

One carrier in a patterned magnetic circuit

An FDT analysis of spin-wave–photon spectra

FDT analysis

Target: *Strong coupling between propagating spin wave and microwave photons in a superconducting resonator*

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Abstract

Li et al. report avoided crossings and directional microwave transmission in superconducting resonators patterned over magnetic films [1]. Their meander-device fitted rates satisfy their strong-coupling criterion; for their quarter-wave device they state it uncertain whether the coupling strictly exceeds the magnon damping rate (SC-008, L32; SC-080, L141; SC-109, L198; SC-237, L310; SC-238, L311; SC-245, L320). Fundamental Density Theory (FDT) reads the resonator and spin-wave modes as two regimes of one carrier. The source’s response equations recover its spectra on the same inputs, including the discrepancy between its exchange-field estimates and its standing-wave separation. Four prospective predictions discriminate additional torsional and isotope inputs: one field-existence test and three conditional constitutive tests. Their observables compare independently prepared paths or samples with conventional responses removed. A film-profile candidate has no supplied constitutive reduction and is retired; one standing-wave reading remains exploratory.

1 Framework

The source status is experimental. Source results retain measured, observed or reported verbs; fits, estimates and reused inputs remain identified as such. Derived, reconstructed and illustrative-assumed describe the analysis’s calculations and test points. The row identifiers SC and U name source claims and numerical inputs; an accompanying L locator is a physical source line. The framework is a true extension of general relativity and quantum mechanics [2]. Its additional sources and constitutive inputs are distinguished from recovering the response model of Li et al. [1].

1.1 One medium, discrete patterns and continuous charts

Matter, energy and spacetime are the omnium, read in different regimes (core 0.1). The circle constant is π ; $g_{\mu\nu}$ denotes the metric throughout. Energy E , mass m , local speed c , response coefficients ε_0, μ_0 , thread length d and kept time t are the entries of one state.

$$E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2. \quad (1)$$

For a confined mode labelled q , $E_q, m_q, \omega_q, T_q, \ell_q$ denote its energy, rest mass, angular frequency, period and Compton length; h and $\hbar$ are the action constants. The massive-leg bridges of core 0.1a

are

$$E_q = m_q c^2 = \hbar \omega_q, \quad T_q = 2\pi/\omega_q = h/E_q, \quad (2)$$

$$\ell_q = cT_q = h/(m_q c). \quad (3)$$

The Compton length is not a support radius R_{box} . Quantised energy at constant speed gives discrete confined masses and null periods and paths at locked ratio $\ell_q/T_q = c = 299792458 \text{ m s}^{-1}$ exactly (U-06). The force ceiling F_{max} , Newton constant G , Planck length ℓ_P and time t_P set the one-sided infima $d_{\text{min}}, t_{\text{min}}$ (core 0.1a, 0.2).

$$F_{\text{max}} = \frac{c^4}{4G}, \quad F_{\text{max}} d_{\text{min}} t_{\text{min}} = \hbar, \quad d_{\text{min}} = c t_{\text{min}} = 2\ell_P, \quad t_{\text{min}} = 2t_P. \quad (4)$$

No pattern occupies the floor; it is no lattice spacing. Continuous chart coordinates describe discrete content. The film, circuit and chip are patterns with their own supports, not contents inserted into a separate spacetime container.

The total gravitating mass-equivalent is $\mathcal{M}$; R_s is its source radius and r_f the field-point atlas radius. The compactness coordinates $\alpha_\sigma, \alpha_{\text{ext}}, \alpha_{\text{int}}$ and gap odds $X_{\text{ext}}, X_{\text{int}}$ name the two branches. The branch tag σ distinguishes exterior and interior; r_c bounds the finite-core chart. Core 0.3 fixes the A-1 atlas and its geometric gap odds X_σ :

$$R_s = \frac{2G\mathcal{M}}{c^2} = \frac{\mathcal{M}c^2}{2F_{\text{max}}}, \quad (5)$$

$$\alpha_\sigma = (R_s/r_f)^\sigma, \quad \sigma = +1 \text{ (exterior)}, \quad \sigma = -1 \text{ (interior)}, \quad (6)$$

$$\alpha_{\text{ext}} = R_s/r_f, \quad R_s < r_f, \quad X_{\text{ext}} = \frac{R_s}{r_f - R_s}, \quad (7)$$

$$\alpha_{\text{int}} = r_f/R_s, \quad r_c \leq r_f < R_s, \quad X_{\text{int}} = \frac{r_f}{R_s - r_f}. \quad (8)$$

The A-2 computational coordinate is β_σ . The chart inventory below names every other application: overlap $X_{\text{ovl}}, \alpha_{\text{ovl}}$, confinement $X_{\text{conf}}, \alpha_{\text{conf}}$, and mode token α_{mode} . The saturation diagnostic α_{core} is schematic only.

Application	Tokens	Warrant and use
A-1	$\alpha_\sigma = X_\sigma/(1 + X_\sigma)$	Geometric compactness; field point, configuration and source are declared separately.
A-2	$\beta_\sigma = \alpha_\sigma/(1 + \alpha_\sigma) = X_\sigma/(1 + 2X_\sigma)$	Geometric computational chart; excluded from force, optical and density slots.
A-3	$X_{\text{ovl}}, \alpha_{\text{ovl}}$	Range token only; no geometric warrant.
A-4	$X_{\text{conf}}, \alpha_{\text{conf}}$	Range token only; no geometric warrant.
A-5	α_{mode}	Naming only; ranks and spaces nothing.
Diagnostic	$\alpha_{\text{core}} \rightarrow 1^-$	Schematic saturation, never an absolute address.

The last three applications carry no assigned value, threshold, ordering, sign or trajectory here. The A-1 seam has $\alpha_{\text{ext}} = 1/2$ at $r_f = 2R_s$ and $\alpha_{\text{int}} = 1/2$ at $r_f = R_s/2$. It is the Clifford torus separating matter and spacetime regimes of the same substance; the fold is the limiting core circle at $\alpha_\sigma \rightarrow 1^-$. Neither end of the atlas is a point; repetition is bounded by the action floor. Core 0.6 distinguishes the mean-density proxy q_σ from the local density ρ_{loc} .

$$q_\sigma = \alpha_\sigma^{3\sigma}, \quad \frac{\partial \rho_{\text{loc}}}{\partial \alpha_\sigma} > 0. \quad (9)$$

The local skin is densest on both branches and the centre dilute; the interior mean proxy increases as its chart coordinate decreases. These statements concern different density objects. The operator Δ denotes a finite change. For an odds increment $x \geq 0$, initial A-1 values $X_{\sigma,0}, \alpha_{\sigma,0}$ obey I-6:

$$X_\sigma = X_{\sigma,0}(1+x), \quad (10)$$

$$\alpha_\sigma(x) = \frac{\alpha_{\sigma,0}(1+x)}{1+\alpha_{\sigma,0}x}, \quad \Delta\alpha_\sigma = \frac{\alpha_{\sigma,0}(1-\alpha_{\sigma,0})x}{1+\alpha_{\sigma,0}x}. \quad (11)$$

The open interval is universal; an interior odds increase reduces the mean-density proxy. Isotope mass increments at fixed geometry enter the locked radius ratio directly, and are not identified with x .

1.2 Force, curvature and the two readings

For two force inputs α_1, α_2 in their declared applications, I-1 gives force F at every scale.

$$F = \frac{c^4}{4G}\alpha_1\alpha_2, \quad 0 < \alpha_1, \alpha_2 < 1, \quad F < F_{\max}. \quad (12)$$

For spherical nonoverlapping monopoles, $R_{b,i}$ is support radius, $R_{s,i}$ source radius, $\mathcal{M}_i$ mass-equivalent, and d their centre separation. The same length symbol denotes a distance throughout; the cross-centre inputs α_i are exterior A-1 evaluations.

$$\alpha_i = R_{s,i}/d, \quad d \geq R_{b,1} + R_{b,2}, \quad R_{b,i} \geq R_{s,i}, \quad (13)$$

$$F = \frac{G\mathcal{M}_1\mathcal{M}_2}{d^2} \leq \frac{F_{\max}}{4}. \quad (14)$$

The last bound belongs only to this pair domain; equality requires equal saturated supports at contact. The chip has nonspherical supports. Its small gravitational compactness is a correct reading of I-1; a rate is reached only through a declared response kernel.

The metric is $g_{\mu\nu}$, the Einstein tensor $G_{\mu\nu}$, stress tensor $T_{\mu\nu}$ and Einstein coupling κ . The angular element $d\mathcal{J}_{\text{ang}}^2$ is the unit-sphere line element; $d\ell_{\text{opt}}^2$ is the optical line element. The three I-3 faces are the branch atlas above, the unchanged field equations and the exact exterior optical metric:

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = \frac{8\pi G}{c^4}, \quad \kappa F_{\max} = 2\pi, \quad (15)$$

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1-\alpha_{\text{ext}})^2} + \frac{r_f^2 d\mathcal{J}_{\text{ang}}^2}{1-\alpha_{\text{ext}}}. \quad (16)$$

The density coordinate is downstream of the stress tensor and the same geometric data. Generalised Snell transport uses this exact optical metric. The remainder notation $O(\cdot)$ denotes asymptotic order. For weak exterior fields, the isotropic optical index n_{iso} and bending angle ϑ_{bend} at impact distance b have the I-3 limit

$$n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2), \quad \vartheta_{\text{bend}} = \frac{4G\mathcal{M}}{bc^2}. \quad (17)$$

The aperture diagnostic $n_{\text{cone}} = (1 - \alpha_{\text{ext}})^{-1/2}$ is distinct from that optical index. Containment means the same curvature from the same stress tensor, data and gauge; relabelling the chart changes no prediction.

The clock factor $\mathbf{m}$ rescales a pattern's kept time and thread length together. Its four-velocity is $\mathcal{U}^\mu$ and rest-space projector $\mathfrak{P}^{\mu\nu}$. Core 0.1c gives the pattern's and bookkeeper's readings:

$$g_{\mu\nu} \mapsto \mathbf{m}^2 g_{\mu\nu}, \quad d/t = c, \quad (18)$$

$$\nabla_{\mathcal{U}} \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \partial_\nu \ln \mathbf{m}, \quad \mathfrak{P}^{\mu\nu} = c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu. \quad (19)$$

Nothing changes on the pattern's own clocks and rulers. Moving at constant speed through a density gradient is gravity, the extended equivalence principle. The slow pattern reads lapse; the stroke reads the optical metric, including the factor of two in weak deflection. A conformal clock factor preserves null paths and is neither compactness, odds nor optical index. For a perfect-form stress tensor, the medium's flow clock factor is temperature at zero net conserved count, or enthalpy per count at conserved count and fixed entropy per count. That statement concerns the medium's flow; the thermal frequency shift of an oscillator it carries comes from its own thermal motion.

The flow speed is v and the falling pattern's time t_{own} . Core 0.1c gives

$$v = c\sqrt{\alpha_{\text{ext}}}, \quad \frac{dr_f}{dt_{\text{own}}} = -v, \quad \frac{d^2r_f}{dt_{\text{own}}^2} = -\frac{GM}{r_f^2}. \quad (20)$$

A rotating source adds swirl: a gyroscope reads half its curl, the leading Lense–Thirring precession. The flow operates at every size. I-3a reads gradients and curvature between neighbours: trace expansion reads density, the antisymmetric bivector reads vorticity, and shear reads the tide. The quadrupole of an extended pattern compares its skin with its centre path. No single spectral shift establishes a density gradient; the tests below pair patterns or reference paths to cancel the conventional deviation.

1.3 Carrier identity, confinement and energy accounting

I-2 identifies photon, graviton, gluon, phonon and magnon as the same spin-one density excitation in different density and geometry regimes. The chain supplies one medium and one propagation speed for these regime readings. The magnon is the stroke through a helical density gradient (P-5). The avoided crossing reads two regime modes of that carrier hybridising. The mode's angular frequency is ω , its complex projection ψ and amplitude A , its helical phase ϕ_h , initial phase ϕ_0 , and imaginary unit i . Cylindrical radius r_{hel} and axial coordinate z_{hel} retain their geometric meanings; $x_{\text{hel}}, y_{\text{hel}}$ are transverse projections and u_{hel} the axial transport speed. The Clifford basis vectors $\mathbf{e}_1, \mathbf{e}_2$ span the phase plane and $\mathbf{B}$ is its unit bivector. I-4 and the cylindrical lift give

$$x_{\text{hel}} = r_{\text{hel}} \cos \phi_h, \quad y_{\text{hel}} = r_{\text{hel}} \sin \phi_h, \quad z_{\text{hel}} = u_{\text{hel}} t, \quad (21)$$

$$\phi_h(t) = \phi_0 - \omega t, \quad \psi = Ae^{i\phi_h}, \quad (22)$$

$$i \longleftrightarrow \mathbf{B} = \mathbf{e}_1 \mathbf{e}_2, \quad e^{\mathbf{B}\phi_h} = \cos \phi_h + \mathbf{B} \sin \phi_h. \quad (23)$$

The exponent is dimensionless; this is a single-angle phase rotor, distinct from a half-angle spinor rotor. Superposition is ignorance of ϕ_0 at observation in a deterministic universe. The phase axis changes while field identity stays fixed. The precession sense and the antenna field's sense read the handed gradient; this identification is the analysis's P-5 reading. I-4's L1, exact in model, is the FDT pair algebra on its declared domain; L2, containment, is GR's reduction to $G\mathcal{M}_1\mathcal{M}_2/d^2$ at leading order of the weak-field, slow-motion expansion, its corrections of relative order $R_{s,i}/d$ and of the bodies' squared speed over c^2 dropped; L3, owed, is the FDT source programme and each claimed divergence. The line-shape identity below belongs to none of these as a new curvature result.

The strand count n labels a braid class; domain Ω , boundary data B , orientation χ and position $\mathfrak{s}$ on the inside/outside chart enter confinement operator D . The eigenmode is u_q ; its mass and frequency are those already defined. I-5 requires the eigenproblem

$$D(\Omega, B; n, \chi, \mathfrak{s})u_q = \omega_q u_q, \quad m_q c^2 = \hbar \omega_q. \quad (24)$$

Changing confinement within one strand class differs from rebraiding to another class. No standing-mode index, wavenumber, occupation or isotope mass number is assigned a strand count. There is no nuclear mass formula, anchor radius or absolute cone address in this calculation.

In I-7, M is rest mass, P pressure, V the specified support volume, T temperature, σ_{SB} the radiation flux coefficient, and a the radiation energy-density coefficient.

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{aT^4V}{c^2}, \quad a = \frac{4\sigma_{\text{SB}}}{c}. \quad (25)$$

Deposited drive energy enters E/c^2 once, whether stored in excited modes or converted into damping heat in the film and resonator. The film's zero-drive magnetic energy and the superconductor's condensation energy each enter E/c^2 once. Expansion work enters PV/c^2 , and radiative energy enters aT^4V/c^2 ; these portions are excluded from the separately counted E . Gravitational binding is read from the geometry and adds no separate local stress term. The source's measurement temperature stays T_{src} : its numerical value is absent (SC-070, L131; SC-280, L325). The positive fixed-geometry radiation partial derivative alone determines neither the total thermal direction of compactness nor an oscillator frequency shift.

1.4 Notation guard

The source's angular mode frequencies are $\omega_c, \omega_m, \omega_{\pm}$, with detuning $\Delta\omega$; ω_0 is its Kittel frequency. Damping rates are $\mathfrak{k}_m, \mathfrak{k}_c, \mathfrak{k}_{\pm}, \mathfrak{k}_{\text{sw}}$; $\mathbf{g}$ is fitted spectral coupling. The gyromagnetic ratio is $\mathbf{g}_m$, thickness $\mathfrak{t}$, signal width $\mathfrak{w}$, antenna width $\mathfrak{w}_a$, group velocity $\mathfrak{v}_g$ and escape time t_{sw} . The spin-wave wavelength is $\mathcal{L}_{\text{sw}}$, wavenumbers $k, k_{\perp}, k_{\perp, \mathbf{p}}$ and standing-mode index $\mathbf{p}$. Saturation magnetisation is M_s , exchange stiffness A_{ex} , applied bias H_{bias} , and generic field H . Scattering parameters are S_{21}, S_{12} , cooperativity C , exchange field H_{ex} , magnetic mode volume V_m and resonator mode volume V_c .

Source use	Manuscript symbol	Reserved distinction
Damping κ	$\mathfrak{k}_m, \mathfrak{k}_c, \mathfrak{k}_{\pm}, \mathfrak{k}_{\text{sw}}$	κ is Einstein coupling.
Gyromagnetic γ ; coupling g	$\mathbf{g}_m; \mathbf{g}$	γ names the photon; $g_{\mu\nu}$ is metric.
Mode n ; thickness t	$\mathbf{p}; \mathfrak{t}$	n is strand count; t time.
Widths l, w ; velocity v_g	$\mathfrak{w}_a, \mathfrak{w}, \mathfrak{v}_g$	l is multipole degree; w probe weight; v flow.
Wavelength λ	$\mathcal{L}_{\text{sw}}$	Resonator design is written quarter-wave.
M_s, A_{ex}	M_s, A_{ex}	M rest mass, A phase amplitude.
Bias subscript B ; S_{21}	H_{bias}, S_{21}	B boundary data, $\mathbf{B}$ bivector, S_i sensitivity.
DE, BV; tesla	DE, BV; T	D confinement operator, T temperature.
Frequencies and wavenumbers	ω with mode labels; k	No source mode is ω_q or a strand-indexed mode.
Permeability; Euler number	$\mu_0; e$	Medium response; exponential only.
Added observables	$H_{\text{deg}}, \mathfrak{t}_{\text{tw}}, \mathfrak{r}_P, T_c$	Degeneracy field, twist rate, standing-mode ratio, transition temperature.

The table's source-symbol column quotes the source dictionary; subsequent equations use only the renamed quantities. The added quantities are defined operationally where used.

2 System chart

The symbol ∞ denotes infinity. A probe weight w defines the sampled average of any interior quantity f over position volume element dV .

$$\langle f \rangle_w = \frac{\int w f dV}{\int w dV}, \quad 0 < \int w dV < \infty. \quad (26)$$

Weights are the specified nonnegative probe-intensity or sensitivity weights; a signed mode overlap is kept separately. The exterior source atlas and the interior chart of each bound configuration are declared independently. The interior skin is at the fold; the laboratory room radius R_{room} comes from data, with no transfer map from the exterior atlas. For the planar YIG film and NbN layer the analysis reads each half-thickness as its room scale; that identification supplies no local density profile.

Pattern or datum	Geometry and chart	Probe or normalisation
Straight circuits (SC-045, SC-049)	3 mm length; 2 and 0.5 μm signal widths; exterior A-1 source	Resonator field over film under signal line for anticrossings and linewidths.
Meander circuit (SC-046, SC-049)	3 mm; 1 μm ; three folds; exterior A-1 source	Same field weight, with its alternating lateral profile.
YIG film (U-03)	Interior A-1 chart; assumed 100 nm thickness; exterior atlas separately	Standing modes weighted through thickness; local skin profile unspecified.
Quasi-Kittel mode (SC-068)	Interior datum	Quarter-wave feedline field over 20 μm width; meander probe width unstated.
Detection path (SC-123)	5 μm coupler spacing, support distance	Transmission path between resonator and coupler.
TEM image	Lamella support	Projection through lamella; no numerical strain or density profile.
Volumes	V_m, V_c, V	Mode and thermal support volumes unprinted; no numerical per-volume inference.
NbN layer (SC-094) and nitrogen-enrichment witness bar	30 nm thickness; interior A-1 chart with half-thickness as room scale from data; exterior A-1 source separately	Current density across the witness bar for T_c ; no local density profile.

The geometry entries trace to (SC-044, L90; SC-045, L93; SC-046, L94; SC-049, L97; SC-068, L129; SC-094, L149; SC-108, L197; SC-123, L204); the YIG thickness is U-03. These dimensions are support sizes, not measurements of interior density. Quarter-wave films originate at Argonne and meander films at KAIST; the source reports better lattice match and lower cryogenic damping for the latter without a numerical strain comparison (SC-052, L103; SC-053, L105; SC-054, L107; SC-055, L108). This separates film provenance from width in comparisons (SA-06, SA-17). The source explains its larger field offset by the smaller signal width and larger excited wavevectors (SC-103, L151; SC-104, L195; SC-105, L196). Those wavevectors remain geometry estimates. Sputtered YIG on YSGG(111) is air annealed at 850 $^{\circ}\text{C}$ for 3 hours and cooled at 30 $^{\circ}\text{C}$ per hour (SC-084, L139; SC-085, L140; SC-086, L141; SC-087, L142). The spacer is 10 nm SiN and resonator layer 30 nm NbN (SC-092, L147; SC-094, L149). The source states that TEM confirmed the YIG films' epitaxial quality and that the dark circular features correspond to strain-induced local lattice rotations and will be discussed elsewhere (SC-088, L143; SC-089, L144; SC-090, L145; SC-091, L146). No resonator frequency, signal-ground gap, film area, substrate thickness or transition temperature is supplied (SC-047, L95; SC-048, L96; SC-051, L99; SC-082, L137; SC-121, L200;

SC-122, L201). This construction map corroborates no invariant: it fixes supports and probes, not a constitutive density relation.

3 Observation map

3.1 Avoided crossings and rates

The source states that the avoided crossing in longitudinal bias indicates coherent coupling of the resonator to the DE spin-wave mode and that the continuous mode traversing the gap corresponds to the film’s quasi-Kittel mode coupled to the feedline (SC-058, L117; SC-060, L119; SC-061, L121; SC-062, L122; SC-063, L123; SC-064, L124; SC-066, L126). Its fitted coupled-mode equation is reproduced below with $\Delta\omega = \omega_c - \omega_m$ (SC-071, L132; SC-072, L133; SC-073, L134; SC-074, L135).

$$\omega_{\pm} = \frac{\omega_c + \omega_m}{2} \pm \sqrt{\frac{\Delta\omega^2}{4} + g^2}. \quad (27)$$

The positive squares and radical restore the dimensionally required frequency norm at this locus. At zero detuning the analysis obtains $\omega_+ - \omega_- = 2g$ by subtraction. The quarter-wave fit gives $g/(2\pi) = 115$ MHz (SC-080, L141); the meander fits give 150 MHz for DE and 100 MHz for BV (SC-237, L310; SC-238, L311). Their peak separation is a fit-derived response, not a force. The source extracts quasi-Kittel magnon damping rates of 70 MHz and 34 MHz, respectively (SC-069, L130; SC-109, L198); it determines the photon damping rate 1.1 MHz for both devices with the magnon mode far detuned (SC-222, L294; SC-223, L295). All these rates use division by 2π . The resonance-field weights are those of the system chart, while the quasi-Kittel probe differs.

The linewidths $\mathfrak{k}_{\pm}$ in the source’s model are (SC-207, L281; SC-208, L285)

$$\begin{aligned} \mathfrak{k}_{\pm} = & \frac{\mathfrak{k}_m}{2} \left(1 \pm \frac{\Delta\omega}{2\sqrt{\Delta\omega^2/4 + g^2}} \right) \\ & + \frac{\mathfrak{k}_c}{2} \left(1 \mp \frac{\Delta\omega}{2\sqrt{\Delta\omega^2/4 + g^2}} \right). \end{aligned} \quad (28)$$

The radical and positive squares here are fixed by a dimensionless mixing fraction. At zero detuning the analysis obtains the mean of the two damping rates; at large positive detuning the plus mode tends to the magnetic rate and the minus mode to the circuit rate, with roles exchanged at negative detuning under the printed convention. Using the source’s least precise two-figure damping inputs gives resonance linewidths of 36 MHz for the wider quarter-wave device and 18 MHz for the meander. The labels of the linewidth branches are the source’s convention; continuity and the fitted spectra determine their experimental assignment. The meander curve reuses the quasi-Kittel damping and fitted spectral coupling; its agreement supports the source’s assumption in-sample (SC-218, L290; SC-219, L291; SC-220, L292; SC-231, L303; SC-232, L304; SC-233, L305; SC-234, L306). For the quarter-wave lower-field branch near 0.15 T, the measured linewidth exceeds that model; the source says this suggests spin-wave damping larger than the Kittel-mode magnon damping, rendering it uncertain whether $g > \mathfrak{k}_m$ is strictly satisfied (SC-240, L313; SC-241, L314; SC-242, L315; SC-243, L316; SC-244, L318; SC-245, L320). For the perpendicular standing spin wave (PSSW) mode in the wider quarter-wave device, the source writes that the magnon-photon coupling strength is smaller than the magnon damping and that in this regime it fits the data using the Purcell-limit damping rate expression, with details in the Supplemental Materials (SC-224, L296; SC-225, L299; SC-226, L300; SC-227, L301; SC-228, L302).

The cooperativity definition is $C = g^2/(\mathfrak{k}_c\mathfrak{k}_m)$ (SC-236, L309). The restored square makes this dimensionless. The source prints 662 and 294 (SC-236, L309; SC-237, L310); central-input

recomputation gives 6.0×10^2 and 2.7×10^2 , limited by the two-figure damping inputs. Rounding intervals include the printed cooperativities if the couplings carry two significant figures; three-figure coupling intervals exclude them. For the meander, both fitted couplings exceed both damping rates, meeting the source’s criterion (SC-237, L310; SC-238, L311; SC-239, L312). These formula substitutions are identity maps and corroborate no invariant; a discrepancy tests the adopted response fit, not the one-carrier ontology.

3.2 Dispersion, sign conventions and overlap

The source describes positive or negative offsets relative to the feedline quasi-Kittel mode as consistent with the respective spin-wave dispersions (SC-035, L80; SC-036, L81; SC-037, L84; SC-038, L85). The source’s DE, BV and Kittel angular frequencies are $\omega_{\text{DE}}, \omega_{\text{BV}}, \omega_0$; the exponential uses Euler’s number. Its dispersion equations read (SC-166, L230; SC-167, L235; SC-168, L240)

$$\omega_{\text{DE}}(k) = g_m \mu_0 \sqrt{H(H + M_s) + \frac{M_s^2}{4}(1 - e^{-2kt})}, \quad (29)$$

$$\omega_{\text{BV}}(k) = g_m \mu_0 \sqrt{H \left(H + M_s \frac{1 - e^{-kt}}{kt} \right)}, \quad (30)$$

$$\omega_0 = g_m \mu_0 \sqrt{H(H + M_s)}. \quad (31)$$

At these three loci the radicals, and the DE magnetisation square, are restored by dimensional consistency; the printed exponential signs remain negative. Multiplication by $\hbar$ converts every dispersion to energy, so no angular frequency is silently treated as energy. The source-selected inputs are $\mu_0 M_s = 0.175 \text{ T}$, $A_{\text{ex}} = 2.6 \text{ pJ m}^{-1}$, $t = 100 \text{ nm}$ and $g_m/(2\pi) = 28 \text{ GHz T}^{-1}$ (U-01–U-04). The estimated wavelengths are 2.8, 5.2 and 6.5 μm , in narrow-quarter-wave, meander and wider-quarter-wave order (SC-216, L294; SC-217, L295); $k = 2\pi/\mathcal{L}_{\text{sw}}$ is their geometric estimate. The later half-wave labels in that list conflict with the quarter-wave design description (SC-042, L87; SC-216, L294; SC-217, L295).

The source’s frequency offsets are $\Delta\omega_{\text{DE}}, \Delta\omega_{\text{BV}}$, each mode minus Kittel (SC-169, L241). Its plotted expression is their difference, although the prose calls it a sum (SC-165, L227; SC-205, L281). For field offsets the analysis defines $\Delta H = H_{\text{Kittel}} - H_{\text{mode}}$ at fixed frequency. This yields positive DE and negative BV offsets in the equations; the earlier source prose uses negative DE and positive BV (SC-101, L149; SC-102, L150; SC-212, L290). The two conventions are displayed together, not merged into a new datum. The source says the curve’s agreement confirms the role of spatially nonuniform modes (SC-172, L244; SC-173, L245; SC-174, L246); with its inputs held fixed the calculation is an identity map and corroborates no invariant. The source’s Fig. 4 caption sends panel (c) to Fig. 2(c) and panel (d) to Fig. 4(b), whereas the text names Fig. 1(e) and Fig. 2(b); it labels the 115 MHz and 150 MHz fit inputs (a) and (b) although the linewidth panels are (c) and (d) (SC-198, L273; SC-199, L274; SC-200, L275; SC-201, L277; SC-202, L278; SC-203, L278; SC-204, L279). Numerical comparisons here use the named devices and printed fit values.

The source’s 43% DE excess is an average over three devices (SC-175, L248; SC-176, L250; SC-177, L251; SC-178, L252); the printed meander pair alone gives 50% by taking its ratio minus unity. The other device pairs are plot-only, so the average has no independent numerical reconstruction here. The source explains that in DE both in-plane and vertical microwave components are orthogonal to magnetisation and can contribute, whereas in BV only the vertical component contributes; it writes that the resulting weaker spin-wave coupling is consistent with previous observations in coplanar waveguide (CPW) resonator systems (SC-179, L253; SC-180, L254; SC-181, L255; SC-182, L256; SC-183, L257; SC-184, L258; SC-185, L260). It emphasises the enhanced coupling strength achieved with the meander resonator design compared to the straight CPW design and writes that this enhancement primarily originates from improved mode overlap between the

spatially alternating microwave field of the meander geometry and the sinusoidal spin wave mode (SC-186, L262; SC-187, L263; SC-188, L264; SC-189, L265; SC-190, L266; SC-191, L267). Its volume argument has $g \sim \sqrt{V_m/V_c}$, with the positive root restored from the stated volume scaling (SC-191, L267; SC-192, L268; SC-193, L270; SC-194, L272; SC-195, L273). It says varying the lateral dimensions of the resonator does not necessarily change the coupling strength, since it is predominantly determined by the mode-volume ratio between the magnon (V_m) and resonator (V_c) systems; in this ratio the lateral area cancels out, leading to only a weak dependence on resonator width (SC-191, L267; SC-192, L268; SC-193, L270; SC-194, L272; SC-195, L273; SC-196, L274; SC-197, L275). The volumes, overlap integral and field computation are unprinted; changing geometry also changes film provenance. This is the source's interpretation of its response comparison, not an independently calibrated density response.

3.3 Standing waves, directional transmission and escape

The source observes an extra mode anticrossing at lower fields in both DE and BV configurations, corresponding to the perpendicular standing spin wave (PSSW) mode; it writes that the coupling strength to this mode is clearly stronger than observed for the straight quarter-wave resonator in Fig. 1(e) and that this enhancement originates from the meander geometry, which increases the microwave field inhomogeneity along the YIG film-thickness direction and thereby improves coupling to the PSSW mode (SC-110, L199; SC-111, L200; SC-112, L201; SC-113, L202; SC-114, L203; SC-115, L204; SC-116, L205; SC-117, L206; SC-118, L207). The field profile compares paths through the extent and supplies the standing-mode weight. The wavenumbers $k_{\perp,0}$ and $k_{\perp,1}$ denote the main and first standing modes. The source exchange-field equation and its one-pinned-surface alternative are (SC-127, L208; SC-128, L210; SC-130, L212; SC-135, L217; SC-137, L220; SC-138, L220)

$$\mu_0 H_{\text{ex}} = \frac{2A_{\text{ex}}}{M_s} (\pi/t)^2, \quad (32)$$

$$k_{\perp,p} = (p + 1/2)\pi/t, \\ \mu_0 H_{\text{ex},1} = \frac{2A_{\text{ex}}}{M_s} (k_{\perp,1}^2 - k_{\perp,0}^2). \quad (33)$$

Positive squares at both field loci follow from the stiffness and magnetisation units. Using U-01–U-03 and the arithmetic convention $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ (U-05) returns the source's 37 mT and 74 mT at its displayed precision, respectively, with AGREE for each (SC-130, SC-138). The observed value is 80 mT, with its configuration and field reference unstated (SC-132, L214). Strong pinning from lattice mismatch is the source's one possibility; its second estimate agrees more closely, not exactly (SC-132, L214; SC-133, L215; SC-134, L216; SC-138, L220; SC-139, L221). No local-density profile has been measured by this subtraction.

The source attributes nonreciprocal detection of the hybrid mode to intrinsic DE nonreciprocity (SC-039, L88; SC-040, L89; SC-041, L90). The parallel detection CPW has the same signal width and gap as the quarter-wave resonator; its separation is 5 μm (SC-119, L198; SC-120, L199; SC-121, L200; SC-122, L201; SC-123, L204). The cited 8.7 μm propagation distance belongs to YIG/GGG elsewhere (SC-124, L205; SC-125, L206; SC-126, L207). In the same field sweep, the lower hybrid branch has stronger S_{21} than S_{12} ; reversal exchanges their order (SC-140, L210; SC-141, L211; SC-142, L212; SC-143, L213; SC-144, L214; SC-145, L215; SC-146, L216; SC-147, L217; SC-148, L218). The source measures 4 dB power isolation at the hybrid peak for both polarities and says that the observed nonreciprocity confirms the role of the Damon-Eshbach spin wave in the hybrid magnonic mode (SC-154, L224; SC-155, L227; SC-156, L228; SC-157, L229). The two degeneracy settings are -0.171 and $+0.187$ T; the source attributes their field difference to hysteresis in the superconducting magnet (SC-149, L220; SC-150, L221; SC-151, L222; SC-152, L223). Their magnitude difference is

16 mT; under the analysis's symmetric true-degeneracy convention, half the signed sum gives an 8 mT common field offset. The reported absence of nonreciprocity in the BV control after a 90 degree rotation refers to supplementary data, and the source also shows that a broad-band spin wave nonreciprocity of 10 dB can be measured under the DE configuration using the same CPW antenna geometry as in Fig. 3(a), without printing its data (SC-158, L231; SC-159, L232; SC-160, L235; SC-161, L237; SC-162, L238; SC-163, L241; SC-164, L242). Neither result adds held-out traces here. The directional path contrast is recovered through the source response; it corroborates no invariant on its own.

The source's escape model defines t_{sw} through effective antenna width and group velocity (SC-249, L328; SC-250, L330).

$$t_{\text{sw}} = \frac{w_a}{v_g}, \quad \frac{\mathfrak{t}_{\text{sw}}}{2\pi} = \frac{v_g}{2\pi w_a}. \quad (34)$$

For the narrow and wider quarter-wave devices and the meander it takes widths 2, 6, 10 μm and calculated velocities 333, 435, 413 m s^{-1} (SC-251, L332; SC-252, L333; SC-253, L334; SC-254, L335; SC-255, L337). Substitution returns 26.5, 11.5, 6.6 MHz, AGREE with the source estimates (SC-256, L338; SC-257, L339; SC-258, L340). Those decimals reproduce nominal estimates; the chosen effective widths establish no experimental error bar. The source interprets these rates below the magnetic damping as intrinsic relaxation dominating (SC-259, L341; SC-260, L342; SC-261, L343). It states that strong coupling without lateral confinement is possible, mainly due to slow group velocity (SC-262, L344; SC-263, L345; SC-264, L346; SC-265, L347); this retains the meander success and quarter-wave uncertainty. Its distant-resonator proposal requires a larger escape fraction before decay, while cited films below 10 MHz damping belong to other work (SC-229, L301; SC-230, L302; SC-266, L348; SC-267, L349; SC-268, L299; SC-269, L300). These escape calculations are identity maps and corroborate no invariant.

4 Cone-position translation

The chip's gravitational window is $\alpha_{\text{ext}} = \mathcal{M}c^2/(2F_{\text{max}}r_f)$ on exterior A-1. It is small at laboratory compactness and has its declared source and field point. No transmission, linewidth, mode fraction, isolation, DE/BV ratio or standing-mode ratio supplies another compactness coordinate. A-4 carries no absolute position for the nuclei or circuit, and A-5 ranks no microwave or magnetic regime. The half-integer standing-wave boundary condition is unrelated to the A-1 seam. The superconducting transition is among I-1a's listed instances on the A-3 chart, whose token supplies no sign or threshold here; the isotope discriminator below uses exterior A-1 and I-7.

For the driven response, $\mathcal{Z}_{\text{drv}}$ is the unnormalised response and $\mathcal{Z}_0$ a positive response scale; $\mathcal{S}_{\text{drv}}$ is its odd contraction, a signed drive entering as a magnitude with its polarity carried by a separate orientation sign into this contraction only (I-6).

$$\mathcal{S}_{\text{drv}} = \frac{\mathcal{Z}_{\text{drv}}}{\mathcal{Z}_0 + \|\mathcal{Z}_{\text{drv}}\|}, \quad \mathcal{Z}_0 > 0. \quad (35)$$

The transmitted signal vanishes at zero drive; its ratio to incident amplitude, $\mathcal{S}_{21}$, need not vanish in that limit. A causal susceptibility maps force to observable; it is not the force law. The source's rate model gives no film-resonator force kernel and no charge-seat or colour-seat force input.

For a standing-mode boundary shift η , define the derived uniform-film coefficient $\mathcal{K}$ and field ratio $\mathfrak{r}_\text{P}$.

$$\mathcal{K} = \frac{2A_{\text{ex}}}{M_s} (\pi/t)^2, \quad (36)$$

$$\mu_0 H_{\text{ex,p}} = \mathcal{K}[(\mathfrak{p} + \eta)^2 - \eta^2], \quad \mathfrak{r}_\text{P} = \frac{(2 + \eta)^2 - \eta^2}{(1 + \eta)^2 - \eta^2}. \quad (37)$$

Both-free, one-pinned and both-pinned surfaces correspond to $\eta = 0, 1/2, 1$ in this conventional idealisation (one-pinned shift: SC-135, L217), giving 4, 3, 8/3 respectively. The main mode's own exchange term $\mathcal{K}\eta^2$ vanishes only for free surfaces. The free second mode is even with zero mean and couples through curvature of the thickness profile; the next odd mode couples through its gradient and gives ratio 9. The independent comparator measures each interface anisotropy and magnetisation, stiffness and anisotropy depth profiles by the U-53 methods. A graded conventional profile moves these fields already. A proposed 20 GHz meander at 2.0 K, on 12 films (U-11, U-36), exceeds the largest required zero-bias frequency: the free third mode gives 11 GHz from U-01–U-04 at two significant figures. Thus all tested modes cross at positive bias in this chosen uniform reference, subject to independent verification of the actual film parameters. The superscript T denotes transpose. For fitted-field covariance matrix Σ_H , gradient $\mathbf{j} = (-\mathbf{r}_P, 1)/H_{\text{ex},1}$, and mean comparator uncertainty s_{comp} , ratio variance is $\mathbf{j}\Sigma_H\mathbf{j}^{\text{T}}$. A common dipolar-offset uncertainty s_{dip} contributes $|1 - \mathbf{r}_P|s_{\text{dip}}/H_{\text{ex},1}$ to the mean, without reduction by film count. Detection and attribution evaluate it at the comparator ratio; exclusion uses the specified alternative ratio (U-47–U-49). This conventional diagnostic determines boundary conditions; it supplies no reduction of the local skin-density ordering to a film profile.

5 Predictions

The tests below are conditional by construction where competing constitutive sensitivities enter. Each observable is obtained independently of the isotope assay or field control used to predict it. DC-4 addresses SA-22 but is retired because the skin-density ordering supplies no reduction to the film's local profile (O-4).

5.1 Shared definitions and decision rules

An enrichment coordinate u is the independently assayed heavy-isotope fraction; its endpoints are u_L, u_H . For observable $\mathcal{O}$ and positive channel variables $\mathbf{r}_i$, S_i is the partial derivative of $\ln \mathcal{O}$ with respect to $\ln \mathbf{r}_i$ with every other listed channel held fixed. For signed strain the derivative is with respect to strain itself, not its logarithm. The finite path, with integrable sensitivities and a positive observable throughout, is

$$\Delta \ln \mathcal{O} = \int_{u_L}^{u_H} \sum_i S_i(u) \frac{d \ln \mathbf{r}_i}{du} du. \quad (38)$$

Two endpoints determine a secant; they determine no local derivative without these constitutive assumptions. The density-channel input is the I-7 increase of exterior A-1 compactness at fixed pressure, radius and packing. The observable sign stays conditional until the sensitivities are established; the discriminator's composite sign is $\text{sgn}(\prod_i S_i)$ along the declared response chain. The following constant sensitivities instantiate the path exactly; they are illustrative assumptions, not inferred from the source spectra.

For Poisson ratio ν , free vibrational log-volume increment ϵ_{free} and clamped out-of-plane strain $\epsilon_{\perp}$, the declared small-strain comparator is

$$\epsilon_{\perp} = \frac{1 + \nu}{3(1 - \nu)} \epsilon_{\text{free}}. \quad (39)$$

It includes zero-point and thermal populations at the test temperature. The substrate fixes in-plane strain; the free linear strain is enhanced out of plane by $(1 + \nu)/(1 - \nu)$. All responses below use this strain derivative, not a hydrostatic derivative or an equal-strain partition. The exponential continuation used for finite test points is a declared model of these small strains.

For two compositions indexed by j , N_j counts independent runs or anneals, s_j^2 is the variance of their net means, and $\mathbf{v}_j = s_j^2/N_j$. The net difference after subtracting the comparator is $\hat{\delta}$. The Welch degrees of freedom are $\mathfrak{f}$, the Student quantile $\mathcal{T}_{p,\mathfrak{f}}$ and normal quantile z_p at probability p . Shared uncertainty is s_{sh} ; the detection threshold is Q .

$$\mathfrak{f} = \frac{(\mathbf{v}_1 + \mathbf{v}_2)^2}{\mathbf{v}_1^2/(N_1 - 1) + \mathbf{v}_2^2/(N_2 - 1)}, \quad Q^2 = \mathcal{T}_{p,\mathfrak{f}}^2(\mathbf{v}_1 + \mathbf{v}_2) + z_p^2 s_{\text{sh}}^2. \quad (40)$$

Within each run the device variance is nested, never counted as independent runs. For measured nuisance differences $\mathfrak{d}_i^{\text{net}}$ and comparator differences $\mathfrak{d}_i^{\text{cmp}}$, define $\mathfrak{d}_i = \mathfrak{d}_i^{\text{net}} + \mathfrak{d}_i^{\text{cmp}}$. Let $\mathbf{a}$ contain the sensitivity-uncertainty contributions $s_{S_i}\mathfrak{d}_i$, the shared calibration contributions $s_{c_i}S_i\mathfrak{d}_i$ and the remaining comparator uncertainty s_0 . Its correlation matrix is $\mathcal{C}$.

$$s_{\text{sh}}^2 = \mathbf{a}^\top \mathcal{C} \mathbf{a}. \quad (41)$$

A calibration shared by several channels acts on their sensitivity-weighted sum before this contraction; separate calibration groups give separate components. Thickness and residual-stress components include the isotope's vibrational strain in $\mathfrak{d}_i^{\text{cmp}}$. Thus netting and comparator calibration cannot be counted as independent errors when they use the same sensitivity. The numerical examples take zero net nuisance difference after adjustment (U-57), retaining nonzero comparator differences and uncertainty in both operations.

Detection is $|\hat{\delta}| > Q$. The bound $\mathfrak{B}_{\text{sys}}$ covers all unmeasured channels shared by a target or gas lot, and in the torsion test the residual-response product and the stage's hand-odd field; it is never divided by replication count. For a positive specified change δ_* , exclusion is $\delta_* - \hat{\delta} > Q + \mathfrak{B}_{\text{sys}}$, and attribution is $|\hat{\delta}| > Q + \mathfrak{B}_{\text{sys}}$. For a negative specified change reverse the contrast before using exclusion. The formulas retain covariance; an instrument resolution is no threshold.

The tables below give the numerical test points and their provenance. The input and output labels refer to the stated tests; every numerical output below is a derived conditional value. Nominal assumption coefficients are specified to two significant figures (U-57); the source constants retain their own digits. Masses use the nominal isotope mass numbers in atomic-mass units, a declared arithmetic approximation with no strand assignment.

Input	Values in the stated order	Provenance
fractions	0.05, 0.95	U-27
iron masses	54, 56	U-21
oxygen masses	16, 18	U-22
nitrogen masses	14, 15	U-23
other masses	89, 93 (Y, Nb)	U-24
Poisson	0.30, 0.25 (YIG, NbN)	U-28
free volumes	-0.00010, -0.00050, -0.00010 (Fe, O, N)	U-29
iron slopes	1.0, 0.50, 0.50, 1.0	U-37
iron comparator	-0.00010, 0.00010	U-40
oxygen slopes	1.0, 0.50	U-38
oxygen comparator	-0.00020, -0.00010, 0.00010, -0.00010	U-41
pairing	0.25	U-30
strain slope	1.0	U-42
nitrogen slope	3.0	U-39

Input	Values in the stated order	Provenance
runs	12, 4 (units per composition, nested devices)	U-33, U-34
scatter	0.0010, 0.0020 (run, device log SD)	U-43
shared	0.10, 0.010, 0.00020 (sensitivity, calibration, remainder)	U-44
correlation	0.25 (shared equicorrelation)	U-45
bound	0.00050 (log units)	U-46
probability	0.05, 0.975 (test size, quantile probability)	U-19
simulation	20000, 260627279 (trials, seed)	U-20
cooldowns	12, 8 (cooldowns, cycles)	U-13
field fit	2.0, 0.25 (μT SD, within-cycle correlation)	U-15
field scatter	1.0, 1.0 (cooldown, cycle μT SD)	U-16
field alternative	10 μT , exclusion target only	U-14
residual	10, 0.010 (μT bound, hand-odd response)	U-17
field bound	0.20, 0.010 (μT stage bound, response SD)	U-18

The co-varying-channel sign table refers to the illustrative sensitivities above; uncalibrated material signs remain conditional.

Test	Positive contributions at the test point	Negative or bounded contributions
T-1	Nonzero magnitude of the additional twist-field contrast	Conventional polarity-even response cancels; residual-response product and stage field are bounded.
T-2	Density input with positive derivative; positive overlap change with positive derivative	Negative magnetic increment and clamped strain with positive derivatives; unresolved target channels bounded.
T-3	Density-induced stiffness increase; negative vibrational magnetisation increment; film and substrate contraction; positive squared-gap pinning increment	Density-induced magnetisation increase and negative vibrational stiffness increment; gas-lot channels bounded.
T-4	Density increase with positive transition derivative	Pairing mass increase with negative derivative; contraction with positive strain derivative; hyperfine and gas-lot channels bounded.

For each simulation independent Gaussian run or anneal errors reset in each composition's unit, with five equal-variance components for stoichiometry, thickness, interface, stress and deposition or anneal conditions. Two nested equal-variance components represent lithography and readout (U-57). The field simulation draws correlated four-state fits within cycles, cycle noise within cooldowns and independent cooldown offsets. The sensitivity and calibration vector is drawn once per simulated experiment with the stated correlation, shared by every unit. The seed is incremented by the test's index. Detection rates below use zero unmeasured bounded shift; attribution uses the worst positive bound and exclusion the worst negative bound at the specified true alternative. These are false-rejection rates under the declared Gaussian model, not empirical instrument performance.

5.2 T-1: torsion and the degeneracy field

DC-1 discriminates SA-08 by introducing the additional P-5 field of material torsion into the longitudinal-bias quarter-wave DE device. Use the source's 2 μm design (SC-057, L115), the chip

clamped at both ends in a nonmagnetic stage and torque train that twists it about the resonator's longitudinal axis, the bias axis of the DE configuration, with 5.0 mm clamp separation and twist rates $\pm 0.10 \text{ rad m}^{-1}$ (U-12), in the source's superconducting magnet (SC-152, L223), at 2.0 K (U-11). The analysis reads the established field as lying along the twist axis and reversing with its hand; P-5 fixes neither its sign nor its magnitude at the film. The observable uses resonator-field weighting under the signal line, comparing the two bias polarities at each hand with magnetisation reversed and port order exchanged.

Let $\mathfrak{h} = \pm 1$ label twist hand, $\mathfrak{b} = \pm 1$ bias polarity, and $\mathcal{H}_{\mathfrak{h}\mathfrak{b}} = \mu_0 H_{\text{deg}}(\mathfrak{h}, \mathfrak{b})$ the positive bias magnitude at zero detuning in Eq. (27). The baseline at fixed hand is $\mathcal{H}_0(\mathfrak{h})$; $\mathcal{B}_{\text{tw}}$ is the signed twist-field projection and $\mathcal{B}_{\text{res}}$ the field unreversed by the sweep. The double contrast is $\mathcal{D}_{\text{tw}}$. For the additive-field test hypothesis, the defining condition and its solution are

$$\omega_{\text{DE}}(\mathcal{H}_{\mathfrak{h}\mathfrak{b}} + \mathfrak{b}[\mathfrak{h}\mathcal{B}_{\text{tw}} + \mathcal{B}_{\text{res}}]; \mathfrak{h}) = \omega_{\text{c}}(\mathfrak{h}),$$

$$\mathcal{H}_{\mathfrak{h}\mathfrak{b}} = \mathcal{H}_0(\mathfrak{h}) - \mathfrak{b}[\mathfrak{h}\mathcal{B}_{\text{tw}} + \mathcal{B}_{\text{res}}], \quad (42)$$

$$\mathcal{D}_{\text{tw}} = \frac{1}{4}(\mathcal{H}_{++} - \mathcal{H}_{-+} - \mathcal{H}_{+-} + \mathcal{H}_{--}) = -\mathcal{B}_{\text{tw}}. \quad (43)$$

The frequency's first argument is expressed as a field in tesla here, with the permeability factor already absorbed. Taking the absolute contrast reads field magnitude without assigning its sign. In the conventional comparator a static twist is time-even; at each hand time reversal maps bias, magnetisation and reversed ports onto each other. Magnetoelastic anisotropy and the resonator's strain response act on both polarity states alike and cancel. The same sweep protocol at both hands cancels a hand-even residual field and magnet hysteresis at first order. The residual crossed with a measured hand-odd response survives at second order, bounded by their product; the stage's own hand-odd field survives directly. Measure the field at the chip position at both stage settings with the clamp released and identical travel (U-50). These controls distinguish an additional field from strain-driven changes of response.

Let s_{cool} be the sample SD of cooldown mean contrasts, N_{cool} their count and $\overline{\mathcal{D}}$ their mean. For specified field magnitude $\mathcal{B}_*$ and relative field-response uncertainty s_{resp} , define Q_{tw} and Q_{ex} .

$$Q_{\text{tw}} = \mathcal{T}_{p, N_{\text{cool}}-1} \frac{s_{\text{cool}}}{\sqrt{N_{\text{cool}}}}, \quad (44)$$

$$Q_{\text{ex}}^2 = Q_{\text{tw}}^2 + (z_p s_{\text{resp}} \mathcal{B}_*)^2. \quad (45)$$

Detection requires $|\overline{\mathcal{D}}| > Q_{\text{tw}}$; attribution adds the two systematic bounds; exclusion of $\mathcal{B}_*$ requires $\mathcal{B}_* - |\overline{\mathcal{D}}| > Q_{\text{ex}} + \mathfrak{B}_{\text{sys}}$. The cooldown is the replication unit, with alternated twist cycles nested inside it. The numerical alternative in the table is solely an exclusion target and assigns no twist-field magnitude. T1 detection threshold: 0.7 μT .

T1 exclusion threshold: 0.73 μT .

T1 systematic bound: 0.3 μT .

T1 simulation: detection, attribution and exclusion reject falsely in 1007, 483, 492 trials, respectively; the corresponding rates are 5.0%, 2.4%, 2.5%.

Under P-5 the prediction is existence: $|\mathcal{D}_{\text{tw}}| > 0$, with no lower positive bound. Define the existence-sign code $\mathcal{E}$ as positive unity for a nonzero twist-field contrast and negative unity for its absence. Its sign-test threshold $Q_{\mathcal{E}}$ is unity in code units; $\mathcal{E}_{\text{FDT}}$, $\mathcal{E}_{\text{conv}}$ denote the field-source and conventional hypotheses, and this encoding assigns no field scale.

$$\mathcal{E}_{\text{FDT}} = +1, \quad \mathcal{E}_{\text{conv}} = -1, \quad Q_{\mathcal{E}} = 1. \quad (46)$$

A finite null measurement excludes specified fields by the physical threshold above; it never excludes every arbitrarily small nonzero field. Detection above the systematic bounds corroborates the additional-source reading of P-5 at the attained precision; a calibrated null at both hands falsifies each specified field magnitude $\mathcal{B}_*$ for which $\mathcal{B}_* - |\overline{\mathcal{D}}| > Q_{\text{ex}} + \mathfrak{B}_{\text{sys}}$ at the tested twist rates.

5.3 T-2: iron enrichment and the meander splitting

DC-2 discriminates SA-28 by holding the meander support geometry fixed while changing the isotope input to exterior A-1 compactness. Use iron-enriched targets on the same substrate, anneal, spacer and NbN stack at 2.0 K (U-11, U-51). Both chosen iron isotopes have zero nuclear spin; residual iron of mass number 57 has spin 1/2 and is assayed; its natural abundance is 2%, while the yttrium hyperfine channel is common (U-25, U-26). The observable is the finite log change of the meander device's DE coupling strength g , independently fitted from the two avoided-crossing branches in the DE configuration, weighted by the meander's field over the film. The reading that increased density raises the film's magnetic energy density is the analysis's I-7 constitutive input.

For a nominal formula-unit mass $\mathcal{A}_{\text{Fe}}(\mathbf{u})$, let $m_Y, m_O, m_{\text{Fe,L}}, m_{\text{Fe,H}}$ be the nominal isotope masses and Λ_{Fe} the finite mass-log span. The formula-unit stoichiometry is an illustrative nominal-composition assumption (U-58), not a composition assay reported by the source.

$$\mathcal{A}_{\text{Fe}}(\mathbf{u}) = 3m_Y + 5[m_{\text{Fe,L}} + \mathbf{u}(m_{\text{Fe,H}} - m_{\text{Fe,L}})] + 12m_O, \quad (47)$$

$$\Lambda_{\text{Fe}} = \ln \frac{\mathcal{A}_{\text{Fe}}(\mathbf{u}_H)}{\mathcal{A}_{\text{Fe}}(\mathbf{u}_L)}. \quad (48)$$

The conditional model uses a density response S_d , magnetic derivative S_M , thickness-strain derivative S_t , overlap derivative S_v and overlap factor $\mathcal{V}$. Each is defined at fixed values of the other channels, pressure, bias configuration and temperature. With comparator magnetic increment $\mathfrak{d}_M$, clamped strain ϵ_{Fe} and overlap increment $\mathfrak{d}_v$, the constant-sensitivity path solves to

$$\Delta \ln g = S_d \Lambda_{\text{Fe}} + S_M \mathfrak{d}_M + S_t \epsilon_{\text{Fe}} + S_v \mathfrak{d}_v. \quad (49)$$

The comparator deletes only the density-response term, keeping vibrational geometry and electronic renormalisation. At the chosen test point the density term is positive, magnetisation and strain terms negative, and overlap positive; outside it each sign follows its independently measured derivative. Fe density span: 0.012.

T2 comparator: 1.9×10^{-5} .

T2 FDT: 0.012.

T2 difference: 0.012.

T2 common uncertainty: 0.0002.

T2 threshold: 0.0013.

The response remains conditional on these sensitivities, not on an overlap value inferred from the same splitting. The printed finite shifts are log changes, not fractional changes.

Alternate depositions from each target, measuring each run's stoichiometry, thickness, interface width, residual stress and argon content independently, and each device's lithographic dimensions (U-51). Use isotope mass spectrometry, reflectometry, interface microscopy, diffraction or curvature for strain, argon assay and dimensional microscopy. Net the run means through the corresponding derivatives, retaining the isotope's vibrational strain in the comparator. Targets set isotope-dependent sputter yield and reflected-argon energy; residual target-wide effects, including assayed residual-spin uncertainty, enter the unmeasured-channel bound if independent measurements do not resolve them. Use Eq. (40) with deposition runs independent and devices nested, Eq. (41) for shared errors, and all three decision rules above. T2 simulation: detection, attribution and exclusion reject falsely in 982, 513, 471 trials, respectively; the corresponding rates are 4.9%, 2.6%, 2.4%. A residual above the threshold plus bounds corroborates the specified I-7 constitutive response; exclusion of its finite path amplitude falsifies that conditional response at the stated precision.

5.4 T-3: oxygen exchange and the standing-wave separation

DC-3 discriminates SA-14 by replacing the air anneal with oxygen-isotope anneals and independently assaying the exchanged profile. Use the source recipe of 850 °C, 3 hours and 30 °C per hour (U-56), common pressure 1.0 bar (U-35), and measurement at 2.0 K (U-11). The two oxygen isotopes have zero spin; residual oxygen of mass number 17 has spin 5/2 and enters the gas-lot control (U-25). SIMS depth profiles of film and substrate's exchanged surface layer determine the path coordinate and weight (U-52). The observable $\mathcal{Y}$ is the positive first-PSSW-to-main-DE field separation at a fixed resonator frequency, weighted by the resonator field through film thickness. The analysis reads the larger response as increased exchange stiffness, with magnetisation competing.

Use field in tesla $\mathcal{B}_s = \mu_0 \mathbf{M}_s$ and squared reduced frequency $\mathcal{F} = (\omega_c/\mathbf{g}_m)^2$; the dispersion correction is $\mathcal{P} = \mathcal{B}_s^2(1 - e^{-2kt})/4$. The positive-field root is $\mathcal{H}(\mathcal{B}_s, \mathcal{P})$ and the DE dipolar offset $\mathcal{D}_{\text{dip}}$. The source dispersion then solves the defining condition exactly:

$$\mathcal{H}(\mathcal{B}_s, \mathcal{P}) = \frac{-\mathcal{B}_s + \sqrt{\mathcal{B}_s^2 + 4(\mathcal{F} - \mathcal{P})}}{2}, \quad (50)$$

$$\mathcal{D}_{\text{dip}} = \mathcal{H}(\mathcal{B}_s, 0) - \mathcal{H}(\mathcal{B}_s, \mathcal{P}), \quad (51)$$

$$\mathcal{Y} = \mu_0 H_{\text{ex},1} - \mathcal{D}_{\text{dip}} > 0. \quad (52)$$

The standing-wave mode uses the Kittel root with its own exchange shift, and the main mode's own exchange shift is subtracted in $H_{\text{ex},1}$. The construction assumes negligible in-plane dipolar correction of that standing mode; independent profile data test this approximation. The baseline coefficient $\mathcal{K}_0$ is the previously defined $\mathcal{K}$ at the light endpoint. The baseline uses the one-pinned-surface model, the source-selected constants and the meander's estimated wavelength. The numerical enrichment profiles are uniform through the probed film (U-59); a measured depth gradient enters the weighted constitutive model explicitly. The chosen bias is 0.18 T (U-10); it fixes the proposed frequency by the DE dispersion with effective main-mode field $\mu_0 H_{\text{bias}} + \mathcal{K}_0/4$, including the pinned main mode's own exchange term, not by an unprinted source resonance. O chosen frequency: 7.4 GHz.

O baseline separation: 71 mT.

Let Λ_O be the log ratio of formula-unit masses $3m_Y + 5m_{\text{Fe,H}} + 12[m_{\text{O,L}} + \mathbf{u}(m_{\text{O,H}} - m_{\text{O,L}})]$. The oxygen masses $m_{\text{O,L}}, m_{\text{O,H}}$ have the meanings in the table. The conditional path gives stiffness log increment $\Lambda_O + \mathfrak{d}_A$ and magnetisation log increment $\Lambda_O/2 + \mathfrak{d}_{\text{M,O}}$ (U-38). Comparator increments $\mathfrak{d}_A, \mathfrak{d}_{\text{M,O}}, \mathfrak{d}_{\text{pin}}, \mathfrak{e}_{\text{sub}}$ are the four oxygen-comparator inputs in order (U-41). The pinning increment multiplies the squared-wavenumber difference; the exchanged-substrate strain adds to the film's clamped strain in thickness. Define $\mathfrak{e}_O$ by Eq. (39). The finite model is

$$\begin{aligned} \mathcal{B}_s(\mathbf{u}_H) &= \mathcal{B}_s(\mathbf{u}_L) e^{\Lambda_O/2 + \mathfrak{d}_{\text{M,O}}}, \\ \mathbf{t}(\mathbf{u}_H) &= \mathbf{t}(\mathbf{u}_L) e^{\mathfrak{e}_O + \mathfrak{e}_{\text{sub}}}, \\ \mu_0 H_{\text{ex},1}(\mathbf{u}_H) &= 2\mathcal{K}_0 e^{\Lambda_O/2 + \mathfrak{d}_A - \mathfrak{d}_{\text{M,O}} - 2(\mathfrak{e}_O + \mathfrak{e}_{\text{sub}}) + \mathfrak{d}_{\text{pin}}}. \end{aligned} \quad (53)$$

Setting the density-channel terms involving Λ_O to zero gives the conventional isotope comparator; all vibrational and pinning terms stay. Substitution into the positive-field roots solves the finite path, including the changing DE dipolar offset. For any channel parameter y , its implicit-root derivative is

$$\frac{\partial \mathcal{H}}{\partial y} = \frac{\partial_y \mathcal{F} - \partial_y \mathcal{P} - \mathcal{H} \partial_y \mathcal{B}_s}{2\mathcal{H} + \mathcal{B}_s}. \quad (54)$$

Thus $S_y = (y/\mathcal{Y})\partial_y \mathcal{Y}$ for a positive multiplicative channel, or $(1/\mathcal{Y})\partial_y \mathcal{Y}$ for strain, retains the field scale factors. At fixed other inputs stiffness raises the exchange separation, magnetisation reduces it through both exchange and the dipolar subtraction, thicker film reduces it, and stronger

squared-gap pinning raises it. The chosen negative substrate strain raises it; a graded substrate or film profile requires its measured sign and weight. O density span: 0.029.

T3 comparator: 0.00088.

T3 FDT: 0.015.

T3 difference: 0.014.

T3 common uncertainty: 0.00019.

T3 threshold: 0.0013.

These shifts are $\Delta \ln \mathcal{Y}$. The response is conditional on the independent stiffness, magnetisation and profile sensitivities.

Alternate anneals, with films nested within each anneal, measuring exchanged fraction, thickness, interface width and residual stress independently (U-34, U-52). Net thickness and pinning-sensitive interface differences per anneal; keep vibrational strain and exchanged-substrate response in the comparator. Gas-lot impurity, residual-spin and mass-dependent anneal-condition channels shared by all anneals enter the common bound where no anneal measures them. Equation (41) uses derivatives of this solved field model, with calibration drawn once across anneals. Apply detection, exclusion and attribution to the mean log-separation contrast using Eq. (40). T3 simulation: detection, attribution and exclusion reject falsely in 902, 469, 438 trials, respectively; the corresponding rates are 4.5%, 2.3%, 2.2%. The finite residual tests the specified I-7 stiffness response; a bound below its path amplitude falsifies that conditional reading without turning the source's unspecified separation into a calibrated exchange field.

5.5 T-4: nitrogen enrichment and the superconducting transition

DC-5 discriminates SA-12 by assaying nitrogen in the resonator layer and measuring its transition independently on a witness bar. Use the 30 nm NbN layer on the same SiN/YIG/YSGG stack (SC-094, L149), reactive sputtering with the two nitrogen gases, and identical pressure, power, thickness and substrate temperature. Four-probe resistance scans from 5 to 20 K define the measurement temperature at each point; the transition criterion is 0.50 of the normal-state resistance, with a chosen light-end test point of 12 K (U-31, U-32). The probe weight is the current density across the witness bar. The transition temperature is T_c , witness resistance $\mathcal{R}(T)$ and normal-state resistance $\mathcal{R}_N$.

$$\mathcal{R}(T_c) = 0.50\mathcal{R}_N. \quad (\text{U-31}) \quad (55)$$

The temperature of this root is independently measured; no mass coordinate is constructed from it. The analysis reads the I-7 larger response as higher transition temperature at fixed other channels.

Let Λ_N be the log ratio of nominal formula masses $m_{\text{Nb}} + m_{\text{N,L}} + \mathbf{u}(m_{\text{N,H}} - m_{\text{N,L}})$, and $\Lambda_{\text{N,atom}}$ the log ratio of the nitrogen atomic means alone. The masses have the table's meanings; ζ_N is the assumed pairing isotope exponent, $S_{\text{d,N}}$ the density derivative and $S_{\epsilon,\text{N}}$ the strain derivative at fixed pairing, geometry, stoichiometry and disorder. For clamped nitrogen strain ϵ_N the finite path solves to

$$\Delta \ln T_c = S_{\text{d,N}}\Lambda_N - \zeta_N\Lambda_{\text{N,atom}} + S_{\epsilon,\text{N}}\epsilon_N. \quad (56)$$

The conventional comparator keeps the pairing and volume terms; the density term is the additional constitutive input. The chosen density contribution is positive, pairing negative, and strain negative with the specified positive strain derivative. The total sign remains conditional until all sensitivities and bounded channels are established. Nitrogen's two stable isotope choices have spins 1 and 1/2 (U-25), so hyperfine changes enter the co-varying channel bound. A-3 supplies no sign or threshold to this A-1 discriminator. N density span: 0.0084.

T4 comparator: -0.016 .

T4 FDT: 0.0095.

T4 difference: 0.025.

T4 common uncertainty: 0.0062.

T4 threshold: 0.012.

These are finite logarithmic temperature changes, not oscillator frequency shifts. The relatively broad shared uncertainty includes the pairing derivative’s uncertainty multiplied by the atomic mass-log span.

Alternate depositions and assay each run’s nitrogen fraction by SIMS, nitrogen content, thickness, residual-resistance ratio and residual stress independently (U-54). Net these nuisance differences, preserving vibrational strain in the comparator. Gas-lot impurity and growth-condition changes, and unresolved hyperfine changes, share the stated bound across every run of a composition. Run scatter and nested witness-device scatter feed Eq. (40); shared netting and comparator errors feed Eq. (41). Apply the same three rules without reducing the gas-lot bound by device count. T4 simulation: detection, attribution and exclusion reject falsely in 973, 809, 494 trials, respectively; the corresponding rates are 4.9%, 4.0%, 2.5%. A residual above threshold plus bound corroborates the specified I-7 transition response; exclusion of its path amplitude falsifies that response at the independently measured temperature and composition.

6 Cross-domain consequences

The torsion comparison changes a source, the isotope comparisons change constitutive inputs, and the standing-wave boundary ratios change neither. This distinction carries the true extension into experimentally different limits. For isotope substitution at fixed support geometry, the independent mass assay fixes the sign of the A-1 source-radius increment. Magnetic energy, vibrational geometry and pairing then transmit that input through separately calibrated sensitivities. The ensemble’s configuration changes while the microwave and standing-wave indices remain on the carrier’s phase axis. P-3 places each nucleus’s saturation diagnostic α_{core} , distinct from its exterior compactness, at the skin of its interior chart, and a material’s load is taken by the interstitial spacetime between the nuclei, never by the nuclei; no measured lattice strain has been inferred from the source’s TEM contrast.

For a coincidence comparison of two independently estimated observables $\widehat{O}_i, \widehat{O}_j$ with uncertainties s_i, s_j , the uncertainty of their difference is s_{ij} .

$$s_{ij} = \sqrt{s_i^2 + s_j^2 - 2 \text{Cov}(\widehat{O}_i, \widehat{O}_j)}. \quad (57)$$

A common calibration is retained in the covariance. A flat ratio identifies a zero differential sensitivity if and only if the multiplicative prefactor is constant over the path; otherwise prefactor drift produces the same ratio. Any length inferred from a field difference requires its independently calibrated field gradient, giving the resolvable length $z_p s_{ij} / |\partial H / \partial d|$ with fields and gradient in consistent units. This comparison is between neighbouring patterns or a pattern and a reference path, as I-3a requires.

The magnon–qubit coupling demonstrations the source cites are background, and the source says its circuit can be extended to the quantum regime with magnetic-field-compatible qubits as single-propagating-magnon detectors (SC-017, L66; SC-018, L67; SC-019, L68; SC-274, L311; SC-275, L313; SC-276, L314; SC-277, L315). That extension, the potential the source highlights for incorporating intrinsic spin-wave nonreciprocity into hybrid magnonic systems, its statement that its results’ extension of cavity magnonics to propagating spin waves at cryogenic temperatures may open new pathways towards quantum information processing with spin waves beyond Kittel modes, and its statements that $\mathfrak{k}_{\text{sw}}$ can be further reduced by increasing w_a , that linewidth broadening due to spin wave bandwidth can be suppressed by increasing the meander layers, and that v_g can be controlled by changing the thickness of the YIG film remain proposals (SC-013, L38; SC-270, L307; SC-271, L308; SC-272, L309; SC-273, L310; SC-274, L311; SC-280, L325; SC-281, L327). I-4

reads their quantum phase through helical transport; no detector or single-magnon operation is attributed to these chips.

7 System-level reading (exploratory)

The scope is one in-sample standing-wave reading, uncounted among the predictions.

X-1: the boundary-conditioned nodal pattern. The observable is the standing-wave separation reported as 80 mT (SC-132, L214), with configuration and reference field unspecified. Its defining condition is Eq. (32); its comparator is the source’s conventional standing-wave calculation. P-2 reads the standing waves as nodal patterns of the driven density field whose geometry is set by the boundary conditions. The analysis identifies the 37 mT estimate with the first free-surface standing mode; the 74 mT calculation retains the source’s one-pinned-surface possibility (SC-130, L212; SC-135, L217; SC-138, L220). The positive-square difference makes the latter twice the former. The film-thickness field profile selects which nodes are visible; index and mode occupation remain at fixed field identity. The source’s closer agreement does not determine either surface anisotropy or a local density profile.

Guards. Every number here reuses the source’s inputs or datum. No mode index, exchange field or ratio defines compactness, overlap odds or a braid class. The local skin-density ordering remains distinct from a film’s exchange-stiffness profile. This identity map corroborates no invariant and supplies no prospective prediction.

8 Convergence statement

On the same stress tensor, geometry, data and gauge, I-3 contains the Einstein equations and their exact optical reading; I-4’s L2 recovers the weak-field, slow-motion expansion at its stated order. For this circuit, setting the additional torsional source and density-response terms to their conventional limits leaves the source’s coupled-mode, dispersion, linewidth, exchange-field and escape-time equations unchanged. They recover, on their own dictionary, the fitted coupling strengths (SC-080, L141; SC-237, L310; SC-238, L311), the fitted magnon and photon damping rates (SC-069, L130; SC-109, L198; SC-222, L294), the offsets set against estimated wavenumbers (SC-216, L294), the estimated radiation-damping rates (SC-256, L338; SC-257, L339; SC-258, L340) and, in-sample, the meander linewidth curve (SC-232, L304). They retain the observed standing-wave separation versus the two calculated estimates, the quarter-wave linewidth excess, and the printed cooperativities beside their central-input recomputation, which returns them within the inputs’ rounding when the coupling strengths carry two figures and not when they carry three. These algebraic identity maps corroborate no invariant.

The source describes hybrid magnonics as cultivating coherent interaction of solid-state magnetic excitations or magnons, its design as enabling coherent coupling between magnons and photons, and cited spin-qubit entanglement as spin-wave-mediated (SA-02; SC-014, L53; SC-015, L55; SC-016, L57; SC-031, L76; SC-032, L77; SC-033, L77; SC-034, L78). FDT reads photon, graviton, gluon, phonon and magnon as one spin-one density excitation, the magnon as its stroke through a helical density gradient, and mediation as a coordinate artefact (I-2, P-5); the splittings, linewidths and transmissions are accepted in full and recovered by the identical response calculation.

9 Status ledger

The table states the outstanding constructions and the test limits once. The effective speed c_{eff} is the cone constitutive speed paired with the aperture diagnostic.

Item	Status and convergence value
O-1	Owed: the finite-core cutoff r_c , the regular core chart, or the outward phase-transfer law.
O-2	Owed: a geometrically constructed odds for the overlap-contrast application (A-3).
O-3	Owed: a canonical odds for the confinement application (A-4).
O-4	Owed: the local profile $\rho_{\text{loc}}(r_f)$ from the omnium constitutive law with boundary, surface and core data, reducing to the measured nuclear profile.
O-5	Owed: multipole and tidal remainders of the closed-form pair expression; the overlap complex.
O-6	Owed: the maps from a charge or a colour seat to an I-1 input; derivation of the A ₂ seat table from cone geometry; the gluon octet's dynamics.
O-8	Owed: the constitutive derivation of $c_{\text{eff}}/n_{\text{cone}}$ and its reconciliation with lensing.
O-9	Owed: the confinement operator D , its domains, boundary data and spectrum; every mass ratio, including the two held-out targets.
Tokens	A-3, A-4 and A-5 are PROVISIONAL; no transferred numerical value, threshold, ordering, sign or trajectory.
T-1	Prospective existence prediction, single-channel source scope; conventional twist contrast zero. No assigned field magnitude: quantitative twist-to-magnetic-moment forms are derived in-manuscript through I-1/I-2 with a declared kernel (P-5), and the analysis declares no kernel.
T-2	Prospective, conditional by construction on independent isotope sensitivities; convergence is the Fe vibrational and electronic comparator.
T-3	Prospective, conditional by construction on stiffness, magnetisation and boundary inputs; convergence is the O anneal comparator.
T-4	Prospective, conditional by construction on density, pairing and strain derivatives; convergence is the N pairing and volume comparator.
X-1	In-sample P-2 reading of the source's standing-wave estimates and datum.
Identity maps	Coupled-mode fit, dispersion offsets, linewidth reuse, central cooperativity arithmetic and escape estimates; no invariant corroborated.
Held-out	Torsional field magnitude, isotope-response functions, independent film depth profile, the proton-to-electron mass ratio and the squared muon-to-electron mass ratio.

10 Conclusions

The avoided crossings are two regime modes of one carrier, read through the source's response equations. Those equations retain their experimental scope: the meander's printed rates meet the stated criterion, while for the quarter-wave device the source leaves it uncertain whether the coupling strictly exceeds the magnon damping rate. Neither rates nor normalised transmission are a static force measurement. Torsion supplies an additional-source test with no assigned amplitude; isotope substitution supplies three finite-path tests with explicit competing channels and shared errors. The standing-wave ratios remain a conventional boundary diagnostic until a local-profile reduction exists.

References

- [1] Y. Li et al., *Strong coupling between propagating spin wave and microwave photons in a superconducting resonator*, arXiv:2606.27279v1 (2026).
- [2] *Core FDT Invariants*, version 5.9, clauses 0.1–0.6, I-1–I-7, P-2–P-5 and O (locked framework).