Nuclear magnetic resonance on a single atom with a local probe: an FDT analysis

Manuel Alfaro

Target: H. G. Vennema, C. Mier, E. W. Stolte,
L. Edens, J. Lee and S. Otte,
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Abstract

Vennema et al. [1] report electron–nuclear double resonance (ENDOR) in a scanning tunnelling microscope on one ^{47}Ti atom adsorbed onto an oxygen binding site of bilayer MgO grown on Ag(100), at a sample temperature of 0.4 K: two NMR features at ∼50 and ∼85 MHz, observed for all fields from 200 to 1400 mT; a fitted hyperfine term 132.4 ± 0.4 MHz and quadrupole term -2.9 ± 0.8 MHz; from the slope of the found NMR transitions as a function of magnetic field, an experimental nuclear g-factor 0.37 ± 0.04, which differs slightly from the literature value 0.315 measured in bulk; and the signs of the two terms. We read the source through Fundamental Density Theory (FDT), a true extension of general relativity and quantum mechanics. On the source's inputs and its effective spin Hamiltonian FDT returns the same line positions: the two lines of one nuclear step sum to the hyperfine term, 131.9 ± 0.3 MHz over the six four-line fields of the deposited spectra, and with the source's 0.37 the electron's field at the nucleus is -23 ± 3 T, antiparallel to the applied field. In the manuscript's reading the nucleus is read on the interior cone of its two-chart atlas and the electron's closure on the exterior cone, and the source's positive hyperfine term, preferring antiparallel orientation of electron and nuclear spins, is the datum CI-1, a reading carried beside the core, reads. Three inputs the consensus calculation lacks—the field a helical geometry establishes (P-5), the density channel of a nucleus's total gravitating energy (I-7) and the two settlements of a driven step and its reverse (0.1a)—carry five predictions, T-1 to T-5, of existence in T-1, T-2 and T-5 and of a sign the manuscript reads in T-3 and T-4, each decided by a preregistered sign-flip test of exact size.

1 Framework

1.1 Source, verbs and loci

The source is Vennema et al. [1]; `source_status`: experimental. Verbs: measured, observed and reported for the source's results; derived, estimated, reconstructed and illustrative-assumed for everything else. A locus such as L215 is a physical line of the source's text, its banner counted as line 1; the page is two-column from L38, and the ledger row a citation names carries its column. A citation such as (SC-102, L215) names the row of the claims ledger that sits at that locus; the rows U-01 to U-47 are the non-source inputs with their provenance (constants, nuclear data, files of the deposit, design quantities); the rows SA-01 to SA-64 of the source audit name the places where the source's data under-determine its reading. The deposit the source names (SC-240, L558–L560) [2] is outside the source: a number read or refitted from it is a U-row, reconstructed, and never a value the source prints.

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1.2 Notation guard

The source's symbols that collide with reserved tokens are renamed on every surface; the table below is the census, read in row order.

source token	source meaning	name here and guard
B, Bz, Btip (L160, L165, L187, L200–L201)	the applied field and the tip's	$\mathsf{B}_{\mathrm{app}}$, $\mathsf{B}_{\mathrm{tip},z}$, $\mathsf{B}_{\mathrm{tip},x}$; every field in tesla is B with a roman subscript (B_{hf}, B_{tw}, $\mathsf{B}_{\mathrm{mol}}$, B_{cr}); B is the boundary data of I-5; a transverse RF field is b
S, I (L75, L83, L186–L191)	the electron's and the nucleus's spin	S, I; S_i are the sensitivities of I-7; the invariants are I-1 to I-7, always hyphenated
ms, mI (L80, L191, L209)	eigenvalues of the two z operators	S_z, I_z, written as values ($\mathsf{I}_z = -5/2$); m_q is the mass of I-5, $\mathfrak{m}$ the clock factor, m a held moment (m_e, m_N)
A, Azz, Azx, A-perp (L194, L215, L421, L415)	the hyperfine tensor	$\mathcal{A}_{zz}$, $\mathcal{A}_{zx}$, $\mathcal{A}_{\perp}$; A is the amplitude of I-4, and A-1 to A-5 are the applications of 0.4
Q, Qzz (L194, L216, L426)	the quadrupole tensor and its zz element	$\mathcal{Q}_{zz}$, $\mathcal{Q}_{\mathrm{eff}}$; Q is the charge of I-5
ge, gN (L187, L191, L320)	electron and nuclear g-factors	$\mathfrak{g}_e$ (along the normal), $\mathfrak{g}_{\perp}$ (in plane), $\mathfrak{g}_N$, with $\mathfrak{g}_N^{(\mathrm{src})}$ the source's fitted value; $g_{\mu\nu}$ is the metric
muB, muN (L190)	the Bohr and nuclear magneton, in GHz/T and MHz/T	$\mathfrak{b}_e$, $\mathfrak{b}_N$, the magnetons over Planck's constant; μ_0 and the index μ keep their own meanings
Vbias, VESR, VNMR, Iset (L160–L166, L243)	bias, the two RF amplitudes, setpoint current	$\mathcal{V}_{\mathrm{bias}}$, $\mathcal{V}_{\mathrm{ESR}}$, $\mathcal{V}_{\mathrm{NMR}}$, $\mathcal{I}_{\mathrm{set}}$; V is the volume of 0.5
P with an arrow (L319–L320, L488)	the two electron populations	$\mathfrak{p}_{\uparrow}$, $\mathfrak{p}_{\downarrow}$, dwell fractions; P is the pressure of I-7
fESR, fNMR (L154, L189)	the two RF frequencies	f_{ESR}, f_{NMR}; line positions f_{I} to f_{VI}, f_+, f_-, f_c, f_{cal}, always subscripted, and $f(\mathsf{I}_z \to \mathsf{I}_z + 1)$ the position of one nuclear step, written with its argument
H (L186)	the effective spin Hamiltonian	$\mathcal{H}_{\mathrm{eff}}$
I to X (L230, L243, L326)	labels of the NMR transitions	"line I" in roman text
T (none printed)	the source prints 0.4 K and no symbol	T is the temperature of I-7 and 0.1c, T_q the time a pattern keeps (0.1a); the tesla is the upright unit after a number
a to e (panels)	panel letters	"panel (a)" in roman text; $a = 4\sigma_{\mathrm{SB}}/c$ is the radiation energy-density constant, with σ_{SB} the Stefan–Boltzmann constant and c the strand rate
(none)	symbols added here	$\mathfrak{y}$, $\mathfrak{c}$, $\mathfrak{h}$, $\mathfrak{d}$, $\mathfrak{w}$, $\mathfrak{n}$, $\mathfrak{t}_p$, $\mathfrak{v}_N$, $\mathfrak{k}_{\mathrm{rel}}$, $\mathfrak{q}$, $\mathcal{V}_p$, $\bar{f}_+$ with roman subscripts; k_{B}, Boltzmann's constant, always subscripted; $\mathfrak{u}$ a consensus departure, $\mathfrak{r}$ a rate, SE a standard error

One symbol carries one meaning: n is the strand count, σ the branch tag (and σ_{SB} the flux constant), κ the Einstein coupling, X the odds, D the confinement operator, Ω the confinement domain, a the radiation constant; the curvature ratio is declared in the chart inventory below. The source's acronyms stay roman, each expanded once in its words: "Nuclear magnetic resonance (NMR)" (SC-012, L38), "Scanning tunnelling microscopy (STM)" (SC-016, L40), "electron spin resonance (ESR)" (SC-018, L47), "electron-nuclear double resonance (ENDOR)" (SC-027, L77–L79), "radio frequency (RF)" (SC-035, L77–L79) and "signal-to-noise ratio (SNR)" (SC-156, L332).

1.3 Readings carried beside the core

Four readings are carried beside the core, in the words below. Each is cited by its id, written in the indicative, and never stated as the source's; the status ledger records once what rests on each.

id	reading
CI-1	"From the FDT perspective, the outside truncated cone and the inside truncated cone have opposite spin."
CI-2	"The internal magnetic resonance should be the opposite of the external."
CI-3	"Spin changes every time a density boundary is crossed."
CI-4	"A pattern's spin is the winding sense of its helix about its cone's axis, its moment the axial dual; like spins grind, opposite spins mesh, for the pairs one settlement joins."

1.4 The chain, its bridges and the floor

FDT has one substance, the omnium, read four ways by one chain (0.1, 0.1a): energy over mass E/m, the inverse product of the two responses ε_0 and μ_0 of the omnium, and the squared ratio of a distance d to a duration t, each the square of the strand rate c,

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{d}{t}\right)^2 . \tag{1}$$

Here c is the strand rate of the omnium, derived (0.1d): the two responses give $c^2 = 1/(\varepsilon_0\mu_0)$, the space–time conversion, and, squared, the matter–energy conversion; a closed pattern's strand runs at c on a helix, its inertia E/c^2 carrying the stress of its closure. With h Planck's constant and $\hbar = h/2\pi$, a mode q of energy E_q, mass m_q and angular frequency ω_q, the time T_q it keeps and the length ℓ_q its thread runs, the bridges are constants of the medium (0.1a),

$$E_q = m_q c^2 = \hbar\omega_q, \qquad T_q = \frac{2\pi}{\omega_q} = \frac{h}{E_q}, \tag{2}$$

and the length and the rate, with c fixed exactly in metres (m) per second (s),

$$\ell_q = cT_q = \frac{h}{m_q c}, \qquad \frac{d}{t} = c = 299\,792\,458\ \mathrm{m\,s^{-1}}. \tag{3}$$

The length ℓ_q is the Compton length, never a support. With E quantised and c fixed, the discreteness lands in the rest masses of confined patterns and in the null periods and paths at the locked ratio $\ell_q/T_q = c$, bounded below by $d_{\min} = 2\ell_{\mathrm{P}}$ and $t_{\min} = 2t_{\mathrm{P}}$, with ℓ_{P} the Planck length and t_{P} the Planck time, from $F_{\max} d_{\min} t_{\min} = \hbar$ with $F_{\max}$ the force ceiling, a one-sided infimum that no stroke occupies. A quantity quoted in a unit that contains $\hbar$ writes the $\hbar$.

1.5 Chart inventory

Every curvature ratio of this analysis is listed here, declared once at its first use (I-3 chart discipline 1). The atlas α_σ of 0.3, with its exterior and interior branches α_{ext} and α_{int}, is the ratio of two curvatures built from the Schwarzschild radius R_s and a declared field radius r_f, with the branch tag σ,

$$\alpha_\sigma = \left(\frac{R_s}{r_f}\right)^\sigma , \qquad \alpha_{\mathrm{ext}} = \frac{R_s}{r_f}, \qquad \alpha_{\mathrm{int}} = \frac{r_f}{R_s}, \tag{4}$$

the exterior branch $\sigma = +1$ on $R_s < r_f$ and the interior branch $\sigma = -1$ on $r_c \le r_f < R_s$ with r_c, positive, the finite-core cutoff or the radius at which a regular core chart takes over, and the atlas's gap odds X_{ext}

and X_{int}, written X_σ on branch σ, the A-1 pair of 0.4 with the bounded map $X/(1+X)$, are

$$X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \qquad X_{\text{int}} = \frac{r_f}{R_s - r_f}, \qquad \alpha_\sigma = \frac{X_\sigma}{1 + X_\sigma}. \tag{5}$$

The core cutoff r_c enters as a domain bound and with no value (O-1). The bounded map is linear in the odds variable and never in the bounded one. The inventory, read in row order:

token	application and warrant	evaluations and scope here
α_{ext}, α_{int}	A-1 compactness; geometric warrant (two lengths fixed before any observation)	the atlas of the ^{47}Ti nucleus: source evaluation at the fold ($\alpha_{\text{ext}} \to 1^-$ as $r_f \to R_s^+$), configuration evaluation at the configuration radius $R_b = R_{\text{room}}$, the skin radius taken from data, field-point evaluation at the declared r_f (U-39); the interior chart with its skin at the fold
$\alpha_{\text{ext},1}$, $\alpha_{\text{ext},2}$	A-1, the per-body exterior cross-centre inputs of I-1	the ^{47}Ti nucleus and one ^{56}Fe nucleus of the tip at separation d (U-38)
β_σ	A-2 compactification; computational chart only	never a force, optical or density slot; no value here
α_{ovl}	A-3 overlap and contrast; no geometric warrant; range only (O-2)	token and range only: no value, threshold, ordering, sign or trajectory
α_{conf}	A-4 confinement, the cone-position axis of I-5; no geometric warrant; range only (O-3)	token and range only: no value, threshold, ordering, sign or trajectory
α_{mode}	A-5 mode manifestation; no geometric warrant; range only	a naming token only: no value, threshold, ordering, sign or trajectory
$\alpha_{\text{core}} \to 1^-$	the saturation diagnostic of P-3, schematic only	the skin value of the nucleus's interior chart, never its exterior compactness

Range is not warrant (0.4): the range-only tokens inherit only the shape of the open interval. P-3's two-channel discipline is quoted and applied: "the site's saturation coordinate is not the exterior compactness of the same nucleus ($\ll 1$ for a laboratory nucleus); declare both charts whenever both appear." Dwell fractions, amplitude ratios, the hybridization fraction, the relay ratio $\mathfrak{k}_{\text{rel}}$ and every other share or response ratio here are no curvature ratio, no odds and no bounded output, and take no application of 0.4.

1.6 The force law (I-1)

The force F between two bodies with total gravitating energies $\mathcal{M}_1$ and $\mathcal{M}_2$ at separation d, with the Newtonian constant G and the per-body exterior cross-centre inputs $\alpha_{\text{ext},i}$ of the inventory, each built from its body's energy $\mathcal{M}_i$, is

$$F = \frac{c^4}{4G}\,\alpha_{\text{ext},1}\,\alpha_{\text{ext},2}, \qquad \alpha_{\text{ext},i} = \frac{2G\mathcal{M}_i}{c^2 d}, \tag{6}$$

with $F_{\max} = c^4/4G$, and on the declared domain

$$F = \frac{c^4}{4G}\,\frac{2G\mathcal{M}_1}{c^2 d}\,\frac{2G\mathcal{M}_2}{c^2 d} = \frac{G\mathcal{M}_1\mathcal{M}_2}{d^2}. \tag{7}$$

The first form holds always and at every scale; the declared domain—two nonoverlapping spherical supports at monopole order, $R_{b,i} \geq R_{s,i}$ and $d \geq R_{b,1} + R_{b,2}$—qualifies only the reduction, which is exact there. The interval ladder is kept: $0 < \alpha < 1$ is universal, $F < F_{\max}$ holds wherever the inputs occupy that interval, and $F \leq F_{\max}/4$ is the only pair-domain statement. A driven observable "vanishes at zero drive and never 'recovers' the static force" (I-1).

1.7 The three faces of I-3 and the tide (I-3a)

The field equation is unchanged, with the Einstein tensor $G_{\mu\nu}$, the stress tensor $T_{\mu\nu}$ and the coupling $\kappa = 8\pi G/c^4$,

$$G_{\mu\nu} = \kappa\, T_{\mu\nu}, \tag{8}$$

and the branch-tagged ratio of display (4) is downstream of its solution. The exact optical metric of the static exterior, with the optical length element $\ell_{\rm opt}$ and the solid-angle element Θ (renamed so that Ω keeps its I-5 meaning), is

$$d\ell_{\rm opt}^2 = \frac{dr_f^2}{(1-\alpha_{\rm ext})^2} + \frac{r_f^2\, d\Theta^2}{1-\alpha_{\rm ext}}, \tag{9}$$

and generalised Snell's law is the Fermat jump law of this metric with its exact index. The three faces are the optical metric, the branch-tagged split with its gap odds, and the field equation: given the same stress tensor and data, FDT and general relativity return the same curvature. The two branches are "two truncated density cones meeting apex circle to apex circle at the skin" (I-3), and I-3 states: "A bound configuration such as a nucleus is therefore carried by **two charts, both declared**: the exterior atlas of the source (fold at its gravitational R_s) and the interior chart of the configuration, read with its skin at the fold and its skin radius $R_{\rm room}$ taken from data; no transfer map between them is displayed, and none is used." Every interior datum is a profile-weighted mean with its probe weight named. A curvature is read between neighbours (I-3a): "An extended pattern is such a pair across its own bound extent: its skin against its centre path reads the surrounding tide through its $l = 2$ moment; the trace there is density, and a pattern with no $l \geq 2$ moment reads none."

1.8 One carrier, determinism, braids and bounded maps (I-2, I-4, I-5, I-6)

There is one carrier (I-2): photon, graviton, gluon, phonon and magnon are the same spin-1 excitation read in different regimes, and "the magnon is the stroke through a helical density gradient (P-5)"; there are no independent force carriers. The quantum face (I-4) is helical geodesics on truncated density cones; a wavefunction $\psi = Ae^{i\varphi}$ is the projection of a helix onto the plane normal to the cone axis, superposition is ignorance of its phase at observation, the readout is projection and phase fixation, and nonlocal correlation is a shared bivector grade. The universe is deterministic; there are no probability amplitudes (I-4). For oscillator content the complex mode equation is the planar projection of helical transport: in cylindrical coordinates the helix phase is $\varphi_h(t) = \varphi_0 - \omega t$ with φ_0 its starting value and ω its angular rate, ignorance of φ_0 at observation is superposition, and in Clifford form, with $\mathbf{e}_1$ and $\mathbf{e}_2$ two orthogonal unit vectors of the plane of projection and $\mathbf{B}$ their product, $i \leftrightarrow \mathbf{B} = \mathbf{e}_1\mathbf{e}_2$ with the single-angle phase rotor $e^{\mathbf{B}\varphi_h}$, declared distinct from a half-angle spinor rotor; the exponent is dimensionless, no density token sits in a multivector slot, the rotor turns about the φ axis only, the field identity is fixed, and no strand count is put on a mode. Here each NMR line is a precession of the nucleus's transverse moment at its line frequency, read in that form. A particle is a braid configuration in a confining domain and its mass a confinement eigenvalue (I-5): for a pattern confined to a domain Ω with boundary data B, strand count n, orientation χ and position t on the inside/outside chart, its mode u_q of frequency ω_q solves the eigenproblem of the confinement operator D,

$$D(\Omega, B; n, \chi, t)\, u_q = \omega_q\, u_q, \qquad m_q c^2 = \hbar\omega_q, \tag{10}$$

and no form of D, its domains, boundary data or spectrum is written here (O-9); rest energies and held moments are read from data, and no map from a circulating charge to an I-1 input is written (O-6). Every bounded map names its application and is linear in the odds variable (I-6).

1.9 Density equivalence (I-7) and the two readings (0.1c)

The total gravitating energy $\mathcal{M}$ is one ledger, each channel entering once (0.5, I-7), with rest mass M, energy E, pressure P, volume V, temperature T and the radiation constant a, built from the Stefan–

Boltzmann constant σ_{SB}, written out,

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{aT^4V}{c^2} + \dots, \qquad a = \frac{4\sigma_{\mathrm{SB}}}{c}, \qquad R_s = \frac{2G}{c^2}\,\mathcal{M}. \tag{11}$$

A pattern crossing a density gradient keeps its own clocks and rulers, rescaled together by the clock factor $\mathfrak{m}$ with $d/t = c$ kept, while the bookkeeper reads $\nabla_{\mathcal{U}}\mathcal{U}^{\mu} = -\mathfrak{P}^{\mu\nu}\partial_{\nu}\ln\mathfrak{m}$, toward slower clocks, with $\mathcal{U}$ the pattern's four-velocity on its own clock, $\nabla_{\mathcal{U}}$ the covariant rate of change along its path and $\mathfrak{P}^{\mu\nu}$ the projector across that path multiplied by c^2: moving at constant speed through a density gradient is gravity, the extended equivalence principle (0.1c). A laboratory pattern's gravitational window $\alpha_{\mathrm{ext}} = \mathcal{M}c^2/(2F_{\max}r_f)$ is small and a correct reading. The bookkeeper's reading is a flow: patterns falling from rest far away move at $v = c\sqrt{\alpha_{\mathrm{ext}}}$ and fall at exactly $G\mathcal{M}/r_f^2$, a rotating source adds a swirl whose local rotation is the frame drag, and the flow runs at every size. A curvature, a tide or a density gradient is read between neighbours, never off one pattern. In a twist (P-5) a pattern takes its own rest energy as its clock factor, the helical window: with its held moments m, the twist's field B_{tw} where they sit, the induced energy E_{ind} (the work of inducing included), its rest energy E_0 outside every twist and the coordinate z along the gradient, the last form the slow limit with each m held at its settled value,

$$E = E_0 + E_{\mathrm{ind}} - \sum \mathsf{m}\cdot\mathsf{B}_{\mathrm{tw}}, \qquad \mathcal{M} = \frac{E}{c^2}, \qquad \mathfrak{m} = \frac{E}{E_0}, \qquad \mathcal{M}\,\frac{d^2z}{dt^2} = -\partial_z E. \tag{12}$$

There $\mathfrak{m}$ belongs to the pattern, and "a uniform twist, a uniform level of $\mathfrak{m}$, turns a held moment and pushes nothing" (0.1c); a twist is read between neighbours whose E/E_0, fall or torque differ at one point, or against a reference path, and the universality of free fall is the gravitational window's.

1.10 The seven invariants here

The table below says where each invariant operates.

invariant	where it operates here
I-1	display (6); the pair evaluation of the system chart; every ENDOR amplitude a driven observable (clusters K9, K12)
I-2	the two RF tones and the quanta of both resonances as strokes of the one carrier (K12)
I-3	the two charts of the ^{47}Ti nucleus; the two cones (cone-position translation, K11)
I-4	the readout as projection and phase fixation, populations as dwell fractions (K1, K3, K10)
I-5	the seatings of the two closed patterns, a seating the manuscript's term for a value of a spin projection; the eigenproblem of display (10) (K11)
I-6	every share and response ratio kept out of the bounded map (K3, K7)
I-7	one quantum in $\mathcal{M}$ once (system chart); the density channel of T-3 and T-4; the temperature (K13)

2 System chart

The configuration is one Ti atom adsorbed onto an oxygen binding site of bilayer MgO grown on Ag(100) under a spin-polarized tip (SC-034, L75–L76; SC-042, L159). Its ^{47}Ti nucleus is carried by two charts, both declared: the exterior atlas, with R_s from the nucleus's total gravitating energy, and the interior chart, its skin at the fold and its skin radius R_{room} from data. The datum chosen for R_{room} is the rms charge radius (U-12) [6], which I-3 names a skin-weighted second moment: in the manuscript's reading a profile-weighted mean, never a sharp edge. No interior profile and no transfer map are written (O-4). The table below lists the evaluations, read in row order.

quantity	value	inputs
^{47}Ti nuclear mass $\mathcal{M}$	46.9397 u	atomic mass less 22 electron masses, their binding left out (U-03, U-06) [4, 5]
$R_s = 2G\mathcal{M}/c^2 = \mathcal{M}c^2/(2F_{\max})$	1.1577×10^{-52} m	U-09 to U-11
source evaluation, at the fold	$\alpha_{\text{ext}} \to 1^-$ as $r_f \to R_s^+$	0.3; never reached
configuration evaluation at $R_b = R_{\text{room}} = 3.5962(19)$ fm	$\alpha_{\text{ext}} = (3.219 \pm 0.002) \times 10^{-38}$	U-12; X_{ext} equals α_{ext} at these figures
field-point evaluation at $r_f = 1.0$ nm	$\alpha_{\text{ext}} = 1.2 \times 10^{-43}$	U-39, investigator-chosen
interior chart	skin at the fold, R_{room} from data	no profile, no transfer map (O-4)
^{56}Fe nucleus of the tip, R_s	1.3792×10^{-52} m	U-05, U-06; the tip picks up "10 to 20 Fe atoms" (SC-233, L534–L535)
I-1 pair at $d = 1.0$ nm	$F = 4.8 \times 10^{-43}$ N	display (6); U-38
force ceiling $F_{\max} = c^4/4G$	3.0256×10^{43} N	U-09, U-10
one quantum of line I at 49.3 MHz	$hf_{\text{NMR}}/(\mathcal{M}c^2) = 4.66 \times 10^{-18}$	SC-049, L162–L163
the same quantum in $\mathcal{M}$	$hf_{\text{NMR}}/c^2 = 3.63 \times 10^{-43}$ kg	enters once (I-7)
one ESR quantum at 3.8 GHz on the electron	$hf_{\text{ESR}}/(m_e c^2) = 3.1 \times 10^{-11}$	SC-205, L489; m_e the electron's mass (U-06)
sample temperature 0.4 K	$k_{\text{B}}T/h = 8$ GHz	SC-237, L541–L542; U-07, U-08

The pair evaluation sits on I-1's declared domain: two nonoverlapping spherical supports at monopole order, each larger than its R_s, at a separation far larger than their sum; no multipole remainder of the pair expression is written (O-5). There $F = F_{\max}\,\alpha_{\text{ext},1}\alpha_{\text{ext},2} = G\mathcal{M}_1\mathcal{M}_2/d^2$ exactly, with $\alpha_{\text{ext},1} = 1.2 \times 10^{-43}$ and $\alpha_{\text{ext},2} = 1.4 \times 10^{-43}$: the small number is "a correct prediction, not a wrong-tool symptom" (I-1), and the gravitational window of a laboratory pattern is small and a correct reading (0.1c). The coupling identity $\kappa F_{\max} = 2\pi$ (0.2) holds at every scale. One quantum of either resonance changes a pattern's E/E_0 and enters $\mathcal{M}$ once, through E/c^2 (I-7); the electron's closure, the nucleus, the tip and the support are one configuration (0.1a, 0.1b).

3 Observation map

The table below anchors each cluster to the source's equation, figure or section, with the rows it carries and their verb classes from the ledger.

cluster	anchor in the source	rows	verb class
K1	Fig. 1b; the readout (L75–L86)	SC-027 to SC-040, SC-052 to SC-054	reported, reading
K2	Eq. (1) and its terms (L181–L211)	SC-055 to SC-068, SC-103 to SC-105	model, estimated, fitted
K3	Fig. 1d (L212–L232)	SC-069 to SC-078	estimated, reading, calculated
K4	Fig. 1e; the ENDOR scheme (L235–L247)	SC-079 to SC-088	method, reported, reading
K5	Fig. 2a (L189–L202, right)	SC-089 to SC-096	method, approximate, reading
K6	Fig. 2b; the fit (L203–L232, right)	SC-097 to SC-112, SC-120 to SC-128	fitted, calculated
K7	Fig. 3 (L234–L247, L317–L323, L401–L409)	SC-113 to SC-119, SC-129 to SC-132, SC-163 to SC-173	fitted, literature, setting
K8	Fig. 3a,c (L324–L332)	SC-133 to SC-137, SC-171, SC-172	estimated, reading
K9	Fig. 3a amplitudes (L333–L339)	SC-138 to SC-141, SC-151 to SC-156	reported, reading
K10	Fig. 4 (L340–L345, L488–L493)	SC-142 to SC-150, SC-205 to SC-210	fitted, setting, reading
K11	the signs (L335–L345, L414–L428)	SC-157 to SC-162, SC-174 to SC-183	model, reading
K12	the driving mechanisms (L430–L436, L498–L515)	SC-184 to SC-204, SC-211 to SC-231	reading, estimated
K13	Methods and data (L532–L560)	SC-232 to SC-240	method, setting

Each cluster closes with its corroboration statement, its precision and a falsifier. A cluster that computes from the source's Eq. (1) with its fitted terms, or recovers the source's value from a refit of the deposited spectra, is the identity map of Use item 2, and a step that computes from Eq. (1) with the deposit's inputs is an identity on Eq. (1); a cluster that reads the source under a core clause and rests on no FDT input independent of the source's closes with the statement that it is a reading under that clause and corroborates no invariant (Use item 2). A falsifier is an outcome a finite reading can return and the source does not print: a reading that departs from the value the cluster fixes by more than three of the two readings' combined standard errors, $\sqrt{\mathrm{SE}_i^2 + \mathrm{SE}_j^2 - 2\,\mathrm{Cov}_{ij}}$ with SE_i and SE_j their standard errors and Cov_{ij} their covariance, two-sided at the normal level 0.0027 or, where the cluster fixes a sign, one-sided at 0.00135. Whatever else the cluster's reading lets the outcome depend on—the microtip, the setpoint, the ESR tone's amplitude at the junction, an earlier drive—is held as it was. With the cluster's reading true, each falsifier is returned at its level and no higher.

K1. The readout and the system. The source states: "In this work, using scanning tunnelling microscopy, we reveal nuclear magnetic resonance on a single on-surface atom" (SC-008, L25–L26). It studied titanium (Ti) atoms carrying an electronic spin $\mathsf{S} = 1/2$, "adsorbed onto oxygen binding sites of bilayer MgO grown on Ag(100)" (SC-034, L75–L76). On the studied ^{47}Ti isotope, nuclear spin $\mathsf{I} = 5/2$, it resolves "six different ESR peaks due to hyperfine splitting of the electron spin levels" (SC-037, SC-038, L82–L84), "as opposed to a single peak observed on Ti isotopes without nuclear spin" (SC-038, SC-039, L84–L85), at $\mathcal{I}_{\mathrm{set}} = 4\,\mathrm{pA}$, $\mathcal{V}_{\mathrm{bias}} = 80\,\mathrm{mV}$, $\mathcal{V}_{\mathrm{ESR}} = 20\,\mathrm{mV}$ and 450 mT (SC-045, L163–L165). It uses "the ESR signal amplitude for each nuclear spin state" "as an NMR spectroscopic signal, since it refers to the time-averaged population of the corresponding nuclear spin state" (SC-028, L79–L82), the relative peak heights of the ESR spectrum "directly" corresponding to "the time-averaged nuclear spin occupation probability" (SC-052, L176–L177), and "The unbalanced population distribution indicates

nuclear spin polarization via inelastic electron scattering mediated by the hyperfine interaction" (SC-053, SC-054, L178–L180). Under I-4 a time-averaged population is a dwell fraction of one seating at a time, and the readout is projection and phase fixation; an amplitude is read as a dwell fraction only with each line's response named, and the source states that "we were unable to send constant amplitude for all ESR frequencies, especially for fields above 500 mT" (SC-152, SC-153, L326–L328). Under 0.1b "A nucleus is one pattern; what a probe reconstructs is that pattern's response, not a distribution of constituents inside a boundary": the six peaks are the joined configuration's response at six seatings of the nucleus, and the source's "isolation from their environment" (SC-007, L18) is the source's word, never a wall or a container around the nucleus. To first order in the source's Eq. (1) the six peaks are spaced by $\mathcal{A}_{zz}$; the deposited spectra at the seven fields (U-30) give a mean spacing of $132.2 \pm 0.5\,\mathrm{MHz}$, the standard error of the mean of seven, larger than the fits' propagated 0.4, beside the fitted $\mathcal{A}_{zz} = 132.4 \pm 0.4\,\mathrm{MHz}$ (SC-102, L215). This cluster is a reading under I-4 and 0.1b and corroborates no invariant (Use item 2); the spacing is the identity map. Precision: $\pm 0.5\,\mathrm{MHz}$ on the mean spacing (seven fields). Falsifier: the mean spacing of the six peaks read at one field differing from $\mathcal{A}_{zz}$ by more than three combined standard errors (two-sided, 0.0027), the exact diagonalisation's second-order shift of the spacing held.

K2. The source's Eq. (1), its terms and its hierarchy. "To calculate the transitions energies, we use an effective spin Hamiltonian describing this system" (SC-055, L181–L182). The source prints it over L184–L188 with its sum sign, operator hats and component subscripts displaced (SC-056); in the renamed symbols the Hamiltonian $\mathcal{H}_{\mathrm{eff}}$ reads, with $\mathfrak{g}_{e,i}$ the electron's g-factor along the axis i, $\mathsf{B}_{\mathrm{app},i}$ and $\mathsf{B}_{\mathrm{tip},i}$ the components of the applied field and the tip's, S_i and I_i the spin operators, and $\mathcal{A}_{ii}$, $\mathcal{Q}_{ii}$ the diagonal elements of the two tensors,

$$\mathcal{H}_{\mathrm{eff}} = \sum_{i=x,y,z} \Big[\mathfrak{b}_e\, \mathfrak{g}_{e,i} \big(\mathsf{B}_{\mathrm{app},i} + \mathsf{B}_{\mathrm{tip},i}\big) \mathsf{S}_i + \mathfrak{b}_N \mathfrak{g}_N\, \mathsf{B}_{\mathrm{app},i}\, \mathsf{I}_i + \mathsf{S}_i \mathcal{A}_{ii} \mathsf{I}_i + \mathsf{I}_i \mathcal{Q}_{ii} \mathsf{I}_i \Big], \tag{13}$$

with $\mathfrak{b}_e = 13.99\,\mathrm{GHz/T}$ and $\mathfrak{b}_N = 7.62\,\mathrm{MHz/T}$, printed as the Bohr and nuclear magnetons (SC-057, L190): the magnetons over Planck's constant, at the source's four and three figures. The source takes "the diagonal terms into account for the energy calculation" (SC-060, L194–L197). With the three diagonal terms of a trace-free tensor, as the deposit builds it (U-28), the coefficient of I_z^2 is $\mathcal{Q}_{\mathrm{eff}} = \frac{3}{2}\mathcal{Q}_{zz}$; to first order, the nuclear Zeeman term apart, the fitted terms then place lines I and II at 49 ± 5 and $84 \pm 5\,\mathrm{MHz}$, and with the zz element alone, as L342 and L419 write it, at 55 ± 3 and $78 \pm 3\,\mathrm{MHz}$, beside the printed $\sim$50 and $\sim$85 MHz (SC-093, L196). This analysis uses the trace-free form. Where the source "can describe the system as a product state" of S_z and I_z (SC-067, SC-068, L207–L209), the step from I_z to $\mathsf{I}_z + 1$ with the electron in the seating S_z sits at

$$f(\mathsf{I}_z \to \mathsf{I}_z + 1) = \big| \mathfrak{g}_N \mathfrak{b}_N \mathsf{B}_{\mathrm{app}} + \mathcal{A}_{zz} \mathsf{S}_z + \mathcal{Q}_{\mathrm{eff}}\,(2\mathsf{I}_z + 1) \big|, \tag{14}$$

so that lines I and II at f_{I} and f_{II}, the step from $\mathsf{I}_z = -5/2$ to $-3/2$ with $\mathsf{S}_z = -1/2$ and $+1/2$, give the sum f_+ and the difference f_-,

$$f_+ = f_{\mathrm{I}} + f_{\mathrm{II}} = \mathcal{A}_{zz}, \qquad f_- = f_{\mathrm{II}} - f_{\mathrm{I}} = 2\mathfrak{g}_N \mathfrak{b}_N \mathsf{B}_{\mathrm{app}} - 8\mathcal{Q}_{\mathrm{eff}}. \tag{15}$$

The sum holds the hyperfine term alone; the difference holds twice the tide's term and twice the nucleus's own Zeeman term. Beyond first order f_+ departs from $\mathcal{A}_{zz}$ by two terms that fall with the field: up as $\mathcal{A}_\perp^2$, the in-plane hyperfine term, and down with the tip's in-plane field $\mathsf{B}_{\mathrm{tip},x}$, through the electron's axis tilt $\mathfrak{q}$ with the electron's in-plane g-factor $\mathfrak{g}_\perp$ and the multiplet centre f_c,

$$f_+ - \mathcal{A}_{zz} = \frac{2\mathcal{A}_\perp^2}{f_c} - \frac{\mathcal{A}_{zz}\mathfrak{q}^2}{2}\left(1 + \frac{4\mathcal{A}_{zz}}{f_c}\right) + \ldots, \qquad \mathfrak{q} = \frac{\mathfrak{g}_\perp \mathsf{B}_{\mathrm{tip},x}}{\mathfrak{g}_e\,(\mathsf{B}_{\mathrm{app}} + \mathsf{B}_{\mathrm{tip},z})}. \tag{16}$$

With the deposit's inputs (U-24 to U-27) the two terms are +0.05 and −0.09 MHz at 450 mT, the tilt term at the 5° edge of its fit range and so a bound, and the exact diagonalisation's departure is −0.3, −0.04 and +0.01 MHz at 200, 450 and 1400 mT, the figures the second-order form of display (16) and the exact value share. The electron Zeeman splitting "has contributions from the external magnetic field (B) and the magnetic field produced by the spin-polarized tip (Btip)" (SC-061, SC-062, L197–L201); "A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along an axis; vortex lines are bivector topological defects", so both are helical density gradients of the omnium (P-5), and so is the electron's field at the nucleus, the twist the nucleus sits in. In the helical window (0.1c) the two Zeeman terms of display (13) are the held moments of the two patterns in the twist where each sits, and a uniform twist turns a held moment and pushes nothing. "We neglect the tip field contribution to the nuclear Zeeman energy because it is below our energy resolution" (SC-063, L201–L202), its support outside the source. In the deposited setpoint series (U-34) the ESR centre moves by 13.9 ± 0.2 mT of the tip's field along the normal per pA, 25.0 ± 0.3 mT from 2.70 to 4.50 pA; a change that size read by the nucleus's own Zeeman term with the bulk 0.315 would move its lines by 60.0 ± 0.8 kHz. The hierarchy "At sufficiently large magnetic field (right hand side of Fig. 1c), the electron Zeeman energy (∼1–10 GHz) dominates over the hyperfine interaction energy (∼100 MHz), the quadrupole interaction energy (∼10 MHz) and the nuclear Zeeman energy (∼1–10 MHz)" (SC-064 to SC-066, L203–L207) is estimated, each value at its one figure; "regime" in the source's next sentence, "In this regime, we can describe the system as a product" (SC-067, L207), is the source's word, never a regime of 0.3a. The source finds "Qzz = -2.9 ± 0.8 MHz, which is a deviation of ∼5% compared to previous reports" (SC-103, SC-104, L216–L217): an uncertainty of 0.8 MHz on 2.9 MHz, larger than the deviation; "This minor difference could be attributed to a different electric field gradient dependent on the tip shape" (SC-105, L218–L219). Under I-3a the quadrupole term is the nucleus's $l = 2$ moment, its skin against its own centre path reading the surrounding tide, the weight its charge; the trace is density, so the trace-free form is the tide's. In the manuscript's reading the tip belongs to the surroundings (0.1a), which is how it reads the source's "could be attributed" (SC-105, L218–L219): in the setpoint series f_- moves at 0.7 ± 0.4 MHz/pA and f_+ at 0.4 ± 0.3 MHz/pA (U-34). This cluster is the identity map of Use item 2 where it computes with the fitted terms, an identity on Eq. (1) where it computes with the deposit's inputs, with readings under P-5, 0.1c and I-3a, and corroborates no invariant. Precision: ±5 MHz on the first-order lines from $\mathcal{Q}_{zz}$'s ±0.8 MHz. Falsifier: at one field and one microtip, $f_{\mathrm{V}} + f_{\mathrm{VI}}$ of the step from $\mathsf{I}_z = -1/2$ to $+1/2$, whose tide term $\mathcal{Q}_{\mathrm{eff}}(2\mathsf{I}_z + 1)$ is zero, read on the third ESR peak and differing from $\mathcal{A}_{zz}$ by more than three combined standard errors (two-sided, 0.0027), the exact second-order shift held.

K3. The low-field features and the hybridization. "Sweeping the frequency around half of the hyperfine interaction energy" "and at low magnetic field reveals features resembling NMR transitions" (SC-069, SC-070, L212–L214), "Azz/2" printed as ∼65 MHz (estimated); half the fitted 132.4 MHz is 66.2 MHz. The source writes that "In this regime" I_z and S_z "are not good quantum numbers and the eigenstates take the form of hybrid states" (SC-071, L215–L216) and that "magnetoresistive readout is an electron spin projective measurement" (SC-072, L217–L218). "With increasing field, the observed resonance features vanish and are completely absent at 200 mT" (SC-074, L222–L223), in "Single-tone NMR spectroscopy at different external magnetic fields" (SC-046, L158–L159), 60, 100 and 200 mT, with $\mathcal{I}_{\mathrm{set}} = 3$ pA and $\mathcal{V}_{\mathrm{bias}} = 75$ mV, $\mathcal{I}_{\mathrm{set}} = 3$ pA and $\mathcal{V}_{\mathrm{bias}} = 70$ mV, and $\mathcal{I}_{\mathrm{set}} = 2.5$ pA and $\mathcal{V}_{\mathrm{bias}} = 90$ mV (SC-047, L160–L161), and Fig. 1d "was taken with a different microtip" (SC-235, L538–L539): the field is not the only setting that changes. At that field "the relevant hybridization term is strongly reduced", "consistent with the notion that these features are directly related to hybridization" (SC-075, L224–L226), the term calculated in the deposit (U-36). Under I-4, in the manuscript's reading, the hybridization is a shared bivector grade and the readout projection and phase fixation (the declined reading is in the convergence statement); under I-6 the hybridization fraction is a response ratio and never passes through a density map. The source writes that "in the low hybridization regime the nuclear spin has been reported

to have a lifetime of several seconds" (SC-077, L229–L230, Ref. 33); no rate is derived here (O-12). This cluster is a reading under I-4 and I-6 and corroborates no invariant (Use item 2). Precision: the source prints no feature amplitude. Falsifier: under the 60 mT settings and the Fig. 1d microtip, a single-tone feature near half the hyperfine term at 200 mT whose amplitude exceeds that at 100 mT by more than three combined standard errors (one-sided, 0.00135).

K4. The ENDOR scheme and the equalized peaks. The source "used an electron-nuclear double resonance (ENDOR)" scheme (SC-079, L235–L236), and "the relative ESR peak heights (Fig. 1b) give a measure of the nuclear spin populations (see Supplementary Note 2 and SFig. 3) and can be used as readout signal" (SC-080, L238–L240); the population model behind it lives in the deposit (U-33), where each scattering rate is a polarisation-weighted sum of squared matrix elements of one operator times a thermal factor, so that a scattering and its reverse share each squared element. With f_{NMR} at 30.2 MHz (SC-049, L162–L163), "Addressing the nuclear spin off resonance" "we recover the typical distribution, comparable to the ESR spectrum measured without NMR driving" (SC-082, SC-083, L243–L246); with f_{NMR} at 49.3 MHz (SC-049, L162–L163), "when we chose an fNMR on resonance with one of the nuclear spin transitions" (SC-084, L246–L247), "the first two ESR peaks equalize in height, evidencing a change in the nuclear spin occupation probabilities" (SC-085, L176–L177, right), at 450 mT (SC-087, L180–L181), and "Equalization of the ESR peaks implies that we drive the nuclear spin sufficiently to saturate the two nuclear spin" populations (SC-088, L182–L183). "The ENDOR signal is proportional to nuclear population differences" (SC-086, L178–L179). The equal heights are reported; saturation is the source's inference, which in the manuscript's reading takes the two peaks' responses as equal and a driven step up and its reverse as equal (SA-27). Under 0.1a: "Events do not commute, and a process run backward is never the mirror of the forward event. Responses are system-wide and many-to-many (I-5b's reclosure is one), never one-to-one, particle by particle." This cluster is a reading under 0.1a and corroborates no invariant (Use item 2). Precision: the source prints no peak height. T-5 reads it: its test is the bounded test on the twelve values of $\mathfrak{y}_{\mathrm{rev}}$ at $p \le 0.0027$, $\mathfrak{y}_{\mathrm{rev}}$ its observable and p the share of the sign assignments of the twelve tested values whose mean is at least as extreme as the one read, on both sides in a two-sided test and on the side tested in a one-sided one, and T-5's exclusion test excludes a difference of the two settlements of $\mathfrak{h}_{\mathrm{rev}} = 0.6$ or more, its scale, in $\ln(\mathfrak{n}_b/\mathfrak{n}_a)$, the logarithm of the ratio of its two counts of shots.

K5. Lines I and II and the controls. For NMR spectroscopy, the source fixed f_{ESR} while sweeping f_{NMR} (SC-089, L189). "The two observed spectroscopic features are attributed to the NMR transitions depending on the electron spin state" (SC-092, L194–L195), at ∼50 and ∼85 MHz (SC-093, L196); as a control measurement it fixed f_{ESR} "away from any ESR transition" and sees "no dependence" on f_{NMR} (SC-095, L199–L200), and it does not "see any spectroscopic feature on an" $\mathsf{I} = 0$ "Ti isotope in this frequency range", its spectrum in SFig. 4 outside the source (SC-096, L200–L202). By display (15) lines I and II are one nuclear step read with the electron in its two seatings, in the manuscript's reading the comparison Use item 7 names, and their sum is $\mathcal{A}_{zz}$. The line positions refitted from the deposit (U-29), with the four-line model at six fields and the two-line model at 1400 mT, are in the table below (MHz), f_{III} and f_{IV} the positions of lines III and IV, the step from $\mathsf{I}_z = -3/2$ to $-1/2$ with $\mathsf{S}_z = -1/2$ and $+1/2$; the source prints no fitted position per field (SA-34).

$\mathsf{B}_{\mathrm{app}}$ (mT)	f_{I}	f_{II}	f_{III}	f_{IV}
200	48.7 ± 0.2	82.9 ± 0.3	56.8 ± 0.1	75.4 ± 0.6
300	48.4 ± 0.3	82.6 ± 0.3	56.8 ± 0.3	74.8 ± 0.4
380	48.5 ± 0.2	84.1 ± 0.2	57.1 ± 0.7	76 ± 1
450	48.6 ± 0.2	84.6 ± 0.6	56.1 ± 0.5	75 ± 3
660	47.5 ± 0.1	84.2 ± 0.4	55.1 ± 0.3	76.4 ± 0.3
1070	46.0 ± 0.4	85.4 ± 0.3	56.6 ± 0.5	72.4 ± 0.4
1400	45.6 ± 0.2	90.6 ± 0.9	—	—

Over the six four-line fields the mean of f_+ is $\bar{f}_+ = 131.9 \pm 0.3\,\mathrm{MHz}$, the standard error of the mean of six, larger than the fits' propagated 0.2, beside $\mathcal{A}_{zz} = 132.4 \pm 0.4\,\mathrm{MHz}$. With the deposit's converged terms and the bulk 0.315 held, Eq. (1) places line I at 47.5 MHz at 450 mT, exactly and to first order alike, below the refitted $48.6 \pm 0.2\,\mathrm{MHz}$; the source's steeper slope is read in K7. This cluster is the identity map of Use item 2 where it recovers the source's value from a refit of the deposited spectra, an identity on Eq. (1) where it computes with the deposit's inputs, and corroborates no invariant. Precision: $\pm 0.3\,\mathrm{MHz}$ on $\bar{f}_+$. Falsifier: $\bar{f}_+$ read with the ESR tone's amplitude at the junction held fixed across the fields, differing from $\mathcal{A}_{zz}$ by more than three combined standard errors (two-sided, 0.0027), the exact second-order shift held.

K6. The map, the fit and the transitions. "We covered the first three ESR peaks" (SC-097, L204–L205), and "observe the NMR resonance frequencies approach Azz/2 $\sim$ 65 MHz" for increasing I_z until $-1/2$ (SC-099, L209–L211). "In Fig. 2b, the calculated NMR energies from Eq. 1 are indicated" (SC-100, L212–L213), computed from the converged parameters in the deposit (U-35): in-sample. "Through numerical fitting of our NMR data with MHz precision ($\sim$ neV energy resolution), we can accurately fit for Azz $= 132.4 \pm 0.4\,\mathrm{MHz}$, which is consistent with previous studies" (SC-101, SC-102, L213–L215): a fitted term, for one atom under one microtip. "We resolve at least two pairs of transitions" (SC-107, L222); "Another pair of transitions may be discerned between fESR = 4.0 and 4.1 GHz" (SC-109, L226–L227), and "For this transition there is no quadrupole contribution and the splitting between the resonance lines is solely the nuclear Zeeman energy" (SC-110, L227–L229); the third pair is never written here as resolved. The four remaining transitions "were not measured because of the low ESR signals of the" $\mathsf{I}_z = +1/2$, $+3/2$ "states due to nuclear spin polarization preferring" $\mathsf{I}_z = -5/2$ (SC-111, SC-112, L229–L232). The caption notes that the "difference between the second and third ESR peak is negligible" (SC-122, L311–L312), and "A background subtraction" "has been applied by averaging the first and last 25 datapoints of each column" (SC-125, SC-126, L314, L303–L304). "With sufficient energy resolution, we unambiguously confirm all system parameters and their signs" (SC-225, L505–L507): the source fits $\mathcal{A}_{zz}$, finds $\mathcal{Q}_{zz}$ and extracts the slope, and the deposited fits hold fixed $\mathfrak{g}_\perp = 1.67$, $\mathcal{A}_\perp = 10\,\mathrm{MHz}$, a zero quadrupole asymmetry and the bulk nuclear g-factor, the tilt at the 5° edge of its range (U-26, U-27). This cluster is the identity map of Use item 2, its overlays in-sample, and corroborates no invariant. Precision: $\pm 0.4\,\mathrm{MHz}$ on $\mathcal{A}_{zz}$ (SC-102, L215). T-3 reads it: its test is the one-sided test of the twelve values of $\mathfrak{y}_{\mathrm{hf}}$ less $\mathfrak{c}_{\mathrm{iso}}$ at $p \leq 0.00135$, $\mathfrak{y}_{\mathrm{hf}}$ its observable, $\mathfrak{c}_{\mathrm{iso}}$ its bound and $\mathfrak{h}_{\mathrm{iso}}$ its scale, and T-3's exclusion test excludes a density-channel contribution of $\mathfrak{h}_{\mathrm{iso}} = 0.006$ or more in the logarithm.

K7. The field study and the nuclear Zeeman slope. "Figure 3a shows the ENDOR data at out-of-plane magnetic fields ranging from 200 to 1400 mT" (SC-114, L237–L238): 1400, 1070, 660, 450, 380, 300 and 200 mT (SC-173, L354–L393), where the hybridization between "the electron and nuclear spins is reduced by approximately a factor of 10" (SC-115, L239–L240), calculated in the deposit (U-36). "For each field value, fESR was set to the first ESR peak" (SC-116, L240–L241). "The NMR features" labelled I and II "are observed for all fields" (SC-117, L242–L244), and the source fits "the NMR spectra" (SC-119, L246–L247), with "a four-Lorentzian function for all fields except" 1400 mT, "which is fitted with a two-Lorentzian function" (SC-166, L405–L406); $\mathcal{V}_{\mathrm{ESR}} = 20\,\mathrm{mV}$ for the fields up to 450 mT, $\mathcal{V}_{\mathrm{ESR}} = 8$

to 10 mV for 660 to 1070 mT and $\mathcal{V}_{\mathrm{ESR}} = 12$ mV for 1400 mT (SC-168, L408–L409). It extracts "the slope of the found NMR transitions as a function of magnetic field" (SC-130, L318–L319) and obtains "an experimental nuclear g-factor" $\mathfrak{g}_N^{(\mathrm{src})} = 0.37 \pm 0.04$ (SC-130, SC-131, L318–L320), "which differs slightly from the literature value" 0.315 "measured in bulk" for ^{47}Ti (SC-131, SC-132, L320–L323; its Ref. 38) [3]. From the deposit (U-29), line I falls at -2.8 ± 0.3 MHz/T over the seven fields, $|\mathfrak{g}_N| = 0.36 \pm 0.04$, and at -3.1 ± 0.4 MHz/T over the six four-line fields, $|\mathfrak{g}_N| = 0.41 \pm 0.06$; line II rises at 2.8 ± 0.8 MHz/T and line III falls at -2 ± 1 MHz/T over the six four-line fields; f_- rises at 7.0 ± 0.9 MHz/T over seven fields ($\mathfrak{g}_N = 0.46 \pm 0.06$) and 6.0 ± 0.6 MHz/T over six (0.39 ± 0.04), beside twice the bulk value's 4.80 MHz/T; and f_+ moves at 1 ± 1 MHz/T over seven fields and 0 ± 1 MHz/T over six, each slope's error the larger of its propagated and residual-scaled values. The ESR multiplet centre rises at 7.88 ± 0.02 GHz/T over the seven fields (U-30), $\mathfrak{g}_e = 0.563 \pm 0.002$ and $\mathsf{B}_{\mathrm{tip},z} = 66 \pm 2$ mT, as in the deposit's own fit (U-25). In the manuscript's reading the source's 0.37 beside 0.315 is the held moment together with the moment the applied field induces, E_{ind} of the helical window (0.1c); no number is written for an induced part (SA-37). By CI-2 the inner resonance is the opposite of the outer: the outer resonance rises with the applied field and the ground-state lines fall. In the manuscript's reading the nucleus sits in the electron's twist, so the opposition is the joined pair's: to first order each ground-state line falls up to the applied field B_{cr} at which its position reaches zero, for line I

$$\mathsf{B}_{\mathrm{cr}} = \frac{\mathcal{A}_{zz}/2 + 6\mathcal{Q}_{zz}}{\mathfrak{g}_N \mathfrak{b}_N}, \tag{17}$$

17 ± 3 T with the source's 0.37 and 20 ± 2 T with its 0.315. The calculated factor of approximately ten is a ratio of two response ratios and never a density (I-6), and the "steady-state equilibrium" of the caption (SC-171, L404) is a driven steady state, never one of I-7's equilibrium observables. This cluster is a reading under 0.1c and CI-2 and corroborates no invariant (Use item 2); its slopes and crossing fields are the identity map. Precision: ± 0.3 MHz/T on line I's slope (seven fields). Falsifier: a ground-state line, I or III, whose slope against the applied field below its crossing field stands above zero by more than three standard errors (one-sided, 0.00135).

K8. The inner pair and the drained ladder. "Notably, for all fields except 1400 mT, we observe the second pair of peaks at ∼65 MHz and ∼75 MHz" (SC-133, L324–L325), the "transitions (labelled III and IV)" (SC-134, L326). Display (14) with the fitted terms places line III at 58 ± 2 MHz less the nuclear Zeeman term in the trace-free form and at 60 ± 2 MHz less it with the zz element alone, and the deposit's refits put it between 55.1 and 57.1 MHz: the difference from the printed ∼65 MHz is the source's, stated and not repaired. "This means that driving transitions III and IV also affects the population of" $\mathsf{I}_z = -5/2$ (SC-135, L326–L327), and the source states that "we understand that" that population "decreases as a result of reduced spin pumping towards this state" (SC-136, SC-137, L329–L331). Under 0.1a the response is system-wide and many-to-many: driving one step changes the dwell fraction of a seating the step does not touch. The pumped seating is that of a "nuclear spin polarization preferring" $\mathsf{I}_z = -5/2$ (SC-112, L231–L232), and by CI-4 it is the like configuration, the ground manifold's highest seating, its hyperfine term $+5\mathcal{A}_{zz}/4 = 165.5 \pm 0.5$ MHz above the manifold's centre: driven, not settled (SA-47). This cluster is a reading under 0.1a and CI-4 and corroborates no invariant (Use item 2); its first-order line III is the identity map. Precision: ± 2 MHz on the first-order line III. Falsifier: with the pump's sense reversed, the tip's polarisation reversed at one setpoint, the first ESR peak remaining higher than the sixth by more than three combined standard errors of the two heights (one-sided, 0.00135).

K9. The amplitude trends and the RF line. For "increasing magnetic fields, we see two effects on the amplitudes of the observed NMR transitions" (SC-138, L333–L334): "First, the inner features (III, IV) become less pronounced as reduced hybridization decreases the efficiency of spin pumping by lowering the flip-flop transition rate (Fig. 3c and SFig. 3)" (SC-139, L334–L336), simulated in the deposit (U-37), and "Second, the higher energy transitions (II, IV) are reduced, consistent with assigning these transitions to

the" $\mathsf{S}_z = +1/2$ "states" (SC-140, SC-141, L337–L338). The source notes that "throughout the studied magnetic field range, the ENDOR measurements shown in Fig. 3a vary in signal strength" (SC-151, L324–L325); the source states that "we were unable to send constant amplitude for all ESR frequencies, especially for fields above 500 mT" (SC-152, SC-153, L326–L328), and "lower VESR reduces the ENDOR signal" (SC-154, L328–L329). Under I-1 every ENDOR amplitude is a driven observable, which "vanishes at zero drive and never 'recovers' the static force"; an amplitude under a drive whose strength varies with field fixes no rate without the drive's calibration. This cluster is a reading under I-1 and corroborates no invariant (Use item 2). Precision: the source prints no amplitude. Falsifier: an ENDOR line amplitude with the NMR tone off, $\mathcal{V}_{\mathrm{NMR}} = 0$ (SC-083, L246), differing from zero by more than three standard errors (two-sided, 0.0027).

K10. The ground-state assignment. "This is further corroborated by measuring ENDOR as a function of VESR, as it is sensitive to both nuclear spin and electron spin populations" (SC-142, L340–L342); the source obtains "the relative population of" $\mathsf{S}_z = -1/2$ and $\mathsf{S}_z = +1/2$ "by comparing the normalized amplitude of the NMR features" I and II "from a four-Lorentzian fit" (SC-144, L343–L345). "For lower VESR, we have less population in the excited electron state" (SC-147, L317–L318, right), "the population ratio" "approaching 1 for increasing VESR (Fig. 4 inset)" (SC-149, L320–L321), and "This removes the ambiguity as to which transitions belong to the electron spin ground state" (SC-150, L321–L323). The ENDOR measurements are obtained with different $\mathcal{V}_{\mathrm{ESR}}$ values, 6, 12 and 19 mV, with f_{ESR} fixed to the first ESR peak at 3.8 GHz (SC-205, L489; SC-210, L491–L492), and "error bars are calculated from the standard deviation of the peak intensity extracted from fitting procedure" (SC-208, L489–L490); "The reduced ratio with decreasing VESR indicates a higher occupation of the" $\mathsf{S}_z = -1/2$ "state" (SC-209, L490–L491). The source prints no value of the ratio; refitted from the deposit (U-32) the amplitude ratio of lines II and I is 0.6 ± 0.2, 0.8 ± 0.1 and 0.9 ± 0.1 at the three amplitudes. Under I-4 the two populations are the dwell fractions $\mathfrak{p}_{\downarrow}$ and $\mathfrak{p}_{\uparrow}$, and their ratio is a share, no curvature ratio; an amplitude ratio reads a population ratio only with the drive and the readout of the two lines equal (SA-41). This cluster is a reading under I-4 and corroborates no invariant (Use item 2); the refitted ratio is the identity map. Precision: ± 0.2 on the ratio at 6 mV. Falsifier: the ratio at 6 mV above that at 19 mV by more than three combined standard errors (one-sided, 0.00135).

K11. The signs. "For 47Ti on oxygen site, previously Azz was taken positive and Qzz negative", "but without consideration of the nuclear Zeeman energy" "this choice is ambiguous" (SC-159, L338–L340). The "electron spin resonance" "transition energies", the bars and the square restored from the flattened print of the source's text, "are given by" $|\mathfrak{b}_e \mathfrak{g}_e \mathsf{B}_{\mathrm{app}}\, \Delta\mathsf{S}_z + \mathcal{A}_{zz} \mathsf{I}_z\, \Delta\mathsf{S}_z + \mathcal{Q}_{zz} \mathsf{I}_z^2|$ (SC-160, L340–L342), and "Under reversal of the sign of Azz" "the nuclear spin state associated with each ESR resonance" becomes $-\mathsf{I}_z$ "and leaves the transition energies unchanged" (SC-161, L342–L344). The "nuclear spin resonance" "transition energies", the bars and the squared projection restored from the same flattening, $|\mathfrak{b}_N \mathfrak{g}_N \mathsf{B}_{\mathrm{app}}\, \Delta\mathsf{I}_z + \mathcal{A}_{zz} \mathsf{S}_z\, \Delta\mathsf{I}_z + \mathcal{Q}_{zz}\, \Delta(\mathsf{I}_z^2)|$ (SC-176, L418–L419), "are no longer invariant under a sign change of A and the corresponding reversal of" $\Delta\mathsf{I}_z$ (SC-177, L420–L421); "The effect of the sign of Azz and Qzz is seen in the direction of the slope of the NMR transitions dependent on the magnetic field (see Supplementary Note 5 and SFig. 10)" (SC-178, SC-179, L421–L423), the Note and SFig. 10 outside the source. "Combining this with the confirmation of the electronic ground state" (SC-179, SC-180, L423–L424), "we unambiguously determine Azz > 0 (preferring antiparallel orientation of electron and nuclear spins)" (SC-181, L425–L426) "and Qzz < 0 (preferring in-plane nuclear charge distribution)" (SC-182, L426–L427). The chain is the slope's direction, the ground-state assignment, the sign convention of Eq. (1), whose nuclear Zeeman term enters with $+\mathfrak{b}_N \mathfrak{g}_N$, and the sign of the ^{47}Ti moment, -0.78848 nuclear magnetons (U-01) [3], so that $\mathfrak{g}_N = 0.31539$ in Eq. (1)'s convention, the moment antiparallel to

the spin. The electron's field B_{hf} at the nucleus for its ground seating is

$$\mathsf{B}_{\mathrm{hf}} = -\frac{\mathcal{A}_{zz}}{2\mathfrak{g}_N \mathfrak{b}_N}, \tag{18}$$

$-23 \pm 3\,\mathrm{T}$ with the source's 0.37 and $-27.6\,\mathrm{T}$ with its 0.315, antiparallel to the applied field: an identity on Eq. (1). In the manuscript's assignment the electron's closure is read on the exterior cone and the nucleus on the interior cone, the two cones of I-3 meeting at the skin; by CI-1 the two have opposite spin, and the skin is the density boundary of CI-3. The datum CI-1 reads is the source's "Azz > 0 (preferring antiparallel orientation of electron and nuclear spins)" (SC-181, L425–L426), by the chain above, and by CI-4 the antiparallel pair is meshed. No derivation of the sign from the two cones is written: it is a held-out target, and no overlap complex is written here (O-5). This cluster is a reading under I-3, P-5, CI-1, CI-3 and CI-4 and corroborates no invariant (Use item 2); its field at the nucleus is the identity map. Precision: $\pm 3\,\mathrm{T}$ on B_{hf}. Falsifier: on a ^{49}Ti atom on the same site, its moment -1.10417 nuclear magnetons (U-02), of the sign of ^{47}Ti's, the ground-state line of its outermost nuclear step rising with the applied field by more than three standard errors of its slope (one-sided, 0.00135).

K12. The driving mechanisms and the outlook. The source writes that "We will consider three interactions, which under modulation can cause $\Delta \mathrm{ms} = \pm 1$ transitions" (SC-184, L430–L431), printed for the electron's projection where L419 writes the nuclear one, a difference that is the source's. "Modulation of the hyperfine interaction would primarily drive flip-flop transitions" (SC-185, L432–L433), "involving also the electron spin, resulting in transition energies in the GHz range for the magnetic fields used in this work" (SC-186, L434–L435), while "the measured NMR remain in MHz frequency range" (SC-187, L436); as the fraction of the in-plane hyperfine term over the field is "very small in our measurement conditions (see SFig. 2), it is unlikely that" (SC-190, L416–L417) "hyperfine mediated flip-flop transitions are the underlying driving mechanism" (SC-191, L418–L419). The source notes that "this does not exclude other forms of hyperfine modulation: the RF electric field could modulate an off-diagonal term" of the form $\mathcal{A}_{zx}\mathsf{S}_z\mathsf{I}_x$ (SC-192, SC-193, L419–L421), with I_x the in-plane nuclear spin. "This could drive the observed transitions while preserving the electronic spin state" (SC-193, SC-194, L421–L423). "Modulation of the quadrupole interaction, possibly induced by the RF field changing the electric field gradient, allows transitions matching the observed NMR frequencies" (SC-196, SC-197, L425–L427), but "would also drive" $\Delta\mathsf{I}_z = \pm 2$ transitions, "which is not well reflected in our measurements" "(see Fig. 2b and Supplementary Note 6 and SFig. 11)" (SC-199, SC-200, L429–L431), the Note and SFig. 11 outside the source, and "This implies that a quadrupole modulation is not the dominant driving mechanism" (SC-201, L431–L432). "Modulation of the Zeeman interaction would describe the observed NMR transitions well based on selection-rule arguments" (SC-202, L433–L434); "an oscillating magnetic field, e.g., from the inhomogeneous stray field of the tip" "would be significantly smaller for the nuclear spin compared to the electron spin" (SC-213, L500–L502). "Unfortunately, the present measurements do not allow us to draw strong conclusions on the driving mechanism" (SC-217, L509–L510), and "We cannot quantify our driving field" (SC-218, L510–L511). The derived reading here: a transverse drive b on the electron at the NMR frequency tilts its axis, and by Eq. (1) the nucleus meets a transverse field $\mathfrak{k}_{\mathrm{rel}}\mathsf{b}$, odd in S_z; to leading order in $\mathcal{A}_{zz}$ over the electron's Zeeman frequency

$$\mathfrak{k}_{\mathrm{rel}} = \frac{\mathcal{A}_\perp}{\mathcal{A}_{zz}}\,\frac{\mathfrak{g}_\perp}{\mathfrak{g}_e}\,\frac{|\mathsf{B}_{\mathrm{hf}}|}{\mathsf{B}_{\mathrm{app}} + \mathsf{B}_{\mathrm{tip},z}}, \tag{19}$$

and the exact matrix element of the source's Hamiltonian with the deposit's inputs is larger at every field: $\mathfrak{k}_{\mathrm{rel}} = 3 \times 10^1$, 1×10^1 and 4 at 200, 450 and 1400 mT, at the one figure of the deposit's fixed $\mathcal{A}_\perp = 10\,\mathrm{MHz}$ (U-26), and the ratio between 200 and 1400 mT is 5.5 in the leading form and 6.2 in the exact one. It is an operator of the form the source names at L421, derived from the source's Hamiltonian with inputs the source does not measure ($\mathcal{A}_\perp$ and $\mathfrak{g}_\perp$ held fixed in the deposit; b none): the manuscript's derived reading, never the mechanism established. The two RF tones and the quanta of both resonances are strokes of

the one carrier (I-2), a quadrupole modulation is a modulation of the tide the nucleus's skin reads against its own centre path, the weight its charge (I-3a), and a Rabi rate is a driven observable read through its response to a calibrated drive (I-1). The source writes that "in the limit of weak hybridization" "the relaxation time is in order of seconds for this system" (SC-228, SC-229, L512–L514, Ref. 33); no rate and no coherence time is derived here (O-12). This cluster is a reading under I-1, I-2 and I-3a, an identity on Eq. (1) where it computes with the deposit's inputs, and corroborates no invariant (Use item 2). Precision: one figure on $\mathfrak{k}_{\mathrm{rel}}$. Falsifier: the ratio of line I's Rabi rates at 200 and 1400 mT, under one transverse drive that enters Eq. (1) through the electron's term alone, as the tip's field does, calibrated at the atom, differing from the relay's exact ratio by more than three combined standard errors (two-sided, 0.0027).

K13. Methods, temperature and the deposit. The "MgO/Ag sample including Ti and Fe surface atoms" was prepared "with the same method described in previous works" (SC-232, L532–L533); the tip picked up Fe atoms (SC-233, L534–L535). "Each ESR-active tip was tested for sufficient nuclear spin polarization dependent on setpoint bias or current at high external field", 1400 mT (SC-234, L535–L537), and all data presented in the main text "were taken with the same microtip, except for Fig. 1d" (SC-235, L538–L539). "All measurements were carried out at a sample temperature of 0.4 K" (SC-237, L541–L542); "We chop the RF signal at 271 Hz" (SC-238, L551); "All RF voltages are expressed in zero-to-peak amplitude" (SC-239, L553–L554); "The raw data generated in this study, as well as the analysis and simulation code, have been deposited in a Zenodo database" (SC-240, L558–L560) [2]. Under I-7 the sample's 0.4 K enters the ledger once, through aT^4V/c^2; $k_{\mathrm{B}}T/h = 8$ GHz there. The Boltzmann parameter of the deposit's six-line ESR model (U-31), 8.7±0.8 mK at 200 mT and 23±2 mK at 1400 mT, is that model's parameter and never I-7's temperature, and never the rate of a clock the medium carries (0.1c). Across the ground manifold's $5\mathcal{A}_{zz}/2 = 331 \pm 1$ MHz, 15.89 ± 0.05 mK, the Boltzmann ratio at the sample's 0.4 K is 0.96: the polarization the source reports is driven. The tip and the support belong to the configuration measured (0.1a). This cluster is a reading under I-7 and 0.1a and corroborates no invariant (Use item 2). Precision: one figure on the sample temperature. Falsifier: with the tip retracted beyond tunnelling for longer than the nuclear lifetime and the seating then read by the single-shot method (U-13) [7], the dwell fraction of $\mathsf{I}_z = -5/2$ above its thermal value at the sample temperature by more than three standard errors (one-sided, 0.00135).

4 Cone-position translation

In the manuscript's assignment the electron's closure is read on the exterior cone of the nucleus's atlas and the nucleus on the interior cone, the two meeting apex circle to apex circle at the skin (I-3); the skin is the density boundary of CI-3, and by CI-1 the two have opposite spin. The electron's closure and the nucleus are closed patterns, and "A nucleus is one pattern" (0.1b). A value of a spin projection is a seating, the manuscript's term: an orientation of a closed pattern in the twist it sits in, with no re-braiding (I-5). The eigenproblem of display (10) carries their rest energies, and no form of D, its domains, boundary data or spectrum is written (O-9). The cone-position axis is the A-4 token α_{conf}, token and range only, with no value, threshold, ordering, sign or trajectory (O-3). The table below translates the source's quantities; every entry of its right column is the manuscript's reading.

source quantity	reading here
$\mathsf{S}_z = \pm 1/2$	the electron's two seatings in the twist where it sits (I-5, 0.1c)
I_z from $-5/2$ to $+5/2$	the nucleus's six seatings in the twist of the applied field and of the electron
$\mathcal{A}_{zz}$	a profile-weighted mean, the weights the electron's spin density with the nucleus's magnetization (I-3)
$\mathcal{Q}_{zz}$	the nucleus's $l = 2$ moment, its skin against its centre path reading the surrounding tide, the weight its charge (I-3a)
the Zeeman terms of display (13)	held moments in the twist, the helical window (0.1c)
a spin state's population	a dwell fraction (I-4): a share, never a curvature ratio (I-6)
the magnetoresistive readout	projection and phase fixation (I-4)
hybridization	a shared bivector grade (I-4)
the GHz signal and the MHz signal, the quanta of both resonances	strokes of the one carrier (I-2)
the nuclear spin polarization preferring $\mathsf{I}_z = -5/2$	by CI-4 the like configuration, the ground manifold's highest seating: driven (0.1a)

5 Predictions

The five blocks below are the divergence candidates DC-1 to DC-5, predictions general relativity and quantum mechanics do not make: in each, FDT supplies what the consensus calculation lacks—an additional source (T-1, T-2: P-5), a constitutive input (T-3, T-4: I-7) or a changed observable dictionary (T-5: 0.1a)—never a curvature ratio, a chart or a branch tag alone. No candidate is retired. Terms common to them: an atom is a Ti atom adsorbed onto an oxygen binding site of the source's bilayer MgO grown on Ag(100) (SA-10), read as the source reads (4 pA, 80 mV, 20 mV, SA-13; the working field the source's 450 mT, SC-045, L163–L165), one microtip per design (SA-62). f_c is the centre of an atom's ESR multiplet, the mean of its fitted peak positions, and f_{cal} half its difference between test fields 1 mT above and 1 mT below the working field; with the deposited slope over the seven fields (U-30), $f_{\mathrm{cal}} = 7.88 \pm 0.02$ MHz. The ^{49}Ti atoms, the helical molecules, the reversal of the applied field and of the bias, the single-shot readout of the nuclear spin (the source's Ref. 33, SC-023, L67–L69) [7] and every calibration named are external to the source (U-41 to U-47).

Decision rules, carried into each block in these words. The sizes are exact: of the $2^{12} = 4096$ equally likely sign assignments of the twelve values, ordered by the mean each gives, $p \le 0.0027$ two-sided admits the 10 most extreme and $p \le 0.00135$ one-sided the 5 most extreme; twelve and both levels are U-40, investigator-chosen. Two tests are common to T-1, T-2 and T-5, for twelve values with a bound $\mathfrak{c}$ and a scale $\mathfrak{h}$. The bounded test: the mean lies outside the interval from $-\mathfrak{c}$ to $+\mathfrak{c}$ and the two-sided test of the values, every one shifted by $\mathfrak{c}$ against the sign of the mean, rejects at $p \le 0.0027$ (size at most 10/4096 wherever the bounded term lies within its bound and no other term enters). The exclusion test: a term of magnitude $\mathfrak{h}$ or more is excluded when $\mathfrak{c} < \mathfrak{h}$, the mean lies between $-(\mathfrak{h} - \mathfrak{c})$ and $+(\mathfrak{h} - \mathfrak{c})$, and the two-sided tests of the values shifted by $+(\mathfrak{h} - \mathfrak{c})$ and by $-(\mathfrak{h} - \mathfrak{c})$ both reject at $p \le 0.0027$ (false exclusion at most 5/4096 on the same condition). Where a design's attributed detection and its exclusion both hold, a term of magnitude smaller than its $\mathfrak{h}$ is detected. Each block's ablation test point is an instantiated effect, never a derived amplitude: with the FDT input in place every one of the twelve values stands at the block's scale above the comparator's value, and with the input ablated at the comparator's value; the threshold is the mean that one sign flip among the twelve equal values the rule tests leaves short of rejection.

5.1 T-1. The twist of a helical molecule at a ^{47}Ti atom

T-1 is candidate DC-1 and discriminates SA-11, where the source "applied an external magnetic field perpendicular to the surface to lift the electron spin degeneracy and drove ESR by applying a radio frequency (RF) voltage to the tunnel junction" (SC-035, L77–L79), a divergence seed. Warrant: P-5, an

additional source, its clauses "A magnetic field is a macroscopic helical density gradient of the omnium" and "Imposing a physical twist (torsion, helical geometry) on a material *is* establishing a magnetic field".

Referent: twelve pairs of ^{47}Ti atoms and, for each atom, one molecule of a helical, closed-shell hydrocarbon, of the hand + or − as imaged, moved by atom manipulation (SA-04) onto the site a fixed number of lattice sites from the atom along a lattice row in a mirror plane of the support, the two molecules of a pair mirror images in that plane (U-41, investigator-chosen). For each pair a fair coin drawn before the manipulation gives one atom a molecule of the hand + and the other one of the hand −. Each atom is read at the two senses of the applied field along the normal, at the two polarities of the bias and, the tip held at one height, at the bias and at half of it, the tip's polarisation following the field, before its molecule is brought and after; f_c is the mean over the two polarities of twice its value at half the bias less its value at the bias; each time a fair coin drawn before the reading sets the order of the two senses (U-42).

Observable: per atom $\mathfrak{d}$, the half difference of f_c between the two senses of the applied field, and its change $\Delta\mathfrak{d}$ on bringing the molecule; per pair the contrast $\mathfrak{y}_{\mathrm{tw}}$, with $\Delta\mathfrak{d}(+)$ and $\Delta\mathfrak{d}(-)$ the changes of the atoms given the hands + and −:

$$\mathfrak{d} = \frac{1}{2}\big[f_c(+\mathsf{B}_{\mathrm{app}}) - f_c(-\mathsf{B}_{\mathrm{app}})\big], \qquad \mathfrak{y}_{\mathrm{tw}} = \frac{1}{2}\big[\Delta\mathfrak{d}(+) - \Delta\mathfrak{d}(-)\big]. \tag{20}$$

FDT input: in the manuscript's reading, a helical molecule is a helical geometry, and by P-5 imposing it establishes a magnetic field; the mirror in that plane, which contains the surface normal, reverses the hand and an axial field's part along the normal and keeps the support, so that part is odd in the hand; a field the geometry establishes does not reverse with the applied field, and $\Delta\mathfrak{d}$ is its Zeeman shift of f_c. P-5's last sentence, "Quantitative twist → moment forms are derived in-manuscript through I-1/I-2 with a declared kernel", assigns neither magnitude nor sense, and no kernel is declared here: an existence prediction. Solved, with $\mathsf{B}_{\mathrm{mol}}$ the field along the normal at the atom that the molecule of the hand + establishes,

$$\Delta\mathfrak{d}(\pm) = \pm\,\mathfrak{g}_e\mathfrak{b}_e\,\mathsf{B}_{\mathrm{mol}}, \qquad \mathfrak{y}_{\mathrm{tw}} = \mathfrak{g}_e\mathfrak{b}_e\,\mathsf{B}_{\mathrm{mol}} \neq 0, \tag{21}$$

and in units of $f_{\mathrm{cal}} = \mathfrak{g}_e\mathfrak{b}_e \times 1\,\mathrm{mT}$ the contrast reads the field in millitesla.

Comparator: zero within the leak. In the consensus calculation (time reversal for an atom beside a molecule that holds no moment, U-17, external) the ESR frequency at no bias is the same at the two senses of the field once the tip's polarisation has reversed, so $\mathfrak{d}$ holds only the part of the tip's field that does not reverse and what the current carries even in the bias and not of first order in it, and $\Delta\mathfrak{d}$ only their change (Use item 7). The leak is the difference of the tip's part of it between the two atoms of a pair, bounded by $\mathfrak{c}_{\mathrm{tw}}$, fixed before the first contrast is read from each atom's $\mathfrak{d}$ before its molecule and the relative change, on bringing it, of the tip's term of its field-even offset, $\frac{1}{2}[f_c(+\mathsf{B}_{\mathrm{app}}) + f_c(-\mathsf{B}_{\mathrm{app}})]$ less the applied field's $f_{\mathrm{cal}}\,\mathsf{B}_{\mathrm{app}}/(1\,\mathrm{mT})$, the tip's two parts taken to change in one proportion (U-43). The first contrast is read only with $\mathfrak{c}_{\mathrm{tw}}$ at or under its design ceiling $0.04\,f_{\mathrm{cal}}$, f_{cal} read before the molecules are brought, the mean over the design's $2 \times 12 = 24$ atoms (U-43, investigator-chosen). Precision criterion: each contrast is a difference of two atoms' changes read under one microtip, so the tip's shared terms cancel and their covariance enters the contrast's variance as $-2\,\mathrm{Cov}$; the sign-flip test reads the twelve contrasts' own scatter, and $\mathfrak{c}_{\mathrm{tw}}$ bounds the part the two atoms do not share. Channels and the sign each gives the contrast:

channel	sign it gives $\mathfrak{y}_{\mathrm{tw}}$
the twist's field (the FDT input)	one sign in every pair
whatever one atom or its place carries and the other does not	a sign the hand coin sets
a drift between the two senses' readings	a sign the order coins set
a spin polarisation the current takes beside a helical molecule (chirality-induced spin selectivity, U-18), odd in the hand and not reversing with the field	nothing of first order in the bias and nothing odd in it in that f_c; the rest for the attribution rule
the leak	inside the interval from $-\mathfrak{c}_{\mathrm{tw}}$ to $+\mathfrak{c}_{\mathrm{tw}}$

Statistic: the twelve contrasts; two-sided sign-flip permutation test. Decision rules ($\mathfrak{c} = \mathfrak{c}_{\mathrm{tw}}$; $\mathfrak{h} = \mathfrak{h}_{\mathrm{tw}} = 0.1\,f_{\mathrm{cal}}$; U-43, investigator-chosen): detection—the bounded test; attribution—a detection is read as the field P-5 assigns to the helical geometry when the bounded test holds, with the detection's sign, on $(4\mathfrak{y}' - \mathfrak{y}_{\mathrm{tw}})/3$, $\mathfrak{y}'$ the contrast at the same height with the bias and the ESR tone's amplitude halved; exclusion—the exclusion test on that combination; inconclusive otherwise. The attributed detection's size is at most 10/4096 and the false exclusion at most 5/4096 wherever the leak lies within $\mathfrak{c}_{\mathrm{tw}}$ in that combination and no other term enters it. Ablation test point: the bound at its ceiling, $\mathfrak{c}_{\mathrm{tw}} = 0.04\,f_{\mathrm{cal}}$; values $\mathfrak{h}_{\mathrm{tw}} = 0.1\,f_{\mathrm{cal}}$ against zero, the bounded test's shifted values $\mathfrak{h}_{\mathrm{tw}} - \mathfrak{c}_{\mathrm{tw}} = 0.06\,f_{\mathrm{cal}}$; threshold $\mathfrak{c}_{\mathrm{tw}} + 5(\mathfrak{h}_{\mathrm{tw}} - \mathfrak{c}_{\mathrm{tw}})/6 = 0.09\,f_{\mathrm{cal}}$. Its exclusion test excludes a field $\mathsf{B}_{\mathrm{mol}}$ whose Zeeman shift of f_c is $0.1\,f_{\mathrm{cal}}$ or more, a field of 0.1 mT or more along the normal.

5.2 T-2. The same twist at a Ti atom without nuclear spin

T-2 is candidate DC-2 and discriminates SA-12, the source's "six different ESR peaks" on the studied ^{47}Ti isotope (SC-037, L82–L83) "as opposed to a single peak observed on Ti isotopes without nuclear spin" (SC-038, SC-039, L84–L85), a divergence seed. Warrant: P-5, an additional source, as T-1. Referent, design, scale, leak, channels and statistic as T-1, on twelve pairs of Ti atoms without nuclear spin (the source's "Ti isotopes without nuclear spin", SC-039, L85), f_c the position of each atom's single ESR peak; contrast $\mathfrak{y}_{\mathrm{tw0}}$, defined as $\mathfrak{y}_{\mathrm{tw}}$ in display (20) on these atoms. FDT input as T-1: in the manuscript's reading, the twist's field lands on the electron's closure whether or not the nucleus holds a moment: an existence prediction. Solved as T-1,

$$\mathfrak{y}_{\mathrm{tw0}} = \mathfrak{g}_e \mathfrak{b}_e \,\mathsf{B}_{\mathrm{mol}} \neq 0, \tag{22}$$

with no hyperfine term to average, since the single peak carries no nuclear seating. Comparator: as T-1's, zero within the leak, with the same bound $\mathfrak{c}_{\mathrm{tw}}$ fixed in the same way. Precision criterion as T-1's. Channels and the sign each gives $\mathfrak{y}_{\mathrm{tw0}}$: those of T-1's table, the hand coin, the order coins and the leak entering alike. Statistic: the twelve contrasts; two-sided sign-flip permutation test. Decision rules as T-1's ($\mathfrak{c} = \mathfrak{c}_{\mathrm{tw}}$; $\mathfrak{h} = \mathfrak{h}_{\mathrm{tw}} = 0.1\,f_{\mathrm{cal}}$), the attribution on $(4\mathfrak{y}' - \mathfrak{y}_{\mathrm{tw0}})/3$ with $\mathfrak{y}'$ this block's contrast at the halved bias and ESR amplitude; the attributed detection's size at most 10/4096, false exclusion at most 5/4096, wherever the leak lies within $\mathfrak{c}_{\mathrm{tw}}$ in this block's combination and no other term enters it. Ablation test point as T-1's: $\mathfrak{c}_{\mathrm{tw}} = 0.04\,f_{\mathrm{cal}}$; values $\mathfrak{h}_{\mathrm{tw}} = 0.1\,f_{\mathrm{cal}}$ against zero, shifted values $0.06\,f_{\mathrm{cal}}$; threshold $0.09\,f_{\mathrm{cal}}$. Its exclusion test excludes a field of 0.1 mT or more along the normal at an atom without nuclear spin.

5.3 T-3. The hyperfine term of two isotopes on one site

T-3 is candidate DC-3 and discriminates SA-30, the source's fitted "Azz = 132.4 ± 0.4 MHz" (SC-102, L215), a divergence seed. Warrant: I-7, a constitutive input, the Discriminator, quoted whole: "For purely density-coupled equilibrium observables, a heavier isotope → larger R_s → higher α → **larger** response—opposite in sign to conventional reduced-mass scaling."

Referent: twelve oxygen sites; onto each, one ^{47}Ti atom and one ^{49}Ti atom, told apart by their six and eight ESR peaks, are moved in turn by atom manipulation (SA-04) and read there under one microtip, the order of the two set by a fair coin drawn before the first is moved (U-44). On each atom, with the ESR tone on its first ESR peak, the two lines of the outermost nuclear step are read as the source reads the NMR features labelled I and II (SA-28, SA-34) at the six fields of the source's four-Lorentzian fits (SA-46); $\bar{f}_+$ is the mean over those fields of the sum of the two positions (display (15)). Observable: $\mathfrak{y}_{\mathrm{hf}}$, the logarithm of the ratio of the two values of $\bar{f}_+$ less $\mathfrak{v}_N$, the logarithm of the ratio of the two nuclear g-factors, with the superscripts (47) and (49) naming the isotope:

$$\mathfrak{y}_{\mathrm{hf}} = \ln \frac{\bar{f}_+^{(49)}}{\bar{f}_+^{(47)}} - \mathfrak{v}_N, \qquad \mathfrak{v}_N = \ln \frac{\mathfrak{g}_N^{(49)}}{\mathfrak{g}_N^{(47)}}, \tag{23}$$

with $\mathfrak{v}_N = (2.7 \pm 0.2) \times 10^{-4}$ from the two isotopes' moments and spins measured in bulk, $-0.78848(1)$ and $-1.10417(1)$ nuclear magnetons, spins 5/2 and 7/2 (U-01, U-02) [3]. For ^{47}Ti the deposited spectra give $\bar{f}_+ = 131.9 \pm 0.3\,\mathrm{MHz}$ (U-29).

FDT input: I-7 Discipline 2 reads: "A sign prediction competing with geometric, electronic or inertial channels is written as a sensitivity-weighted sum $\delta \ln O = \sum_i S_i\, \delta \ln x_i$ with every S_i defined (held-fixed variables stated). The density-channel **input** sign is the FDT discriminator; the **observable** sign is $\mathrm{sgn}(\prod S_i)$", and it is fixed only once the sensitivities are established. The density channel's x is an A-1 α_{ext} of the chart inventory with its object and branch, the exterior compactness of the Ti nucleus, which is P-3's clamp (its skin value $\alpha_{\mathrm{core}} \to 1^-$ is never that compactness); it is moved by the substitution through that nucleus's $\mathcal{M}$ at fixed P, one declared $r_f = 1.0\,\mathrm{nm}$ (U-39) and packing, by I-3 chart discipline 5: "Fixed-geometry perturbations (isotope substitution: P, r, packing all fixed) move compactness only; pressure-trajectory maps do not apply to them." The nuclear masses are the atomic masses 46.9517638 u and 48.9478708 u less 22 electron masses (U-03, U-04, U-06) [4, 5]. In the manuscript's reading "**larger** response" is a larger hyperfine term per unit nuclear g-factor, and $\bar{f}_+$, a splitting of the undriven configuration, is an "equilibrium" observable and not "purely density-coupled": the channel of the two nuclear moments is $\mathfrak{v}_N$, which the observable removes, and its other channels are the other terms of Discipline 2's sum. Solved, with S_{hf} the secant over the step between the two isotopes, the mean of the sensitivity of $\ln \bar{f}_+$ to $\ln \alpha_{\mathrm{ext}}$ at fixed $\mathfrak{g}_N$, I, r_f, P, packing and every term of the consensus departure, and $\mathfrak{u}_{\mathrm{hf}}$ the consensus departure,

$$\mathfrak{y}_{\mathrm{hf}} = S_{\mathrm{hf}}\, \Delta \ln \alpha_{\mathrm{ext}} + \mathfrak{u}_{\mathrm{hf}}, \qquad \Delta \ln \alpha_{\mathrm{ext}} = \ln \frac{\mathcal{M}^{(49)}}{\mathcal{M}^{(47)}} = 0.04165, \tag{24}$$

the input sign positive (the Discriminator) and S_{hf} undeclared, so T-3 is a sign prediction, its one-sided rules taking the observable sign as positive on the manuscript's reading.

Comparator: the consensus departure $\mathfrak{u}_{\mathrm{hf}}$ of the hyperfine term per unit nuclear g-factor between two isotopes on one site—the hyperfine anomaly, from the two nuclei's distributions of magnetization and charge (U-19), the adatom's mass-dependent zero-point term (U-20, external) and Eq. (1)'s terms beyond first order, which differ with the spin (U-24, U-26)—inside the preregistered interval from $-\mathfrak{c}_{\mathrm{iso}}$ to $+\mathfrak{c}_{\mathrm{iso}}$, $\mathfrak{c}_{\mathrm{iso}} = 0.002$ (U-45, investigator-chosen). Precision criterion: each value is a difference of two logarithms read on one site under one microtip, so the site's and the tip's shared terms cancel and enter its variance as $-2\,\mathrm{Cov}$; the moments' last digits put $\pm 0.2 \times 10^{-4}$ on $\mathfrak{v}_N$, far inside $\mathfrak{c}_{\mathrm{iso}}$. Channels and the sign each gives $\mathfrak{y}_{\mathrm{hf}}$:

channel	sign it gives $\mathfrak{y}_{\mathrm{hf}}$
the density channel	its input sign positive (the Discriminator), its sign here the manuscript's reading
the channel of the nuclear moments	none: $\mathfrak{v}_N$ removes it
the consensus departure	of either sign, one sign in every pair, inside the interval
what changes between the two readings (the tip, the site after a manipulation)	a sign the order coin sets

Statistic: the twelve values; one-sided sign-flip permutation tests. Decision rules: detection—the one-sided test of the values less $\mathfrak{c}_{\mathrm{iso}}$ rejects upward at $p \le 0.00135$ (size at most 5/4096 wherever every pair's consensus departure lies in the interval); exclusion—a density-channel contribution of $\mathfrak{h}_{\mathrm{iso}} = 0.006$ or more in the logarithm (U-45, investigator-chosen) is excluded when the one-sided test of the values less $(\mathfrak{h}_{\mathrm{iso}} - \mathfrak{c}_{\mathrm{iso}})$ rejects downward at $p \le 0.00135$ (false exclusion at most 5/4096 on the same condition); inconclusive otherwise; attribution—a detection is read as the density channel when it stands with the interval recomputed from the two isotopes' hyperfine anomaly measured in another configuration, the site's calculated zero-point term and the exact diagonalisation (external). Ablation test point, the comparator's value at the interval's

upper end: values $\mathfrak{c}_{\mathrm{iso}} + \mathfrak{h}_{\mathrm{iso}} = 0.008$ against $\mathfrak{c}_{\mathrm{iso}} = 0.002$, residuals $\mathfrak{h}_{\mathrm{iso}} = 0.006$ against zero; threshold $5\mathfrak{h}_{\mathrm{iso}}/6 = 0.005$. Its exclusion test excludes a density-channel contribution to $\mathfrak{y}_{\mathrm{hf}}$ of 0.006 or more.

5.4 T-4. The outer resonance of two isotopes on one site

T-4 is candidate DC-4 and discriminates SA-05, the source's statement that "nuclear spins of single isotopes are identified by resolving the hyperfine splitting" (SC-019, L52–L54, Ref. 31), a divergence seed. Warrant: I-7, a constitutive input, as T-3. Referent, design, interval, channels and statistic as T-3, on the same twelve sites. On each atom $\mathfrak{w}$ is the slope of a straight-line fit of f_c against the applied field over the source's seven fields (SA-33), the tip's term its offset (SA-19). Observable: $\mathfrak{y}_g$, the logarithm of the ratio of the two slopes,

$$\mathfrak{y}_g = \ln \frac{\mathfrak{w}^{(49)}}{\mathfrak{w}^{(47)}}, \tag{25}$$

with $\mathfrak{w} = 7.88 \pm 0.02\,\mathrm{GHz/T}$ for ^{47}Ti in the deposited spectra (U-30). FDT input as T-3, the electron's closure read on the exterior atlas of the nucleus; in the manuscript's reading "**larger** response" is a larger slope of the outer resonance against the applied field; no channel is removed. Solved, with S_g the secant over the step between the two isotopes, the mean of the sensitivity of $\ln \mathfrak{w}$ to $\ln \alpha_{\mathrm{ext}}$ at fixed I, r_f, P, packing and every term of the consensus departure, and $\mathfrak{u}_g$ the consensus departure,

$$\mathfrak{y}_g = S_g\, \Delta \ln \alpha_{\mathrm{ext}} + \mathfrak{u}_g, \tag{26}$$

the input sign positive and S_g undeclared, so T-4 is a sign prediction, its one-sided rules taking the observable sign as positive on the manuscript's reading. Comparator: the consensus departure $\mathfrak{u}_g$—the consensus dependence of an adatom's electronic g-factor on its isotope (its mass-dependent and zero-point terms, U-20, external) and Eq. (1)'s terms beyond first order (U-24, U-26)—inside the interval from $-\mathfrak{c}_{\mathrm{iso}}$ to $+\mathfrak{c}_{\mathrm{iso}}$. Precision criterion as T-3's: two slopes on one site under one microtip, their shared tip offset removed by each fit and their covariance entering as $-2\,\mathrm{Cov}$. Channels and the sign each gives $\mathfrak{y}_g$: the density channel, its input sign positive, its sign here the manuscript's reading; the consensus departure, of either sign, one sign in every pair, inside the interval; what changes between the two readings, a sign the order coin sets. Statistic: the twelve values; one-sided sign-flip permutation tests. Decision rules as T-3's: detection—the one-sided test of the values less $\mathfrak{c}_{\mathrm{iso}}$ rejects upward at $p \le 0.00135$ (size at most 5/4096 wherever every pair's consensus departure lies in the interval); exclusion—a density-channel contribution of $\mathfrak{h}_{\mathrm{iso}} = 0.006$ or more is excluded when the one-sided test of the values less $(\mathfrak{h}_{\mathrm{iso}} - \mathfrak{c}_{\mathrm{iso}})$ rejects downward at $p \le 0.00135$ (false exclusion at most 5/4096 on the same condition); inconclusive otherwise; attribution—a detection is read as the density channel when it stands with the interval recomputed from the sites' measured changes between the two readings (external). Ablation test point as T-3's: values $\mathfrak{c}_{\mathrm{iso}} + \mathfrak{h}_{\mathrm{iso}} = 0.008$ against $\mathfrak{c}_{\mathrm{iso}} = 0.002$, residuals 0.006 against zero; threshold 0.005. Its exclusion test excludes a density-channel contribution to $\mathfrak{y}_g$ of 0.006 or more.

5.5 T-5. A driven step and its reverse as two settlements

T-5 is candidate DC-5 and discriminates SA-27, the source's "Equalization of the ESR peaks implies that we drive the nuclear spin sufficiently to saturate the two nuclear spin" populations (SC-088, L182–L183), an under-determination the block resolves. Warrant: 0.1a, a changed observable dictionary, its sentence "Events do not commute, and a process run backward is never the mirror of the forward event."

Referent: twelve ^{47}Ti atoms under one microtip. A shot: the nuclear seating read by the single-shot method (the ESR tone on the first ESR peak and on the second in turn; U-13) [7], the NMR tone on line I at an amplitude $\mathcal{V}_p$ for a time $\mathfrak{t}_p$, and the seating read again (U-46). Observable: $\mathfrak{y}_{\mathrm{rev}}$, the mean over the two beginnings of $\ln(\mathfrak{n}_b/\mathfrak{n}_a)$, $\mathfrak{n}_a$ and $\mathfrak{n}_b$ the counts of shots ending in $\mathsf{I}_z = -5/2$ and in $-3/2$, with the

beginning seating as the subscript of each logarithm:

$$\mathfrak{y}_{\mathrm{rev}} = \frac{1}{2}\Big[\ln \frac{\mathfrak{n}_b}{\mathfrak{n}_a}\Big|_{-5/2} + \ln \frac{\mathfrak{n}_b}{\mathfrak{n}_a}\Big|_{-3/2}\Big]. \tag{27}$$

A control shot has the tone at the source's off-resonance 30.2 MHz (SC-049, L162–L163), and the first value is read only where, of 800 control shots that begin in each seating, at least 0.97 end in it. On each atom, beforehand, $\mathfrak{t}_p$ is the shortest at which the two beginnings' $\ln(\mathfrak{n}_b/\mathfrak{n}_a)$ differ by less than 0.1 and $\mathcal{V}_p$ is one from which a doubling at that $\mathfrak{t}_p$ changes $\mathfrak{y}_{\mathrm{rev}}$ by less than 0.05, each read to a counting standard error of 0.02; then shots are taken until $2 \times 800 = 1600$ end in one of the step's two seatings, 800 that began in each (every threshold, 0.02 and 800: U-47, investigator-chosen).

FDT input: 0.1a, in the manuscript's reading: the driven step from $-5/2$ to $-3/2$ and the driven step back are two settlements, each from the state it starts in, so their rates are not bound to the one matrix element the consensus account gives both—an existence prediction, its sense unassigned. Solved for a drive that outruns every other rate, with $\mathfrak{r}_{ab}$ and $\mathfrak{r}_{ba}$ the rates of the driven step from $-5/2$ to $-3/2$ and of the driven step back,

$$\mathfrak{y}_{\mathrm{rev}} = \ln \frac{\mathfrak{r}_{ab}}{\mathfrak{r}_{ba}} \neq 0, \tag{28}$$

and no rate of either settlement is written (O-12). Comparator: $\mathfrak{y}_{\mathrm{rev}}$ inside the preregistered interval from $-\mathfrak{c}_{\mathrm{rev}}$ to $+\mathfrak{c}_{\mathrm{rev}}$, $\mathfrak{c}_{\mathrm{rev}} = 0.2$ (U-47, investigator-chosen). In the consensus account (one matrix element for a driven transition and its reverse, U-21, external) a drive that outruns every other rate makes the two dwell fractions equal, whatever the pump and the relaxation do, so $\mathfrak{y}_{\mathrm{rev}} = 0$. The amplitude criterion bounds what the pump and the relaxation leave ($0.05 \times 4/3$, falling as the inverse square of the amplitude) and the control what a change of seating before the reading and unequal chances of reading the two seatings leave ($\ln(1.03/0.97) = 0.06$); with the tone's off-resonant drive of line III these are terms of one sign in every atom, their sum the comparator's bounded term, inside the interval. Precision criterion: the counting error of each value is symmetric with standard error $2/\sqrt{1600} = 0.05$, and the two beginnings of one atom share its pump, relaxation and reading terms, which the mean over them keeps and the bound $\mathfrak{c}_{\mathrm{rev}}$ carries. Channels and the sign each gives $\mathfrak{y}_{\mathrm{rev}}$:

channel	sign it gives $\mathfrak{y}_{\mathrm{rev}}$
the difference of the two settlements	one sign in every atom
the bounded term	inside the interval
the counting error	symmetric, its standard error $2/\sqrt{1600} = 0.05$

Statistic: the twelve values; two-sided sign-flip permutation test. Decision rules ($\mathfrak{c} = \mathfrak{c}_{\mathrm{rev}}$; $\mathfrak{h} = \mathfrak{h}_{\mathrm{rev}} = 0.6$; U-47, investigator-chosen): detection and inconclusive as T-1's; exclusion—the exclusion test; attribution—a detection is read as the difference of the two settlements when it stands with $\mathcal{V}_p$ and $\mathfrak{t}_p$ doubled. The detection's size is at most 10/4096 and the false exclusion at most 5/4096 wherever the comparator's bounded term lies within $\mathfrak{c}_{\mathrm{rev}}$ and no other term enters. Ablation test point: the bound $\mathfrak{c}_{\mathrm{rev}} = 0.2$; values $\mathfrak{h}_{\mathrm{rev}} = 0.6$ against zero, the bounded test's shifted values $\mathfrak{h}_{\mathrm{rev}} - \mathfrak{c}_{\mathrm{rev}} = 0.4$; threshold $\mathfrak{c}_{\mathrm{rev}} + 5(\mathfrak{h}_{\mathrm{rev}} - \mathfrak{c}_{\mathrm{rev}})/6 = 0.533$. Its exclusion test excludes a difference of the two settlements of 0.6 or more in $\mathfrak{y}_{\mathrm{rev}}$.

6 Cross-domain

The three readings of this analysis run at every size, because the chain is all ratios and selects no size and the form repeats (0.3a). The tide: the nucleus's skin against its own centre path here, read through its $l = 2$ moment, the weight its charge; a planet's skin against its centre path, read through its tidal bulge; and two neighbouring strokes separating in the plane of a pass are one deviation equation, each read between neighbours (I-3a). The twist: the helical window reads the two held moments of this atom

in the twists where they sit, and a magnetized body in a field gradient in the same way; patterns whose held and induced moments, summed per unit E, differ along the gradient fall differently where they sit, while the universality of free fall stays the gravitational window's (0.1c). The settlement: every driven or pumped population of this platform is a system-wide, many-to-many settlement of the state it starts from, as is every reaction network read through 0.1a, and nowhere is a process run backward the forward event's mirror. The pair evaluation of the system chart is the law that holds two planets, evaluated on two nuclei on its declared domain (I-1).

7 System-level reading (exploratory)

Scope: one observable, X-1, uncounted with the predictions; every statement below is an in-sample re-description of the source or a held-out case, and no number is printed.

X-1. The sign of the hyperfine term across joined pairs. Observable: the direction of the field slope of the ground-state NMR line of another adatom of this platform, the source's "Cu" (SC-024, L69–L71), read as the source reads ^{47}Ti's (SA-48). Defining condition: CI-1 read on the two spins, the antiparallel pair the lower; and, set beside it, CI-1 read on the two moments. Comparator: the consensus sign, the sign of the nuclear moment times that of the electron's spin density at the nucleus (U-22, external). ^{47}Ti on oxygen satisfies both readings: its $\mathcal{A}_{zz}$, positive, "preferring antiparallel orientation of electron and nuclear spins" (SC-181, L425–L426), and, its moment antiparallel to its spin (U-01) as the electron's is to its own, its two moments antiparallel. Held-out cases, each external: hydrogen's 1s ground state, the two spins opposite and the two moments parallel (U-16); ^{3}He^{+}'s 1s ground state, the two spins parallel and the two moments parallel (U-14) [8]; and ^{57}Fe in iron, the field at the nucleus opposite to the electron's moment and the two spins parallel (U-15) [9]. Of the two readings of CI-1, hydrogen meets the one on the spins alone, iron the one on the moments alone and the helium ion neither, and the slope of another adatom's ground-state line is where this platform would tell them apart.

Guards. X-1 is not counted with the predictions and carries no number. Its two readings rest on CI-1 alone. No sign is derived from the two cones, and the sign of a pair the settlement does not join is not assigned (O-5).

8 Convergence statement

The limiting map is the identity of Use item 2. On the source's inputs and dictionary—its Eq. (1) with the fitted terms—FDT and the consensus calculation return the same line positions. Every position, sum, difference, crossing field, field at the nucleus and relay ratio computed here from Eq. (1) with the fitted terms and the deposit's inputs (U-24 to U-27), in displays (14) to (19), is an identity on Eq. (1) with those inputs and corroborates no invariant (SA-09, SA-36). The deposited spectra refitted (U-29 to U-34) recover the source's data: six ESR peaks spaced by 132.2 ± 0.5 MHz, the two lines of one nuclear step summing to 131.9 ± 0.3 MHz beside the fitted 132.4 ± 0.4 MHz, line I falling over the seven fields at $|\mathfrak{g}_N| = 0.36 \pm 0.04$ beside the source's 0.37 ± 0.04, and the amplitude ratio of lines II and I rising with the ESR amplitude. The I-1 pair evaluation is $G\mathcal{M}_1\mathcal{M}_2/d^2$ exactly on its domain, and for the same stress tensor and data the curvature is that of general relativity (I-3 containment; I-4's layer L2 at its leading weak-field order). The source's measurements are accepted in full. Six readings are declined, each in two sentences.

The source reads the electron Zeeman splitting as having "contributions from the external magnetic field (B) and the magnetic field produced by the spin-polarized tip (Btip)" (SC-061, SC-062, L197–L201; SA-19), two fields as entities. FDT reads both as one helical density gradient of the omnium where the

electron's closure sits (P-5; Use item 6), and the deposited slope and offset of the outer resonance are accepted in full.

The source reads the features "at low magnetic field" through eigenstates that "take the form of hybrid states" and a "magnetoresistive readout" that "is an electron spin projective measurement" (SC-070, SC-071, SC-072, L213–L218; SA-22). FDT reads the same record deterministically (I-4), the hybridization, in the manuscript's reading, as a shared bivector grade and the readout as projection and phase fixation, and the features and their absence at 200 mT are accepted in full.

The population model behind the readout, in the deposit's `Analysis/rate_equation_functions.py` (U-33), premises that a scattering and its reverse share each squared matrix element of one operator (SC-080, L238–L240; SA-25). FDT reads each pumped or driven step as a settlement of the state it starts from (0.1a: "Events do not commute, and a process run backward is never the mirror of the forward event"), and the relative ESR peak heights are accepted in full; T-5 is where the two readings part.

The source writes that it can "unambiguously determine the signs of the hyperfine and quadrupole interactions, which reveals the nature of the nuclear spin" (SC-157, SC-158, L335–L338; SA-44). FDT reads the signs as properties of the joined configuration, for "There are no intrinsic particle masses or properties" (I-5), and the two signs are accepted in full.

The source reads its $\mathcal{Q}_{zz}$, negative, as "preferring in-plane nuclear charge distribution" (SC-182, L426–L427; SA-49), a distribution inside the nucleus. FDT reads it as the nucleus's $l = 2$ moment, its skin against its own centre path reading the surrounding tide, the weight its charge (I-3a), for "A nucleus is one pattern; what a probe reconstructs is that pattern's response, not a distribution of constituents inside a boundary" (0.1b), and the sign is accepted in full.

The source writes that "Modulation of gN by an RF electric field in the" "junction is arguably weak, as nuclear spins are usually considered to be well shielded from electric fields by the electrons around the nucleus" (SC-204, L436; SC-211, SC-212, L498–L499; SA-55), the electrons a shield around a separate occupant. FDT reads the electron's closure and the nucleus as patterns of one medium, with no container distinct from its contents (0.1b); the argument rests on no measurement of the source, and that "the measured NMR remain in MHz frequency range" (SC-187, L436) "throughout the studied fields" (SC-188, L414–L415) is accepted in full.

A divergence is claimed only in T-1 to T-5, where FDT supplies an additional source (P-5), a constitutive input (I-7) or a changed observable dictionary (0.1a).

9 Status ledger

The table below states, once, every item to be derived that the analysis touches, in the core's words, the held-out targets, the five prediction blocks, the in-sample readings, the range-only tokens and what rests on a reading carried beside the core. The abstract's count of five predictions is read off its T rows.

item	status	where touched; what is left out
O-1	to be derived: the finite-core cutoff r_c, the regular core chart, or the outward phase-transfer law (0.3, I-6)	the interior domain $r_c \le r_f < R_s$; no value of r_c
O-2	to be derived: a geometrically constructed odds for the overlap-contrast application (A-3) (0.4, I-1a)	the A-3 token; no value, threshold, ordering, sign or trajectory
O-3	to be derived: a canonical odds for the confinement application (A-4) (0.4, I-5)	the A-4 token, the cone-position axis; no value, threshold, ordering, sign or trajectory
O-4	to be derived: the local profile $\rho_{\mathrm{loc}}(r_f)$ from the omnium constitutive law with boundary, surface and core data, reducing to the measured nuclear profile (0.6, I-3)	the two charts of the ^{47}Ti nucleus; no profile, no transfer map
O-5	to be derived: multipole and tidal remainders of the closed-form pair expression, and the reading they give of the band $R_H < r_f < R_s$ outside a rotating source's horizon; the overlap complex; how P-4's many-cone settlement signs a pair it does not join (I-1, 0.4, P-4)	the pair evaluation at monopole order, the two cones, the pairs of CI-4, the exploratory guards; no multipole remainder, no overlap complex, no sign of a pair the settlement does not join
O-6	to be derived: the maps from a charge, a circulating charge or a colour seat to an I-1 input; derivation of the A_2 seat table from cone geometry; the gluon octet's dynamics (I-1, I-5a, 0.1c)	the held moments of the helical window, read from data; no map
O-9	to be derived: the confinement operator D, its domains, boundary data and spectrum; every mass ratio, including the two held-out targets (I-5)	the eigenproblem, rest energies and held moments read from data; no form of D, its domains, boundary data or spectrum
O-12	to be derived: the phase-exchange rates Q_a^{ν} and the decay kernel, spectrum, chirality and rates (I-5b, I-6)	the lifetime and the relaxation time the source cites, the rates of T-5; no rate, no coherence time

item	status	where touched; what is left out
held-out	the sign of the hyperfine term from the two cones	no derivation written; the source's sign is the datum
held-out	the two mass ratios of I-5, m_p/m_e and $(m_\mu/m_e)^2$, tests a derived operator must reproduce	never quoted as outputs
T-1	prospective; existence (no twist kernel declared under P-5)	convergence value: $\mathfrak{y}_{\mathrm{tw}}$ zero within the leak $\mathfrak{c}_{\mathrm{tw}}$
T-2	prospective; as T-1, on atoms without nuclear spin	convergence value: $\mathfrak{y}_{\mathrm{tw0}}$ zero within $\mathfrak{c}_{\mathrm{tw}}$
T-3	prospective; a positive density-channel term, its sign the manuscript's reading (S_{hf} undeclared)	convergence value: $\mathfrak{y}_{\mathrm{hf}}$ inside the interval of $\mathfrak{c}_{\mathrm{iso}}$
T-4	prospective; a positive density-channel term, its sign the manuscript's reading (S_g undeclared)	convergence value: $\mathfrak{y}_g$ inside the interval of $\mathfrak{c}_{\mathrm{iso}}$
T-5	prospective; existence, its sense unassigned (no rate written, O-12)	convergence value: $\mathfrak{y}_{\mathrm{rev}}$ zero, inside the interval of $\mathfrak{c}_{\mathrm{rev}}$
in-sample	every line position, sum, difference, crossing field, field at the nucleus and relay ratio from Eq. (1) with the fitted terms and the deposit's inputs (U-24 to U-27); the deposit's overlays (U-35); the refits (U-29 to U-34)	the identity map of the convergence statement
range only	the tokens of A-3, A-4 and A-5	token and range only
CI-1 to CI-4	what rests on them: K7, K8, K11, the cone-position translation, the exploratory section	each given the reading it names

10 Conclusions

The source reports nuclear magnetic resonance on one ^{47}Ti atom on MgO, read through the relative heights of its ESR peaks: the NMR features labelled I and II, observed for all fields from 200 to 1400 mT, a fitted hyperfine term 132.4 ± 0.4 MHz, a fitted quadrupole term -2.9 ± 0.8 MHz and, from the slope of the found NMR transitions as a function of magnetic field, an experimental nuclear g-factor 0.37 ± 0.04, with the signs of the two terms. FDT, a true extension of general relativity and quantum mechanics, recovers all of it on the source's inputs: the sum of the two lines is the hyperfine term, 131.9 ± 0.3 MHz in the deposited spectra, and the electron's field at the nucleus is -23 ± 3 T. In the manuscript's reading the nucleus is read on the interior cone of its two-chart atlas and the electron's closure on the exterior cone, the source's positive hyperfine term, preferring antiparallel orientation of electron and nuclear spins, is the datum CI-1 reads, the pumped seating is driven and not settled, and the quadrupole term is the tide the nucleus's skin reads against its own centre path, the weight its charge. Where FDT supplies what the consensus calculation lacks, it predicts: the field of a helical geometry at an atom with and without nuclear spin (T-1, T-2), a density-channel departure of the hyperfine term and of the outer resonance between ^{49}Ti and ^{47}Ti on one site (T-3, T-4), and a driven step and its reverse read as two settlements (T-5); each is a prediction of existence in T-1, T-2 and T-5 and of a sign the manuscript reads in T-3 and T-4, decided by a sign-flip test of exact size, and the status ledger states what each rests on.

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