

What c Is

A geometric derivation of the space–time and matter–energy conversion
in Fundamental Density Theory

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Fundamental Density Theory

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Abstract

In Fundamental Density Theory c converts the time reading of the one substance into its space reading and, squared, its matter reading into its energy reading. This paper derives it from the geometry of the stroke, the omnium’s one carrier. The omnium’s two response coefficients give the stroke one rate, $\mathfrak{c}^2 = 1/(\varepsilon_0\mu_0)$. Clocks and rulers are closed strokes, so a stroke measured on any pattern’s own clocks and rulers returns that one number, and the uniform motions the stroke admits form the boost group with parameter $\mathfrak{c}$. A closed pattern’s strand runs at $\mathfrak{c}$ on a helix. The unrolled cylinder splits that rate into translation v and circulation u , $v^2 + u^2 = \mathfrak{c}^2$, and because a pattern keeps time by circulating, the right triangle is the interval $\mathfrak{c}^2 d\tau^2 = \mathfrak{c}^2 dt^2 - dx^2$. Away from closed patterns, open strokes keep no time and run at $\mathfrak{c}$, while their turning grades trace a helix of radius $\bar{\lambda} = \lambda/2\pi$ that stands at $\pi/4$ at every frequency: its turning rate ω/k equals its run $\mathfrak{c}$, and its momentum $E/\mathfrak{c}$ at that radius is one quantum $\hbar$. A stationary closed pattern has inertia $E/\mathfrak{c}^2$, so the central invariant $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$ is one rate squared, and $c = \mathfrak{c}$. In the bivector reading $\mathfrak{c}^2$ is an area: the area factor of the strand spinor $\mathbf{w} = v + u\Sigma$. Mass is the bivector grade of the energy spinor, and rest energy and kept time are quadratures of the rectangles spanned by a pattern’s two strokes. Every strand state lies on a velocity three-sphere of radius $\mathfrak{c}$ whose Hopf fibres are the strands. For a pattern held on the exterior branch this sphere is the support of the core: $|z_1|^2 = v^2/\mathfrak{c}^2 = \alpha_{\text{ext}}$ and $|z_2|^2 = u^2/\mathfrak{c}^2 = 1 - \alpha_{\text{ext}}$, its 45° torus is the Clifford torus at $r_f = 2R_s$, and the spinor flip that exchanges translation and circulation is the jacket flip. The floor and the existence of the matter phase make $\mathfrak{c}$ finite and nonzero. The number $299\,792\,458\text{ ms}^{-1}$ counts metres of strand run per second of kept time and has been fixed by definition since 1983; from 1967 to 1983 it measured a ratio of two atomic transition frequencies. Light carries the constant’s name because it was the first open stroke to be measured. A receipt script checks the displayed identities and numbers: 78/78 pass.

1 The constant and its name

The central invariant of FDT,

$$\frac{E}{m} = c^2 = \frac{1}{\varepsilon_0\mu_0} = \left(\frac{d}{t}\right)^2, \quad (1)$$

is one identity read by quantum mechanics, electromagnetism and general relativity (core 0.1). Its constant does two jobs. Read as d/t , it converts the time a pattern keeps into the length its thread runs: metres locked to seconds (0.1a). Read as E/m , it converts the matter reading of the one substance into its energy reading. Throughout the programme c has entered these readings as a given. This paper derives it.

The name the constant carries hides both jobs. “The speed of light in vacuum” records an order of discovery: light was the first massless reading of the stroke to be measured, when Rømer’s timing of the eclipses of Io in 1676 showed that it runs at a finite rate [2]. The letter came from a conversion factor. Weber and Kohlrausch’s constant c of 1856 stood in Weber’s electrodynamic force law as a relative speed and was measured from the ratio of the electrodynamic and electrostatic units of charge at $4.39 \times 10^8 \text{ ms}^{-1}$, in theory $\sqrt{2}$ times the stroke rate, from charges and currents with no light anywhere in the apparatus [3, 4]. Drude gave the letter to electrodynamic waves in 1894 [5]; Einstein took it up in 1907, in the review in which he set out the equivalence of mass and energy [6]. Since 1983 the metre has been defined through c , which makes c in the SI what it is in FDT, a conversion factor fixed by definition [7]. Light moves at c because light is an open stroke; c is not defined by light.

FDT reads the constant by what it does. There is one carrier, the stroke (I-2). Light, the gravitational stroke and every other open reading run at its rate wherever they cross no closed pattern: the gamma rays of GRB 170817A arrived $1.74 \pm 0.05 \text{ s}$ after the gravitational stroke GW170817 and hold the two rates equal to within -3×10^{-15} and $+7 \times 10^{-16}$ [11]. There is no vacuum for the rate to be measured in: ε_0 and μ_0 are response coefficients of the omnium, and c^2 is a property of the medium (0.1).

The derivation proceeds from the geometry of the stroke and of the closed patterns made from it: one rate from the two responses, one rate on the spot, and the group of uniform motions that has it as parameter (§3); the interval as the right triangle of a closed pattern’s strand (§4); open strokes at that rate and their helix at $\pi/4$ (§5); the chain (1) as one rate squared (§6); the square as an area, in the bivector reading (§7); the velocity three-sphere and, for a held pattern, the support of 0.3a (§8); finiteness from the floor and from the matter phase (§9); the floor and the ceiling read through c (§10); and what the number 299 792 458 records (§11). The receipt script of Appendix A checks the displayed identities and numbers; check labels appear in brackets, [R03].

2 Premises, charts and symbols

The derivation uses the locked core v6.0 [1] and nothing else. Each premise is quoted by section and read without its value of c .

- (P1) *One substance.* There is one substance, the omnium, and there are only its patterns; every physical length and time belongs to a pattern, to a pattern-to-probe measurement, or to the floor (0.1, 0.1b).
- (P2) *One carrier.* The omnium carries one elementary disturbance, the stroke: an open helical excitation whose two grades, the vector grade with response coefficient ε_0 and the bivector grade (the twist of P-5) with response coefficient μ_0 , drive each other and turn about the stroke’s run as it advances; the helix is the turning of the grades, and the run is straight (I-2, 0.1, P-5). The circulation laws and the energy and momentum balances of the two grades are the measured record of electromagnetism, read on the omnium.
- (P3) *Matter is closed stroke.* Every massive pattern is a closed braid of strokes, and the closure carries no rate other than the stroke’s, so closed patterns share every symmetry of the stroke equation (I-2, I-5). A closed pattern keeps time by its circulation, and its thread runs a definite length per kept period (0.1a).
- (P4) *Clocks and rulers are patterns.* Clocks and rulers are closed patterns of the omnium. Their kept time and thread length adjust together with the density they cross: no reading on a

pattern's own clocks and rulers registers a common rescaling (0.1a), and moving at constant speed, on them, through a density gradient is gravity (0.1c). A pattern is one pattern of the omnium, closure included (0.1b), and its stress–energy obeys $G_{\mu\nu} = \kappa T_{\mu\nu}$ and is conserved (I-3).

- (P5) *The floor.* Physical strokes are bounded below by the floor, $F_{\max} d_{\min} t_{\min} = \hbar$ with $d_{\min} = \mathfrak{c} t_{\min}$: an infimum, approached and occupied by none (0.1, 0.1a).
- (P6) *The support.* α is the share of the three-sphere, $|z_1|^2 = \alpha_\sigma$, $|z_2|^2 = 1 - \alpha_\sigma$, with the seam $\mathfrak{J} : \alpha \mapsto 1 - \alpha$ and the branch exchange $\mathfrak{i}$ (0.3, 0.3a). The exterior branch carries the flow reading (0.1c).
- (P7) *Planck's quantum.* Energy and frequency are bridged by $\hbar$: a stroke of frequency ω carries $\epsilon = \hbar\omega$, and a pattern's energy and kept period obey $E_q T_q = h$ (0.1a).

The strand rate. $\mathfrak{c}$ denotes the rate at which a stroke runs, measured on the spot. Its value, its independence of the observer, its finiteness and its appearance in the other readings of (1) are not assumed: §§3–9 derive them, and Theorem 1 identifies $\mathfrak{c}$ with the c of (1).

Charts. One density coordinate appears: $\alpha_{\text{ext}} = R_s/r_f$, application A-1 (compactness) on the exterior branch $\sigma = +1$, at field radius r_f of a source of total gravitating energy $\mathcal{M}$, with $R_s = \mathcal{M}\mathfrak{c}^2/(2F_{\max})$ (0.2, 0.3). $\alpha_{\text{int}} = r_f/R_s$ (A-1, $\sigma = -1$) enters through the branch exchange $\mathfrak{i}$ and in Proposition 23. The translation share $\mathfrak{s} = v^2/\mathfrak{c}^2$ of §4, the velocity sphere $S_{\mathfrak{c}}^3$ of §8 and the spinor flip $\mathfrak{f}$ are kinematic: $\mathfrak{s}$ is not an application of the bounded map and occupies no force, optical or density slot, and $\mathfrak{s}$, $S_{\mathfrak{c}}^3$ and $\mathfrak{f}$ coincide with α_{ext} , the support and $\mathfrak{J}$ in one reading only, a pattern held on the exterior branch (Proposition 22). The clock factor $\mathfrak{m}$ of 0.1c is not an α . The threshold $1/2$ is the A-1 seam, $r_f = 2R_s$ on the exterior branch and $r_f = R_s/2$ on the interior branch.

symbol	meaning
$\mathfrak{c}$	strand rate; identified with c in Theorem 1
v, u	translation rate and circulation rate of a strand, $v^2 + u^2 = \mathfrak{c}^2$
ϑ	pitch angle of a strand, measured from its axis: $v = \mathfrak{c} \cos \vartheta$, $u = \mathfrak{c} \sin \vartheta$
$\eta, \gamma; \eta'$	rapidity, $v/\mathfrak{c} = \tanh \eta$, and $\gamma = \cosh \eta = \mathfrak{c}/u$; rapidity of a boost acting on a pattern
t, τ	the bookkeeper's time; the time the pattern keeps
$\mathbf{e}_{\parallel}, \mathbf{e}_{\perp}, \Sigma$	axial and circulating unit vectors of the unrolled strand plane; $\Sigma = \mathbf{e}_{\parallel} \mathbf{e}_{\perp}$, $\Sigma^2 = -1$
$\mathbf{w}, \mathbf{P}$	strand spinor $\mathbf{e}_{\parallel} \mathbf{V}$; energy spinor $(E/\mathfrak{c})\mathbf{w}$
$P_{\pm}$	stroke components $E \pm p\mathfrak{c}$ of a pattern's energy–momentum
$\mathfrak{s}$	translation share $v^2/\mathfrak{c}^2$ (kinematic; not an α)
$\Lambda_C, \Lambda_R, a_{\perp}, \varpi$	strand run per tick; length of a ruler; circulated arc per tick; an outside rate
$\zeta_1, \zeta_2, \mathcal{T}_{\mathfrak{s}}$	coordinates of the velocity sphere $S_{\mathfrak{c}}^3$; its torus $ \zeta_1 ^2 = \mathfrak{s}$
z_1, z_2, T_{α}	coordinates of the support S^3 of 0.3a; its torus $ z_1 ^2 = \alpha$
$\mathfrak{f}, \mathfrak{J}, \mathfrak{i}$	spinor flip; seam (jacket flip); branch exchange of 0.3a
$\mathcal{E}_0; R_h, b_h$	amplitude of a plane stroke; radius and axial advance per radian of a helix

3 One rate: the stroke measured against itself

Proposition 1 (The stroke from the two responses). *In a region crossed by no closed pattern the two grades of a stroke obey*

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \times \mathbf{B} = \varepsilon_0 \mu_0 \partial_t \mathbf{E}, \quad \nabla \cdot \mathbf{E} = 0 = \nabla \cdot \mathbf{B}, \quad (2)$$

and every Cartesian component f of $\mathbf{E}$ and $\mathbf{B}$ obeys the stroke equation

$$\nabla^2 f - \mathfrak{c}^{-2} \partial_t^2 f = 0, \quad \mathfrak{c}^2 = \frac{1}{\varepsilon_0 \mu_0}. \quad (3)$$

The stroke equation is isotropic, because ε_0 and μ_0 are scalars.

Proof. The curl of the first law and the second law give $\nabla \times (\nabla \times \mathbf{E}) = -\varepsilon_0 \mu_0 \partial_t^2 \mathbf{E}$, and $\nabla \times (\nabla \times \mathbf{B}) = \nabla(\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = -\nabla^2 \mathbf{B}$; likewise for $\mathbf{B}$. [R08a, R08e] $\square$

Equation (2) is the measured law of the two grades, read on the omnium, whose response coefficients ε_0 and μ_0 are (0.1). A clock is a closed pattern, and its tick is a length of strand run (P3, P4). A ruler is a closed pattern, and its length is one of its own scales (P1). A measurement of the stroke's rate compares a stroke with both.

Proposition 2 (One rate on the spot). *Let a stroke pass a clock and a ruler at rest together at one place. Let Λ_C be the length of strand the clock runs per tick and Λ_R the ruler's length. The stroke's measured rate is*

$$\mathfrak{c} = \frac{\Lambda_C}{\Lambda_R} \quad \text{rulers per tick}, \quad (4)$$

whatever the rate ϖ at which strokes run there in any outside bookkeeping, and whatever common rescaling the region has undergone.

Proof. The tick lasts $T_C = \Lambda_C / \varpi$. The stroke crosses the ruler in $\Delta t = \Lambda_R / \varpi$, which is $\Delta t / T_C = \Lambda_R / \Lambda_C$ ticks; one ruler per Λ_R / Λ_C ticks is (4). The rate ϖ cancels because the clock is made of the thing it times, and a common rescaling $\Lambda \mapsto \mathfrak{m}\Lambda$ (0.1c) cancels in the ratio. [R02] $\square$

The cancellation is the content of $d/t = c$ on the spot (0.1a): a stroke measured by instruments made of strokes returns the ratio of two of their scales. In three dimensions the same cancellation holds along every direction of the local frame, because the stroke equation is isotropic (Proposition 1). Λ_C and Λ_R are scales of two closed patterns, fixed by their confinement eigenproblems (I-5). The number in (4) is a ratio of them, and §11 shows that the SI number was exactly such a ratio from 1967 to 1983.

Proposition 3 (One rate in uniform motion: the boost group). *The stroke operator $\partial_x^2 - \mathfrak{c}^{-2} \partial_t^2$ is invariant under*

$$x' = \gamma(x - vt), \quad t' = \gamma\left(t - \frac{vx}{\mathfrak{c}^2}\right), \quad \gamma = \left(1 - \frac{v^2}{\mathfrak{c}^2}\right)^{-1/2}, \quad (5)$$

and is not invariant under the Galilean map $x' = x - vt$, $t' = t$. A uniform motion maps the readings of a uniform region linearly, and a linear map that carries strokes to strokes is (5) composed with a rotation and a common rescaling, the rescaling being 1. A clock and a ruler in uniform motion measure every stroke at the rate $\mathfrak{c}$.

Proof. A mode $e^{i(k'x' - \omega't')}$ of the moving frame is the mode $e^{i(kx - \omega t)}$ with $k = \gamma(k' + \omega'v/\mathfrak{c}^2)$ and $\omega = \gamma(\omega' + k'v)$, and $-k^2 + \omega^2/\mathfrak{c}^2 = -k'^2 + \omega'^2/\mathfrak{c}^2$ identically. The modes are complete, so the operator is invariant. Under the Galilean map $\omega = \omega' + k'v$ and $k = k'$, and the form changes. [R01a, R01b] A linear map of (x, ct) that keeps the two stroke lines $x = \pm ct$ sends them to multiples p and q of themselves, and it is the boost of rapidity $\frac{1}{2} \ln(p/q)$ times the common rescaling $\lambda = \sqrt{pq}$. [R01c] The map for $-v$ inverts the map for v , so $\lambda(v)\lambda(-v) = 1$, and isotropy gives $\lambda(v) = \lambda(-v)$; hence $\lambda = 1$. [R01d] A common rescaling is the clock factor of 0.1c, which uniform motion through a

uniform region does not supply. Closed patterns share the symmetry of the stroke (P3), so a clock and a ruler in uniform motion are the image under (5) of a clock and a ruler at rest. Their measurement of a stroke is the image of the resting pair's measurement of the image stroke, which is again a stroke, measured at $\mathfrak{c}$ by Proposition 2. [R07] $\square$

Equation (5) mixes the space reading and the time reading with the coefficient $v/\mathfrak{c}^2$; the combination it keeps is $\mathfrak{c}^2 t^2 - x^2$. In the plane $(x, \mathfrak{c}t)$ it is a hyperbolic rotation by the rapidity η , $\tanh \eta = v/\mathfrak{c}$. This is the sense in which $\mathfrak{c}$ converts time into space: the group of uniform motions that the stroke admits does not act on space and time separately, and $\mathfrak{c}$ is the one number that makes the two readings commensurable. As $\mathfrak{c} \rightarrow \infty$, (5) becomes the Galilean map and the readings untie (Proposition 23), the separation that 0.1a excludes.

4 The strand of a closed pattern: the helix triangle

Take a closed pattern moving at speed v along the unit vector $\mathbf{e}_{\parallel}$ in the bookkeeper's frame, and follow a strand that circulates transverse to $\mathbf{e}_{\parallel}$ in the pattern's own frame. Every closed pattern carries circulation (P3), and by Proposition 3 every clock of a pattern changes rate as this one does; the transverse circulation shows the change geometrically.

Proposition 4 (The strand runs a helix at $\mathfrak{c}$). *In the bookkeeper's frame the strand's velocity is*

$$\mathbf{V} = v \mathbf{e}_{\parallel} + u \mathbf{e}_{\perp}, \quad |\mathbf{V}| = \mathfrak{c}, \quad u = \frac{\mathfrak{c}}{\gamma} = \sqrt{\mathfrak{c}^2 - v^2}, \quad (6)$$

with $\mathbf{e}_{\perp}$ the circulating unit vector transverse to $\mathbf{e}_{\parallel}$. The strand runs a helix whose pitch angle ϑ , measured from the axis, satisfies $v = \mathfrak{c} \cos \vartheta$ and $u = \mathfrak{c} \sin \vartheta$.

Proof. In the pattern's frame the strand moves at $\mathfrak{c}$ transverse to $\mathbf{e}_{\parallel}$, with $x' = 0$ and $(y', z') = \mathfrak{c}t'(\cos \varphi, \sin \varphi)$. The inverse of (5) gives $x = \gamma vt'$ and $t = \gamma t'$ with y, z unchanged. Hence $dx/dt = v$, $|d(y, z)/dt| = \mathfrak{c}/\gamma$, and $|\mathbf{V}|^2 = v^2 + \mathfrak{c}^2/\gamma^2 = \mathfrak{c}^2$. [R04a] $\square$

A helix is a geodesic of its cylinder. Unrolled, the cylinder is a plane and the helix a straight line, the hypotenuse of a right triangle whose legs are the translated length $v dt$ and the circulated length $u dt$ (Figure 1).

Proposition 5 (The interval is the right triangle of the unrolled cylinder). *A pattern keeps time by its circulation, one tick per circulated arc $a_{\perp}$. The arc is transverse, and (5) leaves transverse lengths unchanged, so it is the same arc in every frame, and the kept time accrues as $\mathfrak{c} d\tau = u dt$. Then*

$$\mathfrak{c}^2 d\tau^2 = \mathfrak{c}^2 dt^2 - dx^2, \quad dx = v dt, \quad (7)$$

and the kept time dilates: $d\tau = dt/\gamma$, a tick lasting $\gamma a_{\perp}/\mathfrak{c}$ for the bookkeeper. [R03, R04b]

Proof. Pythagoras on the unrolled cylinder gives $(\mathfrak{c} dt)^2 = (v dt)^2 + (u dt)^2$; substitute $u dt = \mathfrak{c} d\tau$. $\square$

The bookkeeper's interval (7) is the theorem of Pythagoras for the strand, with kept time drawn as circulated length. For a closed pattern, time is circulated strand and space is translated strand, and $\mathfrak{c}$ converts the one into the other. One second of kept time is 299 792 458 m of strand run in circulation.

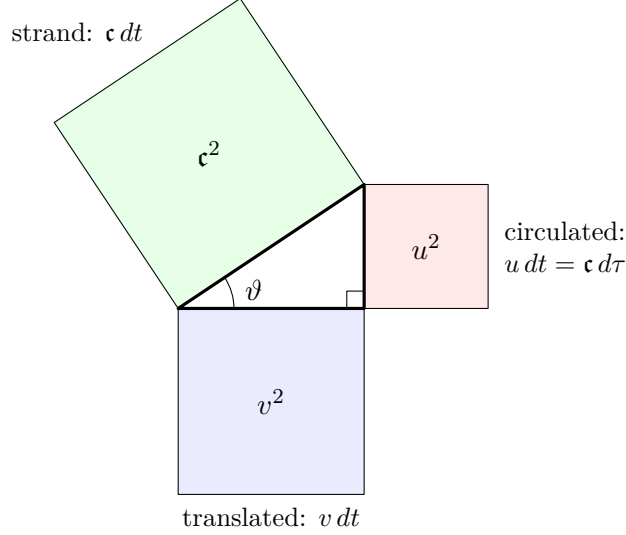


Figure 1: The unrolled strand. Per unit of the bookkeeper's time the strand runs c (the hypotenuse), translating v along the axis and circulating u around it. The circulated leg is the kept time, $u dt = c d\tau$, and the triangle is the interval (7). The squares carry $c^2 = v^2 + u^2$: the strand square partitioned into the translation square and the circulation square. Drawn at $v : u = 3 : 2$.

Proposition 6 (Pitch, rapidity and the Gudermannian). *With $v/c = \tanh \eta$,*

$$\cos \vartheta = \tanh \eta, \quad \sin \vartheta = \operatorname{sech} \eta, \quad \gamma = \cosh \eta, \quad \frac{\pi}{2} - \vartheta = \operatorname{gd} \eta = \arctan(\sinh \eta). \quad (8)$$

Rapidities of successive uniform motions along one line add, $\tanh(\eta_1 + \eta_2) = (\tanh \eta_1 + \tanh \eta_2)/(1 + \tanh \eta_1 \tanh \eta_2)$, and pitches compose through the Gudermannian. [R05]

The circular geometry of the strand and the hyperbolic geometry of the bookkeeper's plane are one geometry, and the Gudermannian carries the one onto the other with no imaginary numbers.

Proposition 7 (Rulers). *Let a ruler's length be fixed by its own stroke round trip in its own kept time, $L_0 = cT_{\text{rt}}/2$. Moving at v along its length, with bookkeeper length L , the round trip lasts $L/(c-v) + L/(c+v) = 2L\gamma^2/c$ for the bookkeeper and $2L\gamma/c$ on the ruler's clock. The same pattern keeps the same round trip, so $L = L_0/\gamma$. [R06]*

Dilation (Proposition 5) and contraction (Proposition 7) are (5) read on a clock and on a ruler. [R07] Every massive pattern is its own stroke clock and its own stroke ruler: matter keeps time and length because matter is closed stroke.

5 Open strokes: the run at c and the helix at $\pi/4$

Proposition 8 (Open strokes). *An open stroke's run does not circulate: it carries the whole rate as translation, $v = c$ and $u = 0$, while its grades turn about the run. It keeps no time, $d\tau = 0$ by (7), and its translation is c in every frame, since (5) carries $x = ct$ onto $x' = ct'$. In a region crossed by no closed pattern every open reading of the one carrier, light and the gravitational stroke among them, runs at c on the spot (I-2); across closed patterns the stroke is read through the medium's aperture (P-4). [R07]*

Proposition 9 (Energy and momentum of a stroke). *For a plane stroke along x , $\mathbf{E} = \mathcal{E}_0 \cos(kx - \omega t) \hat{\mathbf{y}}$ and $\mathbf{B} = (\mathcal{E}_0/\mathbf{c}) \cos(kx - \omega t) \hat{\mathbf{z}}$ with $\omega = \mathbf{c}k$: (i) its two grades carry equal energy, $\varepsilon_0 |\mathbf{E}|^2/2 = |\mathbf{B}|^2/(2\mu_0)$, so $|\mathbf{E}| = \mathbf{c}|\mathbf{B}|$; (ii) its energy balance $\partial_t \varrho_E + \nabla \cdot \mathbf{S} = 0$ has the flux $\mathbf{S} = \mathbf{E} \times \mathbf{B}/\mu_0 = \varrho_E \mathbf{c} \hat{\mathbf{x}}$; (iii) its momentum balance has the density $\mathbf{g} = \varepsilon_0 \mathbf{E} \times \mathbf{B} = \varepsilon_0 \mu_0 \mathbf{S} = (\varrho_E/\mathbf{c}) \hat{\mathbf{x}}$. A stroke carries momentum $1/\mathbf{c}$ per unit energy. [R08b–d]*

Proof. Substitution of the plane stroke, with $\varrho_E = \varepsilon_0 \mathcal{E}_0^2 \cos^2(kx - \omega t)$; the balances hold identically. $\square$

Proposition 10 (The stroke's helix stands at $\pi/4$). *Let a stroke of frequency ω and wavenumber k run along $\mathbf{e}_\parallel$ with its grades turning about the run. Then: (i) its momentum $E/\mathbf{c}$ (Proposition 9) carries its angular momentum, one quantum $\hbar$ (spin 1, I-2), at the radius*

$$\bar{\lambda} = \frac{\hbar \mathbf{c}}{E} = \frac{\mathbf{c}}{\omega} = \frac{\lambda}{2\pi}; \quad (9)$$

(ii) on the cylinder of radius $\bar{\lambda}$ the turning grades trace a helix that advances one wavelength λ per turn around the circumference $2\pi\bar{\lambda} = \lambda$, so its pitch angle is $\pi/4$ at every frequency, and higher frequencies stand on inner cylinders; (iii) in the bookkeeper's time the turning sweeps the cylinder at $\omega\bar{\lambda} = \omega/k = \mathbf{c}$ while the run advances at $\mathbf{c}$: the two legs of the helix are equal, each $\mathbf{c}$, because the stroke equation is non-dispersive, $\omega = \mathbf{c}k$ (Proposition 1); (iv) the phase $kx - \omega t = k(x - \mathbf{c}t)$ is constant along the run, so the stroke keeps no time, and its helix stands at $\pi/4$ in space at a fixed time, in time at a fixed place, and as the line $x = \mathbf{c}t$ in the plane $(x, \mathbf{c}t)$. The helix is the locus of the turning grades, and the transport is the run.

Proof. (i) $E = \hbar\omega$ (P7) and $p = E/\mathbf{c}$ (Proposition 9); $(E/\mathbf{c})\bar{\lambda} = \hbar$ gives $\bar{\lambda} = \hbar\mathbf{c}/E = \mathbf{c}/\omega$. [R31a] (ii) With $\omega = \mathbf{c}k$ the wavelength is $\lambda = 2\pi/k = 2\pi\bar{\lambda}$, so the circumference equals the axial advance per turn and the tangent of the pitch angle is 1. [R31b, R31c] (iii) $\omega\bar{\lambda} = \mathbf{c}$ by (9). [R31d] (iv) Along $x = x_0 + \mathbf{c}t$ the phase is kx_0 ; at a fixed place the grades turn once while $\mathbf{c}t$ advances $\mathbf{c}T = \lambda$. [R31e] $\square$

The angle $\pi/4$ belongs to the open stroke. A closed pattern's phase obeys $\omega^2 = \mathbf{c}^2 k^2 + (m\mathbf{c}^2/\hbar)^2$ (the Remark of §6); its phase rate ω/k and its run $d\omega/dk$ multiply to $\mathbf{c}^2$, so their ratio, $\omega^2/(\mathbf{c}^2 k^2)$, exceeds 1: its turning outruns its transport. At a fixed time its phase still advances one wavelength per turn of a cylinder of radius $1/k$, as every phase does; the open stroke's $\pi/4$ is the equality of its two rates, and only the open stroke has them equal at every frequency. [R31g]

Masslessness is openness. An open stroke's run does not circulate, so it keeps no time and has no frame in which it rests; its grades turn at the full rate on a helix at $\pi/4$, and that turning is its phase, not a clock it keeps. Closedness is mass. A closed stroke circulates, keeps time, and has a frame in which its translation vanishes and its whole rate is circulation. Between the two ends every closed pattern in motion runs the same rate on a helix. Light carries the name of the constant because it was the first open stroke to be measured; the gravitational stroke of GW170817 runs at the same rate to parts in 10^{15} [11], as one carrier requires.

6 The inertia of closed stroke, and the chain

Proposition 11 (The inertia of a closed pattern). *A closed pattern, stationary in its own frame with rest energy E_0 and moving at v , has $E = \gamma E_0$ and $p = \gamma v E_0/\mathbf{c}^2$. Its momentum is $p = Ev/\mathbf{c}^2$, its inertia per unit energy is $1/\mathbf{c}^2$, and with $m \equiv E_0/\mathbf{c}^2$*

$$E_0 = m\mathbf{c}^2, \quad E = \gamma m\mathbf{c}^2, \quad p = \gamma mv, \quad E^2 = (p\mathbf{c})^2 + (m\mathbf{c}^2)^2. \quad (10)$$

For the transverse strand of Proposition 4 the rest energy is the circulating share of the energy, $E_0 = E \sin \vartheta$.

Proof. The pattern is one pattern of the omnium, closure included, and its stress–energy is conserved (P4). Stationary in its own frame, it has $\partial_k T^{ik} = 0$, so $T^{ij} = \partial_k (x'^j T^{ik})$ and the integrated rest-frame stress of the bounded pattern vanishes, $\int T^{ij} d^3 x' = 0$ (von Laue). Integrating the boosted stress–energy over a slice of the bookkeeper’s time gives $E = \gamma(E_0 + \beta^2 S'_{xx})$ and $p\mathbf{c} = \gamma\beta(E_0 + S'_{xx})$, with $\beta = v/\mathbf{c}$ and $S'_{xx} = \int T'^{xx} d^3 x'$; [R09e1] with $S'_{xx} = 0$ these are $E = \gamma E_0$ and $p = \gamma v E_0 / \mathbf{c}^2$. The closure is what makes S'_{xx} vanish. The strands of a ring circulating in a plane containing v carry $S'_{xx} = E_s/2$, the tension $E_s/(2\pi R)$ that holds them carries $-E_s/2$, and the strands alone would give $p\mathbf{c}^2/(Ev) = 3/(2 + \beta^2)$. [R09e2, R09e3] For the strand of Proposition 4 the circulation is transverse and $S'_{xx} = 0$ already for the strands: integrating the strand momentum $(E/2\pi\mathbf{c})(v, u \cos \varphi, u \sin \varphi)/\mathbf{c}$ (Proposition 9) over the circulation gives $(Ev/\mathbf{c}^2, 0, 0)$ directly, and $E_0 = Eu/\mathbf{c} = E \sin \vartheta$. [R09a–c] $\square$

The matter–energy conversion is the square of the space–time conversion because two factors of the strand rate enter the inertia. On the transverse strand one factor $1/\mathbf{c}$ is the stroke’s momentum per unit energy (Proposition 9), and the other is the strand’s translation per unit speed, $v/\mathbf{c}$ per v (Proposition 4); in general both are the coefficient $v/\mathbf{c}^2$ with which (5) mixes energy into momentum. Mass is circulating energy, and inertia is what the circulation shows when the pattern translates.

Remark (The phase). Along the pattern’s path $x = vt$ the phase $(Et - px)/\hbar$ equals $m\mathbf{c}^2\tau/\hbar$. [R30] The phase a pattern carries is its circulation: the projection of the helix onto the plane normal to its axis, the $\psi = Ae^{i\varphi}$ of I-4.

Theorem 1 (The chain is one rate, squared). *For every pattern q , with E_q its rest energy and $\ell_q = \mathbf{c}T_q$ the strand run per kept period,*

$$\frac{E_q}{m_q} = \mathbf{c}^2 = \frac{1}{\varepsilon_0 \mu_0} = \left(\frac{\ell_q}{T_q}\right)^2. \quad (11)$$

With Planck’s quantum $E_q T_q = h$ (P7) the bridges of 0.1a follow: $E_q = m_q \mathbf{c}^2 = \hbar \omega_q$ with $\omega_q = 2\pi/T_q$, and $\ell_q = h/(m_q \mathbf{c})$. The constant of the central invariant is the strand rate,

$$c = \mathbf{c}. \quad (12)$$

Proof. The first equality is Proposition 11, the second Proposition 1, the third the definition of ℓ_q as strand run at the strand rate. [R10] $\square$

Read through (11), the product $\varepsilon_0 \mu_0$ is the inertia per unit energy of every pattern: the electromagnetic reading of the chain is the matter–energy reading, not a coincidence beside it. The measured responses carry it. $\mu_0 = 1.256\,637\,061\,27(20) \times 10^{-6} \text{ N A}^{-2}$ gives $\varepsilon_0 = 1/(\mu_0 c^2) = 8.854\,187\,8188 \times 10^{-12} \text{ F m}^{-1}$ [9], [R10c] and the energy of the gamma rays emitted in neutron capture equals the mass change times c^2 to $(-1.4 \pm 4.4) \times 10^{-7}$ [10]. Density equivalence (I-7) enters here: every energy enters $\mathcal{M}$ exactly once as E/c^2 , and Proposition 11 is the reason the entry is E/c^2 . Sections 7–9 keep the symbol $\mathbf{c}$ for the strand rate.

7 $\mathbf{c}^2$ is an area: the bivector reading

The chain’s constant enters squared, $c^2 = (d/t)^2$, and a square is an area. The geometric algebra of the strand’s own plane shows which area. The plane is the unrolled tangent plane of the cylinder

of Proposition 4: flat, with orthonormal vectors $\mathbf{e}_{\parallel}$ (axis) and $\mathbf{e}_{\perp}$ (circulation) and unit bivector

$$\Sigma = \mathbf{e}_{\parallel}\mathbf{e}_{\perp} = \mathbf{e}_{\parallel} \wedge \mathbf{e}_{\perp}, \quad \Sigma^2 = -1. \quad (13)$$

The product of two vectors is $\mathbf{a}\mathbf{b} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b}$. The square of a vector is the area of the square built on it, and the product of two orthogonal vectors is the oriented area of their rectangle. The reverse $\tilde{\mathbf{x}}$ reverses the order of vector factors; for $\mathbf{x} = a + b\Sigma$, $\tilde{\mathbf{x}} = a - b\Sigma$.

Proposition 12 (The strand square). $\mathbf{V}^2 = \mathbf{c}^2$, and

$$\mathbf{V}^2 = (v \mathbf{e}_{\parallel})^2 + (u \mathbf{e}_{\perp})^2 = v^2 + u^2, \quad (14)$$

because orthogonal vectors anticommute and the cross terms $\pm vu\Sigma$ cancel. The square on the strand rate, of area $\mathbf{c}^2$, is the sum of the translation square v^2 and the circulation square u^2 , and the translation share $\mathfrak{s} = v^2/\mathbf{c}^2 = \cos^2 \vartheta$ is a ratio of two areas (Figure 1). [R11a, R11b]

Proposition 13 (The strand spinor).

$$\mathbf{w} \equiv \mathbf{e}_{\parallel} \mathbf{V} = v + u\Sigma = \mathbf{c} e^{\Sigma \vartheta}, \quad \mathbf{w}\tilde{\mathbf{w}} = \mathbf{c}^2. \quad (15)$$

The map $\mathbf{a} \mapsto \mathbf{a}\mathbf{w}$ turns the plane through ϑ and scales every area by $\mathbf{c}^2$. The strand rate squared is the area factor of the strand's own spinor; translation is its scalar grade and circulation its bivector grade. [R11c, R11d]

Proposition 14 (Mass is the bivector grade). The energy spinor

$$\mathbf{P} \equiv \frac{E}{\mathbf{c}} \mathbf{w} = p\mathbf{c} + m\mathbf{c}^2\Sigma, \quad \mathbf{P}\tilde{\mathbf{P}} = E^2, \quad (16)$$

carries momentum in its scalar grade and rest energy in its bivector grade, with $p\mathbf{c}/E = \cos \vartheta$ and $m\mathbf{c}^2/E = \sin \vartheta$. [R12, R29] A pattern at rest has $\mathbf{P} = m\mathbf{c}^2\Sigma$, a pure bivector: rest energy is circulation, an oriented area. An open stroke has $\mathbf{P} = p\mathbf{c}$, a pure scalar. $E_0 = m\mathbf{c}^2$ states that the energy of a pattern at rest is its inertia times the area of the strand square.

Proposition 15 (The rectangle of the two readings).

$$(v \mathbf{e}_{\parallel})(u \mathbf{e}_{\perp}) = vu \Sigma, \quad vu = \frac{\mathbf{c}^2}{2} \sin 2\vartheta \leq \frac{\mathbf{c}^2}{2}, \quad (17)$$

with equality only at $\vartheta = \pi/4$, where $v = u = \mathbf{c}/\sqrt{2}$ and the three areas coincide: $v^2 = u^2 = vu = \mathbf{c}^2/2$. In energy, $(p\mathbf{c})(m\mathbf{c}^2) = (E^2/2) \sin 2\vartheta \leq E^2/2$, with equality at $p = m\mathbf{c}$, $\gamma = \sqrt{2}$. [R13]

Proposition 16 (The spinor flip keeps the strand square).

$$\mathfrak{f}(\mathbf{w}) \equiv \Sigma \tilde{\mathbf{w}} = u + v\Sigma, \quad \mathfrak{f} \circ \mathfrak{f} = \text{id}, \quad \mathfrak{f}(\mathbf{w}) \widetilde{\mathfrak{f}(\mathbf{w})} = \mathbf{w}\tilde{\mathbf{w}} = \mathbf{c}^2, \quad (18)$$

and the fixed set of $\mathfrak{f}$ is the 45° spinor, $v = u$. $\mathfrak{f}$ exchanges translation and circulation, the space reading and the time reading of the strand, and keeps $\mathbf{c}^2$. [R14]

Remark (Closure is the flip of the run). The spinor flip carries the open run, $\mathbf{w} = \mathbf{c}$, onto the closed pattern at rest, $\mathfrak{f}(\mathbf{c}) = \mathbf{c}\Sigma$: closing a stroke turns its run into circulation (P3). The stroke's helix of Proposition 10, with run $\mathbf{c}$ and turning $\mathbf{c}$, is $\mathbf{c} + \mathbf{c}\Sigma$, and $\mathfrak{f}(\mathbf{c} + \mathbf{c}\Sigma) = \mathbf{c} + \mathbf{c}\Sigma$: exchanging its run and its turning leaves it as it is. [R31f]

Proposition 17 (Quadratures). *Resolve a pattern's energy-momentum along the two strokes of its cone (P-4), $P_{\pm} = E \pm p\mathbf{c}$. Then*

$$(m\mathbf{c}^2)^2 = P_+P_-, \quad (\mathbf{c}\tau)^2 = (\mathbf{c}t - x)(\mathbf{c}t + x). \quad (19)$$

A uniform motion of rapidity η' squeezes the rectangles, $P_{\pm} \mapsto e^{\mp\eta'} P_{\pm}$, and keeps their areas. The rest energy and the kept time are quadratures: each is the side of the square equal to the rectangle spanned by two strokes, and that area is what uniform motion cannot change. [R15]

Proposition 18 (The stroke is the null multivector). *In the algebra of space, with unit pseudoscalar I_3 , $I_3^2 = -1$, let $F = \mathbf{E} + \mathbf{c}I_3\mathbf{B}$, whose bivector part $\mathbf{c}I_3\mathbf{B}$ is the twist. Then*

$$F^2 = (|\mathbf{E}|^2 - \mathbf{c}^2|\mathbf{B}|^2) + 2\mathbf{c}I_3(\mathbf{E} \cdot \mathbf{B}), \quad (20)$$

and a stroke has $F^2 = 0$: its vector grade and its bivector grade balance under $\mathbf{c}$, and $\mathbf{c}^2 = |\mathbf{E}|^2/|\mathbf{B}|^2$ is the ratio of the squares on its two grades. [R16]

Remark (The bookkeeper's split). Written as the paravector $\mathbf{c}t + \mathbf{x}$, an event satisfies $(\mathbf{c}t + \mathbf{x})(\mathbf{c}t - \mathbf{x}) = \mathbf{c}^2t^2 - |\mathbf{x}|^2$. [R17] In the algebra of spacetime the bookkeeper's displacement $\mathbf{x}$ is the bivector spanned by the event and the bookkeeper's own time axis: the space reading is an area spanned with the time reading, and $\mathbf{c}$ is what lets the time grade and the space grade add. Referred to the circulation direction instead of the axis, the strand spinor is $\mathbf{e}_{\perp}\mathbf{V} = u - v\mathbf{\Sigma} = \widetilde{\mathbf{f}(\mathbf{w})}$, with time in the scalar grade and space in the bivector grade: the bookkeeper's split. The two references are exchanged by the spinor flip.

8 The three-sphere measured in $\mathbf{c}$

Definition 1 (The velocity sphere). A strand's state is its pair of rates (v, u) , $v^2 + u^2 = \mathbf{c}^2$, together with two phases: φ_2 , its position around the axis, and φ_1 , its axial position taken modulo the axial advance per turn and scaled to 2π . Write $\zeta_1 = (v/\mathbf{c})e^{i\varphi_1}$ and $\zeta_2 = (u/\mathbf{c})e^{i\varphi_2}$. Then $|\zeta_1|^2 + |\zeta_2|^2 = 1$: the strand states form the unit three-sphere $S_{\mathbf{c}}^3$, the velocity sphere, whose radius read in velocity is $\mathbf{c}$, one radius at every translation share because the stroke has one rate. Its tori are $\mathcal{T}_{\mathbf{s}} = \{|\zeta_1|^2 = \mathbf{s}\}$.

Proposition 19 (The strands are the Hopf fibres of $S_{\mathbf{c}}^3$). *A strand of pitch angle ϑ advances both phases by 2π per turn. With Λ the strand run per turn, its cylinder taken modulo the axial advance per turn is the flat torus with circumferences $\Lambda \cos \vartheta$ and $\Lambda \sin \vartheta$, which is $\mathcal{T}_{\mathbf{s}}$ scaled by $\Lambda/2\pi$ with $\mathbf{s} = \cos^2 \vartheta$, and the strand is the $(1, 1)$ curve*

$$\zeta(\varphi) = e^{i\varphi}(\sqrt{\mathbf{s}}, \sqrt{1 - \mathbf{s}}), \quad (21)$$

a Hopf fibre: a great circle of length 2π at every share. The Hopf flow moves every point of $S_{\mathbf{c}}^3$ at one speed, the strand rate. [R19a–c]

Read in the strand's own Frenet data, the share is a ratio of squared curvatures. A helix of radius R_h and axial advance b_h per radian has curvature $\kappa_h = R_h/(R_h^2 + b_h^2)$ and torsion $\tau_h = b_h/(R_h^2 + b_h^2)$, and with $b_h/R_h = \cot \vartheta$ the translation share is the squared torsion over the squared total curvature: [R19d, R19e]

$$\mathbf{s} = \frac{\tau_h^2}{\kappa_h^2 + \tau_h^2} = \cos^2 \vartheta. \quad (22)$$

Proposition 20 (The 45° strand spans the torus of greatest area). *The area of $\mathcal{T}_s$ in the unit S_c^3 is*

$$A(\mathfrak{s}) = 4\pi^2 \sqrt{\mathfrak{s}(1-\mathfrak{s})} = 4\pi^2 \frac{vu}{c^2}, \quad (23)$$

largest at $\mathfrak{s} = 1/2$, with $A = 2\pi^2$, where the strand stands at 45°, $v = u = c/\sqrt{2}$, and $v^2 = u^2 = vu = c^2/2$ (Proposition 15). [R20]

Proposition 21 (The spinor flip on the velocity sphere). *The map*

$$\mathfrak{f}: (\sqrt{\mathfrak{s}} e^{i\varphi_1}, \sqrt{1-\mathfrak{s}} e^{i\varphi_2}) \mapsto (\sqrt{1-\mathfrak{s}} e^{i\varphi_1}, \sqrt{\mathfrak{s}} e^{i\varphi_2}) \quad (24)$$

exchanges the translation and circulation rates at fixed phases. It is an involution, carries $\mathcal{T}_s$ onto $\mathcal{T}_{1-s}$ and ϑ onto $\pi/2 - \vartheta$, keeps the radius c and the strand square c^2 , carries Hopf fibres to Hopf fibres, and fixes $\mathcal{T}_{1/2}$ pointwise; on the rates it is the spinor flip of Proposition 16. The coordinate swap $(\zeta_1, \zeta_2) \mapsto (\zeta_2, \zeta_1)$ is $\mathfrak{f}$ composed with the exchange of the two phases: it carries $\mathcal{T}_{1/2}$ onto itself and fixes there only the great circle $\zeta_1 = \zeta_2$. [R21a]

Proposition 22 (The support is the velocity sphere of a held pattern). *On the exterior branch, in the flow reading*

$$ds^2 = -c^2 dt_{\text{own}}^2 + (dr_f + v dt_{\text{own}})^2 + r_f^2 d\Omega^2, \quad v = c\sqrt{\alpha_{\text{ext}}}, \quad (25)$$

a pattern held at r_f moves against the flow at the speed v and keeps time at $d\tau/dt_{\text{own}} = \sqrt{1-\alpha_{\text{ext}}}$. Its strand has translation v and circulation $u = c\sqrt{1-\alpha_{\text{ext}}}$, so

$$(|\zeta_1|, |\zeta_2|) = \left(\frac{v}{c}, \frac{u}{c}\right) = (|z_1|, |z_2|), \quad |z_1|^2 + |z_2|^2 = \frac{v^2 + u^2}{c^2} = 1. \quad (26)$$

For a pattern held on the exterior branch the velocity sphere is the support of 0.3a: $\mathfrak{s} = \alpha_{\text{ext}}$ and $\mathcal{T}_s = \mathcal{T}_{\alpha_{\text{ext}}}$; the Hopf fibres of S_c^3 are those of the support; the 45° torus is the Clifford torus, at $r_f = 2R_s$, the A-1 seam; and the spinor flip $\mathfrak{f}$ is the seam $\mathfrak{J}: \alpha \mapsto 1-\alpha$, which fixes the Clifford torus pointwise. The open interval of 0.4 and I-6 reads on the strand as $u > 0$ and $v > 0$: $\alpha_{\text{ext}} \rightarrow 1^-$ is $u \rightarrow 0^+$, and $\alpha_{\text{ext}} \rightarrow 0^+$, approached only as $r_f \rightarrow \infty$, is $v \rightarrow 0^+$. The branch exchange $\mathfrak{i}: r_f \mapsto R_s^2/r_f$ keeps α , fixes the fold and swaps the seams $2R_s \leftrightarrow R_s/2$; it is a different involution. [R18, R21b, R21c, R22]

Proof. With $dr_f = d\Omega = 0$, (25) gives $c^2 d\tau^2 = (c^2 - v^2) dt_{\text{own}}^2$. The held pattern's four-velocity $U_s = (1 - \alpha_{\text{ext}})^{-1/2} \partial_{t_{\text{own}}}$ and the flow's $U_f = \partial_{t_{\text{own}}} - v \partial_{r_f}$ have $g(U_s, U_f) = -c^2(1 - v^2/c^2)^{-1/2}$, the inner product of two patterns in relative motion at v . The substitution $dt_{\text{own}} = dt_S + v dr_f / (c^2 - v^2)$ carries (25) onto $-(c^2 - v^2) dt_S^2 + dr_f^2 / (1 - \alpha_{\text{ext}}) + r_f^2 d\Omega^2$, the static form of the same exterior. The rest follows from Propositions 19–21 with $\mathfrak{s} = \alpha_{\text{ext}}$. $\square$

The hold that keeps the pattern at r_f is the I-1 pair, $F = F_{\text{max}} \alpha_1 \alpha_2 = G\mathcal{M}m/r_f^2$, with $\alpha_1 = R_s/r_f$ of the source and $\alpha_2 = R_{s,m}/r_f$ of the pattern, on I-1's domain: the bookkeeper's reading of the flow's fall at $G\mathcal{M}/r_f^2$ (0.1c). [R24b] For a laboratory pattern the second factor is small, $\alpha_2 = 1.3 \times 10^{-37}$ for 50 GeV at 1 fm (0.1c), and the small number is a correct reading. [R25] On the exterior branch α_{ext} is therefore a ratio of curvatures twice over: of the two circles at R_s and r_f (0.3), and, by (22), of the squared torsion and the squared total curvature of the strand a held pattern runs at r_f . The 45° at which a held pattern's strand stands at the seam is the angle at which every stroke's helix stands (Proposition 10). The flow reading stops at the fold (0.1c, I-3). The kinematic reading of the share on the interior chart is to be derived (D-02).

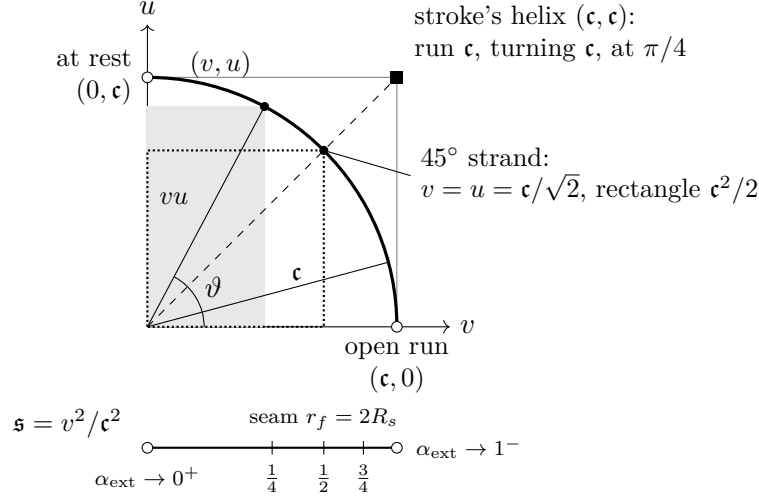


Figure 2: The quarter circle of strand rates, the meridian of the velocity sphere. Every strand state is a point (v, u) at distance c from the origin; the shaded rectangle is the bivector $vu\Sigma$ of Proposition 15, largest for the 45° strand (dotted). The open run $(c, 0)$ keeps no time; a pattern at rest, $(0, c)$, is all circulation. The corner (c, c) on the 45° ray (dashed) is the stroke's helix of Proposition 10, run c along the axis and turning c around it: the spinor flip leaves it fixed and exchanges the open run with the pattern at rest. The scale below gives the translation share $s = v^2/c^2$, aligned with v . For a pattern held on the exterior branch (A-1, $\sigma = +1$) the share is $\alpha_{\text{ext}}(r_f)$: the far field toward the top of the arc, the seam $r_f = 2R_s$ at 45° , and the fold toward the right end. The ends of the scale are limits (0.4, I-6).

9 Finite and nonzero

Proposition 23. $0 < c < \infty$.

Proof. (a) *The floor.* Strokes approaching the floor run at c , so $c = d_{\min}/t_{\min}$, a ratio of two nonzero, finite extents (P5). With $F_{\max}$ held, $d_{\min}^2 = \hbar c / F_{\max}$ and $t_{\min}^2 = \hbar / (F_{\max} c)$: $c \rightarrow 0$ sends $d_{\min} \rightarrow 0$, and $c \rightarrow \infty$ sends $t_{\min} \rightarrow 0$, a vanishing variable of the chain either way (0.1). [R28b]

(b) *The matter phase.* $R_s/r_f = \mathcal{M}c^2/(2F_{\max}r_f)$ (0.2, 0.3). With $F_{\max}$ held, $c \rightarrow 0$ drives $\alpha_{\text{ext}} \rightarrow 0^+$ at every field point, and $c \rightarrow \infty$ carries every field point inside its own fold, where $\alpha_{\text{int}} = r_f/R_s \rightarrow 0^+$; with G held the two limits exchange. In every case every share tends to 0^+ , and the $\alpha > 1/2$ solid torus, the matter phase, empties. Matter exists (0.3a, 0.8). [R28a]

(c) *The readings.* As $c \rightarrow \infty$, (5) becomes the Galilean map: $\gamma \rightarrow 1$ and $t' \rightarrow t$, every pattern's kept time equals the bookkeeper's, and space and time untie [R28c], the separation of the chain's terms that 0.1a excludes. $\square$

10 The floor and the ceiling read through c

With $c = c$ the floor and the ceiling of 0.1a and 0.2 display c as a ratio of the medium's own extremes: [R23, R24a]

$$\frac{d_{\min}}{t_{\min}} = c, \quad d_{\min}^2 = \frac{\hbar c}{F_{\max}}, \quad c^2 = \frac{d_{\min}^2}{t_{\min}^2}, \quad (27)$$

$$\mathcal{M}c^2 = F_{\max} \cdot 2R_s, \quad c^4 = 4GF_{\max}, \quad \kappa F_{\max} = 2\pi. \quad (28)$$

The floor has the aspect ratio c , and c^2 is the floor square per squared floor time. The rest energy of every pattern is the ceiling force acting across its Schwarzschild diameter. Numerically $F_{\max} = 3.0256 \times 10^{43}$ N, $d_{\min} = 2\ell_P = 3.2325 \times 10^{-35}$ m and $t_{\min} = 2t_P = 1.0782 \times 10^{-43}$ s, with $E_{\text{floor}} = F_{\max}d_{\min} = E_P/2$ and $E_{\text{floor}}t_{\min} = \hbar$. In floor units $c = 1$.

11 What the number records

Proposition 2 makes the measured rate a ratio of two patterns' scales, and the history of the SI shows exactly that. From 1960 the metre was 1 650 763.73 wavelengths of the krypton-86 line $2p_{10} - 5d_5$, and from 1967 the second was 9 192 631 770 periods of the caesium-133 ground-state hyperfine transition [7]. From 1967 to 1983, then,

$$c = \frac{9\,192\,631\,770}{1\,650\,763.73} \frac{\nu_{\text{Kr}}}{\nu_{\text{Cs}}} \text{ m s}^{-1}, \quad (29)$$

a ratio of two transition frequencies, which are differences of confinement eigenvalues of two closed patterns, multiplied by two defined numbers. The laser measurement of Evenson et al. gave $299\,792\,456.2(1.1) \text{ m s}^{-1}$, its uncertainty set by the krypton metre [8]; through (29) this is $\nu_{\text{Kr}}/\nu_{\text{Cs}} = 53\,835.1286(2)$, [R26a] and the value fixed in 1983 lies 1.64σ from it. [R26b] In 1983 the metre became “the length of the path travelled by light in vacuum during a time interval of $1/299\,792\,458$ of a second” [7], read in FDT as the length a stroke runs in that time, and the number became a definition: metres locked to seconds through the strand, the $d/t = c$ exactly of 0.1a. In 2019 the Planck constant was fixed as well, which locks the kilogram to the second through $c^2/h = 1.356\,392\,489\,652 \times 10^{50}$ Hz per kilogram: [R27] the bridge $E = mc^2 = h\nu$ of 0.1a, built into the units. The SI is now the central invariant written as a system of measurement.

The number therefore carries no geometric content of its own. It counts metres of strand run per second of kept time. Its dimensionless content from 1967 to 1983 was the eigenvalue ratio $\nu_{\text{Kr}}/\nu_{\text{Cs}}$, which is to be derived from the confinement operator (D-01; core O-9). The geometric content of c is what §§3–9 derive: that it exists and is one; that it is finite and nonzero; that it is the rate of every open stroke away from closed patterns and the circulation of every closed one; that its square is the area of the strand square; and that it is the radius of the velocity sphere, which for a pattern held on the exterior branch is the support.

12 Declined reading and convergence

The standard account reads c as the speed of light in vacuum, a property of one kind of radiation crossing an empty background. FDT reads c as the strand rate of the omnium, the conversion between its space and time readings and, squared, between its matter and energy readings, which every open stroke runs straight where it crosses no closed pattern and every closed pattern runs in circulation (I-2, I-5, 0.1); light is the first open stroke that was measured at that rate, and the vacuum of the standard account is the omnium in its low-density reading.

Convergence. For the same sources and data FDT returns: the boost group with invariant speed c , and with it every measured time dilation and length contraction (Propositions 3–7); local null cones with $d/t = c$ on the spot in every gravitational field (Propositions 2 and 22; I-3 containment); $c^2 = 1/(\varepsilon_0\mu_0)$ with the measured responses (Proposition 1; CODATA 2022); $E = mc^2$ and $E^2 = (pc)^2 + (mc^2)^2$ (Proposition 11; Rainville et al.); equal rates for the gravitational and the electromagnetic stroke (Proposition 8; GW170817 with GRB 170817A); and the SI value (§11). No divergence is claimed: the derivation supplies what c is, not a changed observable.

13 What c is

Every pattern runs at c . An open stroke runs it straight and keeps no time, while its grades turn at the same rate on a helix standing at $\pi/4$. A closed pattern at rest runs it around and keeps time by doing so. A closed pattern in motion runs it on a helix, translating v and circulating u with $v^2 + u^2 = c^2$. Space is translated strand, time is circulated strand, and c converts the one into the other. Mass is the share of energy that circulates and momentum the share that translates, and c^2 is the square they share: the area factor of the strand spinor, and the squared radius of the velocity sphere, which for a pattern held on the exterior branch is the support on which α lives. The name records which open stroke was measured first.

14 Status register

result	rests on	status
One rate from the two responses (Prop. 1)	0.1, P-5	derived
One rate on the spot (Prop. 2)	0.1a, 0.1b, I-5	derived
Boost group with parameter $\mathfrak{c}$; rescaling 1 (Prop. 3)	I-2, I-5, 0.1c	derived
Helix triangle, interval, dilation, contraction (Props. 4–7)	I-5, Prop. 3	derived
Open strokes at $\mathfrak{c}$ away from closed patterns (Prop. 8)	I-2	derived
The stroke's helix at $\pi/4$, radius $\bar{\lambda}$ (Prop. 10)	I-2, P7, Prop. 1	derived
Stroke momentum $1/\mathfrak{c}$ per unit energy (Prop. 9)	0.1, P-5	derived
Inertia of a stationary closed pattern, $E/m = \mathfrak{c}^2$ (Prop. 11)	0.1b, I-3, I-5	derived
The chain as one rate squared; $c = \mathfrak{c}$ (Thm. 1)	Props. 1, 11; 0.1a	derived
Bivector reading (Props. 12–18)	Props. 4, 11; P-4, P-5	derived
Velocity sphere; Hopf fibres; 45° torus; spinor flip (Props. 19–21)	Prop. 4	derived
Support as the velocity sphere of a held pattern (Prop. 22)	0.3a, 0.1c; A-1 exterior branch	derived
$0 < \mathfrak{c} < \infty$ (Prop. 23)	0.1, 0.1a, 0.3a, 0.8	derived
Floor and ceiling read through c (§10)	0.1a, 0.2	derived
$299\,792\,458\text{ m s}^{-1}$	SI, 1983	definition
$\nu_{\text{Kr}}/\nu_{\text{Cs}}$ and every strand scale Λ_q	I-5	to be derived (D-01; core O-9)
Kinematic reading of the share on the interior chart	0.3a, I-3	to be derived (D-02)
The stroke read across closed patterns, the aperture c_{eff}	P-4	to be derived (D-03; core O-8)

invariant	where it enters
I-1	the hold on a pattern at r_f , $F = F_{\max}\alpha_1\alpha_2 = GMm/r_f^2$ (§8); the ceiling $F_{\max}$ fixes the floor (P5) and $\mathcal{M}c^2 = F_{\max} \cdot 2R_s$
I-2	one carrier: one rate for every open stroke away from closed patterns (Prop. 8); spin 1 stands the stroke's helix at radius $\bar{\lambda}$ and $\pi/4$ (Prop. 10); the closure carries no other rate (P3)
I-3	conserved stress–energy for the inertia of a closed pattern (Prop. 11); the exterior metric and its flow reading (Prop. 22)
I-4	the phase a pattern carries is its circulation, the helix projection $\psi = Ae^{i\varphi}$ (§6); a held pattern reads the lapse (Prop. 22)
I-5	matter is closed stroke (Props. 4, 11); mass is rest energy over c^2 , a confinement eigenvalue; no support radius is assigned, and the strand scales stay with the operator D (O-9)
I-6	the open interval of 0.4, read on the strand of a held pattern: $\alpha_{\text{ext}} \rightarrow 1^-$ is $u \rightarrow 0^+$, $\alpha_{\text{ext}} \rightarrow 0^+$ is $v \rightarrow 0^+$ (Prop. 22)
I-7	every energy enters $\mathcal{M}$ once as E/c^2 ; Prop. 11 is why the entry is E/c^2

A Receipt

The script `receipt_c_derivation.py`, in the folder of this paper, checks the displayed identities of §§3–11 symbolically with SymPy and their numbers numerically; its output is `receipt_output.txt`. Run `python3 receipt_c_derivation.py`. Result: **78/78 PASS**.

check	content
R01a-d	stroke dispersion form invariant under (5), not under the Galilean map; linear maps keeping both stroke lines are a boost times a common rescaling; reciprocity and isotropy fix the rescaling at 1
R02	measured rate Λ_C/Λ_R , free of the outside rate and of a common rescaling
R03	unrolled cylinder: $(\mathbf{c} dt)^2 = (v dt)^2 + (u dt)^2$ gives (7)
R04a-b	transverse strand seen moving: velocity $(v, \mathbf{c}/\gamma)$, speed $\mathbf{c}$; tick dilated by γ
R05a-c	$\cos \vartheta = \tanh \eta$, $\sin \vartheta = \operatorname{sech} \eta$, $\gamma = \cosh \eta$; $\pi/2 - \vartheta = \operatorname{gd} \eta$; rapidities add
R06	ruler round trip gives $L = L_0/\gamma$
R07	(5) on a clock and on a stroke: $x' = 0 \Rightarrow t' = t/\gamma$; $x = \mathbf{c}t \mapsto x' = \mathbf{c}t'$
R08a-e	the plane stroke: $(\omega/k)^2 = 1/(\varepsilon_0\mu_0)$; equal energy in the two grades; $\mathbf{g} = \varepsilon_0\mu_0\mathbf{S} = \varrho_E/\mathbf{c}$; energy and momentum balances; $\nabla \times \nabla \times = \nabla \nabla \cdot - \nabla^2$
R09a-c	transverse strand: $p = Ev/\mathbf{c}^2$, $E^2 - (p\mathbf{c})^2 = (Eu/\mathbf{c})^2$, $p = \gamma mv$, $E = \gamma m\mathbf{c}^2$
R09d	free strokes at zero net momentum: the summed four-momentum transforms as one stroke's
R09e1-3	stationary pattern: $E = \gamma(E_0 + \beta^2 S'_{xx})$, $p\mathbf{c} = \gamma\beta(E_0 + S'_{xx})$; ring with and without its closure, $3/(2 + \beta^2)$ against 1; isotropic strokes, $4/3$ at low β
R10a-c	the chain (11); the bridges of 0.1a; CODATA 2022 $\mu_0\varepsilon_0c^2 = 1$
R11a-d	Cl(2): $\Sigma^2 = -1$; $\mathbf{V}^2 = v^2 + u^2$; $\mathbf{w}\tilde{\mathbf{w}} = \det \mathbf{w} = \mathbf{c}^2$; $\mathbf{w} = \mathbf{c}e^{\Sigma\vartheta}$
R12	$\mathbf{P} = p\mathbf{c} + m\mathbf{c}^2\Sigma$, $\mathbf{P}\tilde{\mathbf{P}} = E^2$
R13a-c	$(v\mathbf{e}_{\parallel})(u\mathbf{e}_{\perp}) = vu\Sigma$; $vu = (\mathbf{c}^2/2)\sin 2\vartheta$, largest at $\pi/4$; $(p\mathbf{c})(m\mathbf{c}^2) \leq E^2/2$
R14	$\mathbf{f}(\mathbf{w}) = \Sigma\tilde{\mathbf{w}} = u + v\Sigma$, involution, keeps $\mathbf{c}^2$, fixed set $v = u$
R15a-c	$(m\mathbf{c}^2)^2 = P_+P_-$; boosts squeeze $P_{\pm}$ and keep the area; $(\mathbf{c}\tau)^2 = (\mathbf{c}t - x)(\mathbf{c}t + x)$
R16a-b	Pauli algebra: $I_3 = i$; F^2 as in (20); $F^2 = 0$ for a stroke
R17	paravector: $(\mathbf{c}t + \mathbf{x})(\mathbf{c}t - \mathbf{x}) = \mathbf{c}^2t^2 - \mathbf{x} ^2$
R18a-d	flow form: held pattern's lapse; relative speed v to the flow; carried onto the static form; $ z_1 ^2 = \alpha_{\text{ext}}$, $ z_2 ^2 = 1 - \alpha_{\text{ext}}$
R19a-e	Hopf fibre of unit speed and length 2π ; great circle; pitch torus $= \mathcal{T}_s$; helix curvature and torsion; $\mathbf{s} = \tau_h^2/(\kappa_h^2 + \tau_h^2)$
R20	$A(\mathbf{s}) = 4\pi^2\sqrt{\mathbf{s}(1-\mathbf{s})} = 4\pi^2vu/\mathbf{c}^2$, largest at $1/2$
R21a	modulus exchange: involution, keeps $\varphi_1 - \varphi_2$, fixes $\mathcal{T}_{1/2}$ pointwise; the coordinate swap fixes only $\zeta_1 = \zeta_2$ there
R21b-c	$\mathbf{i}$ keeps α and swaps the seams; exterior seam $r_f = 2R_s$ at $v = u = \mathbf{c}/\sqrt{2}$
R22	$\alpha_{\text{ext}} \rightarrow 1^-$ is $u \rightarrow 0^+$; $\alpha_{\text{ext}} \rightarrow 0^+$ is $v \rightarrow 0^+$
R23a-c	floor: $t_{\min} = 2t_P$, $d_{\min} = 2\ell_P$, $d_{\min}/t_{\min} = \mathbf{c}$, $d_{\min}^2 = \hbar\mathbf{c}/F_{\max}$, $E_{\text{floor}} = E_P/2$; numbers
R24a-b	$\mathcal{M}\mathbf{c}^2 = F_{\max} \cdot 2R_s$, $\mathbf{c}^4 = 4GF_{\max}$, $\kappa F_{\max} = 2\pi$; the I-1 hold $F_{\max}\alpha_1\alpha_2 = G\mathcal{M}m/r_f^2$
R25	pattern window: 50 GeV at 1 fm gives $\alpha = 1.32 \times 10^{-37}$
R26a-b	Evenson's c through (29): $\nu_{\text{Kr}}/\nu_{\text{Cs}} = 53\,835.1286(2)$; the 1983 value at 1.64σ
R27	$\mathbf{c}^2/\hbar = 1.356\,392\,489\,652 \times 10^{50} \text{ Hz kg}^{-1}$
R28a-c	every share $\rightarrow 0^+$ as $\mathbf{c} \rightarrow 0, \infty$ (α_{ext} , or α_{int} inside the fold); floor limits; Galilean limit
R29	$p\mathbf{c}/E = \cos \vartheta$, $m\mathbf{c}^2/E = \sin \vartheta$
R30	along $x = vt$: $(Et - px)/\hbar = m\mathbf{c}^2\tau/\hbar$
R31a-g	the stroke's helix: spin $\hbar$ at radius $\bar{\lambda} = \mathbf{c}/\omega$; circumference = wavelength, $\pi/4$ at every k , in space and in time; turning rate $\omega/k = \mathbf{c} = \text{run}$; phase constant along the run; $\mathbf{f}(\mathbf{c}) = \mathbf{c}\Sigma$ and $\mathbf{f}(\mathbf{c} + \mathbf{c}\Sigma) = \mathbf{c} + \mathbf{c}\Sigma$; a closed pattern's phase rate exceeds its run, $\omega/k > d\omega/dk$, with product $\mathbf{c}^2$

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