L. Adler, "Photon splitting and photon dispersion in a strong magnetic field," *Annals of Physics* **67** (1971) 599–647.

[17] F. Zwicky, "On the red shift of spectral lines through interstellar space," *Proceedings of the National Academy of Sciences* **15** (1929) 773–779.

[18] J. J. Sylvester, "Question 7382," *Mathematical Questions from the Educational Times* **41** (1884) 21.

[19] M. Rauch, "The Lyman alpha forest in the spectra of QSOs," *Annual Review of Astronomy and Astrophysics* **36** (1998) 267–316.

[20] Planck Collaboration, "Planck 2018 results. I. Overview and the cosmological legacy of Planck," *Astronomy & Astrophysics* **641** (2020) A1.

[21] Planck Collaboration, "Planck 2018 results. VI. Cosmological parameters," *Astronomy & Astrophysics* **641** (2020) A6.

[22] D. J. Fixsen, "The temperature of the cosmic microwave background," *Astrophysical Journal* **707** (2009) 916–920.

[23] R. A. Sunyaev and Ya. B. Zeldovich, "The observations of relic radiation as a test of the nature of X-ray radiation from the clusters of galaxies," *Comments on Astrophysics and Space Physics* **4** (1972) 173–178.

[24] P. Noterdaeme, P. Petitjean, R. Srianand, C. Ledoux and S. López, "The evolution of the cosmic microwave background temperature: measurements of $T_{\rm CMB}$ at high redshift from carbon monoxide excitation," *Astronomy & Astrophysics* **526** (2011) L7.

[25] G. Goldhaber *et al.*, "Timescale stretch parameterization of Type Ia supernova B-band light curves," *Astrophysical Journal* **558** (2001) 359–368.

[26] S. Blondin *et al.*, "Time dilation in Type Ia supernova spectra at high redshift," *Astrophysical Journal* **682** (2008) 724–736.

[27] L. M. Lubin and A. Sandage, "The Tolman surface brightness test for the reality of the expansion. IV. A measurement of the Tolman signal and the luminosity evolution of early-type galaxies," *Astronomical Journal* **122** (2001) 1084–1103.

[28] R. Lescanne *et al.*, "Irreversible qubit–photon coupling for the detection of itinerant microwave photons," *Physical Review X* **10** (2020) 021038.

[29] P. Goldreich and W. H. Julian, "Pulsar electrodynamics," *Astrophysical Journal* **157** (1969) 869–880.