The geometry a proof search finds

Google's verified quantum programs, the mathematics that certifies them and the physics that runs them

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Discovering formally verified quantum programs with AI
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Abstract

Shirobokov et al. [1] give teams of Gemini agents a Lean specification, a proof checker and a cost function, with high-level prompts and no problem-specific ideas. The agents return formally verified programs for parallel search, a fixed-point arcsine and three molecular simulations, with gains of up to 2.57 in search rounds and of 2.49 to 290 in fault-tolerant cost. This analysis reads where the savings came from. Every one is a geometric operation: a rotation held where its two shares are equal, a count written as turns of phase and read by interference, a change of frame that makes a physical law local, a system carried as one total, a step settled forward instead of run back. These are the operations Fundamental Density Theory (FDT) assigns to the physical carrier, and Google's search converges on them. Quantum mechanics is the plane projection of helical geodesics, so a search that optimizes the projection finds the helix. In FDT there is no intrinsic quantum advantage. There is geometric advantage: in both of Google's price lists the free operations are geometry, and each verified gain moves work into them. The certificate itself is mathematics, a proposition about operators with no clock, no length and no carrier, and it prices every word of a register whether or not a physical system can occupy it. FDT supplies what the symbols are, and with that the direction of the next saving: the largest scientific gain, 11.8, came from the one specification written on the physical sector. General relativity and quantum mechanics are subsets of FDT, and for every program here FDT returns the quantum record. The framework is published at `https://densitytheory.com` and is used here by reference.

1 Two approaches to one program

1.1 The mathematical approach: Google's certificate

Google poses each problem as a Lean file (Sec. 1.1, Fig. 2). The file states a correctness theorem and a cost bound and leaves typed placeholders for the program, its workspace, its cost and the proofs. Teams of Gemini 4 agents fill the placeholders. They work from high-level optimization prompts, and the researchers' domain knowledge sits in the specification alone (Sec. 1.2, App. C). An external verifier replays every submission in a clean sandbox against the frozen specification and admits the three standard Lean axioms and no others (App. B).

The objects of the proof are operators. Let j index the classical store. A state $\mathcal{B}$ assigns to each store a positive operator $\mathcal{B}(j)$, with total trace at most one, and an assertion $\mathcal{A}$ assigns an effect $\mathcal{A}(j)$ between zero and the identity. The expectation of an assertion, and the correctness of

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a program $\mathcal{C}$ under a precondition $\mathcal{J}$, are (App. A, Eq. (1))

$$\mathcal{V}(\mathcal{B},\mathcal{A}) = \sum_j \operatorname{tr}\big(\mathcal{A}(j)\,\mathcal{B}(j)\big), \qquad \mathcal{V}(\mathcal{B},\mathcal{J}) \le \mathcal{V}\big(\mathcal{C}(\mathcal{B}),\mathcal{A}\big) \ \text{ for every input } \mathcal{B}.$$

Cost is a count attached to the same program: expected parallel oracle rounds for search, and for the four circuits the logical spacetime volume $Q \cdot \mathcal{T}$, peak logical qubits times the number of T gates, with a Toffoli priced at four (App. A, Eq. (3); App. D.1).

An accepted submission is an entailment, exact in its frozen environment. It has no sampling error and no detector threshold: no processor ran the program. The authors state the purpose in those terms: the method lets researchers "design algorithms that today's classical and quantum computers cannot test" (abstract).

1.2 The physical approach: FDT, by reference

Fundamental Density Theory has one substance, the omnium. Matter, energy and spacetime are three readings of it, tied by the chain $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$. General relativity and quantum mechanics are subsets of FDT. On the A-1 compactness chart the branch-tagged curvature ratio α_σ reads the regime: general relativity is $\alpha_\sigma < 1/2$ and reads the spacetime phase, quantum mechanics is $\alpha_\sigma > 1/2$ and reads the matter phase, and the Clifford torus $\alpha_\sigma = 1/2$ is the seam between them (core 0.3a, I-4). The framework, its seven invariants and the derivations that establish this are published at `https://densitytheory.com` [2, 3] and are used here by reference; none is rebuilt in this analysis. Four of its statements carry this analysis.

The helix (I-4, 0.1d). Quantum mechanics is helical geodesics read in projection. The wavefunction $Ae^{i\phi}$ is the projection of a helix onto the plane normal to its axis, ϕ is the helical phase, and the unit i is the unit bivector $\mathbf{\Sigma}$ of that plane, $\mathbf{\Sigma}^2 = -1$. "Superposition" is ignorance of the helical phase at observation. A measurement is a phase fixation at a clamp, the analyzer that reads the helix in projection [4, 5].

The equal-share torus (0.3a, 0.1d). Two shares that sum to one, $|z_1|^2 + |z_2|^2 = 1$, are a point of a three-sphere. The locus of equal shares is the Clifford torus. It is the torus of greatest area, and the exchange of the two shares at fixed phases fixes it point by point. On FDT's support it is the seam $\alpha_\sigma = 1/2$; on the velocity sphere of a strand it is the 45° torus.

The settlement (0.1a). Every step is a system-wide settlement, solved from the state the system is in. Events do not commute, a process run backward is never the mirror of the forward one, and a response is never computed particle by particle or pair by pair.

The pair (I-4). The record of an equal-weight pair of opposite sense is one settlement of the pair and its two clamps, on the projection measure of the clamp that hosts it. Local sign records on that measure return the quantum table cell by cell [6, 7].

Google's letter α names a helium nucleus, a stopping angle and a block-encoding scale. Here α_σ is FDT's compactness alone. A share angle is written ϑ, with shares $\cos^2\vartheta$ and $\sin^2\vartheta$, and a helical or relative phase is written ϕ. Every number is Google's, located in its text in small type; every product shown with its factors is recomputed here.

2 The free column is geometry

Google uses two price lists.

Model	Priced	Free
Query (search)	One parallel round of the oracle	Every unitary independent of the marked set, every measurement, reset and classical branch (App. E.1, Fig. S6)
Fault-tolerant (the four circuits)	The T gate, times peak logical qubits	Clifford gates, measurements and classical feed-forward (App. A; App. D.1)

Both free columns are geometry. The Pauli operators $\mathsf{X}, \mathsf{Y}, \mathsf{Z}$ of one qubit obey the relations of three orthonormal vectors in the geometric algebra of three-dimensional space: each squares to one, and distinct ones anticommute. Their products XY, YZ and ZX are the three unit bivectors, the oriented planes, and each squares to minus one. A one-qubit gate is, up to phase, a rotor $\exp(-\frac{\gamma}{2}\mathsf{B})$ on a unit bivector B: a turn by γ in that plane. The one-qubit Clifford gates are the 24 rotations of the cube whose face normals are $\pm\mathsf{X}, \pm\mathsf{Y}, \pm\mathsf{Z}$. The T gate is the turn by $\pi/4$ about Z, and it carries the cube off itself. Clifford gates on several qubits carry products of Pauli operators to products of Pauli operators, which is why a classical record of the frame tracks them (App. D.1). A measurement is a projection onto an axis of the frame. The free operations are the symmetries of the frame: reflections, quarter turns, permutations, parities and projections. The priced operations are contact with the unknown and a turn finer than the frame's symmetry.

FDT writes its strand in the same algebra. The strand spinor is $c\,e^{\mathbf{\Sigma}\vartheta}$, with $\mathbf{\Sigma}$ the unit bivector of the strand plane (0.1d), and the phase rotor of a helix is $e^{\mathbf{\Sigma}\phi}$ (I-4). The complex matrices of the Lean library and the rotors of the core are one algebra. In the library its unit i is a number. In FDT it is a plane.

This fixes what an advantage is. *Geometric advantage* is the saving that the geometry of the available transformations, correlations and readouts gives an implementation. The standard account reads the gains of a quantum program as a resource intrinsic to quantum amplitudes. FDT reads them as geometric advantage, the geometry of a kept helical phase (I-4); the certified separations and costs are accepted in full. There is no intrinsic quantum advantage. The square-root separation of single-mark search stands, as a separation between two geometries of access: a classical miss removes one address from N, and a coherent query turns the state through a fixed angle in the plane of marked and unmarked (Sec. 2.1; App. E.2).

Read against the two price lists, the gains of Section 3 are of two kinds: work moved from the priced column into the free one, and priced work removed because the physical system, or the form of the law, never required it.

3 Five problems, one geometry

3.1 Parallel search: a rotation held at equal shares

The task is a marked set of known size k among N addresses, with m parallel queries per round on shared workspace, certain output, and cost in expected rounds at large N on the grid of powers of two up to 1,024 for m and k (Sec. 2.1; App. E.1).

Let $|\mathrm{u}\rangle$ and $|\mathrm{m}\rangle$ be the uniform states on the unmarked and the marked addresses. The search state is $z_1|\mathrm{u}\rangle + z_2|\mathrm{m}\rangle$ with $|z_1|^2 + |z_2|^2 = 1$, a point of a three-sphere, and $|z_2|^2$ is the marked share. On one register a Grover step rotates this plane by 2θ, with $\sin\theta = \sqrt{k/N}$. After j steps the angle is $\vartheta = (2j+1)\theta$ and the shares are $\cos^2\vartheta$ and $\sin^2\vartheta$ (App. E.2). In units of the reference cost $\frac{\pi}{4}\sqrt{N/(mk)}$, running the rotation to $\vartheta = \pi/2$ costs one. Stopping early and restarting on a miss costs 0.8786 at its best (Eq. (8)).

The strongest single-register protocol does neither. It turns to $\vartheta = \pi/4$, where the two shares are equal, and stays there. With P the marked projector and $0 < w < 1$ the strength of a weak

check, the two Kraus operators of the check are (Eq. (6))

$$\sqrt{w}\,\mathsf{P} \quad \text{(click)}, \qquad (\mathsf{I}-\mathsf{P}) - \sqrt{1-w}\,\mathsf{P} \quad \text{(no click)}.$$

A click lands on a marked address. A miss damps the marked component slightly and turns its phase by a half turn, and one free diffusion step returns the state to $\pi/4$. The cost is (Eq. (9))

$$\tfrac{1}{2} + \tfrac{1}{\pi} \simeq 0.8183,$$

one half to reach the equal-share angle and $1/\pi$ to wait there for the click.

Google's verified circuits tie this number on the boundaries $m = 1$ and $k = 1$ and carry it to every interior setting: the certified constant for finding one target is 0.818 on all 121 cells of the grid (Figs. S7a, S8b). The published interior protocol runs m unentangled registers, whose success probabilities combine slot by slot, and its constant worsens from 0.942 at $m = 2$ to 0.997 at $m = 1{,}024$ (Eq. (10)). The verified circuit couples all m slots through one reflection about the slot tuples that hold an odd number of targets (App. E.3, Fig. S8a). The m slots become one system with two shares, odd and even, held at the same angle. The gain reaches $0.997/0.818 = 1.22$.

Finding all the targets gains more, up to 2.57 (App. E.3, Fig. S9). The circuit holds a uniform superposition over all sets E of m addresses. One round writes the sign $(-1)^{|E\cap R|}$, with R the targets still unfound: the number of hits enters as that many half turns of phase. A fixed reflection on the Johnson graph of the sets, independent of the targets and free of queries, carries the weight toward the sets that hold every remaining target. One last round records all the hits at once.

The geometry. The search state lives on a three-sphere of two shares. The best certified protocol for one target does not run the full quarter turn. It reaches the Clifford torus of that sphere, the locus $|z_1| = |z_2|$, and holds it: $\frac{1}{2} + \frac{1}{\pi}$ is the price of the equal-share torus, and Google certifies it on the whole grid. This is the form of FDT's support, where the equal-share torus is the seam between the two regimes (0.3a), and of the strand's velocity sphere, where it is the 45° torus on which the product of the two shares is greatest (0.1d). The marked share is a share of the search plane's sphere and is not a compactness. The second move is the settlement's: slot-by-slot search computes the response particle by particle, and the gain comes from treating the slots as one system (0.1a). The third is the helix's: the oracle writes in phase, and everything that turns phase into a record is target-independent geometry, priced at zero.

3.2 Arcsine: the phase circle opened

The task is to add a 25-bit fixed-point arcsine of the input register to an accumulator, with error at most 1.25×10^{-7} on all $2^{24} + 1$ inputs in $[-1, 1]$, and to return every ancilla to zero (App. F.1). The published piecewise-polynomial circuit costs more than $245 \times 440{,}576 = 107{,}941{,}120$, at a looser error (App. F.2, Table S2).

The search ran in two gate sets (App. F.3). In the first, kickmix, a basis input never leaves the form of one basis state with a sign. Its phases are 0 and π, two points of the phase circle, and that is what lets a classical computer test it. Inside it the agents reached $102 \times 4{,}884 = 498{,}168$ and then $100 \times 4{,}152 = 415{,}200$. The circuit they found rests on two symmetries (App. F.4). The arcsine is odd, so a reflection handles the negative half of the domain. Near the endpoint it runs as $\frac{\pi}{2} - \sqrt{2(1-|y|)}$, one form at every scale of $1 - |y|$. The 440 intervals are laid out with widths in proportion to their distance from one, and the group of an interval is the number of leading zeros of $1 - |y|$: a ratio, read by a shift.

The second gate set is the whole of Clifford and T, which only the Lean proof checks. There the agents replaced the bitwise lookup of each coefficient bit by an inner-product block (Fig. S11c). The block puts the coefficient qubit in an equal superposition, writes the products as a relative

phase between the two branches with T and $T^\dagger$ gates, eighth turns of opposite sense, and lets the branches interfere to give the bit. A leftover sign is cancelled by free gates. A block of r products costs $2r + 2$ T gates in place of four per product, and the 41 blocks cost $364 + 230 = 594$. The lookup-and-add stage falls from 1,348 to 894 T gates and the circuit to

$$100 \times (4 \times 776 + 594) = 100 \times 3{,}698 = 369{,}800,$$

eleven per cent below the best signed-state circuit and more than 290 times below the published one (Table S2).

The geometry. After its Hadamard the coefficient qubit sits on the Clifford torus of its own three-sphere, the equator of the Bloch sphere, and its relative phase is the angle around that equator: the phase circle of the helix chart (I-4). Kickmix allows two points of the circle. The verified gain is what the rest of the circle holds, bought with eighth turns in the two senses and read by interference, the projection of the phase onto an axis. The first stage found reflection symmetry and scale invariance, the property FDT's chain has because it is all ratios (0.3, 0.3a). The second found the plane that i names.

3.3 Stopping power: the physical sector and the system's total

The task is the time evolution of a helium nucleus and 218 electrons among 216 fixed protons, to vector error 10^{-2}, on every state antisymmetric under exchange of same-spin electrons, over 500 runs (Sec. 2.3; App. G.1). The baseline costs 5.42×10^{15} per run on 5,650 logical qubits. The verified circuit costs $229{,}357{,}900{,}876{,}653{,}900/500 \simeq 4.59 \times 10^{14}$ per run on 4,338, a gain of 11.8 (App. G.2, G.4).

The number of walk steps scales with λ, the sum of the absolute weights of the Hamiltonian written as a combination of unitaries. In the baseline more than nine tenths of λ comes from the electron–proton and electron–electron terms, and λ pays for every word the register can hold, including configurations no electron system occupies (Sec. 2.3). Three changes remove that cost (App. G.3).

A change of frame. At each step a quantum Fourier transform carries all the electrons from the momentum grid to a real-space grid, where the interactions are applied. In the momentum frame an electron at a place x is a discrete helix: equal weight on every wave number k, and a phase e^{-ikx} that winds at a rate set by the place. The transform reads one helix by its pitch or by its place.

A physical law made local. In real space exclusion is a statement about one grid point: at most two electrons, of opposite spin. A subroutine lets each electron found at a point kick back a phase i in one branch of the combination and $-i$ in the other. For n electrons at the point the average of the two branches is

$$\tfrac{1}{2}\big(e^{i\pi n/2} + e^{-i\pi n/2}\big) = \cos\frac{\pi n}{2} = 1 - n \quad \text{for } n = 0, 1, 2,$$

with magnitude at most one for every n. The Lean proof shows that the admissible states have $n \le 2$ at every point, so each short-range term enters λ once and not once per electron. Together the frame change and the exclusion bound take λ from 1,744,784 to 1.356×10^5 hartree and the walk steps down by a factor 12.8, at 1.42 times the T gates per step.

The system's total. A proton sits off the grid, so the circuit shifts every electron to put it on a grid point. In the plane-wave frame a shift by a is a phase, e^{ika} on one helix and e^{iPa} on the system, with P the total electron momentum. Earlier real-space circuits summed the 218 momenta four times a step. The final one keeps P in a 42-qubit register between steps, because

only the protons and the projectile change it. The sums halve and the T gates per step fall by a tenth.

The geometry. The count n is written as n quarter turns in each sense and read as the projection of the two counter-turning phases onto their common axis: the wavefunction as the plane projection of a helix (I-4), built gate by gate. The shift is the helix's own identity, in which a translation along the axis is a turn of the phase (0.1d). The saved register is the settlement's: the response of 218 electrons is carried as one system's total and is not recomputed particle by particle (0.1a). The largest factor is the sector. A register is a mathematical container with 2^Q words. FDT has no container apart from its patterns (0.1b), and the walk steps fell by an order of magnitude once the proof used the fact that the unphysical words are never occupied.

3.4 Non-Born–Oppenheimer dynamics: settled forward, counted, averaged

The task is one femtosecond of evolution of $C_{23}H_{20}N_3O$ with all 47 nuclei and 187 electrons as quantum coordinates on a real-space grid, to vector error 10^{-2} on every state of the register (Sec. 2.4; App. H.1). The refined baseline costs 4.98×10^{15} on 6,198 qubits. The verified circuit uses $5{,}616 + 417 = 6{,}033$ qubits and costs (App. H.2, H.4)

$$\mathcal{T} = 3{,}326{,}847\,(30{,}116 + 76) + 2 \times 30{,}132 + 245 = 100{,}444{,}225{,}133,$$
$$Q \cdot \mathcal{T} = 6{,}033 \times 100{,}444{,}225{,}133 = 605{,}980{,}010{,}227{,}389,$$

a gain of 8.22. Three changes inside the walk step account for it (App. H.3, Fig. 4e–g).

Settled forward. To apply a pair interaction the baseline swaps the pair's coordinates into fixed slots, acts, and swaps them back. The return is the mirror of the approach and costs as much. The verified circuit copies the coordinates into ancillas and clears the copy with Pauli-X measurements and classically controlled corrections. The return by measurement costs 908 T gates where the forward copy costs 24,194, and routing falls from 46.8 thousand to 25,102 T gates per step.

A law as the mean of threshold records. Both circuits obtain $1/r$ as a mean over branches of equal weight. The baseline's branch test needs a multiplication. The verified circuit compares the rescaled r^2 against three registers in uniform superposition, counts how many are smaller, and looks up one of four coefficients. The binomial weights of a cubic Bernstein polynomial arise from the counting, and $1/r$ is its mean. The cost falls from 19.8 thousand to 3,060.

Charge as a count. A nuclear charge Z acts on an electron as Z unit charges stacked at one point. The baseline prepares unequal weights. The verified circuit prepares an equal superposition and assigns Z of its branches to each nucleus by Boolean logic. The cost falls from 22.5 thousand to 1,256.

The geometry. The first change is the settlement (0.1a): the circuit does not rewind the event, it settles from the state it is in, and the price list charges the projection under four per cent of the mirror. The second is built the way FDT's pair record is built, where a smooth kernel, the cosine, is the mean of sign records over a uniform measure on a clamp's aperture (I-4); here the kernel is $1/r$ and the records are comparisons, a construction the baseline shares and the verified circuit makes cheap. The third replaces fitted weights by equal shares and an integer, the form of FDT's integral ledger (I-5a).

This specification is the one written against physics. To match its baseline it demands correctness on every word of the register, including states that break exchange symmetry, and Google names this as a limit on what the agents were able to optimize (Sec. 3). No saving of the kind found in stopping power was open to the search. Section 6 turns this into a test.

3.5 Compound I: address by place, and the pair's sign

The task is phase estimation of the energy of cytochrome P450 Compound I, in a model of 63 electrons in 58 orbitals with a fixed factorized Hamiltonian, for every eigenstate within 0.1 hartree of the ground state, to root-mean-square error one millihartree (Sec. 2.5; App. I.1). The adjusted baseline costs 1.09×10^{12} on 481 qubits. The verified circuit costs $333 \times 1{,}310{,}839{,}337 = 436{,}509{,}499{,}221$, a gain of 2.49 (App. I.2, I.4).

Address by place. Each term of the Hamiltonian is applied by rotating the orbital frame with 57 plane rotations until the term acts on one orbital. The baseline addresses the chosen term in binary and reads its 57 angles from a table into a register of 247 qubits. The verified circuit gives the terms their own qubits, two terms to a qubit with one more qubit to choose between them, and sets only the chosen one. Each angle bit then arrives as parities of these qubits, computed by Clifford gates with no table. Rotations and their angles fall from 37,688 to 23,636 T gates per step, the inner state preparation falls from 5,888 to 2,264, and the angle register disappears (App. I.3, Table S3).

The pair's sign. The baseline moves spin-down qubits into the place of spin-up ones with four blocks of 58 controlled swaps per step. On two qubits a swap is the identity except on the antisymmetric state $(|01\rangle - |10\rangle)/\sqrt{2}$, whose sign it reverses. The verified circuit changes basis to Bell pairs with Clifford gates, computes the parity of the number of orbitals in that state, applies one controlled sign, and erases the parity by measurement. One parity serves two blocks, and the cost of the swaps halves, from 928 to 464.

The geometry. A binary address is a number to be decoded. A one-hot address is a place, and reading it is a parity. The antisymmetric state is FDT's equal-weight pair of opposite sense, two phases locked a half turn apart (I-4). Exchanging its wings is that half turn: a sign. Section 5.3 gives the pair's record.

4 The convergence ledger

Verified mechanism	Problem and gain	Geometric operation	FDT
Weak loop held at $\vartheta = \pi/4$, cost $\frac{1}{2} + \frac{1}{\pi}$	Search, one target: 0.818 on all cells	The equal-share torus of a three-sphere	0.3a, 0.1d
One reflection about odd-count slot tuples	Search, one target: up to 1.22	m slots as one system	0.1a
Parity sign and target-independent Johnson reflection	Search, all targets: up to 2.57	A count written as half turns; the record made by free geometry	I-4
Odd symmetry; intervals addressed by leading zeros	Arcsine: 107,941,120 to 415,200	Reflection; scale invariance	0.3, 0.3a
Inner-product block: equal superposition, T and $T^\dagger$, interference	Arcsine: 415,200 to 369,800	The phase circle beyond two points, read in projection	I-4, 0.1d
Fourier transform to a real-space grid	Stopping power: opens the next row	Change of frame: a helix read by pitch or by place	I-4
$\pm i$ kickback, $\cos(\pi n/2)$; exclusion proved on admissible states	Stopping power: 12.8 fewer walk steps	A count as quarter turns in two senses; the physical sector	I-4, 0.1b
Total momentum kept in one register	Stopping power: a tenth of the T gates per step	The system's total; translation as phase	0.1a, 0.1d
Copy, then clear by measurement	Non-BO: routing 46.8k to 25.1k	Settled forward, no mirror	0.1a

Verified mechanism	Problem and gain	Geometric operation	FDT
$1/r$ from comparisons against uniform registers	Non-BO: 19.8k to 3.06k	A law as the mean of threshold records over a uniform measure	I-4
Z equal branches per nucleus	Non-BO: 22.5k to 1.26k	Charge as a count on equal shares	I-5a
One-hot term register	Compound I: 96% of the T gates saved per step, with the inner preparation	Address by place	—
Swap blocks as one sign on antisymmetric pairs	Compound I: swaps 928 to 464	The half turn of a locked pair	I-4

Thirteen mechanisms reduce to six operations: equal shares; phase written and read in projection; symmetry and change of frame; the system as one; settlement forward; the physical sector. The prompts held no such list. The specification, the verifier and the cost were the whole of the guidance (Sec. 1.2; App. C), and teams of agents exchanging proofs, notes and critiques arrived at these operations on five unrelated problems (Sec. 2.6, Fig. 5). Some rows were already in the literature the agents drew on, and others the article presents as new: the parity reflection, the Johnson walk and the exclusion bound. Under one cost function the search kept them all. A proof search judged by cost alone converges on the geometry of the helix, because the formalism it optimizes is the helix's projection.

5 Mathematics and physics

5.1 What the certificate is

A certificate is a theorem about operators. That is its strength. It holds for every input in its quantifier, at sizes no machine can test, and it let a search with no problem-specific guidance find the geometry of the ledger. It is also its boundary, in four respects that the source itself records.

Nothing ran. No circuit was executed on a quantum processor. The simulations remain well beyond the resources projected for the first fault-tolerant machines (Sec. 3; App. D.2).

Existence stands in for the program. The 3,326,848 rotation angles of the non-Born–Oppenheimer circuit, and the gate words that approximate them, are fixed by existence proofs. Their cost and error are certified, and the angles have not been computed (App. H.4).

The theorem ends before the observable. The stopping-power and non-Born–Oppenheimer theorems cover time evolution; preparation and readout lie outside the cost. The Compound I theorem takes an eigenstate as input and leaves trial-state preparation uncounted. At one millihartree it leaves the doublet and the quartet unseparated, and the active space itself, already converged classically, leaves the physical spin ordering unresolved (App. G.1, H.1, I.1).

The price list has no clock. $Q \cdot \mathcal{T}$ is proportional to elapsed cycles under one execution model, serial magic-state factories on a planar code. Parallel factories and other architectures price the same program differently (App. D.1).

5.2 What physics supplies

The certificate uses symbols whose referents it has no need to state. FDT states them.

Symbol	In the Lean library	In FDT
i	The imaginary unit	The unit bivector of the plane normal to the helix axis (I-4, 0.1d)
A phase	The argument of a complex number	The helical phase of a strand
A qubit's state	A unit vector in $\mathbb{C}^2$	Two shares on a three-sphere and a phase around its tori
A branch weight	The trace of a branch operator	The measure of the unread helical phase across the hosting clamp's projected aperture, written for the equal-weight pair (I-4)
A measurement	Kraus operators and a store update	A phase fixation at a clamp: one settlement of the system and its clamps (0.1a)
A two-qubit gate	A four-by-four unitary	One settlement of the two patterns (0.1a)
Composition	A matrix product, pulled back in reverse order	Events that do not commute, each settled from the present state (0.1a)
Cost	A count of T gates and qubits	The extent of the held patterns and the time they keep (0.1a)

Three consequences follow, and none is available to the mathematics alone.

The sector. A register of Q qubits has 2^Q words. The certificate quantifies over the words its specification names, and the cost follows the quantifier. Of the three simulations, one quantifies over the physical sector and returned the largest gain, 11.8. One quantifies over every word, and Google names that as a constraint. One fixes its representation in advance, and Google names that too (Sec. 3). The unphysical is what the mathematics overpays for. FDT has only patterns (0.1b), so the physical sector is the whole of its domain from the start.

The clock. Google names its unit a spacetime volume. FDT reads the name literally. Q counts patterns held in phase and $\mathcal{T}$ counts priced turns of phase. On a device each pattern keeps its own time by circulating, with period h/E_q for a mode of energy E_q (0.1a, 0.1d), and a gate lasts a number of those periods. The count becomes a duration when a device supplies the period.

The direction. A proof checker says that a program is right. The ledger says where the next one is. Section 6 uses it.

5.3 The pair and the joint gate

Bell pairs, parities and joint reflections carry most of the ledger. FDT gives the physical account of the smallest of them [6, 7].

Two patterns of opposite sense and equal weight leave their origin with phases locked a half turn apart, ϕ and $\phi + \pi$. The pair and its two clamps are one system, and its record is one settlement on the projection measure of the clamp that hosts it. On the sphere of the helix chart let $\mathbf{a}$ and $\mathbf{b}$ be the unit axes of the two clamps and $\boldsymbol{\eta}$ the shared unit direction. With the first clamp hosting, the density of $\boldsymbol{\eta}$ over solid angle is $|\mathbf{a} \cdot \boldsymbol{\eta}|/(2\pi)$, and the two local sign records are $\mathsf{s} = \operatorname{sgn}(\mathbf{a} \cdot \boldsymbol{\eta})$ and $\mathsf{t} = -\operatorname{sgn}(\mathbf{b} \cdot \boldsymbol{\eta})$. The first sign times the absolute projection is the signed projection, and the integral of $\boldsymbol{\eta} \operatorname{sgn}(\mathbf{b} \cdot \boldsymbol{\eta})$ over the sphere is $2\pi\mathbf{b}$, so the mean of the product is $-\mathbf{a} \cdot \mathbf{b}$ and the four cells are

$$\Pr(\mathsf{s}, \mathsf{t} \mid \mathbf{a}, \mathbf{b}) = \frac{1 - \mathsf{s}\,\mathsf{t}\,\mathbf{a} \cdot \mathbf{b}}{4}.$$

Each wing's marginal is $1/2$ at every remote axis, so no signal passes, and the table is the same whichever clamp hosts. It is the quantum table of the singlet, cell by cell, produced by local records.

The standard account reads the singlet's correlations as a non-local property of an amplitude. FDT reads them as shared bivector grade, one settlement of the pair and its clamps on the host

clamp's projection measure (I-4, 0.1a); the measured table is accepted in full. Bell's inference stands: a measure independent of the clamps, with local sign records, obeys the bound of two. FDT declines that one premise, and Tsirelson's value $2\sqrt{2}$ is the ceiling of the measure a clamp sets.

A joint gate is a joint settlement. With the first wire as control, a controlled-NOT fixes the words that begin with zero and flips the last bit of those that begin with one:

$$\frac{|01\rangle - |10\rangle}{\sqrt{2}} \;\longmapsto\; \frac{|01\rangle - |11\rangle}{\sqrt{2}} = |-\rangle \otimes |1\rangle,$$

with $|-\rangle$ the negative eigenstate of X. Before the gate both marginals are flat. After it the control returns -1 on X with certainty and the target returns -1 on Z with certainty. Turns of either wire alone, and every mixture of such turns, keep the marginals flat (I-4). The gate is one settlement of both patterns, a joint phase fixation that resolves the lock into two definite phases. With a Hadamard it is the standard change of basis to Bell pairs, the Clifford step by which the Compound I circuit turns the question whether a pair is locked into bits, and the price list charges nothing for it.

6 Where the physical reading sends the search

The direction is testable on Google's own pipeline, with its verifier as the judge.

Test S, the sector. Restrict the non-Born–Oppenheimer specification to the exchange-admissible states of the 187 electrons, as the stopping-power specification is restricted, and leave the Hamiltonian, the tolerance, the duration and the cost convention fixed. The certified circuit already meets the restricted specification, so the cost does not rise. FDT's statement is that it falls. The real-space frame is already in place, exclusion at a grid point becomes provable, and the block-encoding scale of 8.045×10^4 hartree drops as λ dropped in stopping power, taking the 3,326,847 walk steps with it (App. H.4). The test passes with a verified circuit below 605,980,010,227,389 whose saving enters through the scale. It fails if a search of the original size returns none.

The rule behind the test is general. Write the specification on the physical sector, leave the frame free, and let counts replace weights. Each clause is a row of the ledger that the search arrived at without being told.

7 Convergence statement

Hold the program, the input, the instruments, the decoder, the Hamiltonian and the cost convention fixed. FDT returns every certified proposition, every expected cost and every integer product of Section 3 unchanged; the map is the identity on the source's semantics. Quantum mechanics is a subset of FDT (I-4), so the record quantum mechanics assigns to each of these programs is FDT's record. For the equal-weight pair the recovery is explicit, cell by cell, through the projection measure of Section 5.3. General relativity is a subset of FDT in the same way, and it enters these programs only through the clock of a run, on which the two return the same elapsed time for the same source and path (I-3). The analysis makes no divergence from the quantum record. It adds no source, no constitutive input and no changed dictionary (core Use 2). What changes is the reading: an amplitude is the projection of a helix, the advantage is geometric, and a specification has a physical sector.

8 Status ledger

Item	Status
Costs, mechanisms and counts	Google's, as reported. Products shown with their factors are recomputed here. The geometric readings are assigned to results already certified and are in-sample.
Test S	Prospective. Judged by Google's verifier on a restricted specification.
Shares	The marked share and a qubit's two shares are shares of a three-sphere. No circuit quantity is an A-1 compactness.
O-17	To be derived: the measure for a pair of unequal weights and for more than two patterns, what selects the host clamp, and how the measure is carried between clamps. It binds every register above two qubits and the dynamics of the joint gate.
O-6	To be derived: the map from a charge to an I-1 force input. The Coulomb terms of the three Hamiltonians are the source's.

9 Conclusions

Google gave a proof checker and a cost to a search with no problem-specific guidance, and the search returned geometry: equal shares, phase, frames, totals, forward settlements and, once, a physical sector. That is the convergence. FDT states its cause. The formalism the search optimized is the plane projection of helical geodesics, so its optima are the helix's symmetries, and the advantage they buy is geometric. The mathematics certifies a program for a machine that has not run it. The physics says what the program's symbols are, which words of its register exist, and where the next order of magnitude lies. General relativity and quantum mechanics are subsets of FDT, and the certified results stand in FDT as they stand in quantum mechanics, with the same records. Test S runs on Google's pipeline.

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