

Sliding tetralayer graphene: geometry, magnetic order and isotope contrasts

FDT analysis

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Abstract

The history-dependent bistability reported by Brandon et al.[1] is experimental evidence that magnetism is geometric, exactly as core P-5 of Fundamental Density Theory (FDT), whose one substance is the omnium, states: “A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along an axis”. FDT is a true extension of general relativity and quantum mechanics. Its bivector reading of the magnetic field makes the reported reversible on–off control of magnetism a laboratory exhibit of geometric magnetism. The stacking assignments remain the authors’ attribution, and the magnetic hysteresis observed in the rhombohedral–Bernal mixed (RB-mixed) state is not observed in the Bernal-dominant (B-dominant) state above the source’s unprinted sensitivity. The same-input calculation returns the source’s switching contours and Hall dictionary. Five prospective predictions add the twist-induced field or the isotope density channel: two handedness comparisons and three finite isotope contrasts. Each is conditional by construction, with a resolved illustrative test point, an explicit consensus limit and covariance-aware decisions. The local carrier-density candidate has no map from the core’s local density. One separate exploratory observable concerns the displacement-odd Hall response.

1 Framework and the measurement dictionary

The source_status is experimental.[1] Measured, observed and reported identify source results; derived, estimated, reconstructed and illustrative-assumed identify the analysis’s algebra, models and design inputs. The symbols SC identify claim loci, SG extraction and sign guards, SA source-weakness register rows, DC divergence candidates, L physical source loci, and U non-source numerical inputs. The locked core supplies the framework statements below.[2] The paper’s charge “carrier” is distinct from the stroke of I-2; its wall pinning is distinct from P-3’s fixed pinning sites. A stacking domain is a pattern’s own support with its own wall data. A stacking phase is distinct from the matter and spacetime phases of the atlas and from helical phase. The source’s “topological”, “chiral” and “polarization” retain their source meanings.

1.1 Notation guard

The table defines the source dictionary before its use. Decorated resistance symbols denote measured voltage-to-current ratios; the undecorated Schwarzschild family keeps its geometric meaning. The source’s charge unit is denoted by e , as in its gate definition.

Source role	Manuscript symbol	Reserved meaning
Carrier density, offset	$\mathbf{n}, \mathbf{n}_0$	n : strand count
Displacement, offset	$\mathcal{D}, \mathcal{D}_0$	D : confinement operator
Applied magnetic field	$\mathbf{B}$	B : confinement boundary data
Temperature, Curie point	T, T_C	$T_{\mu\nu}$: stress tensor; T_q : period
Longitudinal, Hall resistance	$\mathcal{R}_{xx}, \mathcal{R}_{yx}$	R_s : Schwarzschild radius
Mean, difference, branches	$\bar{\mathcal{R}}_{xx}, \Delta\mathcal{R}_{xx}, \mathcal{R}_{xx}^{\text{fw}}, \mathcal{R}_{xx}^{\text{bw}}$	x, y : core map inputs
Stacking free energies	$\mathcal{F}_{\text{rh}}, \mathcal{F}_{\text{Be}}, \Delta\mathcal{F}$	F : force
Calculated energy difference	$\Delta\mathcal{U}, \Delta\mathcal{U}(0, 0)$ (SC-273, L364)	U : pair energy
Work-function difference	$\Delta\Phi$	ϕ : helical phase
Gate voltages	$\mathcal{V}_t, \mathcal{V}_b$	V : volume
Capacitances per area	C_t, C_b	Same source definition
Response coefficient; charge	$\varepsilon_0; e$	Omnium response; source charge unit
Time	t	t : interval or chart position
Resistance unit	Ω	Ω : confinement domain
Twist deviation; susceptibility difference	$\vartheta_{\text{tw}}; \Delta\mathfrak{s}_m$	θ, χ : core coordinates

The census follows the density, field and resistance definitions (SC-056, L89; SC-058, L91; SC-082, L118; SC-264, L349; SC-265, L350), the free-energy and calculation definitions (SC-091, L129; SC-092, L130; SC-273, L364; SC-274, L365), and the field, temperature and time usage (SC-085, L121; SC-086, L122; SC-157, L220; SC-163, L226).

1.2 One substance, two clock readings

The omnium is the one substance; an extended pattern and its support are one configuration, with no separate occupant. The metric $g_{\mu\nu}$ encodes the geometry; its Greek subscripts denote tensor components. Energy E , mass m , distance interval d , time interval t , speed c and the response coefficients ε_0, μ_0 obey core 0.1. For a mode labelled q , $E_q, m_q, \omega_q, T_q, \ell_q$ denote its energy, mass, angular frequency, period and traversal length; h and $\hbar = h/(2\pi)$ are the action constants, and π is the circle constant. Core 0.1 and 0.1a give

$$E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2, \quad (1)$$

$$E_q = m_q c^2 = \hbar\omega_q, \quad T_q = 2\pi/\omega_q = h/E_q, \quad (2)$$

$$\ell_q = cT_q = h/(m_q c). \quad (3)$$

The length in Eq. (3) is a traversal length; a pattern's support radius is independent geometric data. Quantized energy at invariant speed gives discrete rest masses on the massive confinement leg and discrete null periods and paths with locked distance-to-time ratio c . The invariant speed

is $c = 299792458 \text{ m s}^{-1}$ (U-01). Let G denote the gravitational coupling, $F_{\max}$ the force ceiling, $d_{\min}, t_{\min}$ the limiting intervals, and ℓ_P, t_P the Planck length and time. Core 0.1a and 0.2 yield

$$F_{\max} = c^4/(4G), \quad F_{\max} d_{\min} t_{\min} = \hbar, \quad (4)$$

$$d_{\min}^2 = \hbar c / F_{\max} = 4\hbar G / c^3 = 4\ell_P^2, \quad t_{\min} = 2t_P. \quad (5)$$

The floor is an infimum, not a lattice; repetition down the nested supports terminates at this floor. The approximate ceiling is $3.03 \times 10^{43} \text{ N}$ (U-02, three significant figures).

The positive common clock factor $\mathbf{m}$ rescales both intervals. Let $g_{\mu\nu}$ be the metric, $\tilde{g}_{\mu\nu}$ its rescaled form, $\mathcal{U}^\mu$ the four-velocity of a pattern free of every force in $\tilde{g}_{\mu\nu}$, in that pattern's own-clock time, and $\mathfrak{P}^{\mu\nu}$ its velocity-squared orthogonal projector; ∇ denotes the covariant derivative of $g_{\mu\nu}$ and $\mathbf{a}^\mu$ the acceleration with which that pattern is read on $g_{\mu\nu}$. The tensor indices μ, ν label components. Core 0.1c reads

$$\begin{aligned} \tilde{g}_{\mu\nu} &= \mathbf{m}^2 g_{\mu\nu}, & \mathfrak{P}^{\mu\nu} &= c^2 g^{\mu\nu} + \mathcal{U}^\mu \mathcal{U}^\nu, & g_{\mu\nu} \mathcal{U}^\mu \mathcal{U}^\nu &= -c^2, \\ \mathbf{a}^\mu &= \nabla_\mu \mathcal{U}^\mu = -\mathfrak{P}^{\mu\nu} \nabla_\nu \ln \mathbf{m}. \end{aligned} \quad (6)$$

Moving at constant speed, on the pattern's own clocks and rulers, through a density gradient is gravity. Its own clocks and rulers supply no reading of that gradient; the reading compares neighbouring patterns, or a pattern with a reference path, an extended pattern's skin against its own centre path among them. The clock factor is a common conformal factor, independent of the aperture and optical indices; that factor alone does not bend a null stroke. For stress of perfect form, in a zero-net-count medium the clock factor follows temperature, and at fixed entropy per conserved count it follows enthalpy per count. A local thermal-clock shift follows the motion of the atomic pattern through the medium, rather than a reassignment of its own rate to the medium's temperature. Core I-3a separates the relative velocity gradient into trace expansion, antisymmetric vorticity and symmetric traceless shear. The antisymmetric part is the bivector grade whose magnetic projection P-5 names.

1.3 Chart inventory and the three curvature faces

Let $\mathcal{M}$ denote the complete gravitating mass, r_f the independently specified field-point radius, R_s its Schwarzschild radius, and r_c the interior cutoff. The continuous curvature chart describes this discrete content. The sign σ labels the exterior or interior branch, and α_σ is the curvature ratio; X_σ is its gap odds, while β_σ is a computational compactification. The chart inventory fixes every use before evaluation:

Application	Tokens	Warrant and interpretation
A-1	$\alpha_{\text{ext}}, \alpha_{\text{int}}, X_{\text{ext}}, X_{\text{int}}$	Geometric compactness, branch declared; field-point, configuration and source evaluations retain their own r_f .
A-2	β_σ	Computational atlas only; never a force, optical or density input.
A-3	$\alpha_{\text{ovl}}, X_{\text{ovl}}$	PROVISIONAL overlap tokens and range only, no value, threshold, ordering, sign or trajectory.
A-4	$\alpha_{\text{conf}}, X_{\text{conf}}$	PROVISIONAL confinement tokens and range only, no value, threshold, ordering, sign or trajectory.
A-5	α_{mode}	PROVISIONAL mode name and range only, no value, threshold, ordering, sign or trajectory.
Diagnostic	α_{core}	Schematic saturation toward the open upper endpoint; no measured address.

The symbol ∞ denotes the unbounded exterior end. Core 0.3, 0.4 and I-3 give the geometric A-1 split and the A-2 map:

$$R_s = 2GM/c^2 = \mathcal{M}c^2/(2F_{\max}), \quad (7)$$

$$\alpha_\sigma = (R_s/r_f)^\sigma, \quad 0 < \alpha_\sigma < 1, \quad (8)$$

$$\sigma = +1 : R_s < r_f < \infty; \quad \sigma = -1 : r_c \leq r_f < R_s,$$

$$\partial_{r_f} \alpha_\sigma = -\sigma \alpha_\sigma / r_f, \quad \beta_\sigma = \alpha_\sigma / (1 + \alpha_\sigma), \quad (9)$$

$$X_{\text{ext}} = R_s / (r_f - R_s), \quad X_{\text{int}} = r_f / (R_s - r_f). \quad (10)$$

The A-1 Clifford seam is at $\alpha_\sigma = 1/2$: exterior radius $2R_s$, interior radius $R_s/2$. The seam separates the two solid-torus regimes of one support and one substance. The spacetime phase lies below that seam and the matter phase above it, on the named A-1 branch. The fold is the core circle approached as $\alpha_\sigma \rightarrow 1^-$ on both branches, at $r_f = R_s$; the atlas endpoints remain circles. The branch exchange and the support's involution have different domains and do not transport laboratory radii between the two charts of a bound configuration.

Let $d\ell_{\text{opt}}$ denote optical path length, $d\varpi$ the angular line element, n_{iso} the isotropic optical index, n_{cone} the aperture index, and n_{dens} the computational density index. The symbol O denotes the asymptotic remainder. Core I-3 gives the exact exterior optical metric and its weak-field isotropic limit:

$$d\ell_{\text{opt}}^2 = \frac{dr_f^2}{(1 - \alpha_{\text{ext}})^2} + \frac{r_f^2 d\varpi^2}{1 - \alpha_{\text{ext}}}, \quad (11)$$

$$n_{\text{iso}} = 1 + \alpha_{\text{ext}} + O(\alpha_{\text{ext}}^2), \quad n_{\text{cone}} = (1 - \alpha_{\text{ext}})^{-1/2}, \quad (12)$$

$$n_{\text{dens}} = (1 - \beta_\sigma)^{-1/2}.$$

The effective aperture speed c_{eff} is c/n_{cone} (P-4). A jump in the optical index obeys exact Snell refraction; the aperture index is not a substitute lensing index. Let $G_{\mu\nu}$ denote the Einstein tensor, $T_{\mu\nu}$ the full stress tensor and κ the Einstein coupling. The third face, core I-3, is

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \quad \kappa = 8\pi G/c^4, \quad \kappa F_{\max} = 2\pi. \quad (13)$$

The curvature ratio is downstream of this unchanged equation. Containment here means the same curvature from the same stress tensor and data. For the stack's gravitational window the exterior ratio $\alpha_{\text{ext}} = \mathcal{M}c^2/(2F_{\max}r_f)$ of Eqs. (7) and (8) is small and a correct reading. For fall from rest far away, let v denote inward flow speed and t_{own} the local own-clock time. The slow exterior flow of 0.1c is

$$v = c\sqrt{\alpha_{\text{ext}}}, \quad dr_f/dt_{\text{own}} = -v, \quad d^2r_f/dt_{\text{own}}^2 = -GM/r_f^2. \quad (14)$$

A rotating source adds a swirl to the flow; the frame drag a gyroscope reads is the flow's local rotation, half the curl of the swirl, at leading order the Lense–Thirring precession, and it is the vorticity projection of I-3a at every size. The exterior expression stops at its stated branch boundary; its continuation is not an interior construction. The slow lapse is $\sqrt{1 - \alpha_{\text{ext}}}$; the full optical metric retains the spatial contribution as well.

1.4 Force, strokes, phase and confinement

Let α_1, α_2 denote two warranted A-1 exterior source evaluations at the other centre, with masses $\mathcal{M}_1, \mathcal{M}_2$, support radii $R_{b,1}, R_{b,2}$ and separation d . Core I-1 supplies force magnitude F at every

scale:

$$F = \frac{c^4}{4G} \alpha_1 \alpha_2, \quad (15)$$

$$\alpha_i = \frac{2G\mathcal{M}_i}{c^2 d} \quad (i = 1, 2), \quad F = G\mathcal{M}_1\mathcal{M}_2/d^2. \quad (16)$$

The second line requires nonoverlapping spherical monopole supports with $R_{b,i} \geq R_{s,i}$ and $d \geq R_{b,1} + R_{b,2}$. The universal open ratio bound gives $F < F_{\max}$. Only this pair sector gives $F \leq F_{\max}/4$, with equality for equal saturated supports at contact. The universal interval, the compactness seam and this pair inequality are different statements. For overlapping nonspherical layers the source evaluations require multipoles and the overlap complex. A condensed-matter compactness evaluation returns the core's small bare force, of order 10^{-41} N; the binding scale is obtained through a declared response kernel.

Let s be pair separation and s_0 its lower limit; $U(s_0)$ is pair energy relative to infinite separation. For continuous F with an absolutely convergent tail, core I-1 gives

$$U(s_0) = - \int_{s_0}^{\infty} F(s) ds, \quad \frac{dU}{ds_0} = F(s_0), \quad F_{\text{signed}} = - \frac{dU}{ds_0}. \quad (17)$$

Here F_{signed} is the attractive signed component. With an integrable force alone the derivative holds almost everywhere. Let $\mathcal{H}_b(s_0, s)$ be a dimensionless material-response kernel and U_b its binding-energy functional. Assume continuous diagonal values, an absolutely integrable weighted force and a locally uniform integrable bound on its lower-endpoint derivative. The declared kernel then gives the in-manuscript response and its Leibniz boundary term:

$$U_b(s_0) = - \int_{s_0}^{\infty} \mathcal{H}_b(s_0, s) F(s) ds, \quad (18)$$

$$\frac{dU_b}{ds_0} = \mathcal{H}_b(s_0, s_0) F(s_0) - \int_{s_0}^{\infty} \partial_{s_0} \mathcal{H}_b(s_0, s) F(s) ds.$$

Its quantitative material form remains owed through O-5 and O-6; this declared map does not supply a fitted binding energy. Every binding or stacking energy is force times length; a rate and a force have different dimensions. Let u denote delay, j measurement current, τ the voltage response, and $\mathcal{K}$ a causal response kernel with units of resistance per time. For integrable $\mathcal{K}$ and bounded current, a driven realization is

$$\tau(t) = \int_0^{\infty} \mathcal{K}(u) j(t - u) du. \quad (19)$$

The measured voltage vanishes at zero drive; its linear resistance limit stays finite. A susceptibility maps force into an observable and never replaces Eq. (15). The charge-seat map and the nonspherical kernel remain separate from a measured resistance.

I-2 derives the single spin-one stroke from one medium and one invariant local speed. Photon, graviton, gluon, phonon and magnon name that stroke in different regimes; a braided matter pattern belongs to the confinement problem. There is no mediation and no carrier exchange. For the paper's reported "ferromagnet" (SC-014, L38), the core states: "the magnon is the stroke through a helical density gradient (P-5)" (I-2). No magnon spectrum is reported in these Hall measurements.

I-4 reads quantum mechanics as helical geodesics on truncated density cones. Let ψ denote the phase representation, $\mathcal{A}$ its envelope, ϕ its helical phase, i the scalar imaginary unit and ω angular frequency. Let ϕ_0 be the initial phase, $\mathbf{e}_1, \mathbf{e}_2$ orthogonal frame vectors and $\mathbf{B} = \mathbf{e}_1 \mathbf{e}_2$ their oriented

unit bivector. The cylindrical lift of I-4 is

$$\psi = \mathcal{A} \exp(i\phi), \quad \phi = \phi_0 - \omega \mathbf{t}, \quad (20)$$

$$\exp(\mathbf{B}\phi) = \cos \phi + \mathbf{B} \sin \phi, \quad \mathbf{B}^2 = -1. \quad (21)$$

The exponent is dimensionless and its energy is $\hbar\omega$ as in Eq. (2). This is a single-angle cylindrical lift, separate from a spinor half-angle representation. The universe is deterministic; there are no probability amplitudes. Superposition records phase ignorance. The source's quantum oscillations and calibration Landau levels (SC-064, L97; SC-065, L98; SC-266, L351) occupy this phase axis, as do its cited topological flat bands (SC-036, L67). The stackings ABAB and ABCA and the layer count do not supply a strand count or charge assignment.

The confinement domain is Ω , boundary data B , strand count n , geometric parameters χ, t , operator D , eigenpattern u_q and frequency ω_q . Core I-5 supplies

$$D(\Omega, B; n, \chi, t)u_q = \omega_q u_q, \quad m_q c^2 = \hbar\omega_q. \quad (22)$$

The operator is owed in O-9. Changing position at fixed strand count changes mass within a class; changing strand count requires rebraiding. The carbon nuclei in an isotope comparison are matter configurations of this problem. Neither their masses nor a support radius or absolute cone address are calculated from Eq. (22) here.

1.5 Bounded response and the complete mass ledger

For a warranted A-1 branch let α_0 denote its baseline ratio, X_0 its odds, x a nonnegative dimensionless odds-enhancing input, and α' its updated ratio. Core I-6 gives the exact finite update

$$X = X_0(1 + x), \quad \alpha' = \frac{\alpha_0(1 + x)}{1 + \alpha_0 x}, \quad \alpha' - \alpha_0 = \frac{\alpha_0(1 - \alpha_0)x}{1 + \alpha_0 x}. \quad (23)$$

The increment remains inside the open ratio interval. At the mathematical test point $\alpha_0 = 1/4$ on either branch σ , with $x = 1$ (U-44), the updated ratio on that branch is 0.4. No area fraction, resistance ratio, contrast or sign pattern defines a curvature ratio. The gate-driven onset of hysteretic Hall order is an I-1a magnetic-ordering instance on A-3, with no numerical assignment or side on that unwarranted chart. Its thermal onset is not a density crossing; cooling-side ordering uses a separate bounded coherence coordinate.

Let M denote rest-mass contribution, P pressure, V volume, T temperature, a the radiation energy-density constant and σ_{SB} the Stefan–Boltzmann flux constant. Core 0.5 and I-7 require

$$\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \cdots, \quad a = 4\sigma_{\text{SB}}/c. \quad (24)$$

The stacking energy difference enters E/c^2 once; electrostatic energy of gate charge and displacement enters E/c^2 once; magnetic energy enters E/c^2 once. The measurement current's deposited energy and the damping heat it becomes enter the same E/c^2 contribution once, with transfer between those descriptions conserving that entry. Material thermal energy enters E/c^2 ; pressure enters PV/c^2 ; radiation enters aT^4V/c^2 . Binding enters the solved mass aspect, rather than a second local contribution for an energy already counted in E/c^2 . The core gives $a = 7.566 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$ (U-03) and the rounded flux constant $\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ (U-04). Their symbolic relation fixes units; the rounded flux value licenses three significant figures in a numerical recombination.

P-5 states: “Magnetism is geometric—twist induces it. A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along

an axis; vortex lines are bivector topological defects. Imposing a physical twist (torsion, helical geometry) on a material is establishing a magnetic field; mechanical vorticity and magnetic moments are different grade projections of one density-gradient topography, with no carrier exchange (I-2). Quantitative twist $\rightarrow$ moment forms are derived in-manuscript through I-1/I-2 with a declared kernel.” The analysis reads the field of the reported “ferromagnet” (SC-014, L38) as that bivector grade aligned along the stacking axis which the Hall geometry reads. It reads the B-dominant state as having no remanent hysteretic alignment above the source’s unprinted sensitivity; both states retain the applied field and the field of the shared physical twist. The different gate histories select different attributed stacking geometries of the same pattern, and magnetism follows that geometry. The source writes that the switching “is driven by the motion of pre-existing domain walls”, “similar to other slidetronic systems”, and that “Hence, the presence of a mixed-domain structure created by marginal twisting of double bilayer graphene” (Fig. 1b) “is critical for enabling the switching” (SC-069, L102; SC-070, L103; SC-071, L104; SC-072, L105; SC-073, L106), and that for device A “one of the layers underwent an additional global shift and relaxed to rhombohedral and Bernal mixed domains” (SC-243, L324; SC-244, L325), citing Extended Data Fig. 1, outside the main text and Methods, and the source’s Ref. 50. One marginal twist serves both states, and the analysis reads it as P-5’s physical twist, the reading the handedness comparisons carry.

2 System chart and probe weights

A configuration evaluation uses the independent geometric extent of the specified pattern; a source evaluation uses the point where another pattern reads it. The symbol R_{room} denotes an independently measured interior skin scale. The table distinguishes the two charts of each bound configuration used here.

Pattern	Exterior chart	Interior chart
Stack and its nuclei	A-1 exterior, R_s from Eq. (24); independent r_f	Skin at the interior fold; measured support extent, no inferred nuclear address
Isolated rhombohedral domain	A-1 exterior of the domain’s own support and wall	Planar half-width in the place of R_{room} , measured from its walls; no transfer map
Transport device	Spatial response weighting over the stack	No local-domain response inferred from a device resistance
KPFM domain pair	Common-tip potential difference	Each wall and central region resolved through that tip’s lateral kernel

The planar half-width construction is the analysis’s reading of a domain as a bound support, not a conversion of its exterior radius. Let q_σ denote comparison density and ρ_{loc} local density. Core 0.3 and 0.6 distinguish

$$q_\sigma = \alpha_\sigma^{3\sigma}, \quad \rho_{\text{loc}}(\text{skin}) > \rho_{\text{loc}}(\text{centre}). \quad (25)$$

The profile function is owed; comparison density is not that function. The ordering holds on both branches at every scale with an inside, with the centre the dilute region. A scanning carrier-density map therefore requires a further dictionary from local omnium density to the source’s carrier-density coordinate.

All device transport data below carry the same first-order reciprocity weighting. Let $\mathbf{j}_I$ be the current distribution driven by the current contacts, and $\mathbf{j}_V^{\text{rev}}$ the distribution the voltage contacts would drive under reversed field and magnetization, each normalized to its terminal current.

The scalar product weights the isotropic local resistivity, the traceless symmetric dyad weights its anisotropic part, and the cross product along the stacking axis weights its antisymmetric part. All three enter every datum; their mixing changes with contacts and spatial profiles. Let w denote any one of these signed weights, f its local resistivity component, w_+ , w_- its positive and negative parts, and $\mathfrak{w}$ their common integral when the total weight vanishes. For integrable products, define $\langle f \rangle_{\pm}$ by their respective normalized weights; then the observation-region rule has the exact form

$$\int w f = \mathfrak{w}(\langle f \rangle_+ - \langle f \rangle_-), \quad \mathfrak{w} = \int w_+ = \int w_-. \quad (26)$$

The cross product has zero integral in a resistance, and the scalar product in a Hall datum between aligned contacts. For a device symmetric under the mirror exchanging the current contacts, the traceless dyad’s component along the bar axes has zero integral in that Hall datum; its component at forty-five degrees has zero integral in a resistance between voltage contacts exchanged by that mirror. Those components enter by variation between the lobes, not by a uniform domain value. Every KPFM datum instead carries the positive normalized lateral tip kernel specified in the surface-potential block below. These weights prevent a device datum from being read as one domain’s resistance, Hall response or carrier density.

3 Observation map

The source writes “We attribute this hysteretic behavior to a doping-induced sliding phase transition between the Bernal and rhombohedral structures” (SC-060, L93; SC-061, L94; SC-062, L95). It reports a mixed-domain interpretation whose absent clear oscillations are “likely due to the superposition of distinct quantum oscillations” (SC-064, L97; SC-065, L98). Its other region “appears predominantly Bernal” and “may contain small patches of rhombohedral domains and domain walls” (SC-067, L100; SC-068, L101). These are attributed compositions, with supporting Extended Data outside the main text and Methods. The table keeps the source’s precision and verb class.

Source cluster	Datum or reading	FDT use
Fig. 1c, reported switching	$n = -15.5 \times 10^{12} \text{ cm}^{-2}$ and $-4.1 \times 10^{12} \text{ cm}^{-2}$; no repeat uncertainty (SC-059, L92; SC-060, L93)	History-selected support configuration
Extended Data Fig. 2, reported KPFM	Rhombohedral surface potential 22 mV smaller; source’s work-function sign link, no uncertainty (SC-049, L82; SC-050, L83; SC-051, L84)	Common-tip difference, separate sample
Fig. 3, calculated contours	Two parameters besides the tight-binding set; contours “agree well” (SC-149, L207; SC-150, L208; SC-272, L363; SC-273, L364)	Same-input identity map
Fig. 4, observed Hall contrast	At 0.03 T, $ \mathcal{R}_{yx} > 100 \Omega$ in RB-mixed; ordinary Hall effect in B-dominant (SC-157, L220; SC-159, L222; SC-160, L223)	Driven, weighted response
Fig. 4 loops, reported absence	Hysteresis observed in RB-mixed, “No such magnetic hysteresis is observed” in B-dominant (SC-160, L223; SC-161, L224; SC-162, L225; SC-163, L226; SC-164, L227)	Alignment relative to the reference state
Temperature, reported estimate	Curie temperature “around 8 K”, Extended Data outside this paper’s main text and Methods (SC-162, L225; SC-163, L226)	Isotope observable, no reconstructed curve

The flattened density factors at L92, L93, L99, L119 and L179 denote 10^{12} cm^{-2} : the surviving inverse-area unit is the independent dimensional evidence for that reading (SG-01–SG-05). The endpoints are this device’s switching densities, not material constants; the source writes that “the switching behavior depends on the initial domain configuration and the domain wall pinning strength” (SC-087, L125; SC-088, L126). It writes that “A clear area imbalance is observed between the two domains, with the Bernal domain being larger than the rhombohedral one” and that “This is attributed to the energy difference between the two configurations”, “where Bernal stacking is more energetically favorable at charge neutrality under zero displacement field” (SC-051, L84; SC-052, L85; SC-053, L86; SC-054, L87); of its KPFM heterostructure it also writes that “The asymmetry in domain sizes indicates that the sample has relaxed into a mix of rhombohedral and Bernal domains” (SC-253, L336; SC-254, L337). Its time trace reports repeated switching under short gate pulses and calls the switching “highly robust and reproducible” (SC-100, L138; SC-101, L139). Its “This result indicates that the sliding phase transition between Bernal and rhombohedral graphene could be useful as a nonvolatile memory” is an application reading (SC-101, L139; SC-102, L140); no cycle count, switching energy, rate or error rate is printed. The closing claim that this method “will allow” rapid and energy-efficient switching invokes other systems (SC-225, L300; SC-226, L301; SC-227, L302).

The source writes that $\Delta\mathcal{R}_{xx}$ “is almost zero around the top left region of the scan” (SC-122, L174; SC-123, L175) and that “This is because the device remains in the RB-mixed state even with the maximum” $\mathcal{V}_b$ of its third measurement step (SC-123, L175; SC-124, L176). The near-zero difference also permits equal resistances of the two preparations. The source’s dashed boundary is a drawn line without a printed uncertainty, with its bistable-region statement excluding the extreme-hole-density part (SC-127, L179; SC-133, L191; SC-134, L192). Its selected jump lines become constant-free-energy contours “Based on this understanding” (SC-141, L199; SC-142, L200; SC-143, L201; SC-144, L202; SC-145, L203; SC-146, L204). The calculation’s parameters include a KPFM-derived work-function difference and an energy offset determined in the Supplementary Information (SC-273, L364; SC-274, L365; SC-275, L366). Returning those contours from those inputs is the identity map; it corroborates no invariant. The measured jumps and the in-sample fit remain distinct. A constant-free-energy jump line requires its wall barrier to remain constant along that line. The source reports contact-dependent jump magnitudes and signs and says their multi-domain origin is “likely” (SC-135, L193; SC-136, L194; SC-137, L195; SC-138, L196; SC-139, L197; SC-140, L198).

For the Hall maps the RB-mixed preparation first visits the largest-hole-density corner, then follows the gate boundary for each line (SC-165, L232; SC-166, L233; SC-167, L234). The B-dominant preparation first visits zero top and bottom gate voltages (SC-167, L234). The diamond, star and pentagon mark the reported measurement positions, and the pentagon also marks the small-density negative response; their gate coordinates are unprinted (SC-173, L240; SC-174, L241; SC-175, L242; SC-176, L243; SC-177, L244; SC-210, L283; SC-211, L284). The data of both states at the diamond were recorded at exactly the same gate voltage; the source writes that “switching between the two states is achieved solely by the gate-voltage sweep history” (SC-173, L240; SC-179, L248; SC-180, L249). A single nominal calibration does not equate the local carrier densities or displacement fields in differently attributed stackings. The calibration comes from B-dominant Landau levels (SC-266, L351; SC-267, L352).

This comparison corroborates I-3a’s antisymmetric projection, anchored by P-5’s quoted bivector mechanism, as an in-sample geometric reading. Its precision is the source’s negative Hall response at positive displacement field, the bound above one hundred ohms at the stated small field, and the ordinary Hall reference (SC-157, L220; SC-159, L222; SC-160, L223; SC-169, L236). The falsifier is a hysteretic anomalous Hall response in B-dominant as large as in RB-mixed at the

same gate voltages under the reported histories, or its absence in RB-mixed under that preparation. The analysis reads the two states as a comparison pair in the sense of core Use item 7; the reference keeps the shared twist and applied field while the remanent alignment changes. The reported history-dependent bistability (SC-180, L249; SC-181, L250) and reversible on–off control of magnetism (SC-227, L302) are the laboratory exhibit, and the new handedness comparisons isolate the additional physical-twist source.

4 Cone-position translation

The source’s binding distinction is between atomic planes “held together by weak van der Waals interactions” (SC-027, L58; SC-028, L59) and “the robustness of chemical bonds, whose energy scales are many orders of magnitude larger than what can be accessed through gate voltages” (SC-025, L53; SC-026, L54). In I-1 each binding or stacking energy takes the force integral of Eq. (17) with its declared response kernel; Eq. (19) separately maps drive into voltage. In I-5 the closed electronic and nuclear patterns are the stack’s support; the source’s “occupied electrons” is its calculated band-language reading (SC-153, L211). The following translations keep the axes distinct.

Source coordinate	FDT reading	What it fixes
Gate history and wall motion (SC-070, L103; SC-071, L104; SC-072, L105; SC-073, L106)	Boundary data and support geometry	Preparation, not a strand count
Quantum oscillations (SC-064, L97; SC-065, L98)	Helical phase at fixed field identity	Phase-axis response, not an absolute cone address
Isotope mass input	Fixed-geometry A-1 exterior compactness	Mass ratio from independent composition
Interlayer spacing change	Competing geometric path	Independent radius change, not the fixed-geometry mass channel
Hall sign and area fraction	Profile-weighted observable	No constructed curvature ratio
Thermal ordering	Separate bounded coherence coordinate	No compactness crossing inferred from cooling

The pure density-coupled equilibrium discriminator of I-7 reads a heavier isotope as larger R_s , higher exterior compactness at fixed geometry and larger response. That pure-channel sign opposes conventional reduced-mass scaling. Its application to a Curie temperature, susceptibility or potential contrast requires the separate response dictionary and the competing channels below. The phase winding of a band, the stacking sequence and the layer count supply no confinement mass formula.

5 Prospective comparisons

The first pair tests an additional field source; the isotope comparisons test an additional density-coupled constitutive input. The settings below are prospective and illustrative-assumed where U rows are named. Define $\mathbf{p}$ as the two-sided false-rejection probability, z the normal quantile, $\mathbf{t}$ the Student quantile, $\mathbf{t}_{\nu_*}$ its value at residual degrees of freedom ν_* , and SE as standard error with all stated covariance. Use $\mathbf{p} = 0.05$ (U-13); thus $z = 1.96$. The symbols $\mathbf{h}_j$ denote each block’s

total decision threshold, expressed as its effective $z \cdot \text{SE}$ including the stated systematic bound. The index j of $\mathfrak{h}_j$ labels a block only. A finite endpoint difference is a secant; no endpoint pair is reported as a derivative.

5.1 T-1: the hand-odd raw-loop centre

DC-1 discriminates SA-10: the marginal twist that creates the mixed-domain structure supplies no measured hand or twist-field value in the source. This existence prediction is conditional by construction; neither its magnitude nor the sign of the hand-to-field relation is assigned by the core. P-5 states, “Imposing a physical twist (torsion, helical geometry) on a material is establishing a magnetic field” and “Quantitative twist $\rightarrow$ moment forms are derived in-manuscript through I-1/I-2 with a declared kernel.” The analysis reads the physical marginal twist as a field along the stacking axis, reversed by reversing the hand, and reads its effect on switching as that of an applied field. This is an additional source, rather than a new label for the same applied field.

Make mirror devices as device A: cut one bilayer into two and use the shared fabrication sequence (SC-236, L317; SC-237, L318; SC-238, L319; SC-239, L320; SC-240, L321; SC-241, L322; SC-242, L323; SC-243, L324; SC-244, L325). The designed angles are $180^\circ \pm \vartheta_{\text{tw}}$, with $|\vartheta_{\text{tw}}| = 0.03^\circ$ (U-08). The source’s nearly-half-turn device A and small-angle device B do not specify either hand (SC-242, L323; SC-243, L324). Prepare RB-mixed by the source’s largest-hole-density corner and edge path for each line (SC-165, L232; SC-166, L233; SC-167, L234). For each device let $\mathcal{R}_{\text{rem}}$ be half the zero-field Hall-resistance difference after saturation of the two polarities, and let $\mathfrak{g}_*$ denote the device’s own gate pair maximizing its magnitude in the positive-displacement diamond region. Here $\mathfrak{g}$ denotes a gate pair, and “sat” the preceding saturation polarity. With field zeroed at the Hall sensor, define and solve the gate condition by

$$\mathcal{R}_{\text{rem}}(\mathfrak{g}) = \frac{1}{2}[\mathcal{R}_{\text{yx}}(\mathfrak{g}; \text{sat}+) - \mathcal{R}_{\text{yx}}(\mathfrak{g}; \text{sat}-)], \quad \mathfrak{g}_* \in \underset{\mathfrak{g} \text{ in diamond}}{\operatorname{argmax}} |\mathcal{R}_{\text{rem}}(\mathfrak{g})|. \quad (27)$$

Fix these separate gate choices before comparing hands. The zero-field definition removes the linear ordinary Hall contribution; it does not infer a domain value from the device weight. Use $T = 1.5 \text{ K}$ (U-42) and current magnitude 0.5 nA , the source’s anomalous-Hall setting (SC-256, L341; SC-259, L344). Repeat at 0.25 nA and both current polarities (U-11). Keep the current-even part and bound its residual by the amplitude difference; the conventional chiral current-induced magnetization is odd in both current and hand and vanishes with its drive.

Let $\mathbf{B}_{\text{sw}}^{\uparrow, \downarrow}$ denote raw-branch jump locations from the field-derivative peaks, $\mathbf{B}_{\text{ctr}}$ their mean, and $\mathbf{B}_{\text{odd}}$ the hand-odd centre. Let $\mathbf{B}_{\text{tw}}$ denote the field established by the positive hand and $\mathbf{B}_{\text{c}}$ the symmetric coercive magnitude. The switching condition and its solution are

$$\mathbf{B}_{\text{sw}}^{\uparrow, \downarrow} + \mathbf{B}_{\text{tw}} = \pm \mathbf{B}_{\text{c}}, \quad \mathbf{B}_{\text{ctr}} = \frac{1}{2}(\mathbf{B}_{\text{sw}}^{\uparrow} + \mathbf{B}_{\text{sw}}^{\downarrow}) = -\mathbf{B}_{\text{tw}}, \quad (28)$$

$$\mathbf{B}_{\text{odd}} = \frac{1}{2}[\mathbf{B}_{\text{ctr}}(+)-\mathbf{B}_{\text{ctr}}(-)] = -\mathbf{B}_{\text{tw}}. \quad (29)$$

The source anti-symmetrizes Hall resistance in field to remove longitudinal leakover (SC-267, L352; SC-268, L353); that operation centres loops and cannot supply this statistic. A raw-branch contact offset moves a zero crossing, not the jump; its magnetoresistance contribution at the jump is a device-level nuisance. Time reversal leaves the static structural twist unchanged while reversing magnetization and applied field, so the conventional equilibrium loop has zero centre at each hand. Common remanence, training and sweep-rate response cancel between the two hands, the analysis’s neighbour comparison under Use item 7.

Use 12 independent pairs, each together in 2 cooldowns with mounting positions exchanged, and 24 loops per field/current/training condition (U-09, U-10). Interleave hands in one magnet with one training history, read a Hall sensor after each degauss, and report each device’s twist magnitude, rhombohedral area fraction and wall density. Fabrication offsets reset by device; remanence resets by cooldown; position differences reverse on the swap; jump noise resets by loop. Let $\mathfrak{b}_j$ denote a pair’s hand-odd difference averaged over cooldowns, $\bar{\mathfrak{b}}$ its mean over pairs, $\mathfrak{e}_{\mathfrak{b}}^2$ its sample variance and b_{sys} the residual bound. For pair count N_p the decision is

$$\text{SE}(\bar{\mathfrak{b}})^2 = \mathfrak{e}_{\mathfrak{b}}^2/N_p, \quad \mathfrak{h}_1 = t_{N_p-1} \text{SE}(\bar{\mathfrak{b}}) + b_{\text{sys}}. \quad (30)$$

Within each difference, the variance is one quarter of the two centre variances minus twice their covariance; averaging cooldowns retains their shared device covariance. Detection requires $|\bar{\mathfrak{b}}| > \mathfrak{h}_1$. Exclusion of a specified signed field requires its predicted centre to lie outside $[\bar{\mathfrak{b}} - \mathfrak{h}_1, \bar{\mathfrak{b}} + \mathfrak{h}_1]$; exclusion of either sign of a specified magnitude requires $|\bar{\mathfrak{b}}| + \mathfrak{h}_1$ below that magnitude. Attribution requires rejection of zero, inclusion of the predicted centre in that same interval, and the preregistered sensor, swap, current and geometry checks. Thus each rule includes the systematic bound.

The illustrative Gaussian scales are device 0.04 mT, shared cooldown 0.02 mT, position contrast 0.02 mT, loop 0.03 mT and residual bound 0.01 mT (U-15–U-19). The quoted loop scale is the common-mode centre error of its two derivative-peak jump locations. The independent-pair planning variance is device variance divided by twice the pair count, plus loop variance divided by twice the pair count, cooldown count and repeat count; common remanence and the swapped position offset cancel exactly in this planning model. For the illustrative positive-hand field +0.20 mT (U-14), the conditional centre is −0.20 mT and the ablated centre zero. The planned threshold is 0.03 mT, rounded to the one significant figure of the smallest-scale inputs. With seed 251000220 and 10000 null experiments (U-40), the full nested estimator rejects zero in 217 trials, a false-rejection fraction 0.0217. The residual systematic is placed at its positive bound in each null experiment; Gaussian symmetry gives the same marginal error rate for the negative endpoint. The simulated rate is not an empirical bound on device systematics.

5.2 T-2: the field-odd boundary at opposite hands

DC-2 discriminates SA-13: the source’s constant-free-energy interpretation leaves the additional twist field untested. This existence prediction is conditional by construction, with neither hand-to-field sign nor magnitude fixed. P-5 states, “Imposing a physical twist (torsion, helical geometry) on a material is establishing a magnetic field”; the analysis reads that field as adding to the applied field along the stacking axis. Use the T-1 mirror devices, now at Fig. 1c’s $\mathcal{D}/\varepsilon_0 = -0.067 \text{ V/nm}$ (SC-082, L118), each sweep initialized at maximum hole density (SC-112, L155; SC-113, L156). Use $T = 1.5 \text{ K}$ (U-42), applied magnitudes $B_a = 0.02 \text{ T}$ (U-12), both field polarities and zero in alternating order. Measure at 0.5 nA (SC-259, L344) and 0.25 nA (U-11), at both current polarities, retaining the current-even part and bounding its residual by the amplitude difference. Both hands are mounted together in each cooldown in one magnet under one training history; interchange their positions between cooldowns and interleave their loops and density sweeps. Record each device’s measured twist magnitude, rhombohedral area fraction and wall density before comparison. Here too the current-even estimator retains a measured amplitude-difference bound, and the remanent field is read after every degauss.

Let $\mathfrak{n}_{\text{sw}}$ be switching density, $\mathfrak{n}_b$ its field-free boundary, $\mathcal{F}'_{\mathfrak{n}}$ the local density slope of the stacking free-energy difference, and $\mathfrak{k}$ the field-even density coefficient. The slope is nonzero and measured independently; the barrier is assumed field-independent over the sweep. With $\Delta\mathfrak{s}_m$ the susceptibility

difference in the same free-energy normalization, a local quadratic model derives

$$0 = \mathcal{F}'_n(\mathbf{n}_{\text{sw}} - \mathbf{n}_b) - \frac{1}{2}\Delta\mathfrak{s}_m(\mathbf{B}_a + \mathbf{B}_{\text{tw}})^2, \quad (31)$$

$$\mathfrak{k} = \frac{\Delta\mathfrak{s}_m}{2\mathcal{F}'_n}, \quad \mathbf{n}_{\text{sw}} = \mathbf{n}_b + \mathfrak{k}(\mathbf{B}_a + \mathbf{B}_{\text{tw}})^2. \quad (32)$$

The exact quadratic model is the conditional scope; higher field terms are fitted and bounded before using it. Let $\mathbf{n}_{\text{odd}}$ be half the opposite-field difference and $\mathfrak{d}_n$ its hand-odd contrast. Solving Eq. (32) gives

$$\mathbf{n}_{\text{odd}} = 2\mathfrak{k}\mathbf{B}_a\mathbf{B}_{\text{tw}}, \quad \mathfrak{d}_n = 2\mathfrak{k}\mathbf{B}_a\mathbf{B}_{\text{tw}} \quad (33)$$

for equal curvatures and opposite twist fields. Time reversal makes the conventional free energy even in the sample field when no spontaneous order is present at the switching point; the conventional hand contrast is zero. Measure Hall loops at that gate in both states first. If hysteretic, initialize every sweep with one training polarity, repeat with the other, and separate the training-odd term. The common remanent field weights each hand by that device's curvature; curvature mismatch leaves a residual in the pair variance. The position swap removes a fixed position difference even with those fixed unequal curvature weights after both cooldowns are averaged.

Use 12 pairs, 2 cooldowns and 24 sweeps per condition (U-09, U-10). Take one jump-density estimator per sweep; average within device and cooldown before forming each pair contrast. Its standard error retains covariance between opposite-field sweeps and the shared zero-field calibration. Let $\mathfrak{h}_2$ be the pair Student threshold plus the density-systematic bound, with the same degrees of freedom as Eq. (30). Detection requires the mean hand contrast's magnitude above $\mathfrak{h}_2$. Exclusion requires the specified signed prediction outside its interval with half-width $\mathfrak{h}_2$, or, for either sign, the absolute mean plus $\mathfrak{h}_2$ below the specified magnitude. Attribution requires zero rejected and the conditional prediction retained by that interval, with sensor, training, current and geometry checks passed; the same bound enters every rule.

At the illustrative point $\mathfrak{k} = 1 \times 10^{14} \text{ cm}^{-2} \text{ T}^{-2}$ (U-20), its device scatter is 0.10 fraction (U-21). Device density offsets have scale $2 \times 10^8 \text{ cm}^{-2}$, sweep noise $4 \times 10^8 \text{ cm}^{-2}$ and the residual bound is $1 \times 10^8 \text{ cm}^{-2}$ (U-22–U-24). The remanence and position scales are each 0.02 mT (U-16, U-17), drawn per cooldown and fixed pair-position contrast respectively; the odd device offset resets once per fabrication. At the U-14 field, the hand contrast is $8 \times 10^8 \text{ cm}^{-2}$, its ablated value zero, and the planned threshold $2 \times 10^8 \text{ cm}^{-2}$ at one significant figure. The planning pair variance includes half the device-offset variance, sweep variance divided by four times the cooldown and repeat counts, and the curvature-mismatch times remanence contribution. The density systematic is placed at its bounding endpoint in every null experiment. The seeded nested null calculation gives 258 rejections in the U-40 experiments, a fraction 0.0258.

5.3 T-3: the finite isotope change of the Curie point

DC-3 discriminates SA-24: the source reports a Curie temperature “around 8 K” (SC-163, L226), while its temperature curve is in Extended Data and it gives no isotope contrast. This prediction is conditional by construction, using I-7's additional density-coupled constitutive input. The analysis reads its larger response as a higher Curie temperature. The observable sign, including the sign of the product of the channel sensitivities, is conditional. Use tetralayers made as device A from enriched carbon of mass-number inputs 12 and 13 (U-05), carbon-13 fractions 0.01 and 0.99 (U-07). Nuclear spins in units of $\hbar$ are 0 and 1/2 respectively; no heavier stable spin-zero carbon isotope supplies the control (U-06). Make 8 devices per composition, alternating fabrication and independent cooldowns (U-25). Every device's own positive-displacement remanence maximum

is selected by Eq. (27) before comparing compositions. Let T_C be the temperature where the saturated-remanence difference vanishes on the temperature curve; this independently measured endpoint, with its uncertainty from the curve, defines the observable. Temperature offsets and magnet remanence reset at each mounting and enter device scatter. Use the source current setting and its reduced-amplitude, both-polarity control from T-1, with zero field at the sensor during the remanence read.

Let $\mathcal{O}$ denote any one of the positive isotope observables, ξ_i its independently measured channel coordinates, and S_i its logarithmic sensitivity with every other coordinate fixed. Let f denote carbon-13 fraction, f_l and f_h its light- and heavy-composition values, u the symbolic atomic mass unit, $\bar{m}$ the mass-number mixture, and η the finite log response between heavy and light compositions. Core I-7 Discipline 2, applied along the enrichment path, gives

$$S_i = \left. \frac{\partial \ln \mathcal{O}}{\partial \ln \xi_i} \right|_{\xi_{j \neq i}}, \quad \eta = \Delta \ln \mathcal{O} = \int_{f_l}^{f_h} \sum_i S_i(f) \frac{d \ln \xi_i}{df} df, \quad (34)$$

$$\bar{m}(f) = [12(1 - f) + 13f]u, \quad \Delta \ln \alpha_{\text{ext}} = \ln \frac{\bar{m}(f_h)}{\bar{m}(f_l)}. \quad (35)$$

The integers in the mixture are U-05 assumptions, not confinement outputs. The fixed-geometry A-1 support here is the substituted carbon matter pattern, whose complete mass aspect is approximated by the U-05 mass-number input. The compactness equality holds at fixed pressure, packing and radius; the full stack's geometric and material response changes remain separate path channels. For $\mathcal{O} = T_C$, the channel coordinates are mass-driven compactness, interlayer spacing, electron-lattice renormalization, interaction strength, band flatness and domain configuration. Each sensitivity holds all the other listed coordinates fixed; independently fitted compound sensitivities carry the successive interlayer-to-band-to-observable response products. The finite amplitude is the integral in Eq. (34), not one local slope times an enrichment span.

Channel	Signed path input	Response sign
Mass at fixed geometry	Compactness increases	Positive for pure density coupling; Curie reading conditional
Zero-point volume	Heavy isotope smaller volume; anisotropic spacing measured	Interlayer and band sensitivities material-specific
Electron-lattice change	Independently calculated or calibrated path	Material-specific
Interaction, flatness, domains	Independently measured co-variates	Products of held-fixed sensitivities conditional
Unpolarized nuclear spins	Random hyperfine contribution bounded	Either sign within bound

The conventional comparator includes the smaller heavy-isotope zero-point volume, its anisotropic compliance into interlayer spacing, the induced coupling and band changes, and electron-lattice renormalization. The carbon-13 random hyperfine field is bounded independently at each curve's temperature and remanence field; it cannot be omitted by calling carbon substitution nonmagnetic. The fixed-geometry extra density response is removed in the conventional limit while these channels remain.

Measure twist magnitude, rhombohedral area fraction, wall density and gate calibration independently of the Curie observable. Fit their four sensitivities jointly across both compositions with an intercept and composition indicator (U-26). Let $\hat{\eta}$ denote the adjusted contrast, $\mathfrak{c}$ the independently predicted conventional contrast, $\mathfrak{c}_c$ its shared standard uncertainty and b_{hf} the hyperfine

bound. Let $\mathbf{Q}$ denote the design matrix, ϵ_{res}^2 the fitted residual variance and the subscript iso its composition coefficient. The superscript $\mathbf{T}$ denotes matrix transposition. For total device count N_d the decision is

$$\text{SE}(\hat{\eta})^2 = \epsilon_{\text{res}}^2 [(\mathbf{Q}^T \mathbf{Q})^{-1}]_{\text{iso}, \text{iso}}, \quad (36)$$

$$\nu_* = N_d - 6, \quad \mathfrak{h}_3 = \sqrt{[t_{\nu_*} \text{SE}(\hat{\eta})]^2 + (z\epsilon_c)^2 + b_{\text{hf}}}. \quad (37)$$

A shared comparator uncertainty is drawn once per experiment and is independent of the fit; a measured covariance would enter with its signed cross term. Detection requires $|\hat{\eta} - \mathbf{c}| > \mathfrak{h}_3$. Exclusion requires the specified extra density contribution outside the interval centred on $\hat{\eta} - \mathbf{c}$ with half-width $\mathfrak{h}_3$. Attribution requires zero excluded and that extra contribution included, together with the preregistered covariate and hyperfine controls; every rule includes b_{hf} . A null experiment excludes only the specified sensitivity path, not all uncalculated material sensitivities.

At the illustrative constant density sensitivity 1 (U-27), Eq. (35) gives $\Delta \ln \alpha_{\text{ext}} = 0.08$ at one significant figure. The comparator uses spacing change -0.001 with response sensitivity 2 (U-28), and integrated electron–lattice contribution -0.001 (U-29). It is $\mathbf{c} = -0.003$; the conditional total log response is 0.08, with the extra channel removed giving -0.003 . These are log contrasts, not a numerical prediction of a new Curie temperature from the source’s approximate value. Shared comparator uncertainty is 0.002 and hyperfine bound 0.001 (U-30, U-31). Device, cooldown and repeated-reading scales are 0.008, 0.004 and 0.003 in log units (U-32–U-34), with 24 repeated readings (U-10). The illustrative covariates are independent standard normal variables with each response sensitivity 0.002, fitted rather than fixed (U-35). With balanced orthogonal planning covariates the threshold is 0.01, rounded to one significant figure; the actual randomized fit uses its full covariance at each realization. The seeded null simulation draws fabrication once per device, mounting once per cooldown, repeated readings within device and the comparator once per experiment, then refits all four covariates. The hyperfine contribution is placed at its positive bound, the worst signed endpoint, rather than assigned an unlicensed noise distribution. It gives 311 false rejections, fraction 0.0311, in the U-40 experiments.

5.4 T-4: the isotope susceptibility contrast

DC-4 discriminates SA-14: the source writes that “As the switching happens at a constant Free energy difference, it enables us to measure thermodynamic quantities. For example, the magnetic field dependence of the switching boundary allows us to extract the magnetic susceptibility difference between the two structures” (SC-095, L133; SC-096, L134; SC-097, L135; SC-098, L136), citing Extended Data Fig. 5 and the Supplementary Information (SC-099, L137), outside the main text and Methods. This prediction is conditional by construction and reads a larger density-coupled response as a larger magnitude of $\Delta \mathbf{s}_m$. The observable sign, including the sign of the product of the channel sensitivities, is conditional. Use T-3’s enriched tetralayers and alternating fabrication/cooldowns, at T-2’s displacement setting and alternating zero and opposite applied fields. The states are initialized by the maximum-hole-density history before each sweep, with the same Hall-sensor, current-even and training-polarity controls. The temperature is 1.5 K (U-42), field magnitude 0.02 T (U-12), and the per-composition device count is 8 (U-25), with 24 sweeps at each condition (U-10). The sweep is nested in the device; it does not increase the independent device count.

Let $\mathbf{n}_+, \mathbf{n}_-, \mathbf{n}_z$ denote mean boundary densities at the positive, negative and zero fields, and $\hat{\mathbf{e}}$ the estimated curvature. The symbol $\Delta \mathbf{s}_m$ denotes the susceptibility-difference estimate from that curvature and the independent density slope. The defining finite-field condition and solution under

Eq. (32) are

$$\hat{\mathfrak{f}} = \frac{\mathbf{n}_+ + \mathbf{n}_- - 2\mathbf{n}_z}{2B_a^2}, \quad \widehat{\Delta\mathfrak{s}_m} = 2\mathcal{F}'_{\mathbf{n}}\hat{\mathfrak{f}}. \quad (38)$$

The analysis reads the extraction as this field-even curvature, its own dictionary. The field-independent barrier cancels; the density slope does not and is measured for each composition through its own free-energy/work-function relation. A shared zero measurement contributes four times its variance to the numerator, together with the pairwise covariances with both field readings. No source susceptibility magnitude is reconstructed from an unprinted boundary curve.

At a hysteretic switching gate, a trained moment below coercivity contributes a term odd in field and training and is removed by the even estimator. Above coercivity the moment turns and produces a term proportional to the field magnitude, even but nonquadratic. The protocol either stays below that turning field with both training polarities or fits a separate magnitude term using additional field magnitudes before assigning a quadratic curvature. The conventional comparator retains each stacking's band susceptibility, its zero-point spacing change, electron-lattice renormalization, the composition-dependent free-energy slope and the random hyperfine bound at the sweep temperature and fields. All those signs remain material-specific. Equation (34) now uses $\mathcal{O} = |\Delta\mathfrak{s}_m|$ on a path where its magnitude stays nonzero. The held-fixed sensitivities include the measured slope separately from susceptibility and the geometric channel separately from fixed-radius compactness. Thus the slope's change cannot be mistaken for the susceptibility's change.

Fit the four independently measured covariates jointly as in Eq. (36); the device count less six supplies the residual degrees of freedom. Let $\mathfrak{h}_4$ be Eq. (37) with the covariance-propagated susceptibility contrast error and its own comparator uncertainty. Detection requires the adjusted log-magnitude change minus its comparator to exceed $\mathfrak{h}_4$ in magnitude. Exclusion requires the specified finite density-path contribution outside that contrast interval with half-width $\mathfrak{h}_4$. Attribution requires zero excluded and the specified contribution included, with the slope, moment, current and hyperfine controls satisfied; each rule includes the hyperfine bound.

The illustrative susceptibility-slope ratio is 1 (U-43); the density and conventional paths, nuisance scales and comparator/hyperfine terms are U-27–U-35. The conditional log-magnitude change is 0.08; removing the extra channel gives -0.003 . The repeated-reading scale is the density-reading error divided by $2B_a^2|\mathfrak{f}|$, a first-order log-response error; the simulation then takes the exact logarithm of the perturbed nonzero curvature. The three-reading numerator propagates the repeated-reading variance with factor 6, giving a planned threshold 0.01 at one significant figure. The seeded simulation draws each of the three reading means separately, shares the zero term in the even combination, draws device and mounting terms at their reset levels and refits the four covariates, with the hyperfine contribution at its positive bound. It gives 381 false rejections, fraction 0.0381, in the U-40 experiments.

5.5 T-5: the isotope surface-potential contrast

DC-5 discriminates SA-36: the source's KPFM heterostructure is separate from transport, and its reported contrast supplies no isotope dependence. This prediction is conditional by construction; the analysis reads the larger density-coupled response as a larger potential-contrast magnitude, with the observable sign and sensitivity products conditional. Build heterostructures as Fig. 1b: twisted double bilayer graphene on hBN/graphite on the stated substrate (SC-251, L334; SC-252, L335; SC-253, L336; SC-254, L337). Use frequency-modulation KPFM with tip-applied bias as reported (SC-248, L331; SC-249, L332), and the two U-07 compositions in alternating fabrication and imaging sessions. The designed scan setting is 1.5 K and 0 T applied field (U-41); the paper

gives no KPFM temperature. Use 8 independent samples per composition and 4 domain pairs per sample (U-25, U-36), resetting the tip for each sample’s own session. The observable is the finite change of the magnitude of the rhombohedral–Bernal surface-potential contrast, with each pair read within one scan under one tip. The reported 22 mV contrast has no printed uncertainty or identified member of the two KPFM samples (SC-050, L83; SC-252, L335; SC-253, L336). Its work-function sign link is the source’s; this magnitude test needs no assumed sign convention.

Let $\mathbf{v}_\Phi$ denote the true signed pair contrast, $\mathbf{v}_{\text{read}}$ its scanned contrast, w_1, w_2 the two domain widths, and ℓ_{psf} the Gaussian point-spread standard deviation. Let p_{psf} be its full width at half maximum and η_{tip} the contrast attenuation. Here erf is the normalized Gaussian error function; $\hat{\eta}_{\text{tip}}$ denotes the attenuation estimate from independent width and spread measurements. A positive normalized Gaussian kernel, integrated over each centred domain stripe with the opposite background, derives

$$p_{\text{psf}} = 2\sqrt{2\ln 2}\ell_{\text{psf}},$$

$$\eta_{\text{tip}} = \text{erf}\left(\frac{w_1}{2\sqrt{2}\ell_{\text{psf}}}\right) + \text{erf}\left(\frac{w_2}{2\sqrt{2}\ell_{\text{psf}}}\right) - 1, \quad (39)$$

$$\mathbf{v}_{\text{read}} = \eta_{\text{tip}}\mathbf{v}_\Phi, \quad \hat{\mathbf{v}}_\Phi = \mathbf{v}_{\text{read}}/\hat{\eta}_{\text{tip}}. \quad (40)$$

The tip work function acts on both domains alike and cancels in the within-scan difference. Retain a pair only when each measured width exceeds that session’s measured spread and the attenuation is positive. Propagate width and spread uncertainty through division and this selection; the shared session spread makes pairs correlated. The stripe model is the declared conditional lateral kernel; curved boundaries require integration of the measured domain mask with that same independently measured kernel.

Equation (34) now uses $\mathcal{O} = |\mathbf{v}_\Phi|$ on a nonzero-contrast path. The comparator retains each stacking’s zero-point volume acting on its work function, electron–lattice renormalization and the unpolarized hyperfine bound at the specified temperature and zero field. These signs remain material-specific; the fixed-geometry density channel adds separately. Fit twist magnitude, rhombohedral area fraction and wall density jointly across the samples, omitting the transport gate calibration (U-26). The sample is the independent unit; its response is the mean of its corrected pair logarithms, and its domain pairs share both fabrication and tip-session effects. The design matrix of Eq. (36) now has three covariates, with total sample count less five residual degrees of freedom. Let $\mathfrak{h}_5$ use Eq. (37) with that sample-fit error, the independently estimated comparator and the scan’s hyperfine bound. Detection requires the adjusted log-contrast change minus its comparator above $\mathfrak{h}_5$ in magnitude. Exclusion requires a specified density-path contribution outside the interval of half-width $\mathfrak{h}_5$ around that difference. Attribution requires zero excluded and the specified path retained, with the kernel, width-selection, covariate and hyperfine checks passed; all three rules include the bound.

In the illustrative ensemble each width is uniform from 200 to 600 nm (U-37), and each session spread uniform from 30 to 60 nm (U-38). Independent width error has scale 5 nm and shared session-spread error 3 nm (U-39). The log response scales, fitted covariate effects, comparator uncertainty and hyperfine bound are U-27–U-35. The conditional log-contrast change is 0.08, its ablated value -0.003 , and its planned threshold 0.01 at one significant figure. The planning error averages the within-sample reading variance over the domain pairs; kernel uncertainty is retained by the full simulation. The seeded simulation draws both widths of every pair, one true and one measured spread per session, width errors, nested contrast noise, sample and session offsets, shared comparator error and all three fitted covariates, with the hyperfine contribution at its positive

bound. Division and the width cut retain 640000 of 640000 drawn pairs in this broad-domain ensemble. The resulting false-rejection count is 344, fraction 0.0344, in the U-40 experiments. This selection result belongs to the stated width distribution, not to narrower domains.

DC-6, discriminating SA-31, is retired with NO-MAP O-4 because the local-density ordering of Eq. (25) supplies no reduction to a skin-minus-centre carrier-density observable; work-function steps, gate screening, tip gating across a wall and wall states already enter the domain’s conventional electrostatics.

6 Cross-domain consequences

The force law, clock comparison and confinement problem refer to the same medium across scales. The gravitational window of the device uses its complete mass aspect and independent radius, while the Hall reading uses a comparison of the aligned bivector response against a reference geometry. Neither comparison replaces the other or uses a resistance as a curvature coordinate. The same force-to-energy dimensions apply to interlayer binding and to the pair potential; the material response kernel supplies the observed scale. The same action quantum relates a stroke frequency to energy and a confined pattern’s frequency to mass, while their supports and boundary data differ. A carbon isotope changes a supplied mass input at fixed geometry; the ensuing zero-point spacing change is a separate coordinate of the response path. The chart’s compactness seam and the overlap application’s ordering examples therefore remain separate from the Curie-temperature endpoint.

7 System-level reading (exploratory)

This section is an in-sample reading of the measured Hall signs, confined to the source’s marked gate regions and separate from the prospective comparisons.

X-1: displacement-odd anomalous Hall component. The observable is the anomalous Hall response odd under displacement reversal at matched nominal carrier density and fixed training. Let $\mathcal{R}_{\text{AHE}}$ denote that anomalous component and $\mathcal{R}_{\text{odd } \mathcal{D}}$ its displacement-odd part. Its defining condition and the inversion-symmetric comparator are

$$\mathcal{R}_{\text{odd } \mathcal{D}}(\mathcal{D}) = \tfrac{1}{2}[\mathcal{R}_{\text{AHE}}(\mathcal{D}) - \mathcal{R}_{\text{AHE}}(-\mathcal{D})], \quad \mathcal{R}_{\text{odd } \mathcal{D}} = 0 \quad \text{under inversion symmetry.} \quad (41)$$

Inversion changes the displacement sign while preserving the axial Hall response and its training field; this derives the even comparator at fixed preparation. The source reports the opposite Hall sign at negative displacement (SC-187, L259; SC-188, L260) and says the reversal “indicates” an inversion-breaking state (SC-191, L263; SC-192, L264). It calls geometric or hBN asymmetries “unlikely” using absence of moire signatures and mirror symmetry (SC-193, L265; SC-194, L266; SC-195, L267; SC-196, L268); the absence bounds alignment only at the transport sensitivity. The paper “hypothesize[s]” a domain-boundary origin (SC-199, L271; SC-200, L272; SC-201, L273) and, of its proximity-effect hypothesis, writes that “Testing this hypothesis will require further experiments, particularly simultaneous real-space imaging of the domain configuration combined with transport measurements” (SC-206, L278; SC-208, L280; SC-209, L281). The analysis reads the wall as the place where displacement and wall geometry jointly set the sign of the aligned bivector response; geometry alone is displacement-even. P-5 states, “A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along

an axis”; this wall reading applies that projection to the source’s domain-boundary hypothesis. The pentagon’s negative response is instead interpreted by the source through comparison with its cited quarter-metal report (SC-210, L283; SC-211, L284; SC-214, L287; SC-215, L288; SC-216, L289). Its statement “we believe” in two components and its call for more detailed investigation stay attached to that decomposition (SC-219, L292; SC-220, L293; SC-221, L294; SC-222, L295). Only paired displacement points determine the even/odd decomposition; a few signs fix no spatial origin or global amplitude.

Guards. The response retains the three transport weights, nominal B-dominant calibration and stated training. No local domain value, curvature ratio, branch assignment or new prediction count is inferred from these signs.

8 Convergence statement

The limiting map removes the twist field and the extra fixed-geometry density-response sensitivity while retaining the source’s stress, gate and response dictionaries. Let the offsets $\mathbf{n}_0, \mathcal{D}_0$ be the source’s calibration offsets and the gate capacitances retain their per-area units. The source’s definitions are (SC-264, L349; SC-265, L350; SC-266, L351)

$$\mathbf{n} = (C_t \mathcal{V}_t + C_b \mathcal{V}_b)/e - \mathbf{n}_0, \quad \mathcal{D} = (-C_t \mathcal{V}_t + C_b \mathcal{V}_b)/2 - \mathcal{D}_0. \quad (42)$$

The offsets were calibrated using B-dominant Landau levels (SC-266, L351; SC-267, L352). The source’s free-energy account reduces the difference under its reported hole doping (SC-090, L128; SC-091, L129; SC-092, L130; SC-093, L131), treats the switching as at a constant difference (SC-095, L133; SC-096, L134), and draws the jump contours on that understanding (SC-141, L199; SC-142, L200; SC-143, L201; SC-144, L202; SC-145, L203; SC-146, L204). Its calculation integrates a tight-binding band structure with two remaining parameters besides the tight-binding set (SC-271, L362; SC-272, L363; SC-273, L364). One is the KPFM work-function difference; the other is the zero-density, zero-displacement energy offset determined in the Supplementary Information (SC-274, L365; SC-275, L366). Their values and the tight-binding set are unprinted here. Using those same inputs returns that same calculation and the reported “agree well” comparison (SC-149, L207; SC-150, L208); this is the identity map and corroborates no invariant. The map also returns the measured switching endpoints, signs, loops and the source’s field-antisymmetrized resistance, retaining the source’s printed bounds and its absence of any printed uncertainty, repeat statistic or absolute hysteresis sensitivity.

The source names rhombohedral graphite’s “topological flat bands” which “host valley–spin-polarized magnetism” (SC-036, L67; SC-037, L68), and for this device writes “comparing with a previous report [23], we find that the region where we observe AHE corresponds to the spin- and valley-polarized quarter-metal phase of pure rhombohedral tetralayer graphene, but with major distinctions where the magnitude of” $\mathcal{R}_{yx}$ “in our device is significantly larger than that reported in pure rhombohedral tetralayer graphene, and a sign reversal is observed as a function of” $\mathcal{D}$, followed by “This observation suggests that the AHE we observe may originate from a proximity effect from the spin- and valley-polarized quarter-metal in the rhombohedral region to either the domain wall or the adjacent Bernal region” (SC-202, L274; SC-203, L275; SC-204, L276; SC-205, L277; SC-206, L278; SC-207, L279; SC-208, L280). FDT reads this same gate-history-dependent contrast as the stacking patterns’ coherently aligned bivector grade along the Hall axis—P-5’s “A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along an axis”—and declines “the magnetic field as an independent

entity” and “intrinsic particle masses and properties; particles as distinct things” (Use item 6; I-5), with the source’s stacking dependence retained.

The source calls ε_0 “the vacuum permittivity” (SC-082, L118; SC-083, L119). FDT reads ε_0 and μ_0 as response coefficients of the omnium, so c^2 is a medium property of the omnium, with no empty background (0.1), and declines “ c an abstract constant in an empty vacuum” (Use item 6).

The exact Newton reduction of Eq. (16) is I-4’s first algebraic layer. Its second layer recovers the weak-field slow-motion GR data to leading order, with relative corrections of order R_s/d and of the squared relative speed of the two patterns over c^2 . That limiting curvature statement is containment; the numerical gate dictionary is an identity map. The third layer is the source-specific program that supplies the response kernel and boundary data. The new comparisons leave that common-data limit only through the explicitly added physical-twist source or density-coupled isotope sensitivity.

9 Status ledger

The table states the derivational obligations, geometric tokens, in-sample readings and prospective comparisons together.

Item	Status	Delivered content or limiting value
O-1	Owed: finite-core cutoff r_c , regular core chart or outward phase-transfer law	Interior lower boundary named; no value
O-2	Owed: geometrically constructed odds for the overlap-contrast application	A-3 tokens PROVISIONAL; no magnetic threshold assigned
O-3	Owed: canonical odds for the confinement application	A-4 tokens PROVISIONAL; no address
O-4	Owed: local profile $\rho_{\text{loc}}(r_f)$ from the omnium constitutive law with boundary, surface and core data, reducing to the measured nuclear profile	Skin ordering stated; no local carrier-density map
O-5	Owed: multipole and tidal remainders of the closed-form pair expression; overlap complex	Nonspherical source evaluations and material binding kernel
O-6	Owed: maps from a charge or colour seat to an I-1 input; derivation of the seat table from cone geometry; the gluon octet’s dynamics	No force inferred from resistance; colour-sector calibration held out
O-8	Owed: constitutive derivation of $c_{\text{eff}}/n_{\text{cone}}$ and reconciliation with lensing	n_{cone} distinct from optical index
O-9	Owed: confinement operator, domains, boundary data and spectrum; every mass ratio	No nuclear mass formula, anchor radius or absolute address
A-5	PROVISIONAL mode token	Range and name only
T-1	Prospective, conditional by construction	Hand-odd centre; zero in convergence; amplitude kernel O-5/O-6
T-2	Prospective, conditional by construction	Field-odd hand contrast; zero in convergence; amplitude kernel O-5/O-6
T-3	Prospective, conditional by construction	Curie log change; convergence is the conventional channel integral
T-4	Prospective, conditional by construction	Susceptibility log change; convergence retains slope and conventional channels
T-5	Prospective, conditional by construction	Potential log contrast; convergence retains conventional channels and tip correction
Geometric magnetic reading	In-sample	I-3a bivector projection at the two history-selected states
Contour calculation	In-sample identity map	Shared parameters, no invariant corroboration
Displacement-odd reading	In-sample exploratory	Paired signs only; domain-wall origin remains the source’s hypothesis

10 Conclusions

The reported history-dependent bistability is experimental evidence that magnetism is geometric, exactly as core P-5 states: “A magnetic field is a macroscopic helical density gradient of the omnium: the bivector grade of the density field coherently aligned along an axis”. The analysis reads the RB-mixed state’s magnetic alignment along the stacking axis through the core’s bivector projection, with the B-dominant history furnishing the reference at the same gate voltages. The source’s measured switching, signs and loops retain their own dictionary and precision, and its shared-parameter contour calculation remains an identity map. The handedness comparisons isolate the extra twist-established field; the isotope comparisons isolate the added fixed-geometry density channel against geometric, electron–lattice and hyperfine contributions. Their conditional finite amplitudes are resolvable at the stated illustrative points, and their covariance-aware decisions attach to independent fabricated units. A carrier-density sign at a wall awaits a density-to-observable map; the skin ordering alone supplies none.

References

- [1] D. Brandon et al., “Nonvolatile Switching of Magnetism via Gate-Induced Sliding in Tetralayer Graphene,” arXiv:2510.00220v1 (2025), main text and Methods.
- [2] Fundamental Density Theory, *Core FDT invariants*, locked version 5.9.