

The quantum phase of free fall: kept time, the helical window and the geometry of one medium

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October 30, 2026

Target: Or Dobkowski et al.,
“Observation of the quantum phase of free fall and the
consistency with the equivalence principle,”
Science Advances **12**(36), eaec8045, 2 September 2026.
DOI: 10.1126/sciadv.aec8045. Preprint record: arXiv:2502.14535 (v4, 7 December 2025).

Abstract

Dobkowski et al. [1] report thirteen oscillations and approximately 80 rad of phase in a rubidium interferometer that compares a ballistic arm with a magnetically levitated reference (SRC-073, SRC-074). This analysis derives the record from the locked core v6.3 of Fundamental Density Theory (FDT). Both arms are free patterns of one medium. The ballistic arm falls in the Earth’s gravitational window and, through the moment the chip’s twist induces in it, in the twist’s helical window; the reference is held where the Earth’s lapse times its own rest energy is stationary. The phase is the difference of the arms’ actions $-\int E dt_{\text{own}}$ over $\hbar$. In the ideal limit it counts the time the ballistic arm keeps in excess of the reference in the pattern’s own period $T_{\text{Rb}} = \hbar/(\mathcal{M}_{\text{Rb}}c^2) = 5.109 \times 10^{-26}$ s: 12.07 periods, 75.85 rad, at the source’s 2.4 ms worked example, with the twist’s energy along the ballistic path adding 1.56 rad. The second-order Zeeman push that the source books in its effective acceleration is therefore a 2.0% share of the phase, the size of the reported 2-rad residual: the record tests the composed fall, and the universality of free fall is the gravitational window’s. The helical window returns that push, 0.10 m s^{-2} , from the source’s 12.6 G bias and the reference’s levitation alone. The laboratory Lagrangian, the equality of inertial and gravitational mass and source Equations 1–6 follow from the core. The tide between the arms enters at 1.8×10^{-12} of the phase, and the source’s three timing conventions spread its largest example over 2414–2438 μs , a 3.0% phase spread. Five prospective tests isolate duration, levitation, pulse timing, the current-ratio response and, through the $m_f = 0$ pair, the two windows apart: the twist cancels in the sum of the pair’s falls and gravity in their difference. The convergence map recovers the source’s model and data; no divergence is claimed.

1 Framework

source_status: **experimental.** Measured, observed and reported refer to the source record; fitted refers to its in-sample extraction. Derived, reconstructed, estimated and illustrative identify calculations and designs in this analysis. SRC identifiers refer to the associated claims ledger; U identifiers give non-source inputs, and the numerical ones are listed below. The source is Dobkowski et al. [1], with metadata in SRC-001–SRC-007; its preprint record [2] carries the same Equations 1–6 and Methods values. The framework premises are the locked

core v6.3, author lock 2026-10-26 [3]. FDT is a true extension of general relativity and quantum mechanics. Its quantitative consequences below follow from those premises and the source's measured dynamics.

Input	Value and provenance
U-07	^{87}Rb atomic mass 86.909 180 531 u (AME2020), $u = 1.660\,539\,068\,92 \times 10^{-27}$ kg (CODATA 2022): $\mathcal{M}_{\text{Rb}} = 1.443\,161 \times 10^{-25}$ kg, a read confinement eigenvalue, never reconstructed from the phase.
U-16	Exact h , $\hbar$ and c ; CODATA 2022 $G = 6.674\,30 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ and $\mu_B = 9.274\,010\,066 \times 10^{-24} \text{ J T}^{-1}$.
U-17	Earth: $G\mathcal{M}_{\oplus} = 3.986\,004\,418 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ (IERS) and the mean radius $R_{\oplus} = 6.371 \times 10^6 \text{ m}$, taken as the laboratory's Earth-centred radius at four figures.
U-20	Laboratory site: Ben-Gurion University, Be'er Sheva, 31.26° N , about 280 m; WGS84 normal gravity there 9.7934 m s^{-2} .
U-18	^{87}Rb ground-state structure in the Breit–Rabi form: $\nu_{\text{hfs}} = 6.834\,682\,610\,904 \text{ GHz}$, $g_J = 2.002\,331\,13$, $g_I = -0.000\,995\,141\,4$, nuclear spin $3/2$.
U-08, U-19	Future inputs: an independent site gravimeter value; an independent field map of the chip's twist at the arms.
U-09, U-13, U-15	The design multiplier $z_{\text{sig}} = 3$; future joint covariance; future independent residual bounds.

1.1 Force first: the universal law and its exact pair face

Let F denote force magnitude, c the strand rate of the omnium, G Newton's constant and F_{max} the force ceiling. Let α_{i,σ_i} denote the branch-tagged curvature input of body i , where $i = 1, 2$ labels the two bodies and σ_i the branch; the geometric definition and the chart inventory follow below. Core I-1 gives the universal law, at every scale:

$$F = \frac{c^4}{4G} \alpha_{1,\sigma_1} \alpha_{2,\sigma_2}, \quad F_{\text{max}} = \frac{c^4}{4G} = 3.026 \times 10^{43} \text{ N}. \quad (1)$$

Let $\mathcal{M}_i$ be each body's total gravitating energy over c^2 , $R_{s,i}$ its Schwarzschild radius, $R_{b,i}$ its support radius and d_{pair} the cross-centre separation; the unindexed $\mathcal{M}, R_s, R_b$ carry the same meanings for a single source. For the exterior evaluation used here the branch label is ext. Two nonoverlapping spherical supports at monopole order obey

$$\begin{aligned} R_{s,i} &= \frac{2G\mathcal{M}_i}{c^2} = \frac{\mathcal{M}_i c^2}{2F_{\text{max}}}, & R_{b,i} &\geq R_{s,i}, \\ d_{\text{pair}} &\geq R_{b,1} + R_{b,2}, & \alpha_{i,\text{ext}}(d_{\text{pair}}) &= \frac{R_{s,i}}{d_{\text{pair}}}. \end{aligned} \quad (2)$$

Let $\lambda_i = \mathcal{M}_i/d_{\text{pair}}$ be mass per separation and $\kappa = 8\pi G/c^4$ the Einstein coupling. Substitution in Eq. (1) gives the I-1/I-4 reduction:

$$\begin{aligned} F &= \frac{c^4}{4G} \frac{2G\mathcal{M}_1}{c^2 d_{\text{pair}}} \frac{2G\mathcal{M}_2}{c^2 d_{\text{pair}}} = \frac{G\mathcal{M}_1 \mathcal{M}_2}{d_{\text{pair}}^2}, \\ F &= G\lambda_1 \lambda_2 = \frac{2\pi}{\kappa} \alpha_{1,\text{ext}} \alpha_{2,\text{ext}}, & \kappa F_{\text{max}} &= 2\pi. \end{aligned} \quad (3)$$

The cancellation is exact on the declared evaluation domain; the domain qualifies the closed-form reduction, and the universal law stands.

The experiment's pair is the Earth and the rubidium pattern. With the laboratory at the Earth-centred radius $R_\oplus$ and the inputs U-07, U-16 and U-17, the two exterior inputs and the pair face are

$$\begin{aligned} R_{s,\oplus} &= 8.870 \text{ mm}, & \alpha_\oplus &\equiv \alpha_{\oplus,\text{ext}}(R_\oplus) = 1.392 \times 10^{-9}, \\ R_{s,\text{Rb}} &= 2.143 \times 10^{-52} \text{ m}, & \alpha_{\text{Rb}} &\equiv \alpha_{\text{Rb},\text{ext}}(R_\oplus) = 3.364 \times 10^{-59}, \\ F &= F_{\text{max}} \alpha_\oplus \alpha_{\text{Rb}} = \frac{G \mathcal{M}_\oplus \mathcal{M}_{\text{Rb}}}{R_\oplus^2} & &= 1.417 \times 10^{-24} \text{ N}. \end{aligned} \quad (4)$$

This is the rubidium pattern's weight, read by I-1 at monopole order; the small inputs are correct readings of the one law. The source's value $g_{\text{lab}} = g_{\text{eff}} - a_{\text{SOZ}} = 9.81 \text{ m s}^{-2}$, its effective acceleration less its second-order Zeeman push (SRC-164, SRC-165; Sec. 4), is a model value: the source measures no g (SRC-056). The WGS84 normal gravity at the laboratory's site, 9.7934 m s^{-2} (U-20), sits 0.27% below the monopole $G \mathcal{M}_\oplus / R_\oplus^2 = 9.820 \text{ m s}^{-2}$; the site value (U-08) carries the Earth's rotation and multipole remainders, whose pair expression is to be derived (D-03). In place of 9.81 m s^{-2} the site value lowers the ideal phase's magnitude by 0.34%, or 0.26 rad at the worked example of Sec. 3. Force has units of newtons, each curvature ratio is dimensionless, and κ has units of inverse newtons (U-03, U-04).

The interval statements have different logical scopes. The open range is universal for the declared bounded coordinates and gives the strict force ceiling wherever the force inputs occupy it; only the last bound below uses the spherical nonoverlap conditions:

$$\begin{aligned} 0 &< \alpha_{i,\sigma_i} < 1, & F &< F_{\text{max}}, \\ \alpha_{1,\text{ext}} \alpha_{2,\text{ext}} &\leq \frac{R_{s,1} R_{s,2}}{(R_{s,1} + R_{s,2})^2} \leq \frac{1}{4}, & F &\leq \frac{F_{\text{max}}}{4}. \end{aligned} \quad (5)$$

The final equality requires equal gravitational radii, saturated supports and contact at their summed support radii, with the inputs still strictly inside their open intervals. Nonspherical and overlapping evaluations take their own derived source response.

1.2 The simultaneous chain, the strand rate and the floor

Let E and m denote the energy and mass readings of one pattern; d_{pat} and t_{pat} are its stroke length and duration. Let ϵ_0 and μ_0 denote the omnium's electromagnetic response coefficients. Core 0.1 identifies one substance, the omnium:

$$\frac{E}{m} = c^2 = \frac{1}{\epsilon_0 \mu_0} = \left(\frac{d_{\text{pat}}}{t_{\text{pat}}} \right)^2. \quad (6)$$

The rate c is derived (0.1d). The two responses give $c^2 = 1/(\epsilon_0 \mu_0)$, and clocks and rulers are closed patterns of the omnium, so the measured rate is one on the spot and in uniform motion. Let $v_\parallel$ and $v_\perp$ denote a closed pattern's translation and circulation, and t_{own} the time kept on its own clock, for motion along the vertical laboratory coordinate z_{lab} in laboratory time t_{lab} . Its strand runs at c on a helix and keeps time by circulating:

$$v_\parallel^2 + v_\perp^2 = c^2, \quad c dt_{\text{own}} = v_\perp dt_{\text{lab}} \implies c^2 dt_{\text{own}}^2 = c^2 dt_{\text{lab}}^2 - dz_{\text{lab}}^2, \quad \frac{dt_{\text{own}}}{dt_{\text{lab}}} = \sqrt{1 - \frac{v_\parallel^2}{c^2}}. \quad (7)$$

The interval is the strand's right triangle. A stationary closed pattern, closure included, has inertia E/c^2 (von Laue). With Σ the unit bivector of the unrolled strand plane, c^2 is the area

factor of the strand spinor $v_{\parallel} + v_{\perp} \Sigma$, and rest energy is the bivector grade of $pc + mc^2 \Sigma$, with p the momentum: rest energy is circulation. The standard account names c the speed of light in vacuum. FDT reads c as the strand rate of the omnium, the space–time conversion and, squared, the matter–energy conversion; light is the first open stroke measured at that rate (0.1d).

Let $\hbar$ denote the reduced Planck constant and $h = 2\pi\hbar$ Planck’s constant. For mode label q , let $E_q, m_q, \omega_q, T_q, \ell_q$ denote energy, confinement-mode mass, angular frequency, the time the pattern keeps and the length its thread runs in that time. Core 0.1a displays the bridges (U-01, U-02); the right column gives the rubidium pattern’s rest values (U-07, U-16):

$$\begin{aligned} E_q &= m_q c^2 = \hbar \omega_q, & E_0 &= \mathcal{M}_{\text{Rb}} c^2 = 1.297 \times 10^{-8} \text{ J}, \\ T_q &= \frac{2\pi}{\omega_q} = \frac{h}{E_q}, & T_{\text{Rb}} &= \frac{h}{\mathcal{M}_{\text{Rb}} c^2} = 5.109 \times 10^{-26} \text{ s}, \\ \ell_q &= c T_q = \frac{h}{m_q c}, & \omega_{\text{Rb}} &= \frac{E_0}{\hbar} = 1.230 \times 10^{26} \text{ s}^{-1}, \\ \frac{d_{\text{pat}}}{t_{\text{pat}}} &= c = 299\,792\,458 \text{ m s}^{-1} \text{ exactly.} \end{aligned} \tag{8}$$

Energy, mass, kept time and thread length adjust simultaneously, and quantised configuration content changes all these readings by the same stroke. For confined massive modes the mass reading is the rest-mass eigenvalue of I-5. The continuous exterior coordinate labels that content; it imposes no lattice of allowed spacetime intervals. The thread length is the path run in a period, distinct from the support of the pattern.

Let $d_{\min}$ and $t_{\min}$ denote the stroke infima, and define the Planck length and time by $\ell_P = \sqrt{\hbar G / c^3}$ and $t_P = \ell_P / c$. Let $E_P = \sqrt{\hbar c^5 / G}$ be the Planck energy and E_{floor} the ceiling-times-infimum energy. Core 0.1a derives

$$\begin{aligned} F_{\max} d_{\min} t_{\min} &= \hbar, & d_{\min} &= c t_{\min}, \\ d_{\min}^2 &= \frac{\hbar c}{F_{\max}} = \frac{4\hbar G}{c^3} = 4\ell_P^2, & & \\ d_{\min} &= 2\ell_P, & t_{\min} &= 2t_P, \\ E_{\text{floor}} &= F_{\max} d_{\min} = \frac{E_P}{2}, & E_{\text{floor}} t_{\min} &= \hbar. \end{aligned} \tag{9}$$

The force ceiling is never reached, so the floor is the infimum of physical strokes, approached by the least closed pattern and occupied by none, with no spacing rule above it. Let b_{force} be a positive mean force fraction below unity. A one-action stroke at that fraction is longer, not shorter:

$$d_{\text{pat}} = \frac{d_{\min}}{\sqrt{b_{\text{force}}}}, \quad t_{\text{pat}} = \frac{t_{\min}}{\sqrt{b_{\text{force}}}}, \quad 0 < b_{\text{force}} < 1. \tag{10}$$

The dimensions in Eq. (9) are force times length times time, namely action. The factors $\hbar$ and c are indispensable in Eq. (8); angular frequency is not an energy without $\hbar$.

By core 0.1b each excitation is a pattern of the same medium, and each physical support belongs to its pattern. A carrier is a disturbance at the medium’s rate, and Eq. (6) gives one rate to its mass–energy, electromagnetic and medium-stroke readings. These premises derive the I-2 identity: photon, graviton, gluon, phonon and magnon name one spin-1 excitation in different density and geometry regimes. Here the microwave pulses that change the arms’ internal states are open strokes, the chip’s field is the twist, a macroscopic helical density

gradient of the omnium (P-5), and the rubidium atom is one closed pattern, a braided matter configuration outside the carrier-only identity. A closed pattern's phase rate ω/k and its run $d\omega/dk$ multiply to c^2 (I-2): an arm moving at laboratory speed v_{lab} runs at v_{lab} while its phase advances at c^2/v_{lab} , two rates and never the pitch of a helix; $\pi/4$ is the equality of the two rates, which only the open stroke has.

Every step is a system-wide settlement (0.1a): each kick, hold and internal-state change settles the whole prepared cloud with the chip's twist, from the state the system is in, and the source's trajectory images record the settled states. Events do not commute, and a process run backward is never the mirror of the forward event: the launch and closing kicks are two settlements, and the source's $80\mu\text{s}$ kick duration is the average of the two (SRC-167; [2]).

1.3 Notation guard

The table below defines the laboratory notation and keeps the source spellings solely as typographic labels in its first column. Let Θ denote physical temperature, P pressure and V volume; a is the radiation energy-density constant defined below. Let $\mathcal{B}$ denote confinement boundary data, D its operator, Ω its domain and n the helix-count integer. The phase-plane bivector is $\mathbf{B}_{\text{ph}}$ and the twist's magnetic field is B_{mag} ; the bare letter B carries no meaning here. Laboratory time t_{lab} is the bookkeeper's time, t_{own} the time a pattern keeps on its own clock, and $\mathfrak{A}$ a pattern's action. All statistical uncertainties use s with a descriptive subscript; Σ_{cov} denotes a covariance matrix.

Source labels	Definitions used in this analysis
T, 2T, Th, Tkick, Td	τ : ballistic half-duration; 2τ : total duration; τ_h : holding duration; τ_k : kick duration, the average of the two kicks; τ_d : delay.
tau-h, Teff, tauDD	τ_r : holding rise/fall time; τ_{eff} : effective ballistic half-duration; τ_{DD} : decoupling half-interval.
a, akick, g, geff, aSOZ	a_{mag} : holding acceleration; a_k : kick acceleration; g_{lab} : the bookkeeper's gravitational acceleration at the laboratory; a_{SOZ} : the twist's push per unit mass on the $f_{\text{hf}} = 1, m_f = 0$ state; $g_b = g_{\text{eff}} = g_{\text{lab}} + a_{\text{SOZ}}$: the ballistic arm's own fall.
phi, Delta-phi	φ_g : gauge phase; $\Delta\varphi_{\text{br}}$: ballistic-minus-reference phase; Φ_{src} : positive source Equation 5 convention.
B, Bz, Ihold, Ikick, I(t)	B_{mag} and $B_{\text{mag},z}$: the twist's field and its axial component; $J_h, J_k, J(t_{\text{lab}})$: holding, kick and time-dependent currents.
F, mF, R, r	f_{hf}, m_f : hyperfine total angular momentum and projection; $R_{\oplus}$: Earth-centred laboratory radius; r_f : field radius; F_{mag} : calibrated holding force.
P, Pmean, V	p_{out} : output population fraction; $\bar{p}$: mean fraction; V_{vis} : visibility. Pressure and volume keep their separate meanings above.
m, mi, mg	$\mathcal{M}_{\text{Rb}}$: the rubidium pattern's total energy over c^2 , its inertial and gravitational mass alike; m_q : general mode mass; $\mathbf{m}$: a held moment.
sigma, N, norm, temperature	s_{φ} : phase uncertainty; $s_{\text{parameter}}$: parameter uncertainty; N_{at} : atom count; explicit wavefunction norm; Θ_{exp} : effective expansion temperature.
S, z, t	$\mathfrak{A}$: action; z_{lab} : upward position; t_{lab} : laboratory time. The confinement chart argument t_{cfg} is not time.
X, D, Omega, kappa	X_{σ} : geometric odds; D, Ω, κ : confinement operator, domain and Einstein coupling. The source labels R, W, L, gamma receive no additional laboratory meaning.

The source uses two ground-state levels of the same ^{87}Rb pattern, $f_{\text{hf}} = 1, m_f = 0$ for the ballistic arm and $f_{\text{hf}} = 2, m_f = 1$ for the reference (SRC-019, SRC-020, SRC-050). The prepared arms keep that species; a magnetic state change is a change of confinement mode and supplies no elementary braid count for the composite pattern.

1.4 The Alpha Lock, the regimes and the chart inventory

Let $\mathcal{M}$ denote a source's total gravitating energy over c^2 , $R_s = 2G\mathcal{M}/c^2$ its Schwarzschild radius and $r_c > 0$ the finite-core cutoff. Let σ be the branch sign, with ext and int its exterior and interior labels; α_{ext} and α_{int} are the exterior and interior A-1 ratios and α_σ the selected branch. Core 0.3 defines them by the inverse radii of two geometrically fixed circular sections:

$$\begin{aligned}\alpha_{\text{ext}}(r_f) &= \frac{1/r_f}{1/R_s} = \frac{R_s}{r_f}, & R_s < r_f < \infty, & \sigma = +1, \\ \alpha_{\text{int}}(r_f) &= \frac{1/R_s}{1/r_f} = \frac{r_f}{R_s}, & r_c \leq r_f < R_s, & \sigma = -1, \\ \alpha_\sigma(r_f) &= \left(\frac{R_s}{r_f}\right)^\sigma, & \frac{d\alpha_\sigma}{dr_f} &= -\frac{\sigma\alpha_\sigma}{r_f}.\end{aligned}\tag{11}$$

These inputs are fixed by source and evaluation geometry before any observable is consulted. The two reciprocal branches form one atlas: their gradients have opposite signs, and folded α is continuous at the fold and not differentiable there. Both branches approach unity from below at the fold, the exterior ratio approaches zero at infinity, and at fixed positive cutoff $\alpha_{\text{int}} \geq r_c/R_s > 0$; the interior endpoint construction plays no part in this experiment.

Let X_{ext} and X_{int} be the positive gap odds and X_σ the branch-selected odds. Core 0.4, application A-1, gives

$$\begin{aligned}X_{\text{ext}} &= \frac{R_s}{r_f - R_s}, & X_{\text{int}} &= \frac{r_f}{R_s - r_f}, \\ \alpha_\sigma &= \frac{X_\sigma}{1 + X_\sigma}, & X_\sigma &= \frac{\alpha_\sigma}{1 - \alpha_\sigma} > 0.\end{aligned}\tag{12}$$

Odds name this algebra, with no probabilistic reading; the ratio is already bounded by geometry. The computational coordinate $\beta_\sigma = \alpha_\sigma/(1 + \alpha_\sigma)$ of A-2 is a chart record and enters no force, optical or density slot. A-3, A-4 and A-5 are not used in this analysis.

The experiment's density input is the Earth's exterior ratio between the reference height and the ballistic apex. Let z_{apex} be the apex height, $\delta\alpha_\sigma$ a finite change of the A-1 ratio, $x_{\text{odds}} \geq 0$ an odds enhancement and subscript 0 the starting coordinate. Core I-6 gives the exact bounded update by multiplying the odds; from the apex down to the reference height the update is odds-enhancing on the exterior branch:

$$\begin{aligned}X_\sigma &= X_{\sigma,0}(1 + x_{\text{odds}}), & \alpha_\sigma &= \frac{\alpha_{\sigma,0}(1 + x_{\text{odds}})}{1 + \alpha_{\sigma,0}x_{\text{odds}}}, \\ \delta\alpha_\sigma &= \frac{\alpha_{\sigma,0}(1 - \alpha_{\sigma,0})x_{\text{odds}}}{1 + \alpha_{\sigma,0}x_{\text{odds}}}, & x_{\text{odds}} &= \frac{z_{\text{apex}}}{R_\oplus - R_{s,\oplus}}, \\ \alpha_{\oplus,0} &= \alpha_{\oplus,\text{ext}}(R_\oplus + z_{\text{apex}}), & \delta\alpha_\oplus &= \alpha_{\oplus,\text{ext}}(R_\oplus) - \alpha_{\oplus,0} = 1.665 \times 10^{-21}.\end{aligned}\tag{13}$$

The update reproduces the difference of the two A-1 values exactly; the number uses the source's top-arm apex $z_{\text{apex}} = 7.62 \mu\text{m}$ (SRC-164, SRC-168). At leading order $c^2\delta\alpha_\oplus/2 = G\mathcal{M}_\oplus z_{\text{apex}}/R_\oplus^2$: the lapse of Sec. 1.5 reads the excursion as the weight per unit mass times the height. A finite excursion takes this rational form, never a bare linear change.

On the A-1 compactness chart α reads the regime (0.3a): on the exterior branch $\alpha_{\text{ext}} < 1/2$ is the general-relativistic regime, $\alpha_{\text{ext}} = 1/2$ the transition, at the seam $r_f = 2R_s$, and $\alpha_{\text{ext}} > 1/2$ the quantum-mechanical regime. The laboratory radius reads $\alpha_{\oplus} = 1.392 \times 10^{-9}$, deep in the general-relativistic solid torus of the Earth's support, near its $\alpha \rightarrow 0^+$ core circle: the spacetime phase, decompressed substrate. The rubidium pattern is matter, a closed braid read on its own interior chart, in the matter phase $\alpha_{\text{int}} > 1/2$ of its own support (A-1, interior branch, schematic); that chart carries no transfer map to the Earth's exterior atlas (I-3), and none of its values is consulted. The experiment therefore reads a matter pattern's phase, the quantum-mechanical reading, as it runs in the spacetime phase, whose reading is general relativity's: one omnium, one chain, two readings.

The clock factors $\mathbf{m}_j$ and $\mathbf{n}_j$ of Sec. 1.5 are not α s: they are no odds, no bounded output and no application of 0.4, and they enter no force, optical or density slot. The chart inventory is the table below.

Chart	Coordinate	Use in this analysis
A-1 compactness, exterior	$X_{\text{ext}}, \alpha_{\text{ext}}$	The Earth at $R_{\oplus}$ and at the apex; the rubidium pattern's own window in the pair face, Eq. (4); the lapse and the flow.
A-1 regime	α_{ext} against $1/2$, exterior branch	The laboratory in the general-relativistic regime.
A-2 computation	β_{σ}	Chart record only; no slot.
A-3, A-4, A-5	—	Not used.
Interior chart	$\alpha_{\text{int}} > 1/2$; $\alpha_{\text{core, int}} \rightarrow 1^-$	Schematic orientation of the atom's own support: matter phase and skin; no value and no transfer map.
Clock factor	$\mathbf{m}_j, \mathbf{n}_j$ (0.1c)	Not an application of 0.4: the helical window's $E_j/E_{0,j}$ and the composed factor $\mathcal{N}\mathbf{m}_j$ a slow pattern reads.

No transfer map connects these entries in this analysis. Visibility, phase, temperature and calibrated acceleration keep their own physical types.

1.5 The extended equivalence principle and the two windows

Let $\mathbf{m} > 0$ be the clock factor, $\mathcal{U}^\mu$ a pattern's four-velocity in t_{own} , $\nabla_{\mathcal{U}}$ the covariant rate of change along its path and $\mathfrak{P}^{\mu\nu}$ the projector across it. Core 0.1c derives the common rescaling of kept time and thread length and its bookkeeper's reading:

$$d \mapsto \mathbf{m} d, \quad t \mapsto \mathbf{m} t, \quad g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}, \quad (14)$$

$$\nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathbf{m}, \quad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu, \quad g_{\mu\nu} \mathcal{U}^\mu \mathcal{U}^\nu = -c^2.$$

The rescaling keeps $d/t = c$, every ratio of the chain and the null cone. The value of $\ln \mathbf{m}$ at a point is invisible; its gradient is the free fall of one pattern, removed by riding with the pattern; its second derivatives are the deviation of neighbouring patterns, removed by no choice (0.1c, I-3a). This is the extended equivalence principle: moving at constant speed, on the pattern's own clocks and rulers, through a density gradient is gravity. On its own clocks and rulers the ballistic arm meets nothing; the bookkeeper reads it rising, stopping and falling.

The gravitational window. Let $\mathcal{N}(r_f) = \sqrt{1 - \alpha_{\text{ext}}(r_f)}$ be the Earth's lapse on the exterior branch. A slow pattern reads the lapse and a stroke the optical metric, and the factor

of two between their readings is the principle read on a stroke (I-3). Let v_{flow} be the speed of patterns falling from rest far away. At the laboratory radius

$$g = c^2 \frac{d \ln \mathcal{N}}{dr_f} \simeq \frac{c^2}{2} \frac{\alpha_{\text{ext}}}{r_f} = \frac{G\mathcal{M}_{\oplus}}{r_f^2} \quad \text{at leading order,}$$

$$v_{\text{flow}} = c\sqrt{\alpha_{\oplus}} = 11.19 \text{ km s}^{-1}, \quad \frac{d^2 r_f}{dt_{\text{own}}^2} = -\frac{G\mathcal{M}_{\oplus}}{r_f^2} \quad \text{exactly along the flow.} \quad (15)$$

The bookkeeper's reading is a flow at every size: at the laboratory it runs at the Earth's escape speed, and every pattern there that sits in the gravitational window alone is read falling at the flow's acceleration at monopole order, which the site value of g carries together with the Earth's rotation and multipoles (U-08); the ballistic arm adds the twist's push, below. The laboratory floor holds the apparatus against the flow, and the chip's twist holds the reference arm.

The helical window. A pattern whose rest energy depends on where it sits in a twist takes that energy as its clock factor (0.1c). Let E_j be the rest energy of arm or state j where it sits, $E_{0,j}$ its rest energy outside every twist, $E_{\text{ind},j}$ the energy of the moments the twist induces in it, the work of inducing them included, $\mathbf{m}_j$ its held moments and z the coordinate along the twist's gradient. Then

$$E_j = E_{0,j} + E_{\text{ind},j} - \sum \mathbf{m}_j \cdot B_{\text{mag}}, \quad \mathcal{M}_j = \frac{E_j}{c^2}, \quad \mathbf{m}_j = \frac{E_j}{E_{0,j}},$$

$$\mathfrak{A}_j = - \int E_j dt_{\text{own}}, \quad \mathcal{M}_j \ddot{z}_j = -\partial_z E_j = -\partial_z E_{\text{ind},j} + \sum \mathbf{m}_j \cdot \partial_z B_{\text{mag}}. \quad (16)$$

For $E_j > 0$ fixed by the event alone the action is the geodesic action of $\mathbf{m}_j^2 g_{\mu\nu}$ and Eq. (14) holds exactly; the last relation is its slow limit, each held moment at its settled value. A field across the motion makes E_j depend on the velocity; at the arms' $g_b \tau \simeq 1.2 \text{ cm s}^{-1}$ that term enters at $(g_b \tau / c)^2 = 1.6 \times 10^{-21}$, and Eq. (14) holds to that order. The gradient of E_j gives the push and its second derivatives the spring; a uniform level added to E_j changes $\mathcal{M}_j$ and leaves the push unchanged, and a uniform twist turns a held moment and pushes nothing.

Composition, and the two arms. The windows compose: in the gravitational window a pattern is free in $\mathbf{m}_j^2 g_{\mu\nu}$, with $g_{\mu\nu}$ the Earth's metric, and for a slow pattern the lapse where it sits multiplies $E_j / E_{0,j}$, so its weight and the twist's push add. Let $\mathbf{n}_j = \mathcal{N} \mathbf{m}_j$ be the composed factor a slow pattern reads. For the two arms at height z_{lab} , with the reference labelled r and the ballistic arm b,

$$\mathbf{n}_j(z_{\text{lab}}) = \mathcal{N}(R_{\oplus} + z_{\text{lab}}) \frac{E_j(z_{\text{lab}})}{E_{0,j}}, \quad \ddot{z}_j \simeq -c^2 \partial_z \ln \mathbf{n}_j = -g_{\text{lab}} - \frac{\partial_z E_j}{\mathcal{M}_{\text{Rb}}},$$

$$\partial_z \ln[\mathcal{N} E_r] = 0 \iff -\partial_z E_r = \mathcal{M}_{\text{Rb}} g_{\text{lab}}, \quad \ddot{z}_b = -g_b, \quad g_b = g_{\text{lab}} + a_{\text{SOZ}} = 9.91 \text{ m s}^{-2}. \quad (17)$$

The reference is held where the lapse times its own rest energy is stationary: the twist's slope of its rest energy cancels the lapse's, and the twist's push equals the weight $\mathcal{M}_{\text{Rb}} g_{\text{lab}}$, the I-1 face of Eq. (4) carried to the source's g_{lab} . The ballistic state's induced moment puts it in the helical window as well, and it falls at g_b , with $a_{\text{SOZ}} = 0.10 \text{ m s}^{-2}$: the push the source computes in its supplementary text (SRC-165; [2]) and the helical window returns from the source's bias

field, Eq. (43). The source classifies the interferometer as hybrid, a guided reference beside a free-space ballistic arm (SRC-036). FDT reads both arms as free, each on the geodesic of its own composed clock factor, with the guide entering as the twist's term in the reference's rest energy (0.1c).

In the gravitational window, at a pattern's monopole order, $\mathbf{m}$ belongs to the point, and every pattern there falls alike. In the helical window it belongs to the pattern, and patterns whose held and induced moments, summed per unit E , differ along $\partial_z B_{\text{mag}}$ fall differently where they sit. The ballistic arm's extra fall is the helical window's: its second-order Zeeman energy is its E_{ind} . The universality of free fall is the gravitational window's, and it stands. The source assumes that inertial and gravitational mass are equal and that free fall is universal (SRC-025–SRC-027). FDT derives the equality: a stationary closed pattern, closure included, has inertia E/c^2 (0.1d); its gravitating mass is the same total energy over c^2 (0.5); and the lapse multiplies that one E , so weight over inertia is the point's g at monopole order (0.1c).

The source reads the measured phase as the quantum phase of one freely falling object. FDT reads it as the comparison of two free arms of one pattern, each in its own composed clock factor: on its own clocks and rulers the falling arm registers nothing, and the gradient is read between it and the held reference path (0.1c, Use 7).

1.6 Einstein equation, containment and the tide

Let μ, ν label spacetime components and $G_{\mu\nu}$, $T_{\mu\nu}$ and $g_{\mu\nu}$ denote the Einstein tensor, stress tensor and metric. Core I-3 keeps the Einstein field equation unchanged:

$$G_{\mu\nu} = \kappa T_{\mu\nu}. \quad (18)$$

The branch-tagged compactness parameterises an on-shell spherical solution downstream of its stress tensor and boundary data; it is not a tensor quotient or a replacement source equation. Let $d\ell_{\text{opt}}^2$ denote optical length squared and $d\Omega_{\text{ang}}^2$ the solid-angle line element, distinct from the confinement domain. On the exterior branch a stroke reads the exact optical metric $d\ell_{\text{opt}}^2 = dr_f^2/(1 - \alpha_{\text{ext}})^2 + r_f^2 d\Omega_{\text{ang}}^2/(1 - \alpha_{\text{ext}})$, whose Fermat jump law is generalised Snell's law, and a slow pattern reads the lapse. Let Φ_{grav} be the weak-field potential and v_{lab} laboratory speed. Eqs. (7) and (15) give a slow pattern's kept-time rate:

$$\frac{dt_{\text{own}}}{dt_{\text{lab}}} = \mathcal{N} \sqrt{1 - \frac{v_{\text{lab}}^2}{c^2}} = 1 + \frac{\Phi_{\text{grav}}}{c^2} - \frac{v_{\text{lab}}^2}{2c^2} + O\left(\frac{\Phi_{\text{grav}}^2}{c^4}, \frac{v_{\text{lab}}^4}{c^4}, \frac{|\Phi_{\text{grav}}|v_{\text{lab}}^2}{c^4}\right),$$

$$\Phi_{\text{grav}}(z_{\text{lab}}) - \Phi_{\text{grav}}(0) = g_{\text{lab}} z_{\text{lab}} \quad \text{in the locally uniform model.}$$

(19)

This is the containment layer of I-3/I-4, separate from the exact pair algebra; disjoint support alone does not imply weak field. The locally uniform field is the source's analytical model (SRC-094–SRC-096). Given the same source, boundary data, initial data and gauge, FDT and GR return the same curvature, and charts alone supply no observable divergence.

The tide between the arms. Curvature is read between neighbours (I-3a). Slow neighbours on the exterior branch carry, exactly, $(c^2/2)(\alpha_{\text{ext}}/r_f^2) \text{diag}(2, -1, -1)$; let $\Gamma_{\text{td}} = c^2 \alpha_{\oplus}/R_{\oplus}^2 = 2G\mathcal{M}_{\oplus}/R_{\oplus}^3$ be its radial entry at the laboratory. Its potential $-\frac{1}{2}\mathcal{M}_{\text{Rb}}\Gamma_{\text{td}}z_{\text{lab}}^2$ along the ballistic path of Eq. (23) changes the action difference by

$$\delta\mathfrak{A}_{\text{td}} = \frac{2}{15}\mathcal{M}_{\text{Rb}}\Gamma_{\text{td}}g_{\text{b}}^2\tau^5, \quad \frac{\delta\mathfrak{A}_{\text{td}}}{|\Delta\mathfrak{A}_{\text{br}}|} = \frac{2}{5}\Gamma_{\text{td}}\tau^2 = 1.78 \times 10^{-12}, \quad \Gamma_{\text{td}} = 3.083 \times 10^{-6} \text{ s}^{-2}, \quad (20)$$

at $\tau = 1.2 \text{ ms}$, or 1.4×10^{-10} rad of the phase. The interferometer reads the gradient between a falling pattern and a held reference path; the tide between its arms is a fraction 1.8×10^{-12} of that reading.

1.7 Confinement and energy accounting

Let χ denote closure orientation and u_q a confinement eigenfunction; t_{cfg} is position on the configuration chart, not elapsed time. Core I-5 supplies the eigenproblem and the mode-energy bridge:

$$D(\Omega, \mathcal{B}; n, \chi, t_{\text{cfg}})u_q = \omega_q u_q, \quad m_q c^2 = \hbar \omega_q. \quad (21)$$

The operator is to be derived (D-01). The rubidium atom is one closed pattern of the medium (0.1b), and its mass $\mathcal{M}_{\text{Rb}}$ is a confinement eigenvalue read from measurement (U-07). The ground-state levels that the source and test T-5 use are confinement modes of that pattern at fixed braid content: a microwave transition changes the mode, costs $\hbar \Delta \omega = \Delta \mathcal{M} c^2$ and re-braids nothing. The hyperfine splitting $\Delta E_{\text{hfs}} = h \nu_{\text{hfs}} = 4.529 \times 10^{-24} \text{ J}$, 3.49×10^{-16} of E_0 , and the Zeeman structure are read from measurement (U-18). Let R_{box} denote a support radius determined independently of the mass and $\bar{\lambda}_C = \hbar/(m_q c)$ the reduced Compton length: they are different quantities, and identifying them turns a spectral prediction into the mass-length definition.

Let M be the rest contribution to a source's mass aspect, E_{kin} kinetic energy, E_{int} internal energy relative to the chosen rest reference and σ_{SB} the radiation flux constant. Core 0.5/I-7 writes the channel examples as

$$\begin{aligned} a &= \frac{4\sigma_{\text{SB}}}{c}, \\ \mathcal{M} &= M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{a\Theta^4 V}{c^2} + \dots, \\ \frac{a\Theta^4 V}{c^2} &= \frac{4\sigma_{\text{SB}}\Theta^4 V}{c^3}, \quad R_s = \frac{2GM}{c^2}. \end{aligned} \quad (22)$$

Here E collects the designated non-rest energy channels, each once. Temperature is in kelvin, a has units $\text{J m}^{-3} \text{K}^{-4}$, and the radiation entry has units of mass. The covariant parent is the one stress tensor, and the solved mass aspect already carries gravitational binding. The table below states where each channel enters the arms' action.

Channel	Entry and once-only rule
Rest	$E_0 = \mathcal{M}_{\text{Rb}} c^2$, common to both arms; its action $-E_0 \int dt_{\text{own}}$ carries the kept-time difference, which is the phase.
Kinetic	Through the strand triangle, Eq. (7): $+\mathcal{M}_{\text{Rb}} v_{\text{lab}}^2/2$ in the slow limit; never added separately.
Gravitational	Through the lapse, Eq. (19): $-\mathcal{M}_{\text{Rb}} g_{\text{lab}} z_{\text{lab}}$; binding is read from the solved geometry and never added to the stress tensor.
Twist	$E_{\text{ind},j} - \sum \mathbf{m}_j \cdot B_{\text{mag}}$, once, inside the rest energy where the pattern sits; its gradient is the push. The ballistic state's second-order Zeeman energy is inside g_b and is not added again.
Internal state	E_{int} relative to the rest reference; the pulse sequence balances the internal-state dwell (SRC-150, SRC-152).
Pressure and radiation	PV and $a\Theta^4 V$ are stress/energy entries when independently specified; the cloud's 3.3 nK characterises its expansion and supplies no radiation density.

The standard account reads the push on a moment in a field gradient as a magnetic force distinct from motion through a density gradient. FDT reads the field as the helical density gradient of the omnium (P-5) and the push as the bookkeeper’s reading of the pattern’s own rest energy where it sits, the helical window (0.1c). Corroboration of I-1, I-3, I-4 and I-7 is the exact reduction from the arms’ action to the laboratory phase within this accounting; an independently calibrated phase residual outside its declared covariance and source corrections falsifies that joint reading.

2 System chart table

The Earth’s exterior atlas uses its total gravitating source and the Earth-centred laboratory radius; the reference is fixed relative to the Earth (SRC-115–SRC-118). The table separates that geometry from the atomic configuration, the twist and the apparatus lengths. Let R_{room} denote the configuration’s independently measured skin-support radius, whose value is absent here; R_{b} keeps the general support-radius role.

System reading	Declared chart or measurement	Physical use
Earth source	A-1 exterior, $\alpha_{\oplus} = 1.392 \times 10^{-9}$ at $R_{\oplus}$	General-relativistic regime; the lapse, the flow at 11.19 km s^{-1} and the pair face. g_{lab} is the source’s model value (SRC-164, SRC-165).
Atomic source	A-1 exterior, $R_{\text{s,Rb}} = 2.143 \times 10^{-52} \text{ m}$, $\alpha_{\text{Rb}} = 3.364 \times 10^{-59}$ at $R_{\oplus}$	The pattern’s own window in Eq. (4); source radius and support radius remain distinct.
Atomic configuration	Interior chart, skin at its schematic fold, support R_{room} from independent data	Matter phase; densest at the skin, dilute at the centre; no transfer map to the Earth’s exterior atlas.
Twist	The chip’s field, a helical density gradient (P-5)	Holds the reference at $-\partial_z B_{\text{mag}} = 0.303 \text{ T m}^{-1}$ and pushes the ballistic state at $a_{\text{SOZ}} = 0.10 \text{ m s}^{-2}$, Sec. 4.
Apparatus separation	Approximately $113 \mu\text{m}$ below the chip; maximum arm splitting approximately $7.5 \mu\text{m}$	Reported geometry, SRC-019, SRC-020; neither distance is a Schwarzschild radius or a confinement mass address.
Expansion	$\Theta_{\text{exp}} = 3.3 \text{ nK}$, $0.4 \mu\text{m ms}^{-1}$	The cloud’s effective expansion temperature (SRC-137), read in the medium window, Sec. 6.

No interior datum of the atom is measured here, so the observation-region rule of I-3 assigns no profile: an interior datum would be a profile-weighted mean with its probe weight named, never a point coordinate. The core places local density highest at the skin on both branch readings; lower helix-count closures have support toward the dilute interior, and higher rungs require the skin terrain. Corroboration of I-3 and I-6 is the consistent use of the measured apparatus geometry with the declared atlas; a numerical interior profile or a cross-chart assignment drawn from these distances would contradict the observation map before any precision claim arose.

3 Observation map

The source compares output populations with phase models. The rubidium mass is the measured input U-07 and the source’s accelerations are its recorded values, so the phase below is

computed in radians wherever the record supplies its inputs; reported phase amplitudes keep their source provenance.

3.1 From the arms' action to the laboratory phase

Let z_b and z_r be the ballistic and reference centres in the upward laboratory coordinate; subscripts b and r label the arms. Let v_{launch} denote the ballistic launch speed and dots laboratory-time derivatives. Put the ideal apex at $t_{\text{lab}} = 0$, the common endpoints at $t_{\text{lab}} = \pm\tau$ and the reference at zero height. Constant acceleration (SRC-028, SRC-038–SRC-042) at the ballistic arm's own fall gives

$$\begin{aligned} v_{\text{launch}} &= g_b \tau, & z_r &= 0, \\ z_b(t_{\text{lab}}) &= \frac{g_b}{2}(\tau^2 - t_{\text{lab}}^2), & \dot{z}_b &= -g_b t_{\text{lab}}, & z_b(\pm\tau) &= 0. \end{aligned} \quad (23)$$

The end kicks remove the relative momentum. Let $U_j(z_{\text{lab}}) = E_j - E_{0,j}$ be the twist's energy of arm j , locally linear on the arms: $U_r = -\mathcal{M}_{\text{Rb}} g_{\text{lab}} z_{\text{lab}}$ at levitation and $U_b = \mathcal{M}_{\text{Rb}} a_{\text{SOZ}} z_{\text{lab}}$. With Eqs. (7), (16) and (19), each arm's action rate has the slow limit

$$-E_j \frac{dt_{\text{own},j}}{dt_{\text{lab}}} = -E_{0,j} + \frac{\mathcal{M}_{\text{Rb}} \dot{z}_j^2}{2} - \mathcal{M}_{\text{Rb}} g_{\text{lab}} z_j - U_j(z_j) + O(c^{-2}), \quad (24)$$

which is the laboratory Lagrangian plus the rest-energy term. The kinetic entry comes from the strand triangle and the gravitational entry from the lapse; neither is added by hand. Choose the common rest and internal-state reference so that the reference arm's rate vanishes during levitation. The action difference and phase are then

$$\begin{aligned} \mathcal{L}_b &= \frac{\mathcal{M}_{\text{Rb}} \dot{z}_b^2}{2} - \mathcal{M}_{\text{Rb}} g_b z_b = \mathcal{M}_{\text{Rb}} g_b^2 \left(t_{\text{lab}}^2 - \frac{\tau^2}{2} \right), \\ \Delta \mathfrak{A}_{\text{br}} &= \int_{-\tau}^{\tau} \mathcal{L}_b dt_{\text{lab}} = -\frac{\mathcal{M}_{\text{Rb}} g_b^2 \tau^3}{3}, & \Delta \varphi_{\text{br}} &= \frac{\Delta \mathfrak{A}_{\text{br}}}{\hbar} = -\frac{\mathcal{M}_{\text{Rb}} g_b^2 \tau^3}{3\hbar}. \end{aligned} \quad (25)$$

Here Δ keeps the declared ballistic-minus-reference convention.

Kept time and the twist's share. Let Δt_{own} be the ballistic-minus-reference difference of kept time. Splitting Eq. (25) into the rest-energy term and the twist's energy,

$$\begin{aligned} \Delta t_{\text{own}} &= \frac{1}{c^2} \int_{-\tau}^{\tau} \left(g_{\text{lab}} z_b - \frac{\dot{z}_b^2}{2} \right) dt_{\text{lab}} = \frac{(g_{\text{lab}}^2 - a_{\text{SOZ}}^2) \tau^3}{3c^2}, \\ \Delta \varphi_{\text{br}} &= -\frac{E_0}{\hbar} \Delta t_{\text{own}} - \frac{1}{\hbar} \int_{-\tau}^{\tau} U_b(z_b) dt_{\text{lab}} = -2\pi \frac{\Delta t_{\text{own}}}{T_{\text{Rb}}} - \frac{2\mathcal{M}_{\text{Rb}} a_{\text{SOZ}} g_b \tau^3}{3\hbar}. \end{aligned} \quad (26)$$

In the gravitational window alone, $a_{\text{SOZ}} \rightarrow 0$, the phase is $-2\pi \Delta t_{\text{own}}/T_{\text{Rb}}$ with $\Delta t_{\text{own}} = g_{\text{lab}}^2 \tau^3/(3c^2)$. Gaining more time by its height in the lapse than it loses by the strand triangle, the ballistic arm keeps more time than the reference, and the interferometer counts the excess in the pattern's own period T_{Rb} (0.1a). At the source's worked example, $\tau = 1.2 \text{ ms}$, with $g_b = 9.91 \text{ m s}^{-2}$:

$$\begin{aligned} \Delta t_{\text{own}} &= 6.167 \times 10^{-25} \text{ s} = 12.07 T_{\text{Rb}} \quad (75.85 \text{ rad}), & |\Delta \varphi_{\text{br}}| &= 77.41 \text{ rad}, \\ \frac{2\mathcal{M}_{\text{Rb}} a_{\text{SOZ}} g_b \tau^3}{3\hbar} &= \frac{2a_{\text{SOZ}}}{g_b} |\Delta \varphi_{\text{br}}| = 1.56 \text{ rad}. \end{aligned} \quad (27)$$

In the ideal limit the recorded oscillations count these kept periods, with the twist's share adding 2.0% to the count; the finite-pulse terms add 5.7 rad at the worked example, Eq. (32). That share, $2a_{\text{SOZ}}/g_b = 2.0\%$ of the phase, is the size of the reported 2.5% residual (SRC-121): the record tests the composed fall $g_{\text{lab}} + a_{\text{SOZ}}$, with its helical part supplied by the source's field model, and test T-5 reads the two windows apart.

Let $u_{\text{tot}} = 2\tau$ denote total duration. Source Equation 6 writes the levitated phase as the holding force squared (SRC-129); with the twist's share it reads

$$F_{\text{mag}} = \mathcal{M}_{\text{Rb}} g_{\text{lab}}, \quad \Delta\varphi_{\text{br}} = -\frac{F_{\text{mag}}^2 \tau^3}{3\hbar \mathcal{M}_{\text{Rb}}} \left(\frac{g_b}{g_{\text{lab}}} \right)^2 = -\frac{\mathcal{M}_{\text{Rb}} g_b^2 u_{\text{tot}}^3}{24\hbar}. \quad (28)$$

The force-squared face holds as printed where the ballistic arm falls at g_{lab} , and the factor $(g_b/g_{\text{lab}})^2 = 1.020$ is the twist's share. The change from half-duration to total duration accounts for the coefficient change and is not an experimental departure. For dimensional clarity, square brackets denote physical units:

$$[\mathcal{M}_{\text{Rb}} g_b^2 \tau^3] = \text{kg m}^2 \text{s}^{-1}, \quad [\hbar] = \text{kg m}^2 \text{s}^{-1}, \quad \left[\frac{F_{\text{mag}}^2 \tau^3}{\hbar \mathcal{M}_{\text{Rb}}} \right] = 1. \quad (29)$$

Phase is the action difference divided by $\hbar$, and its duration derivative has units of radians per second; neither is a force.

3.2 The frame transformation: one arm carries no reading

Let ψ_{N} and ψ_{E} denote the laboratory and freely falling frame projections and i the imaginary unit; exponentials use Euler's number e . In the gravitational window, source Equation 1 and its gauge phase are (SRC-029, SRC-030; SG-02, SG-03)

$$\begin{aligned} \psi_{\text{N}}(z_{\text{lab}}, t_{\text{lab}}) &= e^{i\varphi_g} \psi_{\text{E}} \left(z_{\text{lab}} + \frac{g_{\text{lab}} t_{\text{lab}}^2}{2}, t_{\text{lab}} \right), \\ \varphi_g(z_{\text{lab}}, t_{\text{lab}}) &= \frac{\mathcal{M}_{\text{Rb}}}{\hbar} \left(-\frac{g_{\text{lab}}^2 t_{\text{lab}}^3}{6} - g_{\text{lab}} z_{\text{lab}} t_{\text{lab}} \right), \\ \varphi_g(0, \tau) - \varphi_g(0, -\tau) &= -\frac{\mathcal{M}_{\text{Rb}} g_{\text{lab}}^2 \tau^3}{3\hbar}. \end{aligned} \quad (30)$$

In the falling coordinates ψ_{E} evolves freely: that is the pattern's own reading, on which nothing changes (0.1c). The factor $e^{i\varphi_g}$ is the bookkeeper's reading of the same arm, and the gauge phase of one arm is never observed. The source notes that in the falling frame the phase is due to the magnetic acceleration (SRC-128). In the gravitational window alone the ballistic arm meets nothing there; with its helical window it meets nothing in the frame falling at g_b , where the reference's relative push $\mathcal{M}_{\text{Rb}} g_b$ carries the whole comparison. Substitution confirms that $e^{i\varphi_g} \psi_{\text{E}}$ solves the laboratory equation with potential energy $\mathcal{M}_{\text{Rb}} g_{\text{lab}} z_{\text{lab}}$ whenever ψ_{E} solves the free one. The interferometer subtracts the two arms at common endpoints, and the endpoint difference is source Equation 2 (SRC-043, SRC-044), equal to Eq. (25) at $g_b = g_{\text{lab}}$, negative sign included. The gauge coefficient is one sixth and the endpoint difference one third; the same endpoints are subtracted once. Corroboration of I-3 and I-4 is this exact equality of the action and frame calculations on their shared ideal domain; a sign-calibrated, closed-arm measurement with independently supplied mass, acceleration and covariance that violates this coefficient falsifies the joint ideal model.

3.3 Finite pulses, phase orientation and derivatives

Let $C_\varphi = \mathcal{M}_{\text{Rb}} g_{\text{b}}^2 / (3\hbar)$ denote the positive ideal phase coefficient, with units s^{-3} . The source prints the coefficient with g ; its analytic model replaces g by g_{eff} throughout Equation 5 and checks the replacement against an explicit a_{SOZ} model to within 0.03 rad [2]. The push a_{SOZ} acts during the hold; outside it each arm spends $55 \mu\text{s}$ of free flight in $f_{\text{hf}} = 2, m_f = 1$ and $16 \mu\text{s}$ in each π pulse (SRC-146–SRC-148). With U-07 and SRC-164,

$$C_\varphi = 4.480 \times 10^{10} \text{ s}^{-3}, \quad \frac{\mathcal{M}_{\text{Rb}} g_{\text{lab}}^2}{3\hbar} = 4.390 \times 10^{10} \text{ s}^{-3}. \quad (31)$$

Let $\mathcal{P}_{\text{time}}$ denote the full time polynomial of source Equation 5 and $\varphi_{\text{reference}}^{\text{src}}$ the source's finite-pulse reference phase. Source Equations 4 and 5 have positive printed prefactors (SRC-062, SRC-068–SRC-070):

$$\begin{aligned} \varphi_{\text{reference}}^{\text{src}} &= 2C_\varphi(\tau_k + \tau_d)^3, \\ \mathcal{P}_{\text{time}}(\tau; \tau_k, \tau_d) &= \tau^3 + \tau^2\tau_k + \tau(\tau_k^2 + \tau_k\tau_d) - \tau_d(\tau_k + \tau_d)^2, \\ \Phi_{\text{src}} &= C_\varphi \mathcal{P}_{\text{time}}. \end{aligned} \quad (32)$$

The source's finite-pulse reference contribution is independent of holding time at fixed kick and delay, and the reference moves during its two free-flight intervals (SRC-061, SRC-063); the full finite-pulse sequence cannot be replaced by a reference stationary at every instant.

Let $\Delta\varphi_{\text{br}}^{\text{rec}}$ denote an opposite-arm reconstruction chosen to continue the ideal ballistic-minus-reference orientation. Define it on the analytical closed-arm family by

$$\Delta\varphi_{\text{br}}^{\text{rec}} = -\Phi_{\text{src}} = -C_\varphi \mathcal{P}_{\text{time}}. \quad (33)$$

This declaration keeps the printed source equation and supplies the orientation used to compare phase curves; its magnitude equals Φ_{src} only where $\mathcal{P}_{\text{time}} \geq 0$. Unwrapping also requires a fixed phase origin and continuity prescription, and a population cosine alone is invariant under reversal of its phase. The source signs in Equations 3–6 are taken as printed.

At fixed $\tau_k, \tau_d, \mathcal{M}_{\text{Rb}}, g_{\text{b}}$, direct differentiation of the entire polynomial gives

$$\begin{aligned} \frac{d\Phi_{\text{src}}}{du_{\text{tot}}} &= \frac{C_\varphi}{2} [3\tau^2 + 2\tau\tau_k + \tau_k^2 + \tau_k\tau_d], \\ \frac{d^2\Phi_{\text{src}}}{du_{\text{tot}}^2} &= \frac{C_\varphi}{4} (6\tau + 2\tau_k), \\ \frac{d^3\Phi_{\text{src}}}{du_{\text{tot}}^3} &= \frac{3C_\varphi}{4}. \end{aligned} \quad (34)$$

These are model derivatives. The source's plotted experimental phase is extracted by a cubic fit, and its numerical-simulation derivative also comes from a fitted cubic (SRC-077, SRC-106–SRC-108); a differentiated fit is not an independently measured derivative.

The two useful limiting maps follow algebraically from Eq. (32):

$$\begin{aligned} \mathcal{P}_{\text{time}}(\tau; 0, \tau_d) &= \tau^3 - \tau_d^3, \\ \lim_{\tau_k, \tau_d \rightarrow 0} \mathcal{P}_{\text{time}} &= \tau^3, \\ \mathcal{P}_{\text{time}}(\tau; 0, \tau_d) - \mathcal{P}_{\text{time}}(0; 0, \tau_d) &= \tau^3. \end{aligned} \quad (35)$$

The first line keeps the delay offset, the second removes both pulse times, and the third subtracts a constant algebraic intercept, not a physical zero-duration experiment; this resolves the source’s zero-kick shorthand (SRC-072). At the worked example, $\tau = 1.2$ ms with $\tau_k = 80 \mu\text{s}$ and $\tau_d = 71 \mu\text{s}$, the ideal term is $C_\varphi \tau^3 = 77.4$ rad and the full polynomial $C_\varphi \mathcal{P}_{\text{time}} = 83.1$ rad; the record’s thirteen oscillations, 81.7 rad (SRC-073, SRC-074), lie between the two. Corroboration of I-4 is the full polynomial and its declared orientation at exact analytical-model precision; a held-out coefficient contrast outside the joint calibration uncertainty falsifies that finite-pulse model.

3.4 Reported observations and calibrations

The table below anchors each observation to a source figure or equation and its ledger category. The model signs above stay explicit when phase magnitudes are compared.

Source anchor	Record	Category and ledger
Figure 2	Thirteen oscillations; approximately 80 rad; 633 cycles of 30 s; reported duration 5.3 h	Observed/reported; SRC-073, 074, 084, 085.
Figure 2 fit	Polynomial envelopes; cubic phase fit excludes the first and last oscillations	In-sample fitted extraction; SRC-075–SRC-078.
Figure 2 precision	Population SEM $1.8 \pm 1.5\%$; visibility from 80% to 20% by $2\tau = 2000 \mu\text{s}$	Reported distinct metrics; SRC-079, SRC-081.
Figure 3B	Approximately 2 rad residual across approximately 80 rad; source fraction approximately 2.5%; statistical phase noise 1.3%	Reported comparison/noise; SRC-120–SRC-123.
Figure 3 bounds	Kick current changed by $\pm 0.5\%$ to construct systematic bounds	Reported control envelope; SRC-119.
Methods, acceleration	$g_{\text{eff}} = 9.91 \text{ m s}^{-2}$, including the second-order Zeeman push on $f_{\text{hf}} = 1, m_f = 0$; the supplementary text splits it as $g + a_{\text{SOZ}}$ with $a_{\text{SOZ}} = 0.10 \text{ m s}^{-2}$ and uses g_{eff} in the analytic model	Reported model input; SRC-164, SRC-165; [2].
Methods, bias	External bias field 12.6 G	Reported apparatus; SRC-163.
Methods, holding	Holding current $J_h = 23.07 \pm 1.49$ mA, calibrated from trajectories	Measured calibration; SRC-169, SRC-170.
Equation 3 propagation	Source compensation inaccuracy 6.5%; phase inaccuracy 0.42%	Reported quadratic propagation; SRC-171, SRC-172.

Let φ_{fit} denote the source’s fitted phase and $\mathbf{c}_{\text{fit},j}$ its polynomial coefficients, with j a non-negative polynomial index. The reconstructed population fit has the source form (SRC-101, SG-14):

$$\begin{aligned}
 p_{\text{out}}(u_{\text{tot}}) &= \bar{p}(u_{\text{tot}}) + \frac{1}{2} V_{\text{vis}}(u_{\text{tot}}) \cos \varphi_{\text{fit}}(u_{\text{tot}}), \\
 \varphi_{\text{fit}}(u_{\text{tot}}) &= \sum_{j=0}^3 \mathbf{c}_{\text{fit},j} u_{\text{tot}}^j.
 \end{aligned} \tag{36}$$

A cubic has four algebraic coefficients. The record supplies neither their joint covariance nor the degrees and covariance of the preceding envelope fits, and the 633 acquisition cycles are not automatically 633 independent phase estimates. Nominal cycle arithmetic recovers the

reported elapsed time without adding precision:

$$\frac{633 \times 30}{3600} \text{ h} \simeq 5.3 \text{ h}. \quad (37)$$

The count is an integer, the cycle time is reported as 30 s, and the result keeps the source's two-digit hour value (U-11).

The blind simulation has no fitted tuning (SRC-120). The separate tuned comparison raises the kick current by 0.15%, changes the idle current from 0.55 to 0.5 mA, and changes $B_{\text{mag},z}$ from 0.33 to 0.35 G, with gauss denoting the field unit (SRC-124–SRC-126). These are different comparisons. The source assigns order 1–2 rad each to trajectory and packet-shape effects and up to approximately 0.25 rad to interactions (SRC-097–SRC-100); its later main-text shape and rotation description gives up to 1 rad (SRC-180). The corrections are source model contributions, not separately measured Gaussian errors or FDT-specific anomalies; Sec. 6 places them in their windows.

For the signed phase at fixed half-duration, let the reference be held at net downward acceleration $g_{\text{lab}} - a_{\text{mag}}$ while the ballistic arm falls at g_{b} . A closed path under net downward acceleration a_{net} has action $-\mathcal{M}_{\text{Rb}} a_{\text{net}}^2 \tau^3 / 3$, so

$$\begin{aligned} \Delta\varphi_{\text{br}}(a_{\text{mag}}) &= -\frac{\mathcal{M}_{\text{Rb}}\tau^3}{3\hbar} [g_{\text{b}}^2 - (g_{\text{lab}} - a_{\text{mag}})^2], \\ \Delta\varphi_{\text{br}}(a_{\text{mag}}) - \Delta\varphi_{\text{br}}(g_{\text{lab}}) &= \frac{\mathcal{M}_{\text{Rb}}\tau^3}{3\hbar} (a_{\text{mag}} - g_{\text{lab}})^2, \\ \frac{\partial\Delta\varphi_{\text{br}}}{\partial a_{\text{mag}}} &= \frac{2\mathcal{M}_{\text{Rb}}\tau^3}{3\hbar} (a_{\text{mag}} - g_{\text{lab}}), \quad \frac{\partial^2\Delta\varphi_{\text{br}}}{\partial a_{\text{mag}}^2} = \frac{2\mathcal{M}_{\text{Rb}}\tau^3}{3\hbar} > 0. \end{aligned} \quad (38)$$

At $g_{\text{b}} = g_{\text{lab}}$ the first line is source Equation 3, $\mathcal{M}_{\text{Rb}}\tau^3 a_{\text{mag}}(a_{\text{mag}} - 2g_{\text{lab}})/(3\hbar)$ (SRC-053–SRC-055). Its minimum is at levitation, where the value carries the ballistic arm's own fall; the twist's share shifts the level and leaves the curvature unchanged. Its negative, and its magnitude on $0 < a_{\text{mag}} < 2g_{\text{lab}}$, have a maximum there: the source calls the extremum a maximum (SRC-173), a word that applies to that magnitude. At $g_{\text{b}} = g_{\text{lab}}$ and fixed a_{mag} the signed phase is linear in g_{lab} , whereas along the levitation family it is quadratic; this specifies the controls behind the source's sensitivity wording (SRC-174).

Let ϵ_{hold} be the fractional holding-acceleration error. Under the source's proportionality of current and acceleration for this calibration (SRC-170 to SRC-172), the fractional phase departure has the squared magnitude

$$\begin{aligned} 100 \frac{1.49}{23.07} \% &\simeq 6.5\%, \\ \frac{|\Delta\varphi_{\text{br}}(a_{\text{mag}}) - \Delta\varphi_{\text{br}}(g_{\text{lab}})|}{|\Delta\varphi_{\text{br}}(g_{\text{lab}})|} &= \epsilon_{\text{hold}}^2, \quad 100(0.065)^2 \% \simeq 0.42\%. \end{aligned} \quad (39)$$

The first result is rounded to two significant figures, below the three figures of the limiting 1.49 mA input; the squared estimate keeps the two figures of the reported 6.5% and holds at $g_{\text{b}} = g_{\text{lab}}$. The kick-current envelope, population SEM, phase noise and residual scale measure different things. Corroboration of I-3 and I-4 is the reported agreement at the source's approximate residual scale; a new falsification uses independently determined covariance and controlled systematics, not a significance invented from these summaries.

3.5 The timing accounts

The source reports $\tau_h = 2272 \mu\text{s}$, $\tau_r = 12 \mu\text{s}$, $\tau_d = 71 \mu\text{s}$ and $\tau_k = 80 \mu\text{s}$ (SRC-149, SRC-156, SRC-158, SRC-167), with $\tau_d = (50 + 16 + 5) \mu\text{s}$ and τ_k the average of the two kicks; its worked example is labelled $2\tau = 2.4 \text{ ms}$ (SRC-140). Three conventions relate the holding duration to 2τ : the main-text definition $\tau_h = 2\tau - 2\tau_d$ (SRC-060); the Methods formula $2\tau_{\text{eff}} = \tau_h + 2\tau_r + 2\tau_d + \tau_k = 2\tau + 80 \mu\text{s}$ (SRC-158, SRC-166); and the supplementary analytic model's $\tau_d = (71 + 6) \mu\text{s}$, half the rise time added, with the main-text definition [2]. Let u_{main} , u_{meth} , u_{model} denote the total duration each gives for the printed settings:

$$\begin{aligned} u_{\text{main}} &= \tau_h + 2\tau_d = 2414 \mu\text{s}, & u_{\text{meth}} &= \tau_h + 2\tau_r + 2\tau_d = 2438 \mu\text{s}, \\ u_{\text{model}} &= \tau_h + 2\left(\tau_d + \frac{\tau_r}{2}\right) = 2426 \mu\text{s}, & \left(\frac{u_{\text{meth}}}{u_{\text{main}}}\right)^3 &= 1.030. \end{aligned} \quad (40)$$

The label 2.4 ms is each of the three at two figures. The ideal phase scales as the cube of the duration, so the conventions spread the largest example's phase by 3.0%, or 2.4 rad at 80 rad , the size of the reported residual (SRC-121). A timing-to-action comparison at the residual's scale therefore takes one measured pulse history; these exact integers audit the printed settings and add no timing precision.

Let z_{top} denote the reconstructed top-arm displacement and z_{bottom} the reference's finite-pulse displacement. With $g_b = 9.91 \text{ m s}^{-2}$ (SRC-164, SRC-165),

$$\begin{aligned} z_{\text{top}} &= \frac{1}{2}g_b \left(\frac{2\tau + \tau_k}{2}\right)^2 = 7.62, 7.71, 7.85 \mu\text{m} \quad \text{for } 2\tau = 2400, 2414, 2438 \mu\text{s}, \\ z_{\text{bottom}} &= \frac{1}{2}g_b(\tau_k + \tau_d)^2 \simeq 0.11 \mu\text{m}. \end{aligned} \quad (41)$$

The reconstructed lengths keep three figures from the three-figure acceleration and the integer settings. The source prints $7.62 \mu\text{m}$ for the top arm and $0.11 \mu\text{m}$ for the reference, leading to its approximately $7.5 \mu\text{m}$ splitting (SRC-164, SRC-168, SRC-157): its third figure comes from the two-figure label. Its Figure 6 caption gives a final residual splitting of order $0.1 \mu\text{m}$ and its Methods $0.2 \mu\text{m}$ (SRC-155, SRC-179); these remain separate reports.

4 Cone-position translation

The two interfering paths are two projections of one prepared closure (I-4). The source's internal labels are confinement modes of one pattern, not new matter species (SRC-019, SRC-020, SRC-050), and its trajectory-calibrated holding current fixes the twist's response at the reference (SRC-169, SRC-170). The common closure's mass enters as U-07 and is never reconstructed from a fitted phase and then used to predict that phase.

4.1 Two free arms in one twist

Both arms are free: each follows the geodesic of $\mathbf{m}_j^2 g_{\mu\nu}$, with $g_{\mu\nu}$ the Earth's metric, Eq. (17). The reference is held where its composed factor $\mathbf{n}_r$ is stationary, the twist's slope of its rest energy cancelling the lapse's; the ballistic arm's $\mathbf{n}_b$ slopes by g_b/c^2 . Let μ_B be the Bohr magneton and $\mu_{2,1}$ the magnitude of the reference state's held moment, which is antiparallel to B_{mag} : $0.5037 \mu_B$ at the 12.6 G operating field (SRC-163) in the Breit–Rabi structure (U-18),

against $0.4998 \mu_B$ at low field. The levitation condition of Eq. (17) fixes the twist's gradient at the reference:

$$-\partial_z |B_{\text{mag}}| = \frac{\mathcal{M}_{\text{Rb}} g_{\text{lab}}}{\mu_{2,1}} = 0.303 \text{ T m}^{-1} = 30.3 \text{ G cm}^{-1}. \quad (42)$$

This is the gradient that the trajectory-calibrated holding current $J_h = 23.07 \pm 1.49 \text{ mA}$ sets at the reference (SRC-169, SRC-170). The ballistic state holds no moment along B_{mag} at first order; the twist induces one, with energy $E_{\text{ind,b}} = -K_q |B_{\text{mag}}|^2$ and $K_q = (g_J - g_I)^2 \mu_B^2 / (4\Delta E_{\text{hfs}}) = 1.905 \times 10^{-23} \text{ J T}^{-2}$ (U-18). In one twist the two pushes stand in the ratio of the moments along $\partial_z B_{\text{mag}}$. With the field magnitude at the arms set by the source's 12.6 G bias (SRC-163),

$$\frac{a_{\text{SOZ}}}{g_{\text{lab}}} = \frac{2K_q |B_{\text{mag}}|}{\mu_{2,1}}, \quad a_{\text{SOZ}} = 0.10 \text{ m s}^{-2} \quad \text{at} \quad |B_{\text{mag}}| = 1.26 \text{ mT}, \quad (43)$$

the push the source computes (SRC-165; [2]); the chip wires' own field at the arms, under half a gauss, leaves two figures. The ballistic arm's extra fall follows from the reference's levitation and the measured ground-state structure, with no parameter left free; test T-5 takes its calibrated value from the field map (U-19). The push is read from the pattern's rest energy, never from a gravitational compactness given to its moment; the map from a circulating charge to an I-1 input is to be derived (D-02).

4.2 The pattern's phase runs on its own time

Let $\mathcal{A}_{\text{mode}}$ be a real nonnegative projection amplitude, φ_0 an initial helical phase, φ_h the evolving helical phase, ϱ_h the cylindrical radius of a geometric representative and z_h its axial coordinate. Let $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ be an orthonormal basis with $\mathbf{e}_3$ along that axis, $\mathbf{x}_h$ the representative position and ψ_h the complex planar projection. The helix phase of arm j advances with the rest energy where the arm sits, per unit of the time it keeps (0.1a), and the I-4 projection reads it:

$$\begin{aligned} \varphi_h &= \varphi_0 - \frac{1}{\hbar} \int E_j dt_{\text{own}} = \varphi_0 + \frac{\mathfrak{A}_j}{\hbar}, \\ \mathbf{x}_h &= \varrho_h \cos \varphi_h \mathbf{e}_1 + \varrho_h \sin \varphi_h \mathbf{e}_2 + z_h \mathbf{e}_3, \quad \psi_h = \mathcal{A}_{\text{mode}} e^{i\varphi_h}. \end{aligned} \quad (44)$$

Outside every twist the phase turns once per kept period T_{Rb} . The interferometer reads the difference of φ_h between the arms at recombination, Eq. (26); written against laboratory time the lift would erase the 75.85 rad kept-time part and keep only the twist's 1.56 rad. The cylindrical representative fixes phase geometry and assigns no unmeasured radius or axial velocity to the atomic configuration.

Let $\mathcal{R}_{\text{ph}}$ denote the single-angle phase rotor, $\mathcal{R}_{\text{spin}}$ a half-angle rotation representative and θ_{rot} its physical rotation angle. The phase-plane bivector obeys

$$\begin{aligned} \mathbf{B}_{\text{ph}} &= \mathbf{e}_1 \mathbf{e}_2, & \mathbf{B}_{\text{ph}}^2 &= -1, \\ \mathcal{R}_{\text{ph}}(\varphi_h) &= e^{\mathbf{B}_{\text{ph}} \varphi_h} = \cos \varphi_h + \mathbf{B}_{\text{ph}} \sin \varphi_h, & \mathcal{R}_{\text{spin}}(\theta_{\text{rot}}) &= e^{-\mathbf{B}_{\text{ph}} \theta_{\text{rot}}/2}. \end{aligned} \quad (45)$$

Both exponents are dimensionless and their angle conventions differ; the phase rotor represents the observed phase orientation, and a half-angle representative assigns no spinorial class to the atom. In the strand reading the rest energy is the bivector grade of $pc + \mathcal{M}_{\text{Rb}} c^2 \Sigma$ (0.1d), so the phase turns with the pattern's circulation.

The standard account reads the interference through probability amplitudes and wavefunction collapse. FDT reads it as deterministic helical phase ignorance, projection and phase fixation, with a shared bivector grade (I-4). The standard account assigns an intrinsic mass to a separate particle. FDT reads mass as the confinement eigenvalue of a pattern with its own support (I-5, 0.1b), with the measured mass as input; both replacements change the reading of the phase record and keep the source-model action and its population projection.

4.3 State response from the helical window

Let $\mathcal{E}_j(B_{\text{mag}})$ be the measured total internal-state energy of state j at field B_{mag} (U-18), with $\mathcal{E}_{f,0}$ the energy of $f_{\text{hf}} = f$ at $m_f = 0$, and let B_{ref} be a fixed reference bias. The helical window's rest energy and its push are

$$\begin{aligned} E_j(z_{\text{lab}}) &= E_{0,j} + U_j(z_{\text{lab}}), & U_j(z_{\text{lab}}) &= \mathcal{E}_j(B_{\text{mag}}(z_{\text{lab}})) - \mathcal{E}_j(B_{\text{ref}}), \\ F_{\text{mag},j} &= -\frac{dU_j}{dz_{\text{lab}}}, & \mathcal{E}_{2,0} + \mathcal{E}_{1,0} &= -\frac{\Delta E_{\text{hfs}}}{4} \quad \text{at every field.} \end{aligned} \quad (46)$$

The reference bias is a uniform level: it changes $\mathcal{M}_j$ and leaves the push unchanged. The last relation is the Breit–Rabi structure of the $m_f = 0$ pair: the two energies sum to a constant, so the twist pushes the pair equally and oppositely at every field, the property test T-5 uses. The ballistic state's second-order Zeeman energy is its E_{ind} , inside g_b (SRC-165); it is not added to a gravitational acceleration and then added again as potential energy.

The translation is summarised in the table below.

Experimental object	FDT reading	Quantitative entry
Prepared atomic state	One closed pattern, with its own boundary data	Mass U-07; Eq. (21).
Two phase paths	Two projections of that preparation, each free in its own composed clock factor	Eqs. (17), (25).
Free-fall phase	Kept-time difference counted in the pattern's own period, plus the twist's share	Eq. (26).
Microwave state change	Confinement-mode change of the same pattern	Calibrated state energies and dwell times (SRC-144–SRC-150).
Magnetic holding	The helical window: the twist's push on the held moment balances the weight	Eqs. (42), (46).
Second-order Zeeman fall	The helical window on the induced moment	a_{SOZ} , Eq. (43).
Output visibility	Fitted projection contrast	Eq. (36) (SRC-081, SRC-101); no compactness assignment.

Corroboration of I-2, I-4 and I-5 is the recovery of the phase projection with a fixed field identity and independently supplied mass, at the precision of the declared dynamics. Failure of the calibrated response to reproduce held-out trajectories and phase contrasts falsifies that response model; a fitted phase alone supplies no microscopic spectrum.

5 Predictions

The following five prospective tests use the ^{87}Rb chip platform. Each is defined together with its independent calibrations, closures, covariance and bounded nuisance inputs. The conventional

comparator is the same-source action and finite-pulse model, and the convergence residual is zero for each test; a distinct residual is assigned to FDT only with an additional source or constitutive response stated.

The superscript T on a vector denotes transpose. Let $\boldsymbol{\vartheta}$ collect a test's measured quantities and independent comparator inputs, $\mathcal{D}$ its residual and $s_{\mathcal{D}}$ its propagated standard uncertainty. Let $\nabla_{\boldsymbol{\vartheta}}\mathcal{D}$ be the gradient with respect to those inputs; each block specifies their contents. Let $z_{\text{sig}} = 3$ be the investigator-chosen design multiplier (U-09). For test index $i = 1, \dots, 5$, $\mathcal{D}_i$ denotes its residual. Let Q be a generic quantity label, not an additional physical coordinate, and $b_Q \geq 0$ a conservative absolute bound on the residual nuisance contribution to quantity Q , in Q 's units, supplied by independent future calibration (U-15). With joint covariance Σ_{cov} , first-order (delta-method) propagation of a nonlinear residual variance and the preregistered residual criterion are

$$\begin{aligned} s_{\mathcal{D}}^2 &\simeq (\nabla_{\boldsymbol{\vartheta}}\mathcal{D})^T \Sigma_{\text{cov}} (\nabla_{\boldsymbol{\vartheta}}\mathcal{D}), \\ |\mathcal{D}| &> z_{\text{sig}} s_{\mathcal{D}} + b_{\mathcal{D}}, \quad z_{\text{sig}} = 3. \end{aligned} \tag{47}$$

The variance equality is exact for fixed-weight linear contrasts, and the design multiplier is not a significance inferred from the source. Calibration correlations are included even when acquisition runs are separate, and bounded nuisance sets are propagated to the residual before testing. Covariance carries probabilistic components, which are not counted again as deterministic bounds. Equation (47) is the conservative symmetric-bound rejection rule; for asymmetric or correlated allowed nuisance sets, rejection minimises the residual discrepancy over that set with the applicable covariance and rejects at that minimum. The zero-bound limit $b_{\mathcal{D}} = 0$ recovers $|\mathcal{D}| > z_{\text{sig}} s_{\mathcal{D}}$. The source supplies no numerical joint covariance (U-13) or independent residual calibration bound (U-15). Let Y denote a generic estimator and Cov statistical covariance. For two correlated estimates Y_1, Y_2 of one quantity, with uncertainties s_{Y_1}, s_{Y_2} , the difference has

$$s_{Y_1 - Y_2} = \sqrt{s_{Y_1}^2 + s_{Y_2}^2 - 2 \text{Cov}(Y_1, Y_2)}. \tag{48}$$

The closed-form resolution scales below hold with uncertainty and bound fixed over the declared design window; design-dependent uncertainties and bounds enter the defining resolution equation before solving, and setting each bound to zero recovers its statistical-only resolution scale.

5.1 T-1: a held-out duration-polynomial contrast

Let $h_{\text{dur}} > 0$ be total-duration spacing and u_0 the first total duration. Let Φ_j be new phase observations at $u_{\text{tot}} = u_0 + j h_{\text{dur}}$, for $j = 0, 1, 2, 3$, with the positive source orientation fixed by a control-phase scan. Each phase is extracted from independently acquired populations at its own duration, not from a common cubic used as its comparator. Keep τ_k, τ_d , field geometry and preparation fixed, and set launch and closing kicks to the measured closure condition at each duration. Let $\mathcal{C}_1$ be the third finite-difference contrast and $\hat{C}_{\varphi}$ the coefficient inferred from it. Equation (32), written in total duration, gives the defining condition and its solution:

$$\begin{aligned} \mathcal{C}_1 &= -\Phi_0 + 3\Phi_1 - 3\Phi_2 + \Phi_3 = \frac{3C_{\varphi}}{4} h_{\text{dur}}^3, \\ \hat{C}_{\varphi} &= \frac{4\mathcal{C}_1}{3h_{\text{dur}}^3}, \\ \mathcal{D}_1 &= \mathcal{C}_1 - \frac{3C_{\varphi}}{4} h_{\text{dur}}^3 = 0. \end{aligned} \tag{49}$$

The constant, linear and quadratic terms cancel exactly. The comparator uses independent $\mathcal{M}_{\text{Rb}}$, g_{lab} and a_{SOZ} ; fitting those inputs from these four phases would remove the test's independent content.

Let $\mathbf{w}_1 = (-1, 3, -3, 1)^T$ be the contrast weights and Σ_Φ the covariance of the four phase estimates; the phase-only contrast variance is $\mathbf{w}_1^T \Sigma_\Phi \mathbf{w}_1$. The complete residual uncertainty in Eq. (47) additionally uses h_{dur} , $\mathcal{M}_{\text{Rb}}$, g_{lab} , a_{SOZ} and their correlations. Let $s_{\mathcal{C}_1}$ denote the contrast uncertainty and h_{min} the spacing that resolves the cubic contrast. The contrast-only bound $b_{\mathcal{C}_1}$ covers phase-contrast nuisance contributions; the full residual bound $b_{\mathcal{D}_1}$ also propagates comparator and calibration nuisance contributions through their allowed set. With that uncertainty and contrast bound fixed over the declared design window, solving the contrast-resolution condition gives

$$\begin{aligned} \frac{3C_\varphi}{4} h_{\text{min}}^3 &= z_{\text{sig}} s_{\mathcal{C}_1} + b_{\mathcal{C}_1}, \\ h_{\text{min}} &= \left(\frac{4(z_{\text{sig}} s_{\mathcal{C}_1} + b_{\mathcal{C}_1})}{3C_\varphi} \right)^{1/3}. \end{aligned} \quad (50)$$

Its units are seconds. A duration-dependent uncertainty and bound are inserted into the defining equation before solving for a design point. This is a single-channel cubic contrast, with measured closure and waveform changes as controlled nuisance inputs. A nonzero residual satisfying Eq. (47) after those controls falsifies the shared finite-pulse prediction; it does not arise from a new chart label.

5.2 T-2: a symmetric, closed levitation scan

Let $\delta a_{\text{scan}} > 0$ be a calibrated acceleration offset and $a_{\pm} = g_{\text{lab}} \pm \delta a_{\text{scan}}$ the two scan settings. Let $\varphi_+, \varphi_{\text{centre}}, \varphi_-$ denote new signed arm-phase estimates at the upper setting, central levitation and lower setting. Use the reference state $f_{\text{hf}} = 2, m_f = 1$ and the ballistic state $f_{\text{hf}} = 1, m_f = 0$ (SRC-019, SRC-020). Fix τ , the gravitational acceleration and the preparation; vary the calibrated holding and kick forces together so that closure is kept. Let η_{path} be time since launch, Δz_{path} the relative arm displacement and $a_{\text{rel}} = g_{\text{b}} - g_{\text{lab}} + a_{\text{mag}}$ the ballistic-minus-reference relative acceleration, downward. For uniform relative acceleration, closure is solved by

$$\begin{aligned} \Delta z_{\text{path}}(\eta_{\text{path}}) &= v_{\text{launch}} \eta_{\text{path}} - \frac{a_{\text{rel}} \eta_{\text{path}}^2}{2}, \\ \Delta z_{\text{path}}(2\tau) &= 0 \quad \iff \quad v_{\text{launch}} = a_{\text{rel}} \tau. \end{aligned} \quad (51)$$

Endpoint position and momentum are verified by trajectories, with the second kick removing the relative velocity. Finite-pulse corrections are calculated from the measured waveform before the ideal scan contrast is compared; a launch fixed at its central value while the holding acceleration changes would not be this experiment.

Let $K_{\text{lev}} = \mathcal{M}_{\text{Rb}} \tau^3 / (3\hbar)$ be the acceleration curvature coefficient, $\mathcal{C}_2$ the symmetric signed contrast and $\hat{K}_{\text{lev}}$ its inferred curvature. Equation (38) gives

$$\begin{aligned} \mathcal{C}_2 &= \frac{\varphi_+ + \varphi_-}{2} - \varphi_{\text{centre}} = K_{\text{lev}} (\delta a_{\text{scan}})^2, \\ \hat{K}_{\text{lev}} &= \frac{\mathcal{C}_2}{(\delta a_{\text{scan}})^2}, \quad \mathcal{D}_2 = \mathcal{C}_2 - K_{\text{lev}} (\delta a_{\text{scan}})^2 = 0. \end{aligned} \quad (52)$$

The contrast is independent of the twist's share on the ballistic arm, which shifts only the level. The defining polynomial has a positive curvature and its minimum at levitation, and

its symmetry isolates the even acceleration channel. Let $s_{\mathcal{C}_2}$ denote the contrast uncertainty and $\delta a_{\min}$ the acceleration offset that resolves it. The contrast-only bound $b_{\mathcal{C}_2}$ covers phase-contrast nuisance contributions. With fixed uncertainty and bound over the design window, the resolvable offset is

$$\delta a_{\min} = \sqrt{\frac{z_{\text{sig}} s_{\mathcal{C}_2} + b_{\mathcal{C}_2}}{K_{\text{lev}}}}, \quad (53)$$

which solves $K_{\text{lev}}(\delta a_{\min})^2 = z_{\text{sig}} s_{\mathcal{C}_2} + b_{\mathcal{C}_2}$ and has acceleration units. The contrast covariance uses weights $(1/2, -1, 1/2)$ on the ordered estimates $(\varphi_+, \varphi_{\text{centre}}, \varphi_-)$. The full residual additionally propagates the calibration of both acceleration settings, central levitation, mass, duration and finite-pulse corrections, and its bound $b_{\mathcal{D}_2}$ propagates the corresponding comparator and calibration nuisance set together with the contrast contributions; a scan-dependent uncertainty and bound enter the defining equation before solving. The conventional comparator is source Equation 3. This is a single even acceleration channel with closure controlled; a violation satisfying Eq. (47) falsifies that model without reversing its signed extremum.

5.3 T-3: independent timing from phase coefficients

Let $\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3$ be coefficients fitted to new phase observations as a polynomial in half-duration, with the pulse widths held fixed; they are distinct from the source's fitted coefficients and from the strand rate c . Let $\hat{\tau}_d$ be a delay reconstructed from those coefficients and τ_d^{time} a separately measured waveform delay on the same ^{87}Rb chip. Under Eq. (32) the nonconstant coefficients obey

$$\begin{aligned} \mathbf{c}_3 &= C_\varphi, & \mathbf{c}_2 &= C_\varphi \tau_k, & \mathbf{c}_1 &= C_\varphi \tau_k (\tau_k + \tau_d), \\ \mathbf{c}_3 \mathbf{c}_1 - \mathbf{c}_2^2 - \mathbf{c}_3 \mathbf{c}_2 \tau_d &= 0, \\ \hat{\tau}_d &= \frac{\mathbf{c}_1}{\mathbf{c}_2} - \frac{\mathbf{c}_2}{\mathbf{c}_3}, & \mathcal{D}_3 &= \hat{\tau}_d - \tau_d^{\text{time}} = 0. \end{aligned} \quad (54)$$

The inversion requires nonzero $\mathbf{c}_2, \mathbf{c}_3$, hence a nonzero kick duration, and it is independent of an additive phase origin and of a common phase-sign reversal. Each coefficient $\mathbf{c}_j$ has units of s^{-j} , with phase in radians understood. The constant term stays part of the finite-pulse model; this contrast's insensitivity to offsets does not discard it. The model's τ_k is the average of the two kick durations (SRC-167), so the inversion returns the delay of that averaged model: the waveform comparator takes the same average, or the polynomial is extended to the two measured kick durations before the inversion.

Let $\mathbf{v}_3 = (\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \tau_d^{\text{time}})$ collect the four inputs and Σ_3 their covariance. The gradient required for the precision criterion is

$$\nabla_{\mathbf{v}_3} \mathcal{D}_3 = \left(\frac{1}{\mathbf{c}_2}, -\frac{\mathbf{c}_1}{\mathbf{c}_2^2} - \frac{1}{\mathbf{c}_3}, \frac{\mathbf{c}_2}{\mathbf{c}_3^2}, -1 \right). \quad (55)$$

The propagated uncertainty has units of seconds, and the resolvable delay residual is $z_{\text{sig}} s_{\mathcal{D}_3} + b_{\mathcal{D}_3}$, not an oscilloscope's resolution alone. This is a single timing-coefficient channel with measured closure and fixed pulse shape. The comparator is independent waveform timing; neither the present paper's in-sample fit nor a timing inferred from these same phase coefficients supplies it. A duration curve fixes a phase coefficient combining mass and acceleration, not those quantities separately, and a nonzero central estimate guarantees only algebraic definability. A timing precision claim requires the joint coefficient inference region, allowed nuisance variation included, to hold both $\mathbf{c}_2$ and $\mathbf{c}_3$ away from zero and the neglected nonlinear terms to be negligible at the claimed precision; otherwise the timing is reported unresolved, or the coefficient relation of Eq. (54) is tested in a joint likelihood with independent waveform timing.

5.4 T-4: a controlled current-ratio response

Let $\mathcal{J}_{\text{ratio}} = J_k/J_h$ be the kick/holding current ratio and η_J its signed fractional displacement from a fixed nominal ratio; let $\epsilon_J > 0$ be a designed symmetric displacement magnitude. On the same chip, interleave measurements at $\eta_J = \pm\epsilon_J$ and measure their trajectories, axial bias and pulse areas. The source identifies this ratio as its leading systematic and a linear-duration phase term (SRC-113, SRC-114); its $\pm 0.5\%$ protocol is a reported envelope, not an uncertainty distribution (SRC-119).

Let $\Phi(u_{\text{tot}}, \eta_J)$ be the newly observed phase and $\mathcal{C}_4(u_{\text{tot}})$ its odd control contrast after independently measured trajectory and state-response corrections. Let $K_J(\eta_J)$ denote the derivative of phase with respect to this control, divided by total duration, on a declared factorised-response family. With a continuously differentiable control response, the exact finite contrast and its defining condition are

$$\begin{aligned}\mathcal{C}_4(u_{\text{tot}}) &= \frac{\Phi(u_{\text{tot}}, \epsilon_J) - \Phi(u_{\text{tot}}, -\epsilon_J)}{2}, \\ \frac{\partial \Phi}{\partial \eta_J} &= u_{\text{tot}} K_J(\eta_J), \\ \mathcal{C}_4(u_{\text{tot}}) &= \frac{u_{\text{tot}}}{2} \int_{-\epsilon_J}^{\epsilon_J} K_J(\eta_J) d\eta_J.\end{aligned}\tag{56}$$

The control integral, not a local derivative times a finite span, determines the amplitude. Let b_J^{cal} be one half of that integral, measured in a separate calibration run, and $\hat{b}_J$ its value inferred from a held-out phase contrast. The solved comparison is

$$\hat{b}_J = \frac{\mathcal{C}_4(u_{\text{tot}})}{u_{\text{tot}}}, \quad \mathcal{D}_4 = \mathcal{C}_4(u_{\text{tot}}) - u_{\text{tot}} b_J^{\text{cal}} = 0.\tag{57}$$

Both response coefficients have units of inverse seconds. Constancy of $\mathcal{C}_4/u_{\text{tot}}$ is equivalent to a constant integrated prefactor on the tested duration family; equality at two durations states only the equality of those two ratios and establishes no derivative law across an unmeasured interval.

The residual covariance includes both phase estimates, the independent calibration, duration, field measurements and the correction estimates; its phase term uses the two-estimate covariance of Eq. (48), divided by four for the half-difference. The residual bound $b_{\mathcal{D}_4}$ propagates the allowed calibration and correction nuisance set. A coefficient mismatch reaches resolution when its absolute phase residual equals $z_{\text{sig}} s_{\mathcal{D}_4} + b_{\mathcal{D}_4}$; dividing by $|u_{\text{tot}}|$ gives the resolvable coefficient change $(z_{\text{sig}} s_{\mathcal{D}_4} + b_{\mathcal{D}_4})/|u_{\text{tot}}|$. The table below gives the co-varying-channel sign bookkeeping.

Channel	Held-fixed condition	Contrast sign
Current ratio	Fixed preparation and measured waveform family	Sign of the independently integrated K_J .
Closure error	Measured final position and momentum	Sign of the calibrated trajectory correction.
Magnetic bias	Fixed nominal bias with measured excursions	Sign of the independently measured state-energy response.
Packet shape	Independent expansion and rotation diagnostics	Sign of its calibrated phase correction.

A ratio-induced signal is not a density coordinate. The conventional comparator is the calibrated linear-duration response of the source model, and a residual beyond Eq. (47) rejects that factorised control model. A distinct microscopic FDT prediction here requires the I-1 reading of the twist's push (D-02).

5.5 T-5: the $m_f = 0$ pair reads the two windows apart

Prepare two runs on the same chip and holding-waveform family. Run A is the source's: reference $f_{\text{hf}} = 2, m_f = 1$, ballistic $f_{\text{hf}} = 1, m_f = 0$. Run B takes the ballistic state $f_{\text{hf}} = 2, m_f = 0$ and holds the reference in $f_{\text{hf}} = 1, m_f = -1$, whose held moment at the 12.6 G operating field is $0.4979 \mu_B$ against $0.5037 \mu_B$ (U-18). Its holding current is recalibrated from trajectories, as the source calibrates J_h , and the ratio of the two calibrated currents, $r = J_{h,B}/J_{h,A} = 1.0116$, is the ratio of the two held moments. By Eq. (46) the twist pushes the two ballistic states oppositely, each in proportion to the gradient its run sets, so where they sit they fall at $g_{\text{lab}} + a_{\text{SOZ}}$ and $g_{\text{lab}} - r a_{\text{SOZ}}$, with a_{SOZ} the push of run A. Let $\hat{C}_A$ and $\hat{C}_B$ be the two runs' cubic coefficients, $k_g = \mathcal{M}_{\text{Rb}}/(3\hbar) = 4.562 \times 10^8 \text{ s m}^{-2}$, $\hat{g}_A = (\hat{C}_A/k_g)^{1/2}$ and $\hat{g}_B = (\hat{C}_B/k_g)^{1/2}$ the falls they read, and $\hat{\rho} = (\hat{C}_B/\hat{C}_A)^{1/2}$. Let $g_{\text{lab}}^{\text{site}}$ be the site gravimeter value (U-08) and $a_{\text{SOZ}}^{\text{cal}}$ run A's push computed from the field map and the measured ground-state structure (U-19, U-18). Then

$$\begin{aligned} \frac{r\hat{g}_A + \hat{g}_B}{1+r} &= g_{\text{lab}}, & \frac{\hat{g}_A - \hat{g}_B}{1+r} &= a_{\text{SOZ}}, & \frac{1 - \hat{\rho}}{r + \hat{\rho}} &= \frac{a_{\text{SOZ}}}{g_{\text{lab}}}, \\ \mathcal{D}_5 &= \frac{r\hat{g}_A + \hat{g}_B}{1+r} - g_{\text{lab}}^{\text{site}} = 0, & \mathcal{D}'_5 &= \frac{1 - \hat{\rho}}{r + \hat{\rho}} - \frac{a_{\text{SOZ}}^{\text{cal}}}{g_{\text{lab}}^{\text{site}}} = 0. \end{aligned} \quad (58)$$

In the weighted sum the twist cancels and the gravitational window remains, where $\mathbf{m}$ belongs to the point; in the difference gravity cancels and the helical window remains, where $\mathbf{m}$ belongs to the pattern. The ratio form carries that difference in units of the fall, free of every common scale of mass, duration and phase origin, and $\mathcal{D}'_5$ uses it. The two ballistic states' masses differ by $\Delta E_{\text{hfs}}/(\mathcal{M}_{\text{Rb}}c^2) = 3.5 \times 10^{-16}$, far below the platform's reach. With $a_{\text{SOZ}} = 0.10 \text{ m s}^{-2}$ the runs read $\hat{C}_B/\hat{C}_A = 0.960$: at the worked example their ideal phases are 77.4 and 74.3 rad, 3.1 rad apart.

The residual uncertainties follow Eq. (47) with the two runs' coefficient covariance. For uncorrelated runs with equal fractional coefficient uncertainty s_C/C and zero bound, $\mathcal{D}'_5$ resolves the helical window at $z_{\text{sig}} = 3$ when

$$\frac{s_C}{C} < \frac{\sqrt{2}(r + \rho)^2}{3(1+r)\rho} \frac{a_{\text{SOZ}}}{g_{\text{lab}}} = 0.97\%. \quad (59)$$

With $\hat{C}$ from the four-point contrast of T-1 inside the source's scan, 2τ from about 0.4 to 2.4 ms in its Figure 2, $h_{\text{dur}} \leq \frac{2}{3} \text{ ms}$ and $\mathcal{C}_1 \leq 9.96 \text{ rad}$, so $s_C/C = \sqrt{20} s_\varphi/\mathcal{C}_1$ and the condition requires a per-duration phase uncertainty $s_\varphi \leq 0.022 \text{ rad}$. A full-scan cubic fit over N durations gives $s_C/C = 1.18 s_\varphi/\sqrt{N}$ and requires $s_\varphi/\sqrt{N} \leq 8.2 \times 10^{-3} \text{ rad}$. One phase sample per cycle at the record's noise of about 1 rad over its 633 cycles gives $s_C/C = 4.7\%$, the size of the source's own 5% prefactor comparison (SRC-110). T-5 therefore takes about 23 times the record's cycles per run at that noise, or a correspondingly lower noise, and the four-point contrast of T-1 stays its held-out check. A residual in $\mathcal{D}_5$ beyond the criterion falsifies the shared gravitational-window model at the platform's precision; a residual in $\mathcal{D}'_5$ falsifies the calibrated twist energy. Neither arises from a chart label. This is the readout of the twist between neighbours, between patterns

whose E/E_0 and fall differ at one point (0.1c, I-3a, Use 7): the twist cancels in the weighted sum and the gravitational window in the difference, and both residuals are zero in FDT and in the source model.

6 Cross-domain consequences

The same action joins force, phase and clock readings. The force has its own I-1 dimensions and source evaluation; the integrated phase has the extra duration and the $\hbar$ bridge. A measured state-energy splitting also sets a mass difference and a clock frequency through Eq. (8): simultaneous readings of one configuration, not separately adjustable sources of phase.

The rest energy is the clock. The phase compares the two arms' own clocks at $\omega_{\text{Rb}} = 1.230 \times 10^{26} \text{ s}^{-1}$, the whole rest energy over $\hbar$. A hyperfine clock carried on an arm runs on the lapse and on its own rest energy where it sits (0.1c). Its kept-time reading is $\Delta E_{\text{hfs}} \Delta t_{\text{own}} / \hbar = 2.6 \times 10^{-14} \text{ rad}$ at the worked example, $\Delta E_{\text{hfs}} / E_0 = 3.49 \times 10^{-16}$ of the rest-energy count. In this twist the $m_f = 0$ transition energy changes along the ballistic path by $2\mathcal{M}_{\text{Rb}} a_{\text{SOZ}} z_{\text{lab}}$, which adds $4\mathcal{M}_{\text{Rb}} a_{\text{SOZ}} g_{\text{b}} \tau^3 / (3\hbar) = 3.1 \text{ rad}$, and its two states fall at $g_{\text{lab}} \pm a_{\text{SOZ}}$. The proper-time form and the internal-energy contribution to the action are the same term, and adding both would double-count that channel. The source's proposed clock extensions read such an internal clock (SRC-131, SRC-132); the present record already reads the rest-energy clock in full, against its 1.3% phase noise of about 1 rad (SRC-122). A clock pair carried on one arm stays on one path only when its two states' held and induced moments, summed per unit E , agree along $\partial_z B_{\text{mag}}$: the pair $f_{\text{hf}} = 1, m_f = 0$ and $f_{\text{hf}} = 2, m_f = 1$ does not, since one twist holds the second state and lets the first fall, and neither does the $m_f = 0$ pair.

The cloud in the medium window. The prepared cloud is a medium of the one omnium. For a stress tensor of perfect form with a conserved count and fixed entropy per count, its patterns are free on clocks and rulers rescaled by its enthalpy per count, and its expansion force is the bookkeeper's reading of that rescaling (0.1c, I-7). That clock factor is the reading of the cloud's flow, never the rate of a clock its atoms carry: an atom's clock runs on the lapse and on its own rest energy where it sits, and its thermal shift is the time dilation of its own thermal motion. The source's model corrections fall in the windows the ideal model omits: the trajectory through the curved field, 1–2 rad, is the twist's spring, the second derivatives of E_j ; the packet's shape, 1–2 rad and up to 1 rad in the main text, combines that spring with the cloud's own expansion; the interactions, up to 0.25 rad, are the medium window (SRC-097–SRC-100, SRC-180). They are source-model contributions, accepted with the record; the 3.3 nK effective temperature characterises the expansion (SRC-137).

Density equivalence. Let $\mathcal{O} > 0$ denote an equilibrium response, $x_i^{\text{ch}} > 0$ its independently specified channel inputs and S_i the logarithmic sensitivity with all other inputs fixed. For a differentiable response along a specified path, the I-7 channel accounting and the exact finite path integral read

$$S_i = \left(\frac{\partial \ln \mathcal{O}}{\partial \ln x_i^{\text{ch}}} \right)_{x_{j \neq i}^{\text{ch}}}, \quad \Delta \ln \mathcal{O} = \int_{\text{path}} \sum_i S_i d \ln x_i^{\text{ch}}. \quad (60)$$

An input sign is not an observable sign until these sensitivities and the co-varying controls are fixed. Core I-7's density-coupled equilibrium isotope discriminator belongs to such a response;

the present phase carries mass linearly through the action and the twist's response, and the experiment keeps one species. At fixed geometry a separately measured positive radiation-energy addition raises the source mass aspect; a thermal trajectory also changes support, internal energy and pressure, so its total direction does not follow from that partial input. Using the 3.3 nK expansion characterisation as a radiation energy density would change the observable dictionary and double-use the preparation information. Corroboration of I-7 is the once-only accounting of Sec. 1; its quantitative falsifier is an independently calibrated internal-energy or twist-energy contrast, not an unmeasured thermal or isotope trajectory.

7 System-level reading (exploratory)

The scope is the apparatus-level reading of the two arms in the record's own parameters; numerical examples in this section are in-sample redescrptions of that record. There is one exploratory observable, X-1.

X-1: kept periods and the twist's share in the record. Let $N_{\text{kept}} = \Delta t_{\text{own}}/T_{\text{Rb}}$ be the number of the pattern's own periods the ballistic arm gains and φ_{tw} the twist's share of the phase. On the record's parameters Eq. (26) gives

$$|\Delta\varphi_{\text{br}}| = 2\pi N_{\text{kept}} + \varphi_{\text{tw}}, \quad \frac{\varphi_{\text{tw}}}{|\Delta\varphi_{\text{br}}|} = \frac{2a_{\text{SOZ}}}{g_{\text{b}}} = 2.0\%, \quad N_{\text{kept}} = 12.07 \quad \text{at} \quad \tau = 1.2 \text{ ms.} \quad (61)$$

Three quantities of the record have one size: the twist's share (1.6 rad), the timing-convention spread (2.4 rad, Eq. (40)) and the reported model-data residual (approximately 2 rad, SRC-121). The comparator is the source's blind simulation with the same inputs, kept separate from its tuned comparison (SRC-120, SRC-124). The reading fixes the scale at which this record tests the gravitational window's universality: about 2.5% of the phase, the size of the helical share.

The apparatus supplies the reference's hold, the ballistic arm the kept-time path and the preparation the state response. I-1 supplies the weight, I-2 the single-medium reading, I-3 the shared source geometry and the lapse, I-4 the phase projection, I-5 the prepared closure, I-6 the geometric chart range and the apex excursion, and I-7 the energy entries. This links the system's controls without turning visibility or the residual into compactness.

Guards. The observable is exploratory and excluded from the prospective prediction count. Its numbers are in-sample redescrptions; its response corrections keep source-model provenance. No fitted α address or cross-chart transfer is introduced, and a distinct quantitative residual requires an additional source, constitutive response or changed observable dictionary with its independent calibration.

8 Convergence statement

FDT agrees with the measured record of general relativity and quantum mechanics without reservation. For this source the common stress tensor, boundary and initial data, gauge, state-response calibration and effective pulse dynamics give the same trajectory action and output populations. The convergence map is explicit: evaluate the Earth's A-1 exterior at the laboratory radius and take the source's g_{lab} , or the site value (U-08); write each arm's

action $\mathfrak{A}_j = - \int E_j dt_{\text{own}}$ on its composed clock factor, Eqs. (16) and (17); take the slow limit, Eq. (24), which is the source's Lagrangian with the twist's energy U_j ; match the measured pulse history; fix the arm orientation and phase origin; apply the same population readout. Within that declared family the finite-pulse reconstruction becomes the ideal phase, and the ideal phase becomes the kept-time count, under

$$\begin{aligned} (\tau_k, \tau_d) &\longrightarrow (0, 0), & a_{\text{mag}} &\longrightarrow g_{\text{lab}} : & \Delta\varphi_{\text{br}}^{\text{rec}} &\longrightarrow -C_\varphi\tau^3 = -\frac{\mathcal{M}_{\text{Rb}}g_{\text{b}}^2u_{\text{tot}}^3}{24\hbar}, \\ a_{\text{SOZ}} &\longrightarrow 0 : & & & \Delta\varphi_{\text{br}}^{\text{rec}} &\longrightarrow -2\pi\frac{\Delta t_{\text{own}}}{T_{\text{Rb}}} = -\frac{\mathcal{M}_{\text{Rb}}g_{\text{lab}}^2u_{\text{tot}}^3}{24\hbar}. \end{aligned} \quad (62)$$

At finite pulses Eq. (32) keeps every duration term. The comparison to the source's full dynamics keeps its reported approximately 2 rad residual and approximately 2.5% scale over approximately 80 rad (SRC-121): that is the source's model-data comparison, and α relabelling adds no residual to it. No divergence is claimed: FDT supplies here no additional source, constitutive input or changed observable dictionary. The record supports derivation of its displayed model and reproduction of its nominal arithmetic; the absent supplementary data and simulation code lie outside the evidence used here (SRC-182–SRC-186).

The source presents the unification of quantum theory and gravity as an open challenge and proposes later gravity-quantisation tests (SRC-011, SRC-134). FDT reads the experiment as one chain read twice: quantum mechanics' reading of a matter pattern's phase and general relativity's reading of the lapse that phase runs on, with the geometry carried by the one quantised carrier (0.1a, 0.3a, I-2). The source's future clock, nanodiamond and collapse proposals keep their prospective category (SRC-131–SRC-135). The measured phase is accepted in full and becomes no measurement of the microscopic spectrum, the floor or unrelated cosmic dynamics.

Corroboration of I-3 and I-4 is the recovered phase on this explicit map at the source's reported precision (SRC-121). The five prospective residuals distinguish failure of a shared calibrated model from a separately specified constitutive departure; each requires an observable, comparator and joint uncertainty beyond the present in-sample extraction.

9 Status ledger

The table below gives programme status and empirical scope. A held-out target is evaluated against independent evidence, never inserted as a result of its own model. Every prospective row uses the covariance and bounded-nuisance rejection rule of Eq. (47), with independent future bounds U-15.

Item	Status and scope
D-01	To be derived: the confinement operator D , its domains, boundary data and spectrum (core register O-9). The ^{87}Rb mass, the hyperfine splitting and the Zeeman structure are read from measurement (U-07, U-18). The proton/electron mass ratio and the squared muon/electron mass ratio stay held-out operator targets, with no benchmark assigned to this experiment.
D-02	To be derived: the map from a circulating charge to an I-1 input (core register O-6). The twist's push is read here from the pattern's rest energy, the helical window of 0.1c.
D-03	To be derived: the multipole and rotation remainders of the Earth's pair expression (core register O-5). The site value (U-08) carries them; the source's g_{lab} is a model value.
T-1	Prospective; single duration-polynomial channel; convergence $\mathcal{D}_1 = 0$. Independent phase samples and mass, acceleration and a_{SOZ} calibration are held-out inputs.
T-2	Prospective; single even acceleration channel with closure controlled; convergence $\mathcal{D}_2 = 0$.
T-3	Prospective; single timing-coefficient channel with nonzero kick; convergence $\mathcal{D}_3 = 0$. Independent waveform delay is the held-out target. Timing precision requires both denominators resolved away from zero over the joint inference region and negligible nonlinear terms; otherwise timing is unresolved or the polynomial relation is tested in a joint likelihood.
T-4	Prospective; measured co-varying-channel signs; convergence $\mathcal{D}_4 = 0$. Independent response calibration is the held-out comparator.
T-5	Prospective; the $m_f = 0$ pair; convergence $\mathcal{D}_5 = \mathcal{D}'_5 = 0$. The site gravimeter and the field map are the held-out comparators (U-08, U-19); resolution at $s_C/C < 0.97\%$ per run, about 23 times the record's cycles at its noise.
Phase record	Reported/in-sample: populations, envelope and cubic-phase extraction, blind-simulation residual and separate tuned comparison; SRC-073–SRC-126. No new fit or significance.
Arithmetic	Reconstructed: acquisition duration, squared calibration response, the three timing conventions and the displacements, Eqs. (37), (39), (40) and (41); the worked-example kept time, twist share and coefficient, Eqs. (27) and (31).
X-1	Exploratory, Sec. 7; one in-sample reading, excluded from the prospective count.

10 Conclusions

The experiment measures the phase between a ballistic arm and an Earth-fixed levitated reference of one rubidium pattern through output populations (SRC-012, SRC-018, SRC-091, SRC-092). Both arms are free patterns of one medium. The reference is held where the Earth's lapse times its own rest energy is stationary; the ballistic arm falls in the gravitational window and, through its induced moment, in the twist's helical window, at $g_{\text{lab}} + a_{\text{SOZ}}$, and the helical window returns that push from the source's bias field and the reference's levitation alone. The arms' action $-\int E dt_{\text{own}}$ over $\hbar$ yields the laboratory Lagrangian, source Equations 1–6 and the finite-pulse polynomial. In the ideal limit the phase counts the time the ballistic arm keeps in excess of the reference in the pattern's own period, twelve periods at the worked example, with the twist adding 2.0%.

The equality of inertial and gravitational mass is derived, the universality of free fall is the gravitational window's, and the second-order Zeeman fall is the helical window's. The record's agreement at 2.5% tests the composed fall, whose helical share has the same size, and the three printed timing conventions spread the largest example's phase by 3.0%. The tide between the arms sits at 1.8×10^{-12} of the phase. Test T-5 reads the two windows apart with the $m_f = 0$ pair: the twist cancels in the weighted sum of the falls and gravity in their difference. The convergence map recovers the source's model and data, and the experiment stands as a laboratory reading of the extended equivalence principle and of the helical window

together.

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