What a spin spiral measures in a diamond sensor

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Abstract

Put et al. [1] refocus a spatially wound spin polarization in diamond. During interaction, the fitted transverse precession rate reverses sign when the winding direction turns from parallel to perpendicular to the selected crystal axis (SC-066, SC-108, SC-109). The finite sample enters twice, through the spins that generate the exchange field and through the spins whose phase the detector collects. The spherical model separates its angular dependence from a radial integral whose low-wave-vector and large-volume limits differ. With the paper's spin definition the legend of the reported twisting fit prints half the classical slope of its one-axis-twisting Hamiltonian (SC-171 through SC-175). Fundamental Density Theory (FDT) is a true extension of general relativity and quantum mechanics: general relativity is FDT in its spacetime regime and quantum mechanics is FDT in its matter regime, so wherever either holds, FDT returns its results. Agreement with the record is therefore the expected outcome, and FDT departs only where it supplies an input the standard account lacks. It reads the applied magnetic field as a macroscopic helical density gradient and recovers the measured record for the same source and data. On the calibrated spin and optical readouts, five proposed comparisons yield zero prospective predictions. In each, every contribution that FDT and the conventional calculation both specify takes one value in both. The torsion field that FDT assigns to a static twist, the load response of the defect levels and the causal aperture enter no number here.

1 Framework

The source status of Put et al. [1] is experimental. Measured and observed refer to instrument readings, and reported preserves the authors' stated result or setting. Fitted identifies a fit to those readings. Derived, estimated, reconstructed and illustrative-assumed distinguish calculation, calibration, inversion and an investigator's choice. The target identification is SC-001 through SC-010. The source gives its supporting data deposit as Ref. 76 (SC-325, SC-326) [6].

1.1 The common rate and the configuration

The omnium is one substance. Its energy E, mass m, distance d and duration t read one state. Its electric and magnetic response coefficients are ε_0 and μ_0. Its strand rate is c. The ordinary

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and reduced action quanta are h and $\hbar$, with π the circle constant. For mode label q, write $E_q, m_q, \omega_q, T_q, \ell_q$ for rest energy, confinement mass, angular rate, period and distance the thread runs in a period. The matter leg uses a massive mode. The two electromagnetic response equations give a wave equation with coefficient $1/(\varepsilon_0\mu_0)$. The bridges of 0.1a and 0.1d give the following identity with the exact strand rate (U-01), whose value counts metres of strand run per second of kept time.

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0\mu_0} = \left(\frac{d}{t}\right)^2, \qquad c = 299\,792\,458\ \mathrm{m\,s^{-1}}.$$

$$E_q = m_q c^2 = \hbar\omega_q, \qquad T_q = 2\pi/\omega_q = h/E_q, \qquad \ell_q = cT_q = h/(m_q c).$$

For an open stroke the period and wavelength step with energy at the fixed ratio c. The Compton thread length of a massive mode and the support of its closed pattern are independently defined. The force ceiling $F_{\max}$ times a floor length and duration gives $\hbar$. Their ratio is c. Write $\ell_{\mathrm{P}}, t_{\mathrm{P}}$ for the Planck length and time. The resulting length and time infima are $2\ell_{\mathrm{P}}$ and $2t_{\mathrm{P}}$. No stroke occupies the infimum (0.1a). Every step is one system-wide, many-to-many settlement of the state the system is in, and events do not commute.

Write v for the translation speed of a closed pattern and u for its strand's circulation speed. The helix gives $v^2 + u^2 = c^2$. Include closure stress in the inertia. For a stationary, localized closed pattern with vanishing first moment of the surface traction in its rest frame, stress-energy conservation gives zero integrated spatial stress in its rest frame, and a boost gives inertia per energy $1/c^2$ (0.1d). The translation share v^2/c^2 is kinematic. It equals the exterior compactness only for a pattern held on the exterior branch in the flow reading. An open stroke transports energy along its straight run at c and keeps no time. Its grades turn at angular rate ω on a helix of radius c/ω, advancing a wavelength per turn at pitch angle $\pi/4$. For a massive phase of axial wavenumber k_{mode}, the dispersion $\omega^2 = c^2k_{\mathrm{mode}}^2 + (mc^2/\hbar)^2$ puts every term in inverse seconds squared. The phase rate ω/k_{mode} exceeds the run $d\omega/dk_{\mathrm{mode}}$ and their product is c^2. Their equality in the open stroke has no bearing on the prepared NV cone angle.

1.2 Chart inventory and force

Let $\mathcal{M}$ be total gravitating mass, G the gravitational coupling, R_s its Schwarzschild radius, r_f the field radius, R_{b} the material support radius and r_c the lower radius of the interior atlas. Every α_σ below has branch σ, with α_{ext} and α_{int} the exterior and interior ratios. Their positive gap odds are X_{ext} and X_{int}. Definition 0.3 takes the ratio of circular curvatures fixed by the two lengths.

$$R_s = 2G\mathcal{M}/c^2, \qquad \alpha_\sigma(r_f) = (R_s/r_f)^\sigma, \qquad \partial_{r_f}\alpha_\sigma = -\sigma\alpha_\sigma/r_f.$$

$$\alpha_{\mathrm{ext}} = R_s/r_f, \quad X_{\mathrm{ext}} = \frac{R_s}{r_f - R_s}; \qquad \alpha_{\mathrm{int}} = r_f/R_s, \quad X_{\mathrm{int}} = \frac{r_f}{R_s - r_f}.$$

The exterior domain has $r_f > R_s$ and $\sigma = +1$. The interior domain has $r_c \le r_f < R_s$ and $\sigma = -1$ (O-1). These are reciprocal parameterizations. Both approach unity from below at the fold. Simultaneous dilation of both lengths leaves each ratio fixed. At a fixed source the atlas is continuous over discrete pattern content. A change of spin direction at fixed mass and radius leaves this coordinate fixed.

Application	Geometric warrant and use
A-1	Compactness. At the field point use the specified r_f, at the support evaluate at R_b on its declared branch, and at the other body's centre use the actual separation. The source fixes R_s. Positive gap odds X give $X/(1+X)$ exactly, with no calibration.
A-2	The computational coordinate $\beta_\sigma = \alpha_\sigma/(1+\alpha_\sigma)$ has geometric warrant and stays out of every force, optical and density slot.
A-3	Overlap-contrast token α_ovl, open interval only (O-2).
A-4	Confinement-position token α_conf, open interval only (O-3).
A-5	Mode name α_mode, open interval only. It orders no carrier modes.
Skin	$\alpha_\mathrm{core} \to 1^-$ is a schematic skin limit on the configuration's interior chart. The laboratory spin image supplies no value.

A-3 through A-5 lack geometric warrant for numerical assignments. On A-1 the ratio is already bounded by the domain. Odds is algebra here. For an odds-enhancing factor $1+x$ with $x \geq 0$, write b_0 for the initial A-1 value and b for the updated one at fixed branch. I-6 gives $b = b_0(1+x)/(1+b_0x)$, equivalently $b - b_0 = b_0(1-b_0)x/(1+b_0x)$. The enclosed-mean comparison density scales as $\alpha_\sigma^{3\sigma}$ at fixed mass and shape. Inside the body that comparison is a dilution proxy. Local density rises toward the skin and is dilute at the centre (0.6). No local material profile is inferred here (O-4).

Write F for the force magnitude and $\alpha_\mathrm{left}, \alpha_\mathrm{right}$ for its two density inputs, each on its declared chart. In the pair evaluation let $\mathcal{M}_1, \mathcal{M}_2$ be the two masses, d_centres their centre separation, and $\alpha_{1,\mathrm{ext}}, \alpha_{2,\mathrm{ext}}$ their A-1 exterior cross-centre inputs. I-1 gives the universal law $F = (c^4/4G)\alpha_\mathrm{left}\alpha_\mathrm{right}$. The monopole evaluation is

$$F_\mathrm{max} = \frac{c^4}{4G}, \qquad F = F_\mathrm{max}\alpha_{1,\mathrm{ext}}\alpha_{2,\mathrm{ext}} = \frac{G\mathcal{M}_1\mathcal{M}_2}{d_\mathrm{centres}^2}.$$

The universal law is the force ceiling times its two density inputs at all scales. The last equality uses two nonoverlapping spherical monopoles, each support at least its own Schwarzschild radius, and separation at least the sum of the support radii. The open interval is universal. Its inputs give $F < F_\mathrm{max}$. On the pair domain the sum of the two compactnesses is at most unity, so their product is at most one quarter. Equality in $F \leq F_\mathrm{max}/4$ requires equal saturated supports in contact. Outside that evaluation no multipole or overlap remainder is calculated here (O-5). The small gravitational compactness of a laboratory device is its correct gravitational reading. Its measured spin rate requires a response with frequency dimensions. Between two defect moments the universal law acts at the input pair that a circulating charge maps to, and their coupling energy is its open line integral; that map enters no number here (O-6), and the source's dipolar Hamiltonian carries the measured coupling.

1.3 Gravitational, medium and helical readings

Let $G_{\mu\nu}$ denote the Einstein tensor, $T_{\mu\nu}$ the full stress tensor, κ the Einstein coupling, $g_{\mu\nu}$ the metric, $d\ell_\mathrm{opt}$ an optical length and $d\ell_\mathrm{ang}$ the unit sphere's angular line element. Greek indices μ, ν denote spacetime components. I-3 leaves the tensor equation unchanged. The spherical exterior optical metric in areal radius follows from the solution.

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G/c^4, \qquad d\ell_\mathrm{opt}^2 = \frac{dr_f^2}{(1-\alpha_\mathrm{ext})^2} + \frac{r_f^2\, d\ell_\mathrm{ang}^2}{1-\alpha_\mathrm{ext}}.$$

At an interface let $\mathfrak{n}_{\text{in}}, \mathfrak{n}_{\text{out}}$ be the exact optical indices and $\theta_{\text{in}}, \theta_{\text{out}}$ the ray angles to the normal. Fermat's law gives $\mathfrak{n}_{\text{in}} \sin\theta_{\text{in}} = \mathfrak{n}_{\text{out}} \sin\theta_{\text{out}}$. Write O for asymptotic order. The weak-field isotropic index begins at $1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2)$. The lapse alone gives half the leading bend. The radial ruler supplies the other half. The P-4 aperture index is this metric's index across the density gradient (O-8) [8] and has no numerical role in the device optics. For identical stress tensor, cosmological terms, initial and boundary data and gauge the curvature is the same. The exact pair algebra belongs to I-4 layer L1. Layer L2 recovers the Newtonian acceleration from GR to leading order in compactness and speed squared over c^2 in the weak, slow limit. An additional observable response needs an independent source, constitutive law or measurement dictionary, the L3 question.

Let $\mathfrak{m}$ denote a positive clock factor, $\mathcal{U}$ a pattern's four-velocity measured in its own time and $\mathfrak{P}^{\mu\nu}$ the transverse projector multiplied by the squared strand rate. Covariant differentiation along the pattern is $\nabla_{\mathcal{U}}$. Its kept time and thread length rescale together. Motion at constant speed on its own clocks and rulers through a density gradient is gravity (0.1c). The bookkeeper's acceleration follows from the conformal metric.

$$g_{\mu\nu} \mapsto \mathfrak{m}^2 g_{\mu\nu}, \qquad \nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathfrak{m}, \qquad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu.$$

The conformal change preserves null paths. A slow pattern in the exterior reads the lapse. A stroke reads the optical metric. Curvature is read between neighbours or against a reference path. An extended pattern compares skin and centre through its quadrupole moment. The trace reads density (I-3a). Free fall from rest far away has inward flow speed $c\sqrt{\alpha_{\text{ext}}}$ and acceleration magnitude $G\mathcal{M}/r_f^2$ along the flow. Rotation adds a swirl. To leading Lense–Thirring order a gyroscope measures half the spatial curl of the swirl of the bookkeeper's flow. The free-fall four-velocity is irrotational. A material medium has its own vorticity. These readings apply at every size.

For a perfect stress tensor and zero net conserved count, temperature T sets the flow's clock factor up to a constant reference. With conserved count and fixed entropy per count it is the enthalpy per count. Neither prescription gives the transition gap of a carried clock. That clock samples the local lapse and its own dynamics, including the time dilation of its thermal motion. In the helical window let E_0 be the moving pattern's rest energy outside a twist, E_{ind} the induced-moment energy including the work of induction, j a site index, m_j the held moment of site j at its settled value and $\mathbf{B}_j$ the total twist field at site j. The longitudinal coordinate is z. For positive, event-fixed rest energy the pattern's clock factor determines its conformal equation of motion (0.1c). The force equation shown here is its slow-motion limit:

$$E = E_0 + E_{\text{ind}} - \sum_j \mathsf{m}_j \cdot \mathbf{B}_j, \qquad \mathfrak{m} = E/E_0, \qquad \mathcal{M}\, d^2z/dt^2 = -\partial_z E.$$

The whole moving body has rest mass $\mathcal{M}$ equal to its rest energy divided by the squared strand rate. The longitudinal acceleration is $-\partial_z E/\mathcal{M}$. At fixed held moments and external settings the derivative includes the gradient of the induced energy with its induction work. Compare the preparations at the same event, along the same gradient direction. Their slow accelerations differ when their net projected energy gradients per whole-body rest energy differ. Gravitational monopole free fall remains universal. A uniform twist turns a held moment and gives no translational push. A spatially uniform addition to the rest energy changes the inertia and leaves the magnetic force unchanged. A neighbouring pattern, a different moment preparation or an independent path provides the reference.

Let M be the baseline rest contribution, P pressure, V volume, σ_{SB} the thermal flux constant and a the radiation energy-density constant. In the following I-7 ledger E_{add} denotes disjoint stored internal energy increments. The stress tensor defines the total.

$$\mathcal{M} = M + \frac{E_{\mathrm{add}} + PV + aT^4V}{c^2} + \cdots, \qquad a = \frac{4\sigma_{\mathrm{SB}}}{c}.$$

Choose a boundary around the diamond, its elastic support and the portion of the electromagnetic field under consideration. Count stored strain, thermal, spin and field energies once. A work term represented by a stored elastic energy has already entered. Equilibrium radiation contributes aT^4V where thermal radiation occupies that volume. The driven laser and outgoing fluorescence enter through actual field energy and boundary flux. External mechanical and electromagnetic work and exported heat and light balance changes inside. Gravitational binding is read from the solved mass aspect. The spatially varying spin polarization fixes none of these totals.

1.4 Carrier, closure and spin coherence

One medium and one strand rate give the I-2 carrier identity: photon, graviton, gluon, phonon and magnon are the open spin-one excitation in different geometries. The optical pump and fluorescence occupy the optical window, each photon one unbound strand. A magnetic field is a macroscopic helical density gradient (P-5). The pulsed spin texture records a controlled distribution of internal phases. Its spatial pitch has no relation to the open stroke's pitch angle. The matter participating in the transition consists of closed patterns. For confinement domain Ω, its boundary data, integer strand count n and specified configuration, let D denote the confinement operator and u_q an eigenmode. The eigenproblem $Du_q = \omega_q u_q$ specifies the mode cost. Mass is a confinement eigenvalue. This experiment measures no confinement size for the defect mode, and no form of D, its domains, boundary data or spectrum is written here (O-9). A change of confinement at fixed n preserves class. A change of n changes class by rebraiding. A microwave rotation of a defect subspace assigns neither class nor a new strand count.

For the quantum face I-4 let ϕ_h be the phase of a projected helix, ϕ_0 its initial value and $\mathcal{B}$ the oriented unit bivector of the plane spanned by orthogonal unit vectors $\mathbf{e}_1, \mathbf{e}_2$. In cylindrical coordinates the phase rotor obeys

$$\phi_h(t) = \phi_0 - \omega t, \qquad \mathcal{B} = \mathbf{e}_1\mathbf{e}_2, \qquad e^{\mathcal{B}\phi_h} = \cos\phi_h + \mathcal{B}\sin\phi_h.$$

The bivector squares to minus unity. This is a single-angle phase rotor. A spinor rotation uses a half-angle. Phase ignorance at observation is superposition on this reading. The ensemble Fourier first moment below requires site preparation, evolution, collection and a phase reference. Its two quadratures are experimentally distinct components of one complex number. A single microscopic phase cannot be reconstructed from that number.

1.5 Notation guard

In the tables below a printed token in the first column belongs to Put et al. The last column fixes the name used here (SG-01 through SG-44). The ensemble size is N. Site indices are i, j, with operators $\mathsf{s}_i, \mathsf{s}_j$. Hats indicate unit directions. Superscripts on spin components refer to the preparation frame. The scalar k denotes a spatial wave-vector magnitude in the spin measurement. $\mathbf{k}$ denotes its vector. The phase convention is positive for winding and negative for the analysis winding.

Printed	Quantity	Name and units here
Alpha	Group, decay power	i_{group}, p_{dip}, respectively.
Gamma	Moment response, spiral decay	g_{NV} in rad/s/T, Γ_{sp} the reconstructed e-folding rate in inverse time.
Phi	Precession angle	Φ_{sp} in radians for the complex phase.
Omega	Shape	Omega-shaped microwave loop.
a	Spacing, radial cutoff	ℓ_{sep}, ℓ_{lower}. The spherical cutoff can be set to the spacing.
S	Site, collective, Fourier spin	s_j, $\mathsf{s}_{\mathrm{coll}}$, $\mathsf{s}_{\mathbf{k}}$, all dimensionless spins and their sums.
B	Coil, exchange field	$\mathbf{b}_{\mathrm{coil}}$ in tesla, $\mathbf{b}_{\mathrm{ex}}$ in energy units.
D, R^*	Spatial dimension, sphere radius	d_{sp}, ℓ_{pol}.
T, T'	Gradient durations	$\tau_{\mathrm{w}}, \tau_{\mathrm{u}}$ for winding and analysis.
Calligraphic T	Schematic torque	Torque on the central spin.
W_{typ}	On-site disorder	w_{dis} in the stated frequency convention.
X	Laboratory axis label	Preparation-frame x axis or microwave pulse X axis.

Printed	Quantity	Name and units here
Q, Q'	Wave vectors	$\mathbf{k}_{\mathrm{w}}, \mathbf{k}_{\mathrm{u}}$ in radians per length.
Theta	Preparation cone angle	ϑ_{p}, distinct from the direction of propagation.
Chi	Twisting rate, exchange integral and kernel	χ_{tw} (angular rate), J_{sp} (energy), K_{sp} (energy per contributing spin).
Plain and script A	Angular polynomial and coupling	P_2 is dimensionless. J_{ang} is an angle-dependent energy times volume.
G	Shell weight	g_{shell}, dimensionless.
J family	Wire currents, dipolar scale, pair and typical coupling	i_{coil} with wire index (A), J_{dip} (energy times volume), J_{pair} (energy), J_{typical} in the reported units.
J in Fig. 9	Gradient normalizer	J_{fig9}, the coupling reference for an energy difference over a stated interval.
Rho	Coarse and uniform polarization density	$\rho_{\mathrm{spin}}, \rho_{\mathrm{uniform}}$, spins per volume. Optical weights are separate.
Omega, lower case	Angular precession slope	ω_{sp}, radians per unit time.
r family	Site, pair, radial shell	$\mathbf{x}_{\mathrm{site},j}$, $\mathbf{d}_{\mathrm{pair},ij}$, s_{shell}, all lengths.
Tau	Pump time, normalized pump time, pulse gap	t_{pump}, $t_{\mathrm{pump}}/t_{\mathrm{sat}}$ and t_{gap}. t_{sat} is the calibrated saturation time.

2 System chart

Optical pumping initializes the electronic ground-state spins of the NV centres in the measurement volume. Repeated resonance scans align the bias field to the selected crystallographic axis $\hat{\boldsymbol{\eta}}$ (SC-014, SC-015, SC-284, SC-285). The two-level manifold uses magnetic quantum numbers $0, -1$ of the ground-state triplet (SC-032, SC-036, SC-037). The bias, drive and detection volume belong to the apparatus. The chart entries below specify what each physical length measures.

Object	Evaluation and accessible information
Device exterior	A-1 at the field point for the full gravitating source. Coil field, exchange coupling and drive power alone do not specify the source mass.
Bound configuration	Separate interior chart with its skin at its own fold and its physical skin radius fixed by geometry when measured. No transfer to the gravitational fold of the exterior atlas.
Bulk polarization	Disk-shaped support bounded axially by growth and transversely by pumping (SC-115, SC-246). Fourier imaging measures a weighted marginal. Its edge is not a material skin.
Beam polarization	Triangular beam with a finite illuminated support and nonuniform pumping. The physical width, the axial extent of the beam and the distribution of polarized sites are distinct (SC-018, SC-019, SC-129, SC-236).
Discrete model	Positions, density, pair cutoff and collection volume refer to separate parts of the spin forward calculation (SC-299 through SC-308).

An interior reading averages the response with its named probe weight. One projection leaves the directions it integrates over undetermined even at perfect signal to noise. A local omnium density, a torsion field and the site's confinement address cannot be extracted from an optical marginal.

3 Observation map

The map below anchors each reading to Put et al. Each comparison with a fresh reading uses independent settings or a held-out acquisition at fixed preparation, device geometry, probe weights, calibration and previous drive history. Its uncertainty includes correlations between the forecast and the new reading. The finite-error rule in the Predictions section applies in the physical units of each comparison, including uncertainty in the ancillary data. A printed precision or nominal image resolution cannot supply this uncertainty.

Source anchor	Instrument or estimator	Scope
Eq. (3), Sec. VIII H	ESR frequencies and fitted gradients	Spatial derivative of an averaged field. Beam inversion uses the calibration detunings (SC-025, SC-026, SC-281).
Fig. 2, Sec. III	Measured Fourier quadratures	Each analysis wave vector selects one complex Fourier component. Sweeping that vector and inverse-transforming the quadratures reconstructs the weighted projection (SC-057 through SC-071).
Eqs. (2), (4)–(7)	Uniform control and angular slopes	Collective symmetry, angular averaging and a fitted precession rate have distinct premises (SC-078 through SC-084, SC-099, SC-110).
Fig. 4, Eqs. (8)–(12)	Fitted bulk rate, beam amplitude and reconstructed profiles	Distinct ordinates, finite-size model and angular normalization (SC-113 through SC-134, SC-146 through SC-162).
Fig. 5, Eq. (13)	Tomography, fitted slopes and decay	Angular normalization, restricted Ising comparison and scaling hypothesis (SC-163 through SC-204).

3.1 Calibration before winding

A 532 nm green pump prepares the selected spins. The red fluorescence is collected through the same objective in a confocal setup under ambient conditions (SC-257, SC-259, SC-033). The field samples in Fig. 1(e) average over a 200 nm radius (SC-026). The bulk inversion fits ESR spectra for all groups, fits a local quadratic field to obtain in-plane derivatives and infers out-of-plane components from Maxwell constraints (SC-273 through SC-277). The beam calibration compares detunings with a finite-element model initialized by confocal positioning. It adjusts the beam translation and field scale, shifting the inferred position by less than 0.6 µm (SC-278 through SC-283). Those detunings enter the fit. A held-out test needs a separate scan.

For the selected axis define $\mathbf{g}_B$ as the spatial gradient of the projected coil field. Let y_{tog} denote the microwave toggling sign and t' a time of integration. The winding coefficient is g_{NV} in angular frequency per tesla. With the signed coil current retained, Eq. (3) and the winding prescription (SC-042, SC-052 through SC-055) give

$$\mathbf{g}_B = \nabla(\mathbf{b}_{\mathrm{coil}} \cdot \hat{\boldsymbol{\eta}}), \qquad \mathbf{k}_{\mathrm{w}} = g_{\mathrm{NV}} \int_0^{\tau_{\mathrm{w}}} y_{\mathrm{tog}}(t')\, \mathbf{g}_B(t')\, dt'.$$

The result has units of radians per metre. The inverse-analysis waveform defines $\mathbf{k}_{\mathrm{u}}$ with the sign convention in the coherence below. A positive longitudinal Hamiltonian convention gives the Zeeman energy coefficient $\hbar g_{\mathrm{NV}} \mathbf{g}_B \cdot \mathbf{x}_{\mathrm{site},j}$ for s_j^z. The paper gives $J_{\mathrm{dip}} = \hbar\, 2\pi (52\,\mathrm{MHz\,nm}^3)$ (SC-038). At the reported maximum of approximately 2.2 mT/µm (SC-045, SC-046), Put et al. compare typical neighbours separated along the gradient and state that their Zeeman shift difference exceeds the dipolar interaction by at least one order of magnitude (SC-047, SC-048). Compare $g_{\mathrm{NV}}|\mathbf{g}_B|\ell_{\mathrm{sep}}$ and $J_{\mathrm{dip}}/(\hbar \ell_{\mathrm{sep}}^3)$, both in rad/s. For a numerical Fig. 9 ordinate specify the separation vector $\mathbf{l}_{\mathrm{cal}}$ and use $\hbar g_{\mathrm{NV}} |\mathbf{g}_B \cdot \mathbf{l}_{\mathrm{cal}}| / J_{\mathrm{fig9}}$ with the reference coupling in energy units (SC-286). A field gradient and a frequency alone have a dimensionful ratio.

The bulk density of 246 ppb per group is estimated from the XY16 decay timescale of 21.5 µs using a simulation-based conversion and assuming dipolar dominance (SC-264 through SC-267). The independently characterized beam has approximately 3.8 ppm per group (SC-270). Typical spacings are about 28 nm and 11 nm, respectively (SC-020, SC-021). The source's open-boundary discrete truncated Wigner simulation uses 1572 random bulk spins in a cylinder of diameter 500 nm and height 185 nm, and 6750 spins for the beam (SC-299 through SC-303). The latter's model length is twice its 300 nm width (SC-304, SC-305). Its ultraviolet cutoff is 0.2 times the typical spacing, reported as 6 nm in bulk and 2.2 nm in the beam (SC-306 through SC-308). Imaging resolution is reported below 20 nm (SC-205). A cutoff, an interspin spacing, an imaging resolution and an uncertainty on thickness answer four different questions. The field calibration is an in-sample reconstruction and corroborates no invariant (Use item 2). A fresh resonance difference across a known baseline tests its signed prediction in hertz beyond the combined uncertainty.

3.2 Refocusing and the uniform control

A global microwave rotation about the preparation y axis sets the cone before winding. The two equal winding halves have opposite gradient polarities and a microwave half-turn between them (SC-052, SC-053). The static site detuning integrates to zero while the switched gradient adds. The spins enter the quench at the common cone angle ϑ_{p} and at site-dependent azimuths. Periodic driving for the chosen quench duration changes the interaction, and inverse winding

followed by the global quadrature measurement selects the final mode (SC-050 through SC-063). The native bulk uniform-mode control uses XY-16-type decoupling (SC-079). For SU(2) dynamics (SC-082), the bulk uses cXY8-DROID-vXY4-Mirror with a cosine envelope, a 40 ns half-turn and 10 ns spacing (SC-292, SC-293, SC-311). The beam uses cXY4-DROID-vXY4-Sym with a Gaussian envelope, a 30 ns half-turn and 30 ns spacing (SC-294, SC-295, SC-318). The buffers per gradient edge are 50 ns in bulk and 10 ns in the beam (SC-296 through SC-298). These intervals belong to control timing.

Let $\mathcal{C}$ be the complex detected transverse moment, ϖ_j each site's real weighted transverse amplitude, ϕ_{ref} the common prepared phase relative to the analysis reference and i the imaginary unit. Summing site phases immediately after inverse winding at zero quench gives the overlap derived from SC-057 through SC-062.

$$\mathcal{C} = e^{\mathrm{i}\phi_{\mathrm{ref}}} \sum_j \varpi_j e^{\mathrm{i}(\mathbf{k}_{\mathrm{w}}-\mathbf{k}_{\mathrm{u}})\cdot\mathbf{x}_{\mathrm{site},j}}.$$

For each nonlinear waveform, integrate its actual signed coil field projected on the selected axis at each site, weighted by the microwave toggling sign, and multiply by g_{NV}. Add the two physically signed site phases. In the subtraction convention above, the analysis phase is the negative of the signed unwinding phase. At equal profiles the spatial phases cancel. A common phase rotates the two quadratures and leaves the magnitude unchanged. For the equatorial bulk spiral of pitch 0.242 µm, the measured magnitude revives near matched wave vectors, with approximate width $2\pi/(0.4\,\mu\mathrm{m})$ and a Gaussian fit (SC-061, SC-065 through SC-069). The width reflects finite polarized extent. Winding loss during that preparation, coherent phase evolution during a quench and decay of contrast during the quench have separate controls. The authors prepare two antipodal pairs to suppress spurious global rotations and report the antipode-averaged imaginary Fourier component after unwinding (SC-072 through SC-076).

Let $\mathbf{d}_{\mathrm{pair}}$ be a pair displacement, $\hat{\mathbf{d}}_{\mathrm{pair}}$ its direction, superscript T a transpose and I the spatial identity tensor. After taking out a positive prefactor, Eq. (1) prints $3\mathsf{I} - \hat{\mathbf{d}}_{\mathrm{pair}}\hat{\mathbf{d}}_{\mathrm{pair}}^{\mathsf{T}}$ (SC-030, SC-031). Its trace is 8, whereas the magnetic tensor $\mathsf{I} - 3\hat{\mathbf{d}}_{\mathrm{pair}}\hat{\mathbf{d}}_{\mathrm{pair}}^{\mathsf{T}}$ has trace zero. A nonzero overall scalar leaves this discrepancy. The effective-model comparison therefore starts at the independently stated Eqs. (2) and (4). For pair distance $d_{\mathrm{pair},ij} = |\mathbf{d}_{\mathrm{pair},ij}|$ and direction cosine $\xi = \hat{\mathbf{d}}_{\mathrm{pair}} \cdot \hat{\boldsymbol{\eta}}$, the angular polynomial is $P_2(\xi) = (3\xi^2 - 1)/2$ (SC-040). Let H_{nat} and H_{iso} be the native and engineered spin energies. With the pair dependence of each angular factor understood, SC-035, SC-081 and SC-082 give

$$H_{\mathrm{nat}} = \sum_{i<j} \frac{2J_{\mathrm{dip}}P_2(\xi)}{d_{\mathrm{pair},ij}^3}\left(\mathsf{s}_i \cdot \mathsf{s}_j - 2\mathsf{s}_i^z\mathsf{s}_j^z\right), \quad H_{\mathrm{iso}} = \sum_{i<j} \frac{J_{\mathrm{ang}}(\xi)}{d_{\mathrm{pair},ij}^3}\mathsf{s}_i \cdot \mathsf{s}_j.$$

Here $J_{\mathrm{ang}}(\xi) = 2J_{\mathrm{dip}}P_2(\xi)/3$ (SC-082). The native Hamiltonian conserves the axial collective component. Its angular factor integrates to zero with an isotropic weight. Neither fact sets every site's transverse torque to zero in a finite disk. Every pair scalar product in the engineered Hamiltonian commutes with each collective spin component. The two commutators rotating its two factors are equal and opposite. This holds for arbitrary scalar position-dependent coefficients. A fully aligned product is a triplet on each pair and thus stationary up to phase. At finite wave vector the position phases weight the two commutators differently. In the no-spiral control the reported antipode-averaged y signal has no observed time dependence. The finite-wave-vector traces evolve (SC-077 through SC-087). This cluster is a reading under I-4 and corroborates no

invariant (Use item 2). A held-out no-quench quadrature at a known mismatch tests the complex overlap, in normalized spin units, after propagating the uncertainties of both waveforms and the optical weight.

3.3 Direction, geometry and the radial integral

The central-spin construction of Eqs. (5)–(7) begins with the SU(2) interaction (SC-088, SC-089, SC-090 through SC-098). Set the central spin's azimuth to zero and call its neighbours' relative azimuths ζ_j. For central site i and neighbour j, let $\mathfrak{h}_j$ be the angular coupling $J_{\mathrm{ang}}(\xi)$ for that pair divided by the cube of its separation, times the magnitude of the neighbour's transverse spin polarization. In transverse complex notation their contribution to $\mathbf{b}_{\mathrm{ex}}$ is proportional to $\sum_j \mathfrak{h}_j(\cos\zeta_j + \mathrm{i}\sin\zeta_j)$. Setting an origin for the phase leaves the weighted sine sum intact. Inversion of the displacement with equal weights cancels it, as does a suitable ensemble average. Retain both sums in a finite beam until the preparation and collection weights justify that reduction. The exchange field here carries energy units. Dividing by the signed energy per tesla yields its magnetic-field equivalent. In the cosine reduction of Eq. (7), the per-spin kernel is $K_{\mathrm{sp}} = J_{\mathrm{ang}}(\xi)[1-\cos(\mathbf{k}\cdot\mathbf{d}_{\mathrm{pair}})]/(2s_{\mathrm{shell}}^3)$, integrated against $\rho_{\mathrm{spin}}\, d^3\mathbf{d}_{\mathrm{pair}}$ (SC-101 through SC-104). Only the exchange-field component perpendicular to the central spin drives its torque. Precession of the detected mode requires the weighted sum of changes at the emitting sites.

Put et al. change the current ratio on the bulk wires and observe opposite directions for parallel and perpendicular propagation at fixed preparation (SC-107 through SC-111). This is distinct from changing the hemisphere of the initial spins (SC-086, SC-112). In Fig. 3(k) the abscissa is $\arccos(\hat{\mathbf{k}}\cdot\hat{\boldsymbol{\eta}})$ in degrees and the magnitude is $2\pi/(0.242\,\mu\mathrm{m})$ (SC-110). Let $\xi_{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\boldsymbol{\eta}}$ and let $\omega_{\parallel}$ be the angular-rate prefactor of the solid line, its value at parallel winding. Its model is $\omega_{\mathrm{sp}} = \omega_{\parallel} P_2(\xi_{\mathbf{k}})$ (SC-111). The normalized angular factors are 1 for parallel winding and $-1/2$ for perpendicular winding, so their signed ratio is -2. The ordinates are the prefactor and its negative half. The zero between them is at $\arccos(1/\sqrt{3})$. The native interaction gives faster early-time precession and the engineered interaction eventually gives the larger amplitude (SC-135, SC-136). The authors attribute the first difference to Floquet rescaling of the exchange term by 1/3 and the second to conservation of the summed transverse spin components, which protects the spiral order from local relaxation (SC-137, SC-138). Their continuous easy-axis to easy-plane XXZ sweep is in Supplemental Fig. S3 (SC-139) [2].

For the ideal sphere put $s = s_{\mathrm{shell}}$ and fix $k = |\mathbf{k}|$. The normalized angular average of a plane wave against the traceless second Legendre harmonic retains only its degree-two component. Write j_1, j_2 for the spherical Bessel functions of orders one and two. The plane-wave coefficient contributes i^2. The angular average of the constant harmonic with P_2 vanishes. With $d\omega_{\mathrm{solid}}$ the solid angle on the displacement sphere, the remaining integral is

$$\int \frac{d\omega_{\mathrm{solid}}}{4\pi} P_2(\hat{\mathbf{d}}_{\mathrm{pair}}\cdot\hat{\boldsymbol{\eta}}) e^{\mathrm{i}\mathbf{k}\cdot\mathbf{d}_{\mathrm{pair}}} = -j_2(ks)P_2(\xi_{\mathbf{k}}), \qquad |\mathbf{d}_{\mathrm{pair}}| = s.$$

Consequently $1-\cos(\mathbf{k}\cdot\mathbf{d}_{\mathrm{pair}})$ gives $+j_2(ks)P_2(\xi_{\mathbf{k}})$. This is the radial weight $g_{\mathrm{shell}} = j_2$ of the unhalved angular filter in source Eq. (9) (SC-149, SC-150). For Eq. (7) there is a further half (SC-104). Write $x_{\mathrm{rad}} = ks$ for the dimensionless shell radius, with endpoints $x_{\mathrm{lo}} = k\ell_{\mathrm{lower}}$ and $x_{\mathrm{hi}} = k\ell_{\mathrm{pol}}$, and I_{rad} for the reduced integral. For a positive lower cutoff, the Bessel recurrence and differentiation give

$$\frac{d}{dx_{\mathrm{rad}}}\left[\frac{j_1(x_{\mathrm{rad}})}{x_{\mathrm{rad}}}\right] = -\frac{j_2(x_{\mathrm{rad}})}{x_{\mathrm{rad}}}, \qquad I_{\mathrm{rad}} = \int_{x_{\mathrm{lo}}}^{x_{\mathrm{hi}}} \frac{j_2(x_{\mathrm{rad}})}{x_{\mathrm{rad}}}\, dx_{\mathrm{rad}} = \frac{j_1(x_{\mathrm{lo}})}{x_{\mathrm{lo}}} - \frac{j_1(x_{\mathrm{hi}})}{x_{\mathrm{hi}}}.$$

The recurrence follows directly by differentiating $j_1(x_{\mathrm{rad}})/x_{\mathrm{rad}} = (\sin x_{\mathrm{rad}} - x_{\mathrm{rad}} \cos x_{\mathrm{rad}})/x_{\mathrm{rad}}^3$. The primitive tends to $1/3$ at zero and falls as the inverse square of its argument at infinity. The integrand is integrable at both ends. At small k and fixed cutoffs, expansion of the primitive gives

$$I_{\mathrm{rad}} = (x_{\mathrm{hi}}^2 - x_{\mathrm{lo}}^2)/30 + O(x_{\mathrm{hi}}^4 + x_{\mathrm{lo}}^4).$$

For $x_{\mathrm{lo}} \ll 1 \ll x_{\mathrm{hi}}$ the integral approaches $1/3$. When both limits grow at fixed positive lower radius, the two primitive values both decay. Eq. (8) starts the radius at zero, while Eq. (10) introduces a positive lower cutoff along with the variable substitution (SC-146, SC-151 through SC-158). Replacing the upper limit by infinity before taking small k changes the answer. With uniform density, solid angle, angular coupling and Eq. (7)'s half restored, the energy is $J_{\mathrm{sp}} = 4\pi\rho_{\mathrm{uniform}} J_{\mathrm{ang}}(\xi_{\mathbf{k}}) I_{\mathrm{rad}}/2$. Additional torque and polarization factors enter the measured angular slope after division by $\hbar$.

The contact term gives an independent normalization check (SC-159 through SC-162). For nonzero axial separation s, the second axial derivative of $1/s$ is $2/s^3$, while $P_2(1)/s^3 = 1/s^3$. Take the spherical principal value at the origin. Let δ denote the Dirac distribution. The resulting extension of $P_2(\hat{\mathbf{d}}_{\mathrm{pair}} \cdot \hat{\boldsymbol{\eta}})/s^3$ equals half of $(\hat{\boldsymbol{\eta}} \cdot \nabla)^2(1/s) + (4\pi/3)\delta^{(3)}(\mathbf{d}_{\mathrm{pair}})$, where the superscript on $\delta^{(3)}$ is the dimension. Indeed the Hessian away from the origin gives the traceless angular tensor and its trace at the origin is the Coulomb Laplacian. With transform of $1/s$ equal to $4\pi/k^2$, the half-normalized transform for nonzero k is $-4\pi P_2(\xi_{\mathbf{k}})/3$. Eq. (11) omits the common half. The intermediate expression in Eq. (12) is twice its printed endpoint. The directional reversal survives that normalization check.

In Fig. 4(c) the fitted early-time bulk rate rises, saturates and falls, with a marker at $2\pi/(0.185\,\mu\mathrm{m})$ (SC-113, SC-114, SC-119). A separate Fourier image along a different axis is consistent with the 185 nm layer thickness under a rectangular-profile fit. The grown thickness is independently specified (SC-116 through SC-118, SC-246). Fig. 4(e) concerns the *maximum normalized amplitude* in beam NV group one, with the width marker at $2\pi/(0.3\,\mu\mathrm{m})$ (SC-120 through SC-123). The beam width sets this wave-vector marker. Fig. 4(e) gives no reference amplitude for the ordinate's normalization. Its inset has $t_{\mathrm{pump}}/t_{\mathrm{sat}}$ of 3.6 and 11, both shorter than 26 on the main trace. The latter of the two inset traces approaches saturation at smaller wave vector (SC-124 through SC-126). The solid curve imports the ancillary winding loss (SC-127). Fig. 4(f) resolves a dip in one projection. The authors state that it arises from nanophotonic interference of the pump light, confirmed by electric-field simulations. The two lobes for short pumping and their merger at longer pumping belong to simulated cross sections (SC-129 through SC-134). Overlap of acquisition errors between a correction and an outcome requires their joint covariance. A reciprocal-size marker supplies a model scale. This cluster reconstructs the source model and corroborates no invariant (Use item 2). A fresh signed angular slope at a held-out direction tests the weighted-kernel forecast in rad/s, at fixed k, pump and detector settings. Calibrate the model amplitude on other directions.

3.4 What the twisting fit determines

Write N for the number of effective spin-half sites, and p_z for the Bloch polarization. The collective component is the sum of the site components, so $p_z = 2\langle \mathsf{s}_{\mathrm{coll}}^z \rangle / N$ (SC-036, SC-171, U-10). Fig. 5 uses spin tomography after unwinding. The maximum quench in panel (a) is 289.44 µs. Panels (b)–(d) use improved drive and gradient homogeneity at $2\pi/(0.242\,\mu\mathrm{m})$ (SC-165 through SC-167). Phase in panel (b) is in degrees. Eq. (13) defines $\Phi_{\mathrm{sp}} = \arg \mathcal{C}$ wherever

the magnitude is positive. Unwrap the phase, multiply degrees by $\pi/180$, and fit a slope against quench time over a stated window. The authors report linear fits in this panel without specifying each fit window. Dividing an angular slope by 2π converts it to cycles per unit time. A global phase reference changes the intercept.

The one-axis-twisting Hamiltonian H_{tw}, in angular-frequency units, is $H_{\mathrm{tw}}/\hbar = \chi_{\mathrm{tw}}(\mathsf{s}^z_{\mathrm{coll}})^2/N$ (SC-172) [3]. Differentiating with respect to the collective axial component gives the classical transverse phase slope.

$$\omega_{\mathrm{sp}} = \frac{2\chi_{\mathrm{tw}}}{N}\langle \mathsf{s}^z_{\mathrm{coll}}\rangle = \chi_{\mathrm{tw}} p_z.$$

For fully polarized spins at preparation, $p_z = \cos\vartheta_{\mathrm{p}}$. The fitted phase slopes exhibit a cosine dependence. Put et al. report the effective twisting parameter 8.51(2) kHz (SC-169 through SC-175). Their Fig. 5(c) legend multiplies the same parameter by $\langle \mathsf{s}^z_{\mathrm{coll}}\rangle/N$. With the stated spin definition that expression is half the classical slope of the printed Hamiltonian (SA-20). Preserving the fitted parameter and the printed curve therefore requires stating which spin normalization is plotted. Fig. 5(b) plots phase in degrees against quench time in microseconds, and its slopes put the ordinate of Fig. 5(c) in thousands of radians per second, so the reported parameter is an angular rate and its cyclic value is that rate divided by 2π. Refitting the angular points or assigning full confidence coverage to individual slopes needs the complete fit windows and raw replicate covariances, which are unspecified (SC-100).

Fig. 5(d)'s purple curve multiplies simulated precession by measured decoherence (SC-176, SC-177). It and the beam's winding-loss curve are empirically corrected using different ancillary readings. In Fig. 5(f) the spiral amplitude at $\vartheta_{\mathrm{p}} = 45^\circ$ decays more slowly at longer wavelength. The authors use stretched-exponential fits (SC-184, SC-185, SC-191). For a reconstruction with initial amplitude A_{sp} and shape exponent $b_{\mathrm{dec}} > 0$, write the envelope as $A_{\mathrm{sp}}\exp[-(\Gamma_{\mathrm{sp}}t)^{b_{\mathrm{dec}}}]$. Here Γ_{sp} is the inverse e-folding time. Identifying it with the authors' fitted rate requires their fit prescription. Panel (g) marks the extrinsic decay, and (h) divides the angular precession rate by the authors' fitted decay rate, printed as gamma (SC-186, SC-187, SC-194). For the reconstructed ratio, let $e_{\mathrm{rate}}, e_{\mathrm{decay}}$ be the two fit fluctuations. Their variance operator is Var. At first order the ratio has variance $\mathrm{Var}[e_{\mathrm{rate}} - (\omega_{\mathrm{sp}}/\Gamma_{\mathrm{sp}})e_{\mathrm{decay}}]/\Gamma^2_{\mathrm{sp}}$. Their covariance enters the variance of this difference. In a long-wavelength spiral nearby spin pairs are approximately collinear. Under SU(2), the authors argue, these pairs do not contribute appreciably to many-body relaxation. They identify slow hydrodynamic modes protected up to the macroscopic spin-transport timescale, set by the spiral wavelength and the transport universality class (SC-180, SC-181, SC-182, SC-183). The observed amplitude is an ensemble signal.

For dilute positionally disordered *two-dimensional Ising* spins at unity polarization, the plotted theoretical maximum is $\sqrt{3/2} \simeq 1.22$ (SC-188 through SC-190) [5, 2]. The native point falls below it and engineered ratios exceed it over part of the wave-vector sweep (SC-192, SC-193). The cited analysis predicts that isolated, disordered dipolar systems can, in principle, achieve an ideal $O(\sqrt{N})$ quality factor due to the separation of timescales between spiral relaxation and exchange-driven twisting (SC-198 through SC-200) [4, 2]. For three-dimensional dipolar interactions it expects size-independent twisting at wave vector tuned to inverse system size and parametrically slower relaxation in larger systems owing to emergent spin hydrodynamics (SC-201 through SC-204). The reported scan varies wave vector on a fixed specimen. Long coherence and high-fidelity readout enter the authors' outlook for entanglement generation and squeezing (SC-206 through SC-208). A first moment gives no spin variance. A metrological comparison requires independent spin variance, contrast, detector-noise calibration and a phase-sensitivity

reference. This cluster reproduces a fit and corroborates no invariant (Use item 2). A held-out unwrapped angular slope tests the cosine law in rad/s with independent preparation and calibration, including the covariance of its fitted amplitude.

4 Cone-position translation

A spiral is prepared by setting the azimuth of a different closed pattern at each site. Moving along the helix in a reconstructed spin image means sampling a different site. It does not trace the path of a photon, the trajectory of one defect or a displacement on the confinement axis. P-5 reads the applied field as an aligned helical density gradient. I-4 supplies the interpretation of the transverse complex components, while the pulse propagator and the detector define their experimental values. The dipolar polynomial takes both signs at one A-1 compactness. P-4 assigns a sign to a cone pair that a global settlement actually joins. The angle between a winding vector and a crystal axis identifies no such pair.

The observation of phases tests the spin response to a field in tesla. A static force reaches a driven rate only through a causal response kernel with units converting a force history to angular frequency and with a specified time normalization (I-1). Neither inverse pitch nor polarization density has those units. A static torsion test therefore needs a spatial field response at the actual probes. An optical test needs a path-dependent phase. A load test needs the two level shifts and the local clock-ratio response. These distinctions fix which comparisons survive the ablation of their FDT input.

5 Predictions

Let $\mathbf{y}$ be the vector of an active reading, a reference reading and their independently calibrated conventional forecasts in that order. A reading already includes its predeclared polarity or order contrast. Let $\mathbf{w}$ be the signed contrast vector, Σ the joint covariance of the four estimates, $\mathcal{R}_{\text{test}}$ their residual, $s_{\text{test}} > 0$ its standard uncertainty and b_{test} a proposed signed residual mean. For each independently predeclared comparison (U-11 through U-14),

$$\mathbf{w} = (1, -1, -1, 1)^{\mathsf{T}}, \quad \mathcal{R}_{\text{test}} = \mathbf{w}^{\mathsf{T}}\mathbf{y}, \quad s_{\text{test}}^2 = \mathbf{w}^{\mathsf{T}}\Sigma\mathbf{w}.$$

Exclude the specified mean when $|\mathcal{R}_{\text{test}} - b_{\text{test}}| > 3s_{\text{test}}$. In a calibrated linear range with jointly Gaussian errors and known covariance the exact two-sided size per test is $\operatorname{erfc}(3/\sqrt{2})$. Shared drift, region-to-region correlations and forecast uncertainties enter the contraction. When averaging, use the covariance of the averaged vector, retaining calibrations shared across acquisitions. Phases require an independently established unwrapped linear range. For estimated covariance, nonlinear fits, residual drift or wrapped phases calibrate the criterion from the actual replication and noise model. Selecting the largest of five discrepancies requires its joint selection law. The rule excludes a specified residual at the held-out setting. It gives no threshold in tesla or strain until the apparatus has been calibrated.

The disposition of each candidate concerns only its explicitly calculable contribution. In each sentence the entries are, in order, the residual mean with the stated FDT input, the residual mean with its conventional value, their difference and the threshold, in units of the standard uncertainty.

DC-1 (SA-01) is retired for the shared projected Zeeman response, $(0, 0, 0, 3)$. P-5 reads a static twist as a magnetic field, so at zero external bias FDT places a torsion-odd field where the conventional time-reversal-invariant equilibrium places none; the twist-to-field kernel that sizes it is to be derived (D-01), and the comparison holds no FDT residual mean to test.
DC-2 (SA-02) is retired for the shared ordered spin/readout map, $(0, 0, 0, 3)$. Each order is its own system-wide settlement, and the core names no objective that adds a history-to-quadrature term.
DC-3 (SA-03) is retired for the matched rest-energy and support calculation, $(0, 0, 0, 3)$, including a finite preparation-dependent deflection; at one energy the helical window returns the conventional gradient force.
DC-4 (SA-04) is retired for a stipulated common additive level shift alone, $(0, 0, 0, 3)$. The full load response carries the two level sensitivities and the local clock-ratio response, and no form of D or its spectrum enters here (O-9).
DC-5 (SA-05) is retired for the common material phase integral and interfaces, $(0, 0, 0, 3)$. The aperture index is an index of the gravitational optical metric that both calculations carry (O-8), so it cancels from the loaded-diamond contrast.

6 Cross-domain comparison

A magnetic Ramsey phase samples a field integrated over time. Static deflection includes the negative spatial derivative of the magnetic energy at the measured reference position divided by the calibrated stiffness, with populations and external settings held fixed. An instantaneous referenced Ramsey frequency samples the local clock ratio times a level-energy *difference* divided by h, and an optical phase samples a wave-vector integral along a path. Even when the same load is applied to all four instruments, their linear responses have different units and different coefficients. Treat a hydrostatic pressure and the strain it induces with a constitutive relation, or specify independent load coordinates. Counting both as independently applied deformations would duplicate the input. For a positive response and positive independent controls, define each sensitivity as the partial derivative of the logarithm of the response with respect to the logarithm of its control, holding the other controls fixed. Integrate each sensitivity against its logarithmic control differential along the path and sum the integrals. The final response is the initial response times the exponential of that sum. Subtract the initial response to obtain the signed change. For any differentiable path on which a response or control reaches zero or takes negative values, integrate the ordinary total differential of the response in the independent controls. Propagate the joint uncertainty of the resulting change.

At equal calibrated total magnetic fields, the Zeeman shift is shared. At equal mechanical energy landscapes, the push follows the same gradient. The load on the support enters the total stress tensor once. Assigning that total to a particular NV level would add a state-dependent material law. Likewise, a local propagation constant in strained diamond already contains its ordinary dispersion and polarization dependence. An additional aperture reading needs an independent material prescription for the difference between the two arm phases. These facts allow large conventional signals without a theory separation.

7 System-level reading (exploratory)

The comparison concerns three-dimensional nonnegative polarization profiles with the same observed short-axis marginal in the beam. The sphere in Eq. (8) replaces the device by a uniform source seen from its centre. For a whole-sphere detector of uniform weight and polarization, pairs of displacement length s have an additional weight equal to the fractional overlap of two translated spheres. Adding the two spherical-cap volumes gives $1 - 3s/(4\ell_{\mathrm{pol}}) + s^3/(16\ell_{\mathrm{pol}}^3)$ up to the diameter and zero beyond it. This is already a different forward calculation.

X-1. Projection-compatible exchange spread. The observable is the difference of two normalized, double-averaged exchange energies. Its defining condition is exact agreement with the same measured short-axis marginal. Its comparator uses the same angular kernel, microscopic cutoff and optical weights. The following bounded integral evaluates the spread without digitizing unknown structure along the beam (SC-129 through SC-134). Let M_{col} be the nonnegative measured column profile normalized to unit integral. Take uniform optical collection on its support for this family (U-20, U-21). Place the conditional transverse distribution inside the measured triangular section, of maximum transverse diameter W_{beam}. For a nonuniform independently known positive collection weight divide the constructed column by that weight before normalizing to its observed column integral. Unknown collection weights add freedom.

In the unresolved long direction use either one slab or two translated copies with equal shares about the same midpoint. Both fit inside the beam. Let ℓ_{split} be the separation of their centres, h_{slab} each slab's thickness and r_{uv} a positive pair exclusion length. Impose $W_{\mathrm{beam}}/\ell_{\mathrm{split}} \le 1/4$, $r_{\mathrm{uv}}/\ell_{\mathrm{split}} \le 1/4$ and $h_{\mathrm{slab}} \le r_{\mathrm{uv}}^2/(100\ell_{\mathrm{split}})$ as illustrative shape restrictions (U-23). Use a uniform slab density times the same normalized transverse column distribution in every slab. Any extra emitter weighting by transverse collection or analysis sensitivity is identical on translated copies. Contributing spins carry the preparation weight. The two normalized site measures are denoted $dP_{\mathrm{e}}, dP_{\mathrm{s}}$ and can differ. For this illustrative orientation take the wave vector and quantization axis along the separation, with $k\ell_{\mathrm{split}} = \pi$. This chosen alignment and the exclusion length are inputs to the example. The device calculation uses its independently determined projections. The measured marginal alone supplies no preferred axial placement.

Write Δz for axial pair displacement and s for pair distance. Measure the response in units of the angular energy–volume prefactor at parallel alignment divided by ℓ_{split}^3, per contributing spin. Only pairs with $s \ge r_{\mathrm{uv}}$ enter the double integral $\iint$. The integrated dimensionless kernel is

$$\iint_{s \ge r_{\mathrm{uv}}} dP_{\mathrm{e}}\, dP_{\mathrm{s}}\, \frac{\ell_{\mathrm{split}}^3}{s^3} P_2(\Delta z/s) \frac{1 - \cos(\pi \Delta z/\ell_{\mathrm{split}})}{2}.$$

Denote the integral for a same-slab pair by B_{in} and its average for the two cross-slab orders by B_{cross}. The one-slab result is B_{in}. The two-slab result is $(B_{\mathrm{in}} + B_{\mathrm{cross}})/2$. Thus the family spread is exactly $(B_{\mathrm{cross}} - B_{\mathrm{in}})/2$ for every allowed marginal. For a real phase ζ, using $1 - \cos\zeta \le \zeta^2/2$ gives $|B_{\mathrm{in}}| \le \pi^2 \ell_{\mathrm{split}} h_{\mathrm{slab}}^2/(4r_{\mathrm{uv}}^3)$. For cross pairs, $|\Delta z|/\ell_{\mathrm{split}}$ lies between 0.99 and 1.01. The lower bound for the angular cosine squared is $0.99^2/(0.99^2 + 0.25^2)$, the phase filter is at least $(1+\cos(0.01\pi))/2$, and $s/\ell_{\mathrm{split}} \le \sqrt{1.01^2 + 0.25^2}$. The cross kernel is at most $1/0.99^3$. Combining the bounds and rounding outward places the spread between 0.40 and 0.52 in the illustrative units. For every admissible column profile in this family, this interval bounds the two-slab response minus the one-slab response. The absolute energy scale requires the actual spin density and collection calibration.

Guards. The example keeps the projection and the boundary fixed while redistributing polarization along the beam's long axis, which the short-axis marginal integrates over. All values here describe the stipulated in-sample family. The span is neither an experimental uncertainty nor a force. The crystal skin, optical support and ultraviolet cutoff have different definitions. A held-out image along the beam constrains the slab separation and shares through its own transfer function and error covariance.

8 Convergence statement

At the same source, pulse waveforms, stress tensor and boundary data, use the propagator of the whole ensemble under the selected sum of pair couplings, including ordinary open-system channels, followed by inverse winding and the calibrated optical projection. This returns the complex spin coherence and the same fitted rates as the consensus calculation. Supplying a measured ancillary decay to both calculations returns the same empirically corrected curve. None of these operations takes the curvature ratio α_σ, a chart or a branch tag as an input.

Put et al. observe a clear revival in the coherence magnitude centered at matched winding and unwinding without an intermediate quench, providing evidence of reversible dynamics under the gradient field (SA-09, SC-066 through SC-069). FDT accepts the measured refocusing in full and reads the unwinding as a new settlement of the whole state it starts from (0.1a), which restores the site phases without running the winding's microscopic events and environmental records backward.

Put et al. approximate quantum spin-spin interactions by classical magnetic fields acting on each spin, first focusing on the SU(2)-symmetric Hamiltonian, and conclude that the observed spin precessions in Fig. 2(h) and Fig. 2(i) are driven by the dipolar exchange field because angular averaging removes the Ising contribution in their three-dimensional dipolar ensemble (SA-10, SC-088, SC-089, SC-095, SC-096). FDT reads the precession of the detected mode as the settlement of the whole ensemble at every step, with the applied field a macroscopic helical density gradient (P-5), the site-field approximation (with its finite-device weighting conditions) the consensus reduction of the ensemble propagator above, and the quadratures the two components of the weighted sum of the sites' projected helices on the quantum face (I-4); the measured rates are accepted in full as the record of that system-wide settlement (0.1a). Where FDT, general relativity and quantum mechanics converge, FDT reproduces their results; where FDT declines the standard account's assumptions, the magnetic field as an entity apart from the omnium, microscopic reversibility and probability amplitudes (Use item 6), it is under no obligation to reproduce them.

8.1 Static torsion and magnetic phase

Modify a beam with two mapped NV regions and hold each torsional polarity at rest. Take both polarities in random order within each matched comparison. For an equal-half echo the toggling integral of a static projected field vanishes at each site, even for an inhomogeneous field. Axis rotation and motion through the switched coil field can survive. Use a separately referenced DC-sensitive Ramsey interval for a static-field measurement. Define Δ_{sites} as active minus reference and $\mathcal{O}_{\mathrm{odd}}$ as half the difference of opposite settled polarities. With the actual axes at the two regions and $\mathbf{B}$ the local total magnetic field sampled along each probe history, the signed direct phase is $\mathcal{O}_{\mathrm{odd}}\Delta_{\mathrm{sites}}\int g_{\mathrm{NV}}(\mathbf{B}\cdot\hat{\boldsymbol{\eta}})\,dt$ in radians. Calibrate crystal rotation, displacement, strain,

permitted magnetoelastic response and field projection on independent settings. Holding the total field fixed fixes that phase in both calculations.

For a separate control, null the external bias at a target with no selected magnetic order. Characterize leakage, remanence and probe backaction independently. Leave the target-generated field free. In conventional time-reversal-invariant equilibrium static strain is even and magnetization is odd. The equilibrium mean magnetization vanishes for either torsional polarity. At finite bias propagate the measured susceptibility and leakage for each polarity through the external probe geometry. An independently frozen additional profile with fixed normalization and nonzero differential projection supplies a testable phase. With an investigator-specified profile a finite null excludes only the specified amplitude and projection under the uncertainty rule. P-5 reads the twist as a field; its twist-to-field kernel is to be derived (D-01) and enters no phase forecast here. The circulating-charge map (O-6) is a separate input. Put et al.'s experiment supplies the winding measurement, not this static torsion comparison (SC-052 through SC-054).

8.2 Order contrast with an open spin system

Prepare an equatorial bulk spiral without quenching. Compare two permutations of labelled collinear gradient-area blocks, matched in duration and site-resolved integrated phase. At each matched acquisition randomize which order runs first, acquire both and monitor a neighbouring reference region. Match initialization, optical weights and the microwave phase reference. Read the same referenced transverse quadrature for both orders. In the calibrated phase-only limit equal site phases, coherence amplitudes and weights give the same complex sum. For the open system compute each order's ordinary propagator and readout, retaining control transients, bath history and order-dependent attenuation. Commuting phase rotations alone do not imply equal open-system signals. Subtract the orders within each description and then subtract the two contrasts. Equal response maps give equal order contrasts even for finite ordinary order dependence. Each order is its own system-wide settlement (0.1a), and with no objective named it adds no history-to-quadrature term. This test returns a normalized quadrature difference.

8.3 Exchanged preparation on elastic supports

Mount two matched diamond sensors in mapped gradients. Exchange the zero- and minus-one-level populations, randomizing which sensor receives each preparation. Measure settled populations, relaxation, support stiffness, susceptibilities, temperature and actual gradients independently. Let E_{mag} include the coupling of the held moments at their settled values and the induced energy including its induction work at fixed settings, x_{mech} the displacement coordinate, x_{ref} the measured reference position and K_{eff} the calibrated effective stiffness. In the linear range the incremental magnetic force is $-\partial_{x_{\mathrm{mech}}} E_{\mathrm{mag}}(x_{\mathrm{ref}})$ at fixed populations. Its exchanged-state difference divided by K_{eff} gives the change in static deflection in metres. Include state-dependent support and gravity offsets that survive the exchange. Equivalently minimize support plus magnetic and gravitational energy for each preparation on the stable branch. Inertia in an acceleration experiment is the rest energy of the entire moving body divided by c^2 in both descriptions. At the same energy gradient and stiffness the helical reading and the conventional calculation return the same response.

8.4 Loaded gap and optical path

In neighbouring bulk sensing regions prepare zero spatial winding and use Ramsey spectroscopy on the zero-to-minus-one transition. Apply a measured load to one region, randomize loaded/unloaded order, and reference both regions to the same clock. Match optical and microwave controls and monitor temperature, stress, strain and field. Let $E_{\mathrm{hi}}, E_{\mathrm{lo}}$ be the ordered energies and C_{clk} the local-to-reference clock ratio. The observable per region is $C_{\mathrm{clk}}(E_{\mathrm{hi}} - E_{\mathrm{lo}})/h$ in hertz. A shared additive energy cancels from each gap. Each region's frequency differential is $[C_{\mathrm{clk}}(dE_{\mathrm{hi}} - dE_{\mathrm{lo}}) + (E_{\mathrm{hi}} - E_{\mathrm{lo}})\, dC_{\mathrm{clk}}]/h$. Determine each level's partial responses to each independent mechanical coordinate, temperature and magnetic field, holding the others fixed. Propagate their measured load-induced changes with each region's clock-ratio calibration against the common reference. Subtract the reference-region frequency differential from the active one and propagate their joint covariance, including shared controls. For a finite change between two settled conditions, hold each setting fixed during its Ramsey interrogation and compare the two stationary frequency products. If the same coherent levels accumulate only dynamical phase while conditions vary during an interrogation, integrate each instantaneous referenced frequency over common reference-clock time to obtain its accumulated phase in cycles. After removing the clock-ratio term, the gap contribution to the frequency differential is $C_{\mathrm{clk}}\, d(E_{\mathrm{hi}} - E_{\mathrm{lo}})/h$. Multiply this contribution by h/C_{clk} to recover the gap differential. Use independent mechanical coordinates or a constrained stress–strain relation. I-7 counts the device energy. It prescribes neither equality of the shifts nor a distinct gap response. The medium's flow factor does not give the carried defect's clock shift.

Add a weak independently stabilized interferometric probe distinct from the preparation beam. Measure the load contrast between neighbouring diamond paths, exchange arm assignments in random order within each matched acquisition, and keep probe frequency fixed. For each path supply its own geometry, stress, temperature, pump conditions, mode, complex polarization-resolved dielectric response and interface data. The signed phase in radians is the active-minus-reference difference of the real propagation-constant integral plus the interface phases. Both calculations include electrodynamics in diamond and the same gravitational metric and boundaries. Equal integrals and interface phases give equal signed contrasts even for finite load-induced phase. A held-out extra phase requires a distinct load-to-optical-phase law. The aperture index is an index of that shared gravitational metric (O-8) and adds no material number here. The conformal clock factor preserves null paths.

9 Status ledger

Each O-row names what has been derived of the item, with its source, and what remains to be derived; the visible-universe analysis prints the results of the two derivation notes [10].

Item	Status and content
O-1	Derived in part [8]: once a stroke is chosen the floor fixes the core cutoff, at the floor length $2\ell_{\mathrm{P}}$ for the radial stroke, half of it for a stroke across the core and $1/2\pi$ of it for one around it; a uniform interior attains the bound that holds the mass inside r_f to α_{int}^3 of the mass inside the fold when the density does not fall outward. To be derived: the choice of stroke, the regular core chart and the outward phase-transfer law.

Item	Status and content
O-2	Derived in part [8]: a flank-to-gap odds, the sum of two profiles' flank widths, each from crest to the shoulder where the second derivative changes sign, over half the gap between the facing crests. For identical profiles it equals 2, the 2/3 of I-1a, exactly where the midpoint turns from valley to ridge, and no density value, volume mean, reference-density ratio or circulation enters it. To be derived: the profile (O-4), the unequal pair and the transfer to the instances of I-1a. Every increasing function of this odds that keeps 2 at the crossing passes the same test; the core fixes no member, and A-3 carries its open range only.
O-3	To be derived: a canonical odds for the confinement application. The scaling, spectral, section-ratio and floor-ratio routes select none [7, 8].
O-4	Derived in part [8]: the fold stress law, by which a regular static approach to the fold carries a radial stress equal to minus the energy density, and radial tension inside every static configuration that approaches saturation with its stresses bounded by its energy density. To be derived: the two constitutive relations among the energy density and the radial and tangential stresses, and the reduction to the measured nuclear profile.
O-5	Derived in part [7, 8], at layer L2: the multipole expansion of the pair law for supplied sources, with bounds on its remainder after quadrupole order; the attraction of two hollow supports through their whole overlap, never above its contact value; the spin remainders of the pair law for two bodies on a common axis; the width of the band outside a rotating source's fold, the Schwarzschild radius of the rotational share of its mass; and the attraction, in the closed form of I-1, of a pair the settlement does not join. To be derived: the overlap beyond L2, which pairs a settlement joins and the map from a rotation to a flow direction.
O-6	To be derived: the maps from a charge, a circulating charge or a colour seat to an I-1 input, the seat table from cone geometry and the gluon octet's dynamics. Derived [9]: the magnitude that any per-body charge map linear in the charge returns, fixed by Coulomb's law read through I-1.
O-7	The SNO neutral-current total flux agrees with the standard solar model and is data. Derived in part [7, 8]: one solar energy account with a fission part and a fusion part, each contribution counted once; the map from source spectra to detector counts; and, from the measured proton–proton-chain fluxes, a fission heat share below 0.17 of the photon luminosity at 95 per cent confidence in the stationary account. To be derived: the settlement's computation against the SNO flux and the FDT computation of every other neutrino anomaly.
O-8	Derived [8] on every chart whose metric is in hand: the bookkeeper reads the stroke's electric and magnetic responses as one tensor, whose index across a density gradient in the static exterior's areal chart is the aperture index $(1-\alpha_{\mathrm{ext}})^{-1/2}$ of P-4 and whose index along it is the square; the pair is the exact optical metric, and the leading deflection is twice the Schwarzschild radius over the impact parameter. To be derived: the interior instance, which needs the interior lapse (O-1, O-4), and the response beyond first order in the fields.
O-9	To be derived: the confinement operator, its domains, boundary data and spectrum, and every mass ratio.
A-3, A-4, A-5	Range only: overlap, confinement and mode names. The flank-to-gap odds of O-2 assigns A-3 no value.
Echo and field	In-sample: ESR fits, beam inversion, winding overlap, projection and refocusing.
Spin dynamics	In-sample: site-field interpretation, angular fits, corrected curves, spherical crossover, twisting, fitted decay and quality factor.
Profiles	In-sample: illustrative exchange spread at a common marginal.
Held-out targets	New ESR baseline, unmatched quadrature, directional and cone-angle slopes, second projection and calibrated differential sensor readings.
Divergence	No candidate has a distinct mapped residual beyond the predeclared threshold.

Item	Status and content
D-01	To be derived: the twist-to-field kernel of P-5 for a static torsion, which sizes the torsion-odd field of the zero-bias torsion control. Derived [9]: its 2-form setting for flowing patterns, one flow 1-form per kind of pattern whose vorticity carries a mechanical and a magnetic grade, with the London moment as the condensate limit.

10 Conclusions

The refocusing experiment fixes a Fourier overlap. During the quench a disordered spin ensemble responds to an angularly modulated dipolar kernel over a finite polarized volume. Its directional sign and the order of its radial limits follow explicitly. A measured short-axis profile leaves the polarization along the beam's long axis free, and that freedom moves even the simplest two-site average. The reported twisting fit concerns a mean phase, and readout variance or metrological gain requires further observables. FDT recovers every reading of the experiment from the same source and data. In the proposed modified devices, ordinary magnetic, elastic, spectroscopic and optical responses already survive the comparison. The zero-bias torsion control is where FDT's reading of a twist as a field meets the conventional time-reversal zero, and the twist-to-field kernel (D-01) turns that reading into a signed differential prediction in its own units.

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