

The Premises of Yang–Mills

Frozen geometry, an expansion about the empty configuration
and a chosen group, read against one global settlement
and the gauge structure of the support and the seats

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Fundamental Density Theory

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Core v6.3 (author lock 2026-10-26) · companion to *Collective Geometry and the Yang–Mills Gap*

Abstract

Yang–Mills theory is the frame transport of one carrier. Its connection is a bivector-valued one-form, its curvature the holonomy of the carrier’s frame per unit area, and its self-coupling the non-commutation of rotations in different planes. Everything the Clay formulation adds to that geometry is a premise: a metric that does not answer to energy, no gravitational scale, infinite volume, a ground with nothing in it, a dynamics organised as vertices joined by propagators around an empty configuration, and a group chosen from a list. This paper audits the twelve premises of the problem against the locked core of Fundamental Density Theory (FDT), a true extension of general relativity and quantum mechanics, and states what FDT obtains in the place of each. FDT keeps the geometry and replaces the premises: every step is one system-wide settlement of matter, energy and spacetime, every transition a many-to-many reclosure, and the gauge structure is read off the support of the atlas and the three seats of the three-strand closure. Three results carry the weight. The gap’s power series in the coupling about the empty configuration is identically zero (Proposition 1), and the settlement is many to many: the Borromean three-strand closure is a three-body invariant that no pairwise linking number detects (Proposition 2), and the confining network of three sources is a Steiner minimum that no sum of pair terms reproduces, with $1 \leq L_Y/(\mathcal{P}/2) \leq 2/\sqrt{3}$ (Proposition 3). Under the seat–support premise that nothing mixes the two, the Clifford algebra of the support and the three seats returns the Standard Model’s group $S(U(3) \times U(2)) \cong (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$, its hypercharge as the relative phase of the seats and the support, and one chiral, anomaly-free generation as the even half of the spinor module (Theorems 7 and 8). The stage the problem requires is the endpoint $R_s = 0$ that the open interval excludes, and the one step that reaches it, deleting G , also deletes the floor $2\ell_P$, $2t_P$ that bounds every stroke: the frozen geometry and the ultraviolet problem are manufactured by the same premise (Proposition 9). The gap problem changes with its premises. It is posed on the stable closed modes of the settlement in a background, where, under the neutrino-minimum premise of the companion paper, the least mode is the free two-strand neutrino and the gap is its confinement cost, and on the closure sector of a closed pattern’s own region, where the step between the two lowest confinement eigenvalues is the edge, its operator to be derived. The record is met: no free fractional charge, closure to white on the seat-lattice premises of the core’s register; a neutron electric dipole moment below 1.8×10^{-26} e cm, the test of $\theta = 0$ in the seat sector, which is to be derived; a partial lifetime of $p \rightarrow e^+ \pi^0$ above 2.4×10^{34} years, with no transformation that joins seats to support; and a ground with a measured rest frame, the 369.82 km s^{-1} dipole of the microwave background. The consensus theory’s own lattice computation places the confining term of three static sources on the minimal network with the tension of the quark–antiquark string, a test

of the network reading, whose tension is to be derived. A receipt script checks the displayed identities and numbers: **97/97** pass.

1 Yang–Mills as frame transport

A carrier’s phase is a rotation. Core I-4 reads $\psi = Ae^{i\varphi}$ as the projection of a helix onto the plane normal to its axis; in the Clifford algebra of that plane, i is the plane’s unit bivector and $e^{i\varphi}$ its rotor. With $e_ie_j + e_j e_i = 2\delta_{ij}$, a bivector $e_{ij} = e_i e_j$ ($i \neq j$) squares to -1 , the rotor $R = \exp(-\varphi e_{ij}/2)$ turns the (i, j) plane by φ through $v \mapsto RvR$, and bivectors of planes that share an axis do not commute: [YP01a–d]

$$[e_{12}, e_{23}] = 2e_{13}. \quad (1)$$

A Yang–Mills connection is a bivector-valued one-form: it states how the carrier’s frame turns along a displacement. With $\nabla_{\mathcal{A}} = d + \mathcal{A}$ and a change of frame R ,

$$\mathcal{A}' = R^{-1}\mathcal{A}R + R^{-1}dR, \quad \mathcal{F} = d\mathcal{A} + \mathcal{A} \wedge \mathcal{A}, \quad \mathcal{F}' = R^{-1}\mathcal{F}R. \quad (2)$$

$\mathcal{F}$ is the holonomy of the frame per unit area: transport around a small loop returns the frame turned by $\mathcal{F}$ over the enclosed area. The term $\mathcal{A} \wedge \mathcal{A}$ is the composition law of rotations in different planes. [YP02]

One carrier has two presentations here (I-2). Free, its frame turns in one transverse plane; the rotations commute, the field equation is linear, and the presentation is the photon. Standing inside a three-strand closure, its frame turns under the eight bivector generators of the closure’s seat surface, the realisation of $\mathfrak{su}(3)$ constructed in Section 5; those rotations do not commute, and the presentation is the gluon (I-2, I-5). The self-coupling is the non-commutation of the one carrier’s frame rotations on the seat surface, and nothing is exchanged (I-2, I-4, I-5).

The Euclidean functional

$$S_{\text{YM}}[\mathcal{A}] = \frac{1}{4g^2} \int \mathcal{F}_{\mu\nu}^a \mathcal{F}^{a\mu\nu} d^4x \quad (3)$$

is the integral of a local density, and its stationary points are the connections with $\nabla_{\mathcal{A}} \star \mathcal{F} = 0$ at every point; its least configurations in each topological sector, the self-dual and anti-self-dual ones with action $8\pi^2|k_{\text{top}}|/g^2$, are global objects, labelled by an integer that no local variation changes. [YP03] What organises the theory about the empty configuration is not the functional; it is the premises. The Clay statement fixes them [4, pp. 5–6]: for every compact simple gauge group G , prove that a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$, existence including axiomatic properties at least as strong as those of Wightman or of Osterwalder and Schrader [5, 6]. The theory’s data are a Hilbert space carrying a unitary representation of the Poincaré group of Minkowski space $(\mathbb{R}^4, \eta)$; positive energy; a vacuum Ω_η , invariant and unique up to a phase, with $H\Omega_\eta = 0$; and the gap, $\text{spec}(H) \subset \{0\} \cup [\Delta, \infty)$. Its correlation functions are to agree at short distances with the predictions of asymptotic freedom and perturbative renormalisation theory [4, p. 6].

The argument runs as an audit. Section 2 states the premises this paper takes from the core and from its companions; Section 3 names the twelve premises of the problem as the problem uses them, the core clause that replaces each and what FDT obtains in its place. Section 4 reads the dynamics as one settlement and proves one proposition that the expansion about the empty configuration cannot hold and two that no sum of pair terms holds; Section 5 reads the gauge structure off the support of the atlas and the seats. Sections 6–11 carry the remaining premises; Section 12 states the gap in their place. The record, the tests, the declined readings and the convergence statement close the paper. The confinement theorems are those of *Confinement and the mass gap* [3] and the

domain argument that of *Collective geometry and the Yang–Mills gap* [2]; no dataset and no fitted number enters a derivation. The receipt script of Appendix A checks the displayed identities and numbers; check labels appear in brackets, [YP01a].

2 Premises, charts and symbols

The derivation uses the locked core v6.3 [1]; each premise is quoted by section.

- (P1) *One substance, only patterns.* There is one substance, the omnium: matter = energy = spacetime. There are no separate particles and no boxes; a room, cell or domain is a pattern’s own support with its own boundary data, never a vessel (0.1, 0.1b).
- (P2) *The chain is one state, and the floor.* The four terms of $E/m = c^2 = 1/(\varepsilon_0\mu_0) = (d/t)^2$ adjust together, with the bridges $E_q = m_q c^2 = \hbar\omega_q$, $T_q = 2\pi/\omega_q = h/E_q$, $\ell_q = cT_q = h/(m_q c)$, the Compton length and never the support R_{box} , and $d/t = c$ exactly on the spot. The floor $F_{\text{max}} d_{\text{min}} t_{\text{min}} = \hbar$, $d_{\text{min}} = c t_{\text{min}}$, gives $d_{\text{min}} = 2\ell_P$ and $t_{\text{min}} = 2t_P$: the infimum of physical strokes, approached and never occupied, a minimum that places no lattice (0.1a, 0.1d).
- (P3) *Every step is a system-wide settlement.* The omnium settles matter, energy and spacetime by one global optimisation, solved at every step from the state the system is in. Events do not commute, a process run backward is never the mirror of the forward event, and responses are system-wide and many-to-many, never one-to-one, particle by particle. The core names no objective for the optimisation; an analysis that names one writes it as its own reading (0.1a).
- (P4) *The force and its pair domain.* $F = F_{\text{max}}\alpha_1\alpha_2$, $F_{\text{max}} = c^4/4G$, with exterior cross-centre inputs $\alpha_i(d) = R_{s,i}/d$, is exact on two nonoverlapping spherical supports at monopole order, where it returns GM_1M_2/d^2 ; other configurations take derived evaluations, the multipole and overlap remainders of O-5 (I-1). The pair face (L1), the weak-field slow-motion containment (L2) and the source programme with each claimed divergence, to be derived in the core’s register (L3), are three layers, never merged (I-4).
- (P5) *One carrier.* Photon = graviton = gluon = phonon = magnon: one spin-1 excitation of the density field, read in different density and geometry regimes. The gluon is the stroke standing inside a closed pattern’s own region; there are no independent carriers and no mediation (I-2).
- (P6) *The geometry.* $G_{\mu\nu} = \kappa T_{\mu\nu}$ stands unchanged, $\kappa = 8\pi G/c^4$, $\kappa F_{\text{max}} = 2\pi$, with $T_{\mu\nu}$ the one total source; for the same source and data FDT and general relativity return the same curvature (I-3). Clocks and rulers are closed patterns adjusting their own dimensions to keep the chain, so their adjustment is the geometry and no background metric lies beneath them (0.1a, 0.1c).
- (P7) *The atlas, its seam and its two phases.* $\alpha_\sigma = (R_s/r_f)^\sigma$ is a ratio of two curvatures, branch-tagged, on the open interval $0 < \alpha_\sigma < 1$ (0.3, I-6). On the A-1 chart it is the share of the three-sphere, $|z_1|^2 = \alpha_\sigma$, $|z_2|^2 = 1 - \alpha_\sigma$; the Clifford torus $|z_1| = |z_2| = 1/\sqrt{2}$ is $\alpha_\sigma = 1/2$, the seam, at $r_f = 2R_s$ on the exterior branch and $r_f = R_s/2$ on the interior branch, and it splits the support into the general-relativistic solid torus $\alpha_\sigma < 1/2$, the spacetime phase, and the quantum-mechanical solid torus $\alpha_\sigma > 1/2$, the matter phase; the ends $\alpha_\sigma \rightarrow 0^+$ and $\alpha_\sigma \rightarrow 1^-$ are the two Hopf core circles. The seam is $\mathfrak{J} : \alpha_\sigma \mapsto 1 - \alpha_\sigma$, the exchange of the two moduli at fixed phases, which fixes the Clifford torus pointwise; the branch exchange

$i : r_f \mapsto R_s^2/r_f$ keeps α_σ and fixes the fold $r_f = R_s$. The three-sphere is the shape of the atlas, never a container (0.3a).

- (P8) *The quantum face.* Quantum mechanics is helical geodesics on truncated density cones; $\psi = Ae^{i\varphi}$ is the projection of a helix onto the plane normal to the cone axis, and the dynamics is deterministic (I-4).
- (P9) *Closures and their books.* Matter is braided photon helices; the helix count n fixes the class, confinement fixes the mass, $D(\Omega, B; n, \chi, \tau_{\text{conf}}) u_q = \omega_q u_q$, $m_q c^2 = \hbar \omega_q$, with χ the orientation, τ_{conf} the position on the inside/outside chart (the core's t , renamed so that t stays a duration) and the operator D the core's register item O-9; colour is the SU(3) of the internal nodal surface of the $n = 3$ closures (I-5). On three resolved integral seat sectors with the common mode quotiented, a primitive positive integral pairing and refinement and target independence, the four premises of O-10, the seat lattice is $K_\cap = A_2$, and a configuration closes on its own exactly when $\ell_{\text{total}} \in K_\cap \mathbb{Z}^2$ (I-5a). A closed mode is stable exactly when $\langle f | T | i \rangle = 0$ for every final state at or below threshold that keeps every exact book; beta, $2 \rightarrow 3+3+2$, is one many-to-many reclosure; the fermion class is $\nu_F = B_s \bmod 2$, never $n \bmod 2$ (I-5b).
- (P10) *Density equivalence.* $\mathcal{M} = M + E/c^2 + PV/c^2 + aT^4V/c^2 + \dots$ with $a = 4\sigma_{\text{SB}}/c$; every channel enters once, and gravitational binding is read from the solved geometry (I-7, 0.5).
- (P11) *Nodal architecture and many cones.* Observed structure is nodal: the loci where the transverse density gradient vanishes are the density geodesics, into which excitations lock (P-2). In a system of cones the selection rule signs the pairs one settlement joins, never pair by pair, and how the settlement signs a pair it does not join is O-5 (P-4).
- (P12) *The background.* Each universal domain is a bounded configuration read on the same atlas as a nucleus, and the substrate's cumulative standing-wave state is the domain's own (0.1b); the cosmic microwave background is its cosmological reading (0.8), and at quantum scale the same state reads as quantum fluctuations [3]. A declared background state is written $\mathcal{B}$.
- (P13) *The neutrino minimum* [3]. In every admitted background $\mathcal{B}$, a free $n = 2$ mode attains the least energy $E_{2,\text{min}}[\mathcal{B}] = \hbar \omega_{\nu,\text{min}}[\mathcal{B}] > 0$ over the stable closed sector, at positive frequency.
- (P14) *The links of the two closures.* The neutral two-strand closure's exact link is the Hopf pair; the charged three-strand closure's leading branch is Borromean.
- (P15) *Crossing chirality.* Crossing chirality is the crossing of the Clifford torus between the two solid tori: a parity-odd term requires an internal crossing operator.
- (P16) *The seat-support premises.* The three seat phases of the three-strand closure span $\mathbb{C}^3$ and the support of the atlas spans $\mathbb{C}^2$, each with its complex structure; the omnium supplies the seat triple and the support pair, and nothing that mixes them; the common phase of the five slots, seats and support together, is removed, this paper's extension to the seat-support space of the common mode that I-5a quotients on the three seats.

(P1)–(P12) quote the core, and (P12)'s quantum-scale reading is stated in [3]. (P13) is the neutrino-minimum premise of [3], to be derived there from the spectrum of D (D-01 of [3]). (P14)–(P16) are the author's readings of the two closures, of chirality and of the seat-support structure; each is used where it is named.

Charts. Every α in this paper is the A-1 compactness coordinate (0.4): the exterior inputs $\alpha_{i,\text{ext}}$ and $\alpha_{i|j}$ of I-1, the aggregate α_{ext} of a region, and the share $\alpha_\sigma = |z_1|^2$ of 0.3a, whose seam $\alpha_\sigma = 1/2$ lies at $r_f = 2R_s$ on the exterior branch and at $r_f = R_s/2$ on the interior branch. A-2 does not enter. A-3 (overlap) carries the 2/3 of I-1a, and its geometric odds are the core’s register item O-2; A-4 (confinement position) is the cone-position axis of I-5, and its canonical odds are O-3. A-5 names the carrier’s presentations, photon and gluon, and ranks nothing. No threshold transfers between applications: the A-1 1/2 and the A-3 2/3 are separate numbers (0.4). A bound configuration is carried on two charts, both declared: the exterior atlas of its source, with its fold at the source’s gravitational R_s , and its own interior chart, read with its skin at the fold and its skin radius R_{room} taken from data, with no transfer map between them (I-3). No saturation diagnostic $\alpha_{\text{core}} \rightarrow 1^-$ enters.

symbol	meaning
$e_i, e_{ij}, R, \tilde{R}$	Clifford vectors; bivectors $e_i e_j$; a rotor and its reverse
$\mathcal{A}, \mathcal{F}, g, G$	connection, curvature, Yang–Mills coupling (the metric is always written $g_{\mu\nu}$ or η), gauge group
$\Lambda_{\text{YM}}, \mu, b_0, k_{\text{top}}$	the Yang–Mills scale, the renormalisation scale, the one-loop coefficient, the sector index
$\theta, \theta_q, \bar{\theta}$	the vacuum angle, the phase of the quark-mass determinant, $\bar{\theta} = \theta + \theta_q$
c_Δ	the pure number $\Delta/\Lambda_{\text{YM}}$ (sans-serif; c is the strand rate)
$z_1, z_2, \alpha_\sigma, \mathbb{T}_{\alpha_\sigma}$	coordinates of the support $\mathbb{C}^2$; the share $ z_1 ^2$; the torus $\{ z_1 ^2 = \alpha_\sigma\} \subset S^3$
$\mathcal{H}, \mathfrak{J}, \mathbf{i}, w$	mean curvature of $\mathbb{T}_{\alpha_\sigma}$; the seam; the branch exchange; the Weyl element of $\text{SU}(2)$
$J, J_s, J_z, \mathbb{W}$	complex structures of the support, the seats and the support in $\mathbb{W} = \mathbb{C}^3 \oplus \mathbb{C}^2$
Y, T_3, Q	hypercharge, isospin and charge readings (in units of e ; T_3 is never a period)
$N_s, o_1, o_2, N_z, N_{\text{occ}}$	occupied seat slots; support occupations; $N_z = o_1 + o_2$; total occupation
$\mathbf{a}_k, \mathbf{a}_k^\dagger$	slot operators of the spinor module (sans-serif; the radiation constant is a)
$\mathbf{t}, \mathfrak{d}$	triality and doubletness
$K_\cap, \Lambda_{\text{rel}}, \mathbf{b}_i, \ell$	seat lattice, relative lattice, its basis, intersection data (I-5a)
$L_Y, \mathcal{P}, S_\Delta, \varsigma$	minimal network length, perimeter, area of a triangle of sources; string tension (σ is the branch tag)
$\mathcal{B}, \mathcal{S}[\mathcal{B}], \Delta_m, T_{\text{sup}}, E_{2,\text{min}}$	background; its stable positive-frequency closed modes; their mass infimum; their period supremum; the least energy
$\mathbb{T}, \Omega_\eta, E_{\text{base}}$	the transition operator of I-5b; the Clay vacuum; the lowest eigenvalue of a closed pattern’s own region
C, q, D, Ω, B	a closure, a mode label, the confinement operator, a support and its boundary data (I-5)
χ, τ_{conf}	the orientation and the position on the inside/outside chart (I-5; t is always a duration)
$T, T_q, T_{C,q}$	the temperature in aT^4V/c^2 ; the time a pattern keeps; a mode’s period

3 The twelve premises

Each row names a premise as the problem uses it, the core clause that replaces it, and what FDT obtains in its place. The premises are numbered (i)–(xii); the sections that follow carry the derivations.

	premise	FDT clause	what FDT obtains
(i)	a fixed Minkowski metric; geometry does not answer to energy	$G_{\mu\nu} = \kappa T_{\mu\nu}$, no background metric (I-3, 0.1a)	only the empty state fits the stage; it is posed at $R_s = 0$, the end the exterior branch approaches and never reaches (I-6)
(ii)	no gravitational scale; strokes of every length	the floor $2\ell_P$, $2t_P$ from $F_{\max} dt = \hbar$ (0.1a)	$G \rightarrow 0^+$ deletes the floor and manufactures the ultraviolet problem; finitely many carrier modes per bounded support
(iii)	infinite volume $\mathbb{R}^3$	bounded universal domains (0.8)	a positive floor over infinite volume is infinite energy; the far field is read at the $\alpha_{\text{ext}} \rightarrow 0^+$ core circle (0.3a)
(iv)	a unique, Poincaré-invariant, empty vacuum, $H\Omega_\eta = 0$	no empty state (0.1); the background $\mathcal{B}$ (0.8)	the gap belongs to a background; the measured ground carries a rest frame
(v)	vertices and propagators around $\mathcal{A} = 0$	every step one system-wide settlement; many-to-many reclosure (0.1a, I-5b)	the gap has the zero series; the Borromean three-strand closure is a three-body invariant; the confining network is a Steiner minimum
(vi)	virtual exchange; running by screening clouds	one carrier, no mediation (I-2); I-1a; I-6	the one-loop pole is a divergence at a finite scale, excluded; asymptotic freedom is the repulsive branch of I-1a
(vii)	free asymptotic gluons; separate asymptotic particles	the gluon is the carrier standing in a closure (I-2); only patterns (0.1b)	no free gluon state; observables are settlements of whole configurations
(viii)	a gauge group chosen; every compact simple G	the support S^3 (0.3a), the three seats, $K_\cap = A_2$ (I-5a), and the seat-support premises (P16)	$S(U(3) \times U(2)) \cong (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$, hypercharge, one generation
(ix)	a potential with gauge redundancy; a global gauge slice	loop and closure data (I-4, I-5)	no slice is chosen; the Gribov–Singer obstruction never arises
(x)	a free coupling g ; a scale by dimensional transmutation	no coupling in I-1; supports (I-5); seat maps (O-6)	under (P13), the gap’s value has a source: the neutrino’s confinement cost in $\mathcal{B}$
(xi)	a free vacuum angle θ	crossing chirality (P15)	$\theta = 0$ in the seat sector, to be derived (D-04); the neutron moment, which reads $\bar{\theta}$, is its test (T-3)
(xii)	confinement as a dynamical theorem still to be proved	closure (I-5); triality zero (I-5a)	closure to white is exact algebra on the premises of O-10; the singlet among triality-zero states (O-6) and the cost (O-9) are the open questions

4 One settlement, many to many

Core 0.1a fixes the dynamics: the omnium settles matter, energy and spacetime by one global optimisation, never one term at a time, and solves it at every step from the state the system is in (P3). Take a complete configuration of a domain: its metric $g_{\mu\nu}$, its total source $T_{\mu\nu}^{(\Omega)}$ with every channel entering once (0.5), its closures with their integral labels (I-5, I-5a), their helical phases (I-4), and the background $\mathcal{B}$ (0.8). The core names no objective for the optimisation. This paper reads it through one closure functional of the domain, as its own reading: the settled configuration is a

stationary point of that functional over the admissible set, every exact book held—four-momentum, charge, fermion class, angular momentum and every derived turning class (I-5b). The functional, its per-event response operators and the hyperedge transition kernel, the decay kernel of the core's register item O-12, are to be derived (D-01). Every variation carries its geometric response. At fixed metric I-3 fixes the source, $T_{\mu\nu} = G_{\mu\nu}[g]/\kappa$, so a family that changes the source at fixed metric violates I-3 and is no deformation of the omnium [2, Theorem 7].

The pair law is a face of the settlement. I-1's $F = F_{\max}\alpha_{1,\text{ext}}\alpha_{2,\text{ext}}$ is exact on its declared domain, two nonoverlapping spherical supports at monopole order, where it returns $G\mathcal{M}_1\mathcal{M}_2/d^2$. A resolved configuration supplies its whole matrix of exterior A-1 inputs, $\alpha_{i|j} = R_s[\mathcal{M}_i]/d_{ij}$, to one response problem at once, with the multipole and overlap remainders of O-5; reading the pair outputs as independent and additive is the weak-field, slow-motion reduction (I-4, layer L2), and the aggregate α_{ext} of a region is recovered only after its interior is solved. The signs settle the same way: in a system of cones the selection rule signs the pairs one settlement joins, never pair by pair from a table of every pair (P-4).

Transitions are hyperedges. A closed mode is stable exactly when $\langle f|\mathbf{T}|i\rangle = 0$ for every final state at or below threshold that keeps every exact book; decay is loop unwinding through the one channel left open, every book balancing across the event (I-5b). Beta is the exhibit: $2 \rightarrow 3 + 3 + 2$, one invariant-preserving many-to-many reclosure. The transition structure is a directed hypergraph whose edges carry a set of closures to a set of closures, and each edge re-solves the domain; the edges are directed because a process run backward is never the mirror of the forward event, and their composition is ordered because events do not commute (0.1a). The process is one hyperedge of the settlement, with no internal lines (I-2, I-5b).

The problem organises the dynamics about the empty configuration: a power series in g about $\mathcal{A} = 0$, built from local vertices of three and four lines joined by propagators. Proposition 1 shows what that expansion cannot hold; Propositions 2 and 3 show what no sum of pair terms holds.

Proposition 1 (the gap has the zero series). *For $\text{SU}(N)$ let $b_0 = 11N/3$, let the coupling run at one loop with the renormalisation scale μ as $\mu dg/d\mu = -b_0g^3/16\pi^2$, and let $\Lambda_{\text{YM}} = \mu \exp(-8\pi^2/b_0g^2)$. Then $\mu d\Lambda_{\text{YM}}/d\mu = 0$; in a theory of the kind the Clay statement asks for, whose one scale is Λ_{YM} , every energy of the spectrum is a pure multiple of Λ_{YM} , the gap $\Delta = c_\Delta\Lambda_{\text{YM}}$ with c_Δ a pure number among them; and at fixed μ*

$$\lim_{g \rightarrow 0^+} \frac{d^j \Delta}{dg^j} = 0 \quad \text{for every } j \geq 0. \quad (4)$$

The perturbative expansion is a power series in g about the empty configuration; the gap's series is identically zero.

Proof. Differentiating with the running gives $\mu d\Lambda_{\text{YM}}/d\mu = \Lambda_{\text{YM}}[1 + (16\pi^2/b_0g^3)(-b_0g^3/16\pi^2)] = 0$. [YP20b] The classical functional (3) carries no dimensionful parameter, so the one invariant scale is Λ_{YM} and every energy of the spectrum is a pure multiple of it. Each derivative of $\exp(-8\pi^2/b_0g^2)$ is a polynomial in $1/g$ times that exponential, and $g^{-j} \exp(-8\pi^2/b_0g^2) \rightarrow 0$ as $g \rightarrow 0^+$ for every j . [YP20a] The two-loop factor is a power of g and changes none of this. [YP20c] $\square$

The Clay problem asks for the one quantity its own organising expansion assigns zero at every order. The self-dual and anti-self-dual sectors enter as $\exp(-8\pi^2|k_{\text{top}}|/g^2)$ and are absent at every order as well. The tension of the confining network of Proposition 3 shares the zero series: in the same theory $\varsigma = c_\varsigma\Lambda_{\text{YM}}^2/\hbar c$ with c_ς a pure number, and by the same argument its series about

the empty configuration is identically zero, [YP20f] so the linear term ςL_Y enters at no order. The expansion's own three-body terms are of another kind: for three static sources they begin at two loops, as powers of the coupling [14]. The gap belongs to the whole solution, which is the object 0.1a names.

Proposition 2 (the Borromean three-strand closure is a three-body invariant). *Three closed loops can be inseparable with every pairwise linking number zero: the Borromean rings, the closure of the three-braid $(s_1 s_2^{-1})^3$ in the standard generators s_1, s_2 of B_3 , are Brunnian, and their linking is Milnor's triple invariant $\bar{\mu}(123) = \pm 1$ [7], a three-body quantity that no pairwise quantity detects. Any two fibres of one Hopf fibration link once (the Hopf link, the closure of s_1^2 in B_2), so three fibres of one fibration are pairwise linked and never Borromean.*

Proof. The closure of $(s_1 s_2^{-1})^3$ has three components, and each pair of components meets in crossings whose signs sum to zero, so every pairwise linking number vanishes; [YP23a] the closure of s_1^2 has two components with linking number one. [YP23b] Removing any one Borromean ring leaves two unlinked circles (Brunnian), and $\bar{\mu}(123) = \pm 1$ is Milnor's [7]. Two distinct fibres of the Hopf map $S^3 \rightarrow S^2$ are great circles whose linking number is ± 1 . The numerical Gauss integrals return 0.000 for each pair of the Borromean rings and 1.000 for each pair of three Hopf fibres. [YP22a–c] $\square$

The neutral two-strand closure's exact link is the Hopf pair (P14), native to the Clifford torus, whose $(1, 1)$ circles are Hopf fibres; the charged three-strand closure's leading branch is Borromean (P14) and lives off that family of circles. On its Borromean leading branch the closure that holds three seats together is a three-body invariant, and a decomposition into pairs does not contain it. Linking is blind to it pair by pair, as I-5b states of the photon ban.

Proposition 3 (the confining network is a Steiner minimum). *For three sources at $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ let*

$$L_Y = \min_{\mathbf{x}} \sum_{i=1}^3 |\mathbf{x} - \mathbf{q}_i|, \quad (5)$$

the minimal network: three segments meeting at 120° when every angle of the triangle is below 120° , the two shorter sides otherwise. No function f of one separation gives $L_Y = f(|\mathbf{q}_1 - \mathbf{q}_2|) + f(|\mathbf{q}_2 - \mathbf{q}_3|) + f(|\mathbf{q}_1 - \mathbf{q}_3|)$ for all configurations, and $1 \leq L_Y/(\mathcal{P}/2) \leq 2/\sqrt{3}$ with $\mathcal{P}$ the perimeter.

Proof. The equilateral family forces $f(s) = s/\sqrt{3}$, so pair additivity would give $L_Y = \mathcal{P}/\sqrt{3}$ on every triangle. [YP21a] The triangle with two unit sides enclosing 150° has $L_Y = 2$, while $\mathcal{P}/\sqrt{3} = (2 + \sqrt{2 + \sqrt{3}})/\sqrt{3} = 2.270$. [YP21b] For the bounds, with sides $s_1 \leq s_2 \leq s_3$ and area S_Δ : summing the triangle inequality $|\mathbf{x} - \mathbf{q}_i| + |\mathbf{x} - \mathbf{q}_j| \geq |\mathbf{q}_i - \mathbf{q}_j|$ over the three pairs gives $L_Y \geq \mathcal{P}/2$. Below 120° , $L_Y^2 = \frac{1}{2} \sum_i s_i^2 + 2\sqrt{3} S_\Delta$, [YP21c] and the Hadwiger–Finsler inequality $2 \sum_{i < j} s_i s_j - \sum_i s_i^2 \geq 4\sqrt{3} S_\Delta$, with $\sum_{i < j} (s_i - s_j)^2 \geq 0$, gives $4 \sum_{i < j} s_i s_j - \sum_i s_i^2 \geq 12\sqrt{3} S_\Delta$, which is $L_Y \leq \mathcal{P}/\sqrt{3}$. [YP21d] At 120° and above, $L_Y = s_1 + s_2$ and $s_3 \geq \frac{\sqrt{3}}{2}(s_1 + s_2)$ give $L_Y/(\mathcal{P}/2) \leq 4/(2 + \sqrt{3}) < 2/\sqrt{3}$. [YP21e, YP21f] $\square$

The lattice computation of the consensus theory measures the confining term of three static sources on this minimal length. Its potential is a constant, a two-body Coulomb term with half the quark–antiquark coefficient and the linear term $\varsigma_{3Q} L_Y$ with $\varsigma_{3Q} \simeq \varsigma_{Q\bar{Q}}$ [8]; for triangles whose largest angle is below 120° the potentials fall on one curve as a function of L_Y , with the tension of the quark–antiquark string, and carry a force that no superposition of two-body forces reproduces [9]. Proposition 3 bounds the two-body form: half the perimeter, a sum of pair terms with half the

tension, is the lower bound of the network, $\varsigma\mathcal{P}/2 \leq \varsigma L_Y \leq \varsigma\mathcal{P}/\sqrt{3}$, with $L_Y/(\mathcal{P}/2)$ fixed by the shape. At short separation the computed potential follows that bound, below the network [10, 11], and beyond about 0.8 fm between sources its slope passes to that of the network [11]. FDT reads the network as the density geodesics of the standing carrier in a three-seat closure, the loci of P-2 where the transverse density gradient vanishes: three lines meeting at one junction. Three lines of one tension balance at the junction only where their three pulls cancel, at 120° , which is the condition that the network's length is stationary over the junction's position; with one tension the network's length is the Steiner minimum (5). The tension, its equality across two- and three-seat closures, $\varsigma_{3Q} = \varsigma_Q\bar{Q}$, and the passage from the bound to the network near 0.8 fm are to be derived (D-06).

The same holds for stiffness. Minimising over the responding coordinates of one pattern lowers its stiffness by $\mathbf{B}\mathbf{D}^{-1}\mathbf{B}^* \geq 0$, with $\mathbf{D} > 0$ the stiffness of the responding coordinates and $\mathbf{B}$ their coupling to the frozen ones (sans-serif; D is the confinement operator and B its boundary data), so a positive frozen block decides nothing and a negative one stays negative [2, Lemma 10], while a response tied to the source by a constraint carries a multiplier term of no universal sign [2, Lemma 8]; the stability of a pattern is the ledger form of I-5b. Pairs are faces of the settlement: exact on two nonoverlapping spherical supports at monopole order (I-1), and the lower bound of the confining network; the expansion about the empty configuration assigns the gap and the tension zero at every order. The settlement is the object, and its transitions are many to many.

5 The gauge structure of the support and the seats

Yang–Mills takes its group as an input, and the Clay statement ranges over all of them. FDT reads the group off two structures: the support $S^3 \subset \mathbb{C}^2$ with share $|z_1|^2 = \alpha_\sigma$ (0.3a), and the three seats of the three-strand closure with the seat lattice $K_\cap = A_2$ (I-5a). Under the seat–support premises (P16) the Clifford algebra of the two returns the Standard Model's group, its hypercharge and one generation of its content. The three-sphere is the shape of the atlas, never a container (0.3a).

5.1 The share and the seam

Write $z_1 = x_1 + ix_2$, $z_2 = x_3 + ix_4$ and take the Clifford algebra $\text{Cl}(4, 0)$ of $\mathbb{R}^4 = \mathbb{C}^2$. The bivectors e_{12} and e_{34} turn the phases of z_1 and z_2 independently and keep $|z_1|^2$ and $|z_2|^2$ fixed; they generate the phase torus $\mathbb{T}^2$. [YP04a] The map $z \mapsto |z_1|^2$ is, up to sign and a factor of two, the first component of the moment map of that torus action; on S^3 the two components sum to a constant, so over each value $0 < \alpha_\sigma < 1$ lies one torus $\mathbb{T}_{\alpha_\sigma} = \{|z_1|^2 = \alpha_\sigma\}$, and over the ends $\alpha_\sigma \rightarrow 1^-$ and $\alpha_\sigma \rightarrow 0^+$ lie the two Hopf core circles $\{z_2 = 0\}$ and $\{z_1 = 0\}$. The open interval of I-6 is the interior of the moment segment, and its ends are circles (0.3a). The pairs $(\frac{1}{2}|z_k|^2, \arg z_k)$ are action–angle coordinates of the symplectic form $dx_1 \wedge dx_2 + dx_3 \wedge dx_4$ of $\mathbb{C}^2$: the share is twice the action conjugate to the phase of z_1 . [YP04c]

Proposition 4 (the seam is the one minimal torus). *Every $\mathbb{T}_{\alpha_\sigma}$ is flat. In the unit three-sphere, with the unit normal $(\sqrt{1-\alpha_\sigma} e^{i\varphi_1}, -\sqrt{\alpha_\sigma} e^{i\varphi_2})$, its principal curvatures are $-\sqrt{(1-\alpha_\sigma)/\alpha_\sigma}$ and $\sqrt{\alpha_\sigma/(1-\alpha_\sigma)}$, and its mean curvature and area are*

$$\mathcal{H}(\alpha_\sigma) = \frac{\alpha_\sigma - \frac{1}{2}}{\sqrt{\alpha_\sigma(1-\alpha_\sigma)}}, \quad |\mathbb{T}_{\alpha_\sigma}| = 4\pi^2 \sqrt{\alpha_\sigma(1-\alpha_\sigma)}. \quad (6)$$

The mean curvature vanishes, and the area is stationary and maximal, only at $\alpha_\sigma = \frac{1}{2}$.

Proof. With $z = (\sqrt{\alpha_\sigma} e^{iu}, \sqrt{1-\alpha_\sigma} e^{iv})$ the induced metric is $\text{diag}(\alpha_\sigma, 1-\alpha_\sigma)$, constant, so the torus is flat; the product of the principal curvatures is -1 , and Gauss's equation in the unit sphere gives intrinsic curvature $1 + (-1) = 0$. The second fundamental form with the displayed normal gives the curvatures, their mean and the area $4\pi^2 \sqrt{\alpha_\sigma(1-\alpha_\sigma)}$, whose derivative vanishes only at $\alpha_\sigma = \frac{1}{2}$, a maximum. [YP08a-g] $\square$

The seam of 0.3a (A-1; $r_f = 2R_s$ on the exterior branch, $r_f = R_s/2$ on the interior branch) is the one minimal torus of the family, the Clifford torus, with principal curvatures ∓ 1 , and every embedded minimal torus in S^3 is congruent to it [15]. The support's own area functional is stationary exactly at the seam.

The seam map is $\mathfrak{J} : \alpha_\sigma \mapsto 1 - \alpha_\sigma$, the exchange of the two moduli at fixed phases (0.3a),

$$\mathfrak{J} : (\sqrt{\alpha_\sigma} e^{i\varphi_1}, \sqrt{1-\alpha_\sigma} e^{i\varphi_2}) \mapsto (\sqrt{1-\alpha_\sigma} e^{i\varphi_1}, \sqrt{\alpha_\sigma} e^{i\varphi_2}). \quad (7)$$

It carries $\mathbb{T}_{\alpha_\sigma}$ onto $\mathbb{T}_{1-\alpha_\sigma}$, carries Hopf fibres to Hopf fibres, and fixes the Clifford torus pointwise: the jacket flip. [YP07a, YP07b] The coordinate swap $(z_1, z_2) \mapsto (z_2, z_1)$ is $\mathfrak{J}$ composed with the exchange of the two phases. It also carries $\mathbb{T}_{\alpha_\sigma}$ onto $\mathbb{T}_{1-\alpha_\sigma}$ and the seam onto itself; it keeps the orientation of $\mathbb{R}^4$ ($\det = +1$) and reverses the seam's own ($\det = -1$ on its two angles), trading meridian for longitude, and on the seam it fixes only the great circle $z_1 = z_2$, one Hopf fibre. [YP07c-e] The branch exchange $\mathfrak{i}$ is a third map: it keeps α_σ and fixes the fold (0.3a).

5.2 The rotors of the support

The six bivectors of $\text{Cl}(4, 0)$ split into two commuting triples,

$$\mathfrak{su}(2)_\pm = \text{span}\{e_{12} \pm e_{34}, e_{13} \mp e_{24}, e_{14} \pm e_{23}\}, \quad [\mathfrak{su}(2)_+, \mathfrak{su}(2)_-] = 0. \quad (8)$$

The complex structure of the support is $J = e_{12} + e_{34}$, and its commutant among the bivectors is

$$\mathfrak{u}(2) = \mathfrak{u}(1)_J \oplus \mathfrak{su}(2)_-, \quad (9)$$

four generators. [YP05a-c] $\mathfrak{u}(1)_J$ turns z_1 and z_2 together; its orbits are the Hopf fibres, the $(1, 1)$ circles that rule every $\mathbb{T}_{\alpha_\sigma}$ and rule the seam. $\mathfrak{su}(2)_-$ is the rotor algebra of the support: $S^3 \cong \text{Spin}(3) \cong \text{SU}(2)$ is the unit group of the even algebra $\text{Cl}^+(3, 0)$, the quaternions, and $\text{SU}(2)$ acts on S^3 freely and transitively. Its maximal torus keeps the share, and the rest of the group does not. [YP04b] The share of a configuration is fixed by its two curvatures on its branch (0.3), and no rotor moves a configuration on the atlas: the rotors act on the support pair as the seat-support module carries it (Sections 5.4–5.6), on states and their readings, so no rotor carries a closure of the matter phase into the spacetime phase (0.3a).

Proposition 5 (the Weyl element). *The Weyl element*

$$w = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} : (z_1, z_2) \mapsto (-z_2, z_1) \quad (10)$$

acts on the share as $\alpha_\sigma \mapsto 1 - \alpha_\sigma$, the action of $\mathfrak{J}$ on the share, and fixes no point of S^3 . Every element of $\text{SU}(2)$ that exchanges the two support lines is $w \text{diag}(e^{i\varphi}, e^{-i\varphi})$, and its square is -1 , the nontrivial element of the centre $\{\pm 1\}$ of $\text{SU}(2)$.

Proof. $|(-z_2)|^2 = |z_2|^2 = 1 - \alpha_\sigma$; $w(z_1, z_2) = (-z_2, z_1)$ forces $z_1 = -z_2 = -z_1$, so $z = 0 \notin S^3$; and $(w \text{diag}(e^{i\varphi}, e^{-i\varphi}))^2 = -1$ for every φ . [YP06a-d] $\square$

5.3 The seats

Core I-5a sets the seat lattice on three resolved integral seat sectors with the common mode quotiented, a primitive positive integral pairing and refinement and target independence, the four premises of O-10: $\Lambda_{\text{rel}} = \{y \in \mathbb{Z}^3 : y_1 + y_2 + y_3 = 0\}$, with basis $\mathbf{b}_1 = \epsilon_1 - \epsilon_2$, $\mathbf{b}_2 = \epsilon_2 - \epsilon_3$ (ϵ_k the unit vectors of $\mathbb{Z}^3$), and

$$K_{\cap} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = A_2, \quad \det K_{\cap} = 3, \quad \text{coker } K_{\cap} = \mathbb{Z}_3. \quad (11)$$

Proposition 6 (the seat algebra). *$K_{\cap}$ is the Cartan matrix of $\mathfrak{su}(3)$. On the three seat phases, $\mathbb{C}^3 = \mathbb{R}^6$, the realification $X = X_r + iX_i \mapsto \begin{pmatrix} X_r & -X_i \\ X_i & X_r \end{pmatrix}$ carries $\mathfrak{u}(3)$ onto the commutant of the realified i among the fifteen bivectors of $\text{Cl}(6,0)$, and removing the seats' common mode, the one I-5a quotients, leaves $\mathfrak{su}(3)$: rank two, eight generators. Its simply connected group $\text{SU}(3)$ has centre $\mathbb{Z}_3 = \text{coker } K_{\cap}$, the triality of I-5a.*

Proof. The basis gives $2(\mathbf{b}_i, \mathbf{b}_j)/(\mathbf{b}_j, \mathbf{b}_j) = K_{\cap}$, the Cartan matrix of the roots $\epsilon_i - \epsilon_j$ of $\mathfrak{su}(3)$ on $\mathbb{C}^3$; the Smith form of $K_{\cap}$ is $\text{diag}(1, 3)$. [YP10a–c] The commutant of the realified i in $\mathfrak{so}(6)$ has dimension nine. [YP11] $\square$

The eight generators of $\mathfrak{su}(3)$ are bivectors that turn the three seat phases among themselves on the seat surface of the three-strand closure, the internal nodal surface whose $\text{SU}(3)$ is colour (I-5). They are internal directions of the closure, never strands and never spatial dimensions.

5.4 The seat–support algebra and its group

Put the two structures together: $\mathbb{W} = \mathbb{C}_{\text{seats}}^3 \oplus \mathbb{C}_{\text{support}}^2 = \mathbb{R}^{10}$, with $\text{Cl}(10,0)$ and its forty-five bivectors $\mathfrak{so}(10)$. Two complex structures act, J_s on the seats and J_z on the support.

Theorem 7 (the seat–support group). *The commutants are*

$$\text{Comm}(J_s + J_z) = \mathfrak{u}(5) \quad (25), \quad \text{Comm}(J_s) \cap \text{Comm}(J_z) = \mathfrak{u}(3) \oplus \mathfrak{u}(2) \quad (13), \quad (12)$$

and removing the five-slot common mode, the complex trace generated by $J_s + J_z$, leaves

$$\mathfrak{s}(\mathfrak{u}(3) \oplus \mathfrak{u}(2)) = \mathfrak{su}(3) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)_Y \quad (8 + 3 + 1 = 12), \quad (13)$$

whose centre is spanned by iY with

$$Y = \text{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \quad (14)$$

The lift $\widehat{M} = \frac{1}{4} \sum_{ij} M_{ij} e_i e_j$ carries each of the twelve generators to a bivector of $\text{Cl}(10,0)$ and keeps every bracket, $[\widehat{M}, \widehat{N}] = \widehat{[M, N]}$. The group of unitary maps of $\mathbb{W}$ that keep the split and have determinant one is

$$\mathbf{G}_{\text{SM}} = \text{S}(\text{U}(3) \times \text{U}(2)) \cong \frac{\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)}{\mathbb{Z}_6}, \quad (\gamma_3, \gamma_2, \lambda) \mapsto \text{diag}(\lambda^{-2}\gamma_3, \lambda^3\gamma_2), \quad (15)$$

with kernel $\{(\lambda^2 \mathbf{1}, \lambda^{-3} \mathbf{1}, \lambda) : \lambda^6 = 1\} \cong \mathbb{Z}_6$.

Proof. The commutant dimensions, the twelve anti-Hermitian traceless generators commuting with J_s and J_z , their closure, the centre and the bracket-keeping lift are computed exactly. [YP12a–f, YP13a–b] The map (15) has determinant $\lambda^{-6}\lambda^6 = 1$; it is onto, since for $(A, B) \in \text{S}(\text{U}(3) \times \text{U}(2))$ any λ with $\lambda^6 = 1/\det A$ gives $\gamma_3 = \lambda^2 A$ and $\gamma_2 = \lambda^{-3} B$ of determinant one; and $\lambda^{-2}\gamma_3 = \mathbf{1}$, $\lambda^3\gamma_2 = \mathbf{1}$ force $\lambda^6 = 1$. [YP14a–c] $\square$

Hypercharge is the traceless relative phase of the seats and the support, $3y_s + 2y_z = 0$, normalised so that a support slot reads $y_z = \frac{1}{2}$: the five-slot common phase is removed, and the seats' common phase stays in hypercharge as $y_s = -\frac{1}{3}$. At the level of the group the determinant condition, not a quotient, removes the five-slot phase. The $\mathbb{Z}_6 = \mathbb{Z}_3 \times \mathbb{Z}_2$ is the product of the two centres: $\mathbb{Z}_3$ the centre of the seat group $\text{SU}(3)$, the triality of I-5a, and $\mathbb{Z}_2 = \{\pm 1\}$ the centre of the support's $\text{SU}(2)$, the kernel of $\text{Spin}(3) \rightarrow \text{SO}(3)$. [YP14d] The denominators of hypercharge are the two counts, three seats and two support coordinates, and the gluing condition

$$Y + \frac{\mathfrak{t}}{3} + \frac{\mathfrak{d}}{2} \in \mathbb{Z} \quad (\mathfrak{t} \text{ triality, } \mathfrak{d} \text{ doubletness}) \quad (16)$$

holds on every fermion, every conjugate, the doublet scalar and every gauge field of the record. [YP19]

The algebra of Theorems 7 and 8 is that of the $\text{SU}(5)$ and $\text{Spin}(10)$ constructions [16, 17, 18, 19]: G_{SM} is the subgroup of $\text{SU}(5)$ that keeps the split $\mathbb{C}^3 \oplus \mathbb{C}^2$, and $\bigwedge \mathbb{C}^5$ carries one generation with its conjugates. FDT supplies the two summands, the three seats and the support pair, and the premise (P16) that nothing mixes them; grand unification adds the generators that mix them.

5.5 One generation from the module

The spinor module of $\text{Cl}(10, 0)$ is the exterior algebra $\bigwedge \mathbb{C}^5$ on the five slots, thirty-two states, with the Clifford vectors acting as $\mathbf{a}_k + \mathbf{a}_k^\dagger$ and $i(\mathbf{a}_k^\dagger - \mathbf{a}_k)$. [YP15a, YP15b] A traceless element U of $\mathfrak{u}(5)$ lifts to the derivation $\sum_{kl} U_{kl} \mathbf{a}_k^\dagger \mathbf{a}_l$ with no offset; the common mode $i\mathbf{1}$ alone lifts with one, to $i(N_{\text{occ}} - \frac{5}{2})$, N_{occ} the total occupation. [YP15c, YP15d] Removing the five-slot common mode is what lets hypercharge act as a pure reading of occupation. On a state with N_s occupied seat slots and support occupations $o_1, o_2 \in \{0, 1\}$, $N_z = o_1 + o_2$, hypercharge, isospin and the charge embedding $Q = T_3 + Y$ (O-10) read [YP15e, YP16a]

$$Y = -\frac{N_s}{3} + \frac{N_z}{2}, \quad T_3 = \frac{o_1 - o_2}{2}, \quad Q = T_3 + Y = o_1 - \frac{N_s}{3}. \quad (17)$$

The book is integral: the occupations (N_s, o_1, o_2) . Hypercharge is a reading with denominators three and two, the seat count and the support count; charge is a reading with denominator three, integral exactly when $N_s \equiv 0 \pmod{3}$: triality zero, closure to white (I-5a). [YP16b, YP16c] The slots are seat and support labels. They are not strands: N_s is not the helix count n , and an occupied slot is not a closure. Which closures carry which slots is to be derived (D-02).

Theorem 8 (one generation). *The volume element of $\text{Cl}(10, 0)$ acts on $\bigwedge \mathbb{C}^5$ as $i(-1)^{N_{\text{occ}}}$, the occupation parity: it commutes with every bivector and anticommutes with every vector, and its two eigenspaces are the two hands. The even half is one generation of the record together with the singlet ν^c , as left-handed fields:*

occupied slots	$(\text{SU}(3), \text{SU}(2))_Y$	states	Q	name
none	$(1, 1)_0$	1	0	ν^c
two seats	$(\bar{3}, 1)_{-2/3}$	3	$-\frac{2}{3}$	u^c
one seat, one support	$(3, 2)_{1/6}$	6	$\frac{2}{3}, -\frac{1}{3}$	$(u, d)_L$
both support	$(1, 1)_1$	1	1	e^c
three seats, one support	$(1, 2)_{-1/2}$	2	0, -1	$(\nu, e)_L$
two seats, both support	$(\bar{3}, 1)_{1/3}$	3	$\frac{1}{3}$	d^c

The odd half carries the conjugates, and the two halves are inequivalent under G_{SM} : the content is chiral. Every gauge anomaly and the mixed gravitational anomaly cancel on the even half:

$\sum Y^3 = \sum Y = 0$, the $SU(3)^2U(1)$, $SU(2)^2U(1)$ and $SU(3)^3$ sums vanish, and the doublets number four, so the global $SU(2)$ anomaly, present for an odd number of doublets [21], is absent.

Proof. On the Fock space of the five slots the volume element equals $i(-1)^{N_{\text{occ}}}$, commutes with the twelve lifts and anticommutes with the ten vectors. [YP15f, YP15g] The readings (17) on the sixteen even states give the table, [YP17a] the odd states give the conjugate representations, [YP17b] and the anomaly sums are computed exactly. [YP18a–f] $\square$

5.6 Doublets, and the two pairings

The doublets are exactly the states with one support slot occupied. The centre $-1 \in SU(2)$ acts as $(-1)^{N_z}$, so doubletness $\mathfrak{d} = N_z \bmod 2$ is the centre character of the support's $SU(2)$, and the two members of a doublet are exchanged by the Weyl element, which acts on the share as $\mathfrak{J}$ does (Proposition 5). Its members are module states, not positions on the atlas, and the identification of the doublet with a crossing pair of (P15) is to be derived (D-05). The same- n pairings of I-5 are one closure read from the free room and from the nuclear room, related by the branch exchange $\mathfrak{i}$. Core 0.3a separates the two involutions: $\mathfrak{i}$ keeps α_σ and fixes the fold, $\mathfrak{J}$ sends α_σ to $1 - \alpha_\sigma$ and fixes the seam. Two involutions give two pairings. The Weyl element exchanges the module states read as ν and e , and those read as u and d . As closures, ν and e are braids of different helix count ($n = 2$ and $n = 3$), related by re-braiding and never by a smooth shift (I-5), and u and d are sub-strands of the $n = 3$ braid; a path from 1 to w in $SU(2)$ is a path of module states, never a path between closures, and which closures carry which slots is to be derived (D-02). Neutrino and neutron at $n = 2$, and electron and proton at $n = 3$, are $\mathfrak{i}$ -pairs; no pair is both.

The doublet scalar of the record carries the quantum numbers $(1, 2)_{1/2}$ of the support pair itself, the singly occupied support slots, and the set of its least-energy values in the standard account's potential is a three-sphere, the support's shape. FDT reads every mass as confinement cost (I-5; [3]): the measured 125 GeV state and the W and Z masses are accepted as data, and their confinement costs are to be derived (D-03).

5.7 What the chosen group becomes

(viii) A chosen group. The Clay statement ranges over every compact simple G . The omnium supplies the seat triple and the support pair, and nothing that mixes them (P16). Two simple factors act on FDT structures, $SU(3)$ on the seats and $SU(2)$ on the support, glued to the relative phase by the $\mathbb{Z}_6$ of the two counts; a group that mixes the two structures, or exceeds them, has nothing to act on. Beyond $SU(3)$ and $SU(2)$, the universality clause of the Clay statement concerns groups with no FDT structure to act on. Because no element of G_{SM} carries a seat slot into a support slot, the leptoquark transformations of $SU(5)$ and $\text{Spin}(10)$ have no FDT realisation, and no transformation of the group joins the proton's seats to the support; whether a reclosure of the proton to $e^+\pi^0$ is open is the ledger's verdict (I-5b), whose kernel is the core's register item O-12, and the record bounds its partial lifetime above 2.4×10^{34} yr [20].

6 The stage

(i) Frozen geometry. The Clay data fix the Minkowski metric η ; it enters through Poincaré covariance and is not a variable of the theory. On $(\mathbb{R}^4, \eta)$, $G_{\mu\nu} \equiv 0$, so I-3 admits only $T_{\mu\nu} \equiv 0$. Every state with energy $E > 0$ has $\mathcal{M} \geq E/c^2 > 0$, $R_s = 2GM/c^2 > 0$ and a geometry that is not η ; the one state with $T_{\mu\nu} \equiv 0$ is the empty state, and the omnium has none (0.1) [2, Theorem 5].

The stage is $R_s = 0$ at every field radius: no source, no fold and no branch of the atlas, the end the exterior branch approaches as $\alpha_{\text{ext}} \rightarrow 0^+$ and never reaches (I-6). If general relativity is ignored, the Clay problem statement does not apply to reality [2]. Clocks and rulers are closed patterns adjusting their own dimensions to keep the chain; their adjustment is the geometry (0.1a).

(ii) **No gravitational scale.** Yang–Mills is posed with G absent.

Proposition 9 (one limit, two premises). *The floor of 0.1a is*

$$F_{\text{max}} d_{\text{min}} t_{\text{min}} = \hbar, \quad d_{\text{min}} = c t_{\text{min}} \implies d_{\text{min}} = 2\sqrt{\frac{\hbar G}{c^3}} = 2\ell_P, \quad t_{\text{min}} = 2t_P, \quad (18)$$

and

$$\lim_{G \rightarrow 0^+} d_{\text{min}} = 0, \quad \lim_{G \rightarrow 0^+} F_{\text{max}} = \infty, \quad \lim_{G \rightarrow 0^+} R_s = 0 \text{ for every } \mathcal{M}. \quad (19)$$

Deleting G deletes the floor, and the same step freezes the geometry: one limit produces (i) and (ii) together.

Proof. $d_{\text{min}}^2 = \hbar c / F_{\text{max}} = 4\hbar G / c^3$, and the three limits are read off $d_{\text{min}} \propto \sqrt{G}$, $F_{\text{max}} = c^4 / 4G$ and $R_s = 2G\mathcal{M} / c^2$. [YP24a–d] $\square$

The continuum limit, the demand that the theory hold for strokes of every length down to zero and the step no construction has completed in four dimensions, is then posed on strokes below $2\ell_P$, which the chain excludes. With G kept, every stroke respects the floor, and the floor is a minimum that places no lattice (0.1a). The ultraviolet question becomes resolution independence over resolutions bounded below by $2\ell_P$. The floor’s energy is $E_{\text{floor}} = F_{\text{max}} d_{\text{min}} = E_P / 2$, with $E_{\text{floor}} t_{\text{min}} = \hbar$ (0.1a). [YP24e]

Proposition 10 (finitely many carrier modes per bounded support). *A mode’s period exceeds t_{min} [2, Proposition 15], so $\omega < 2\pi / t_{\text{min}}$ and its wavenumber $k < 2\pi / d_{\text{min}} = \pi / \ell_P$. A bounded support has a discrete spectrum, and the carrier’s modes below that bound are finite in number: Weyl’s count $\mathcal{N} \simeq 2V k_{\text{max}}^3 / 6\pi^2$, with two transverse polarisations, gives 1.0×10^{60} for a support of radius 1 fm, 1.0×10^{105} for 1 m and 1.0×10^{183} for 10^{26} m. [YP24f–i]*

The continuum measure of the existence clause lives on degrees of freedom the chain excludes; the settlement of a bounded domain has finitely many carrier modes. Nothing below $2\ell_P$, $2t_P$ is a stroke of anything (0.1a).

(iii) **Infinite volume.** The gap must survive $V \rightarrow \infty$ on $\mathbb{R}^3$. Every admissible region of the omnium carries energy, and with the density floor of the spacetime phase, D-3 of [2], a geometry of infinite volume carries infinite energy, while the Clay vacuum assigns zero to the same volume; no subtraction of a finite constant relates the two [2, Theorem 13 and Corollary 14]. The setting of an FDT spectral statement is a bounded support within a universal domain, a bounded configuration read on the same atlas as a nucleus (0.8). The far field is read at the $\alpha_{\text{ext}} \rightarrow 0^+$ core circle (0.3a), never as a region of flat, infinite space.

7 The ground

(iv) **An empty, invariant vacuum.** The Clay data require a vacuum invariant under the Poincaré group, unique up to a phase, with $H\Omega_\eta = 0$. The omnium has no empty state (0.1); the lowest pattern of a closed pattern’s own region has $E_{\text{base}} > 0$ and keeps the period $\hbar / E_{\text{base}}$, which the convention $H\Omega_\eta = 0$ sets to infinity [2, Proposition 15]. The ground of a domain is the background $\mathcal{B}$

(P12): the substrate’s cumulative standing-wave state, read at cosmological scale as the microwave background and at quantum scale as quantum fluctuations. The measured background carries a frame: its dipole, $3362.08 \pm 0.99 \mu\text{K}$, reads a solar-system velocity of $369.82 \pm 0.11 \text{ km s}^{-1}$ relative to it [22]. [YP27] The ground of the observed domain is not Poincaré invariant. The laws keep their local covariance— $d/t = c$ on the spot (0.1a, 0.1d), the density cone is the local light cone (P-4), and no lattice selects a frame (0.1a)—and the frame belongs to the state. No variable of the chain vanishes and nothing is virtual (0.1, I-2).

The gap therefore belongs to a background. Write $\mathcal{S}[\mathcal{B}]$ for the stable, positive-frequency closed modes (C, q) admitted in $\mathcal{B}$, membership the ledger’s verdict of I-5b, with periods $T_{C,q}[\mathcal{B}] = h/E_{C,q}[\mathcal{B}]$ (0.1a), and

$$\Delta_m[\mathcal{B}] = \inf_{(C,q) \in \mathcal{S}[\mathcal{B}]} m_{C,q}[\mathcal{B}], \quad T_{\text{sup}}[\mathcal{B}] = \sup_{(C,q) \in \mathcal{S}[\mathcal{B}]} T_{C,q}[\mathcal{B}]. \quad (20)$$

Lemma 11 (the exact criterion [3]). *For every background, $\Delta_m[\mathcal{B}] = h/(c^2 T_{\text{sup}}[\mathcal{B}])$, zero exactly when $T_{\text{sup}}[\mathcal{B}] = \infty$. Under (P13), $\Delta_m[\mathcal{B}] = E_{2,\text{min}}[\mathcal{B}]/c^2 > 0$.*

Proof. The mass of a mode is $m_{C,q} = h/(c^2 T_{C,q})$ in its period $T_{C,q}$, a positive, continuous, strictly decreasing function of the period, so its infimum over $\mathcal{S}[\mathcal{B}]$ is its value at the period supremum; attainment is not needed. [YP25a, YP25b] Under (P13) a free $n = 2$ mode attains the least energy over $\mathcal{S}[\mathcal{B}]$ at positive frequency; divided by c^2 , its energy is the infimum of mass. $\square$

The criterion puts the whole gap in the bound on admitted periods. A gap that belongs to its background is a quantity the Clay formulation cannot express.

8 The expansion

(v) Vertices and propagators around the empty configuration. The Clay statement requires the correlation functions to agree at short distances with asymptotic freedom and perturbative renormalisation [4, p. 6]: the perturbative expansion about the empty configuration is written into the problem. Section 4 carries the reading: the gap has the zero series (Proposition 1), the Borromean three-strand closure is a three-body invariant (Proposition 2), and the confining network is a Steiner minimum no sum of pair terms reproduces (Proposition 3).

(vi) Virtual exchange and the running coupling. FDT reads the scale dependence of the effective coupling with no exchange (I-2). The measured decrease of the effective coupling at short separation, the scaling of deep-inelastic data [34], is the repulsive branch of I-1a: above the A-3 threshold $2/3$ the overlap of two density distributions forms a ridge, and the ridge prevents collapse toward the upper end of the open interval (I-1a, I-6: asymptotic freedom, nuclear saturation, the maximum-density shell of a horizon). The one-loop coupling

$$g^2(\mu) = \frac{8\pi^2}{b_0 \ln(\mu/\Lambda_{\text{YM}})} \quad (21)$$

diverges as $\mu \rightarrow \Lambda_{\text{YM}}^+$: a divergence at a finite control value, which I-6 and 0.8 exclude for every physical quantity. [YP20d, YP20e] The pole marks where the expansion stops, never a property of the omnium.

(vii) Free asymptotic states. The perturbative S-matrix takes free gluons as its asymptotic states, and confinement removes them: the organising framework assumes the states the physics forbids. In FDT the gluon presentation is the carrier standing inside a closed pattern’s own region

(I-2); it has no free state by definition, and its free counterpart is radiation. Observables are settlements of whole configurations, the ledger’s hyperedges between closures that exist (I-5b), and the pair S-matrix is their two-body face. The premise that a symmetry acts on separate asymptotic particles as a tensor product also carries the Coleman–Mandula theorem [24], whose conclusion is a direct product of the Poincaré group with an internal group. The seat–support group meets that form: it acts on the seat–support module, the seats and the support pair, and never mixes them with the directions of spacetime.

9 The redundancy

(ix) Gauge slices. Quantisation on the potential requires a slice through each gauge orbit. Slices meet orbits more than once [25], and for a non-abelian group no continuous global slice exists: the space of connections is a nontrivial bundle over its orbit space [26]. Every Faddeev–Popov and BRST construction sits on a slice that does not exist globally. FDT’s physical data are loop data: the rotor a frame returns with around a closed loop (I-4) and the integral labels of closures (I-5, I-5a). The holonomies of all loops determine the connection up to a change of frame [27], and no slice is ever chosen. The fermion class is loop data of the same kind: Levi-Civita transport of a frame along a Hopf fibre of the unit $S^3 \cong \text{SU}(2)$, read in its left-invariant framing, turns it by 2π per circuit, [YP29] so its spin class counts fibre circuits mod 2, the shape $\nu_F = B_s \bmod 2$ requires (I-5b). The obstruction to a slice is the topology of the changes of frame themselves: the space of connections is contractible and the group of changes of frame is not [26].

10 The parameters

(x) A free coupling and a scale from nowhere. Yang–Mills carries one free dimensionless coupling g , traded by dimensional transmutation for one free scale Λ_{YM} ; the Clay statement asks for $\Delta > 0$ and no value, because the theory fixes only the pure number $\Delta/\Lambda_{\text{YM}}$. FDT’s force law has no coupling constant: $F = F_{\text{max}}\alpha_{1,\text{ext}}\alpha_{2,\text{ext}}$ with geometric A-1 inputs on its declared domain (I-1). The maps from a charge, a circulating charge or a colour seat to an I-1 input are the core’s register item O-6; couplings are readings of those maps, never inputs. The scale of a confined mode is its support, $m_{nq} \simeq \hbar\xi_{nq}(B)/(cR_{\text{box}})$ in the scale-similar reduction, with R_{box} fixed by geometry and never the Compton length (I-5). Under (P13) the scale of the stable-sector gap is the neutrino’s confinement cost in its background, $\Delta_m[\mathcal{B}] = E_{2,\text{min}}[\mathcal{B}]/c^2$ (Lemma 11). The number the Clay statement leaves out has a named source.

(xi) A free vacuum angle. The Euclidean functional admits θk_{top} , with

$$k_{\text{top}} = \frac{1}{32\pi^2} \int \mathcal{F}_{\mu\nu}^a \tilde{\mathcal{F}}^{a\mu\nu} d^4x \quad (22)$$

the sector index: a total derivative whose integral labels the classes $\pi_3(\text{G}) = \mathbb{Z}$ of maps from the three-sphere at infinity into the group, generated by an $\text{SU}(2)$ subgroup, itself a three-sphere; the term violates P and CP. Nothing in Yang–Mills fixes θ . The record bounds the neutron’s electric dipole moment, $|d_n| < 1.8 \times 10^{-26} \text{ e cm}$ [28], and with it $\bar{\theta} = \theta + \theta_q$, the vacuum angle together with the phase θ_q of the quark-mass determinant: $|\bar{\theta}| \lesssim 10^{-10}$ [30]. [YP28] FDT’s parity account is crossing chirality (P15): a parity-odd term requires an internal crossing operator. The seat sector is transport on the seat surface of one closure, and the absence of an internal crossing operator on that surface is to be derived (D-04); with it the seat sector’s parity-odd term vanishes and θ has no seat: $\theta = 0$ in the seat sector. The neutron moment reads $\bar{\theta}$, so $\theta = 0$ meets it with $\theta_q = 0$, the

phase of the quark-mass determinant with the masses read as confinement costs (I-5), which is to be derived with D-04.

11 Confinement

(xii) Confinement as a theorem still to be proved. The consensus requires confinement to follow from the dynamics: the physical states are $SU(3)$ -invariant [4, p. 3], and in the pure theory the Wilson loop obeys an area law with a string tension [12]; its proof lies outside the Clay statement, among the extensions the problem description names [4, p. 6]. FDT reads confinement as topological closure (I-5), and closure to white is exact algebra: a configuration closes on its own exactly when $\ell_{\text{total}} \in K \cap \mathbb{Z}^2$, triality zero (I-5a). The proton $(2, -1)$, the neutron $(1, -2)$, the meson $(1, 1)$ and $\Delta^{++} (3, 0)$ close; a single u , uu and ud do not. [YP10d, YP10e] The module returns the same condition: the charge reading is integral exactly when $N_s \equiv 0 \pmod{3}$. Triality zero holds for the octet as for the singlet: which triality-zero configurations are singlets, the octet-plus-singlet counting, rests on the A_2 table from cone geometry, the core's register item O-6. On the four premises of the seat lattice, the core's register item O-10, closure to white is the physics of confinement; the dynamical question is the cost of a closure, the confinement operator D and its spectrum (O-9).

12 The FDT gap statement

Three statements replace the Clay gap, each a statement about the settlement.

Confined-mode mass [3]. Every admitted closed mode has $m_{C,q} = \hbar\omega_{C,q}/c^2 > 0$ (I-5). The gluon presentation's mass is its whole confinement cost; the carrier has no intrinsic mass to add.

Stable-sector gap [3]. Under (P13),

$$\Delta_m[\mathcal{B}] = \frac{E_{2,\min}[\mathcal{B}]}{c^2} = \frac{\hbar\omega_{\nu,\min}[\mathcal{B}]}{c^2} = \frac{h}{c^2 T_{\text{sup}}[\mathcal{B}]} > 0, \quad (23)$$

and the least mode is the free two-strand neutrino.

The closure-sector edge [2]. On the closure sector of a closed pattern's own region, the relative generator $H_{\text{rel}} = H_{\text{phys}} - E_{\text{base}}I$, with H_{phys} the generator of the region's internal dynamics, has its lower edge at $\epsilon_* = \hbar(\omega_1 - \omega_0)$, the step between the two lowest confinement eigenvalues of the region in which the carrier stands; for the $n = 3$ closures, the class that carries colour, the confinement operator, its domain and boundary data and the estimate uniform over the admissible closures are to be derived (D-1 of [2]).

Membership of the stable sector is the ledger's verdict on the global solution, $\langle f | T | i \rangle = 0$ for every final state at or below threshold that keeps every exact book (I-5b), so the gap is a statement about the settlement's hyperedges: every candidate decay of the least mode is a many-to-many reclosure, and none is open. Four-momentum leaves the least mode no lighter closed final state under (P13), and its fermion class ν_F forbids a purely bosonic one (I-5b); ν_F on the exact framed closures is the core's register item O-11. Free radiation carries no gap; the gap belongs to closures [3].

The Clay requirements map as follows.

Clay requirement	FDT	in its place
unitary Poincaré representation on a fixed $(\mathbb{R}^4, \eta)$	declined	the metric answers to energy (I-3); the laws keep local covariance (0.1a, P-4)
a vacuum invariant and unique up to a phase, $H\Omega_\eta = 0$	declined	the ground is $\mathcal{B}$, whose energy enters $\mathcal{M}$ once as a non-rest contribution of $T_{\mu\nu}^{(\Omega)}$; the lowest pattern of a closed pattern's own region has $E_{\text{base}} > 0$ (0.1, 0.5, 0.8, I-7)
every compact simple $\mathbf{G}$	replaced	$\mathbf{G}_{\text{SM}} = \text{S}(\text{U}(3) \times \text{U}(2))$ from the seats and the support under (P16) (0.3a, I-5a; Theorem 7)
continuum limit to zero length	replaced	resolutions bounded below by $2\ell_P$; finitely many modes per bounded support (0.1a; Proposition 10)
infinite volume	replaced	bounded supports within universal domains (0.8)
short-distance agreement with asymptotic freedom and perturbative renormalisation	accepted, read	the record accepted and read, the running on the repulsive branch of I-1a; the settlement's short-distance correlators are to be derived (D-01)
$\Delta > 0$ above the vacuum	replaced	$\Delta_m[\mathcal{B}] > 0$ on the stable closed sector, the neutrino's confinement cost (P13); the edge ϵ_* on the closure sector, its operator and uniform estimate to be derived (D-1 of [2])
confinement, an extension of the problem [4, p. 6]	read	closure to white, triality zero (I-5, I-5a), exact algebra on the premises of core O-10; the singlet among triality-zero states (O-6)

13 Energy accounting

I-7 counts each channel once:

$$\mathcal{M} = M + \frac{E}{c^2} + \frac{PV}{c^2} + \frac{aT^4V}{c^2} + \cdots, \quad a = \frac{4\sigma_{\text{SB}}}{c}. \quad (24)$$

The Yang–Mills stress is a channel of the one $T_{\mu\nu}^{(\Omega)}$ (0.5) and sources curvature (I-3). A confined mode's energy enters $\mathcal{M}$ once, as the rest contribution of its state; the background's energy enters once, as a non-rest contribution of the one $T_{\mu\nu}^{(\Omega)}$ (0.5), its radiative term aT^4V/c^2 in the observed domain, and is never subtracted; a confined mode's cost $E_{C,q}[\mathcal{B}]$ depends on the background through which modes are admitted and what they cost, and the background's own energy is never added to that cost a second time [3]; gravitational binding is read from the solved geometry. The Clay normalisation $H\Omega_\eta = 0$ discards a gravitating channel [2], and the settlement keeps it.

14 The record and the tests

what the record shows	the measurement, or the consensus computation where marked	where it is read
no free fractional charge	searches for fractionally charged particles in bulk matter, at accelerators and in cosmic rays find none [33]	closure to white, (xii); Theorem 8
three static sources on the minimal network	consensus computation, pure gauge: the linear term $\varsigma_{3Q}L_Y$ with $\varsigma_{3Q} \simeq \varsigma_{Q\bar{Q}}$ beside a two-body Coulomb term, and a force no two-body superposition reproduces [8, 9]; the two-body form at short separation [10, 11], the slope of the minimal network beyond about 0.8 fm [11]	Proposition 3; D-06
the group and one generation	every field of the record satisfies the gluing condition, and one generation is free of gauge and gravitational anomalies [34]	Theorems 7 and 8 [YP18, YP19]
no strong-sector parity violation	$ d_n < 1.8 \times 10^{-26} e \text{ cm}$ [28], so $ \bar{\theta} \lesssim 10^{-10}$ [30]	(xi); D-04
no proton decay to $e^+\pi^0$	partial lifetime above $2.4 \times 10^{34} \text{ yr}$ [20]	(viii)
a ground with a rest frame	the microwave dipole, $3362.08 \pm 0.99 \mu\text{K}$, $369.82 \pm 0.11 \text{ km s}^{-1}$ [22]	(iv) [YP27]
asymptotic freedom	the scaling of deep-inelastic data [34]	(vi); I-1a
the gluon's confined mode	consensus computation, pure gauge: a glueball spectrum with its lightest scalar near 1.7 GeV [31, 32]	confined-mode mass (Section 12); a test for D (O-9)
neutrino masses	nonzero mass-squared splittings from oscillations; the absolute scale bounded above [34]	(P13); Lemma 11

Each test below names the measurement that would end a result, a premise or a reading of this paper.

- T-1 *Closure to white.* A replicated free fractional charge ends closure to white (I-5a) as the reading of confinement.
- T-2 *The neutrino minimum.* A stable closed mode lighter than the lightest neutrino ends (P13) and the stable-sector gap (23); a resolved nonzero lightest mass, from beta-endpoint, double-beta or cosmological data, meets it.
- T-3 *The seat sector's angle.* A strong-sector parity-odd contribution to an electric dipole moment, a nonzero $\bar{\theta}$, ends the reading of (xi).
- T-4 *The split.* A measured vector state of quantum numbers $(3, 2)_{-5/6}$, the transformation of $\text{SU}(5)$ that joins a seat slot to a support slot, ends the seat–support premises (P16) and the reading of Theorem 7 as the gauge group; a proton decay alone does not, since its reclosure is the ledger's verdict (I-5b), whose kernel is the core's register item O-12.
- T-5 *The content.* A chiral fermion outside the gluing condition (16) ends the module reading of Theorem 8.
- T-6 *The network.* A measured hadron record that requires the confining energy of three sources to depart at large separation from one tension on the minimal network ends the reading of Proposition 3; the lattice potentials of the consensus theory without dynamical quarks [8, 9] are its comparator. With dynamical quarks the potential saturates beyond the screening

separation, as the quark–antiquark string does [13], which the settlement reads as a reclosure into separate closures.

Seven pieces sit outside the derivation and leave every result in it unchanged:

- D-01 *The settlement.* The closure functional of a domain over its complete configurations, this paper’s reading of the optimisation of 0.1a, its per-event response operators, the hyperedge transition kernel, the decay kernel of the core’s register item O-12, and the settlement’s short-distance correlators, which the record of asymptotic freedom tests, are to be derived.
- D-02 *Slots to closures.* The coupling of the seat–support connection to closures: which closures carry seat and support slots, the mixing of the fibre phase with the relative phase, and the Weinberg angle, are to be derived.
- D-03 *The support pair’s costs.* The confinement costs of the support-pair state at 125 GeV and of the W and Z states are to be derived (core O-9).
- D-04 *The seat sector’s angle.* The absence of an internal crossing operator on the seat surface, with it $\theta = 0$ in the seat sector, and the vanishing of the quark-mass phase θ_q that the neutron moment reads beside it, are to be derived.
- D-05 *The doublet.* The doublet as a crossing pair of (P15): doubletness $\mathfrak{d}$, the centre character of the support’s $SU(2)$, read as the crossing book, and the two hands of the module, the eigenspaces of its volume element, read as crossing chirality, are to be derived; charge parity $n \bmod 2$, the fermion class ν_F and $\mathfrak{d}$ stay three books until then.
- D-06 *The tension.* The tension of the standing-carrier network, its equality across two- and three-seat closures, $\varsigma_{3Q} = \varsigma_{Q\bar{Q}}$, and the passage from the two-body form at short separation to the network near 0.8 fm, are to be derived.
- D-07 *Generations.* The count of generations in two books, never conflated (I-5): the lepton rungs e, μ, τ at $n = 3$ through the open fork, and the quark rungs $n = 3, 5, 7, 9$ of the n -ladder, is to be derived.

15 Declined readings and convergence

The standard account reads the self-coupling as gluons interacting with gluons through cubic and quartic vertices. FDT reads it as the non-commutation of the one carrier’s frame rotations on the seat surface; nothing is exchanged (I-2, I-4, I-5).

The standard account builds every process from local vertices of three and four lines joined by propagators and reads the internal lines as virtual particles. FDT reads the process as one hyperedge of the settlement with no internal lines (I-2, I-5b).

The standard account reads the Y-shaped flux of three static sources as a gluon string with a junction. FDT reads it as the density geodesics of the standing carrier in a three-seat closure, three lines of one tension balancing at 120° (P-2, Proposition 3).

Grand unification reads the Standard Model’s group as the remnant of a larger symmetry broken at a high scale. FDT reads it as the symmetry of two distinct structures, three seats and the support pair, joined by one relative phase; the carrier is one from the outset, and nothing is broken to reach the group (I-2, P16, Theorem 7).

The standard account generates the W , Z and fermion masses from the doublet’s vacuum value. FDT reads every mass as confinement cost (I-5), and the measured masses are accepted as data:

the costs at 125 GeV and of W and Z are to be derived (D-03), and the fermion masses are the spectrum of the core's register item O-9.

The standard account reads η as an arena on which the field's clocks and rulers are read. FDT reads clocks and rulers as closed patterns adjusting their own dimensions to keep the chain; their adjustment is the geometry (0.1a).

The standard account reads the theory's short-distance behaviour as a limit to be controlled by renormalisation. FDT reads it as the floor: nothing below $2\ell_P$, $2t_P$ is a stroke of anything (0.1a, Proposition 9).

The standard account reads the microwave background as a thermal bath on an empty, invariant vacuum and reads quantum fluctuations as virtual pairs in it. FDT reads both as the one background $\mathcal{B}$: no variable of the chain vanishes and nothing is virtual (0.1, 0.8, I-2).

The Osterwalder–Schrader route defines the theory through a probability measure on Euclidean configurations. FDT reads the dynamics as one deterministic settlement, the helical phase fixed and no amplitude attached (I-4), and the fluctuation content that measure encodes as $\mathcal{B}$.

The standard account reads the scale dependence of the effective coupling as screening and antiscreening by clouds of virtual pairs. FDT reads it with no exchange: asymptotic freedom is the repulsive branch of I-1a (I-2).

The standard account reads the gluon as a gauge boson massless in its Lagrangian, with an infrared mass scale generated dynamically. FDT reads the gluon as the one carrier standing in a closed pattern, whose mass is that standing mode's confinement cost (I-2, I-5).

Convergence. For the same source and data, FDT returns the measured record. Under the seat-support premises (P16), the group that acts faithfully on every field of the record, $S(U(3) \times U(2))$, its hypercharges, its chirality and one anomaly-free generation are returned by the Clifford algebra of the support and the three seats, with the $\mathbb{Z}_6$ of the gluing condition (Theorems 7 and 8). No free fractional charge is observed [33]; closure to white is triality zero, and the charge reading of every closed configuration is integral. The consensus theory's lattice computations are read beside the record as computations, never counted as data: the potential of three static sources carries the linear term on the minimal connecting length with $\varsigma_{3Q} \simeq \varsigma_{Q\bar{Q}}$ and a force beyond any two-body superposition [8, 9], with the two-body form at short separation [10, 11], a test of the network reading (D-06); the glueball spectrum, with its lightest scalar near 1.7 GeV [31, 32], is a test for the confinement operator of O-9, never its output. The deep-inelastic scaling record [34] is accepted in full and read on the repulsive branch of I-1a. Classical well-posedness on Minkowski space is recovered as the $\alpha_{\text{ext}} \rightarrow 0^+$ reading of the coupled solution: for 1 GeV in 1 fm, $\alpha_{\text{ext}}(1 \text{ fm}) = 2.65 \times 10^{-39}$ [2]. [YP26] The background's measured dipole is the frame of $\mathcal{B}$; freely propagating light stays massless, and the gap belongs to closures. Oscillation experiments measure nonzero mass-squared splittings, and beta-endpoint and cosmological analyses bound the absolute scale from above; (P13) places a strictly positive confinement cost at the bottom of the stable sector and prints no number, so every measured bound is met. The observables FDT adds each rest on an input this paper names: $\theta = 0$ in the seat sector (D-04), against the Standard Model, where $\bar{\theta}$ is a free parameter, and the Peccei–Quinn extension, which reaches $\bar{\theta} = 0$ through an axion [29]; and a strictly positive mass for the lightest neutrino (P13), where the consensus account allows zero. Against grand unification FDT reads the proton with no transformation that joins seats to support, since no element of G_{SM} does (P16), a reading the Standard Model shares; whether a reclosure of the proton to $e^+\pi^0$ is open is the ledger's verdict (I-5b), whose kernel is the core's register item O-12. The measured neutron moment and proton lifetime already sit where these statements place them.

16 The premises of Yang–Mills

Yang–Mills is the frame transport of the one carrier. Its premises freeze the geometry, empty the ground, delete the floor, choose the group and expand the settlement about an empty configuration. FDT keeps the transport and replaces the premises: one settlement, many to many; a group fixed, under (P16), by three seats and the support pair of the atlas; a gap that is the least confinement cost of the stable closed sector in its background, and on a closed pattern’s own region the step between its two lowest confinement eigenvalues. The frozen stage and the ultraviolet problem are one premise, the deletion of G , and the gap’s zero series is the expansion’s own verdict on itself. Where a premise departs from nature, FDT is under no obligation to reproduce it.

17 Status register

result	rests on	status
Frame transport: the bracket (1), covariance (2), holonomy	I-4	derived
The settlement as a stationary point of a closure functional	0.1a	this paper’s reading; the functional to be derived (D-01)
The pair law as a face of the settlement; the signs of a many-cone system	I-1, I-4, P-4	derived on the pair domain; core O-5
Transitions as directed hyperedges	0.1a, I-5b	derived
The zero series of the gap (Proposition 1)	the Clay data	derived
The Borromean three-strand closure (Proposition 2)	(P14)	derived
The Steiner minimum and its bounds (Proposition 3)	—	derived
The network as density geodesics of one tension	P-2	this paper’s reading; the 120° balance derived from one tension; the tension to be derived (D-06)
The seam the one minimal torus (Proposition 4); $\mathfrak{J}$, the swap and $\mathbf{i}$	0.3a	derived
The rotors of the support and the Weyl element (Proposition 5)	0.3a	derived
The seat algebra (Proposition 6)	I-5a	derived as algebra; its premises core O-10; the A_2 table from cone geometry and the octet’s dynamics core O-6
The seat–support group, hypercharge and $\mathbb{Z}_6$ (Theorem 7)	(P16), I-5a	derived
One generation, chirality, anomalies (Theorem 8)	Theorem 7	derived; the charge embedding $Q = T_3 + Y$ core O-10; slots to closures (D-02)
The doublet as a crossing pair	(P15)	to be derived (D-05)
One limit, two premises (Proposition 9)	0.1a, 0.2	derived
Finitely many carrier modes (Proposition 10)	0.1a; [2]	derived

result	rests on	status
Infinite volume carries infinite energy	0.8; [2]	derived in [2], with its floor D-3 there
The exact criterion (Lemma 11)	0.1a, I-5b	derived
The stable-sector gap (23)	(P13)	derived under (P13); (P13) to be derived in [3]
The closure-sector edge ϵ_*	I-5; [2]	to be derived (D-1 of [2]; core O-9, O-6)
$\theta = 0$ in the seat sector	(P15)	to be derived (D-04)
Confinement as closure to white	I-5, I-5a	derived as algebra; its premises core O-10; the singlet among triality-zero states core O-6
The support pair's costs at 125 GeV and of W, Z	I-5	to be derived (D-03; core O-9)
Generations in two books	I-5	to be derived (D-07)

The core's register items this paper names are O-2 and O-3 (the odds of A-3 and A-4), O-5 (the remainders of the pair expression, the overlap complex and the many-cone signing), O-6 (the seat maps, the A_2 table from cone geometry and the gluon octet's dynamics), O-9 (the confinement operator and its spectrum, with m_p/m_e and $(m_\mu/m_e)^2$ held-out targets it must reproduce), O-10 (the four premises of the seat lattice, the U(1) embedding and its unit), O-11 (ν_F on the exact framed closures), O-12 (the decay kernel and rates) and O-15 (the domain-merger kinematics and the derivation of the standing-wave state).

invariant	where it enters
I-1	the pair law exact on its domain and a face of the settlement (Section 4); no coupling constant, couplings as readings of the O-6 maps, (x)
I-2	one carrier: photon and gluon as presentations; no virtual exchange, no internal lines; the gluon's free counterpart is radiation, (vii)
I-3	the field equation kept: the frozen background admits only the empty state, (i); containment for the fixed-background results
I-4	the phase as a rotor; frame transport and holonomy data, (ix); the deterministic settlement (Section 15); the L2 layer of pair additivity
I-5	mass as confinement cost; colour on the seat surface of the $n = 3$ closures; the same- n pairings; the vacuity guard, (x); the three gap statements
I-6	the open interval: $R_s = 0$ excluded, (i); the one-loop pole excluded, (vi); the Hopf core circles as the ends of the moment segment
I-7	every channel once (24); the background's energy kept; $H\Omega_\eta = 0$ discards a channel

A Receipt

The script `receipt_ym_premises.py`, in the folder of this paper, checks the displayed identities symbolically with SymPy, in exact rationals wherever the statement is algebraic, and the displayed numbers numerically: the spinor module on the thirty-two states of the Fock space of five slots, the Steiner network against direct minimisation, linking by braid closure and by Gauss integrals, and the transport of a frame along a Hopf fibre by integration of the Levi-Civita equation. Its output

is `receipt_output.txt`; run it with `python3 receipt_ym_premises.py`. Result: **97/97 PASS**.

check	content
YP01a-d	$e_i e_j + e_j e_i = 2\delta_{ij}$ in $\text{Cl}(4)$, $\text{Cl}(6)$, $\text{Cl}(10)$; every bivector squares to -1 ; $R = \exp(-\varphi e_{12}/2)$ turns the plane by φ ; the bracket (1)
YP02	the covariance (2) for a non-abelian change of frame $R(x, y) \in \text{SU}(2)$
YP03	the self-dual pattern: $\mathcal{F} = \star \mathcal{F}$, $\mathcal{F}_{\mu\nu}^a \mathcal{F}_{\mu\nu}^a = 192\rho^4/(x^2 + \rho^2)^4$, integral $32\pi^2$, $k_{\text{top}} = 1$, action $8\pi^2/g^2$
YP04a-c	the phase torus keeps both moduli; the maximal torus of the rotor group keeps the share and the rest moves it; action-angle pairs
YP05a-c	$\mathfrak{so}(4) = \mathfrak{su}(2)_+ \oplus \mathfrak{su}(2)_-$ (8); the commutant of J (9)
YP06a-d	the Weyl element: in $\text{SU}(2)$; $\alpha_\sigma \mapsto 1 - \alpha_\sigma$; no fixed point on S^3 ; every exchanging element squares to -1
YP07a-e	$\mathfrak{J}$ carries $\mathbb{T}_{\alpha_\sigma}$ onto $\mathbb{T}_{1-\alpha_\sigma}$ and fibres to fibres, and fixes the Clifford torus pointwise; the swap has $\det = +1$ on $\mathbb{R}^4$, $\det = -1$ on the seam's angles, and fixes there the circle $z_1 = z_2$
YP08a-g	the torus family (6): metric, normal, principal curvatures, mean curvature, flatness, the area and its one maximum at the seam, curvatures ∓ 1 there
YP10a-e	$K_\cap$ the Cartan matrix of $\mathfrak{su}(3)$; $\det K_\cap = 3$; Smith form $\text{diag}(1, 3)$; the I-5a readings and the closure list
YP11	the commutant of the realified i in $\mathfrak{so}(6)$ is $\mathfrak{u}(3)$
YP12a-f	commutants 25 and 13 (12); the twelve generators (13): independence, closure, the centre spanned by iY (14)
YP13a-b	the lift keeps all 78 brackets; every lift is a bivector
YP14a-d	the $\mathbb{Z}_6$ of (15): kernel, determinant, surjectivity, the product of the two centres
YP15a-g	the module: anticommutation, the Clifford relations, lifts without offset, the common mode $i(N_{\text{occ}} - \frac{5}{2})$, hypercharge as occupation, the volume element $i(-1)^{N_{\text{occ}}}$, chirality
YP16a-c	the readings (17) on the thirty-two states; integral charge exactly at triality zero; the gluing condition on every state
YP17a-b	the even half is one generation; the odd half carries the conjugates; the content is chiral
YP18a-f	$\sum Y = \sum Y^3 = 0$; the $\text{SU}(3)^2\text{U}(1)$, $\text{SU}(2)^2\text{U}(1)$ and $\text{SU}(3)^3$ sums vanish; four doublets
YP19	the gluing condition (16) on the eighteen rows of the record's fields and conjugates, and on ν^c and its conjugate
YP20a-f	the zero series (4) for $j \leq 8$; $\mu d\Lambda_{\text{YM}}/d\mu = 0$; the two-loop factor; the one-loop coupling (21) and its pole; $b_0 = 11$ for $\text{SU}(3)$; the zero series of a tension $\propto \Lambda_{\text{YM}}^2$
YP21a-f	the Steiner network (5): the equilateral family; the 150° triangle, 2 against 2.270; the closed form on 200 triangles; Hadwiger-Finsler; the case at 120° and above; $1 \leq L_Y/(\mathcal{P}/2) \leq 2/\sqrt{3}$ on 4000 triangles
YP22a-c	Gauss integrals: the Borromean rings pairwise 0.000; the Hopf link 1.000; three Hopf fibres pairwise 1.000, one sign
YP23a-b	braid closures: $(s_1 s_2^{-1})^3$ has three components, pairwise unlinked; s_1^2 the Hopf link
YP24a-i	the floor (18); the limits (19); $E_{\text{floor}} = E_P/2$; $k_{\text{max}} = \pi/\ell_P$; the Weyl counts 1.0×10^{60} , 1.0×10^{105} , 1.0×10^{183}
YP25a-b	the exact criterion of Lemma 11
YP26	1 GeV in 1 fm: $\alpha_{\text{ext}}(1 \text{ fm}) = 2.65 \times 10^{-39}$
YP27	the dipole: $c \Delta T/T_0 = 369.81 \text{ km s}^{-1}$ from $3362.08 \text{ } \mu\text{K}$ on $T_0 = 2.7255 \text{ K}$ [23], within the quoted 369.82 ± 0.11
YP28	$ \bar{\theta} \lesssim 10^{-10}$ from $ d_n < 1.8 \times 10^{-26} \text{ e cm}$ and $d_n = 2.4 \times 10^{-16} \bar{\theta} \text{ e cm}$ [30], a coefficient known to 40–50%: the bound runs from 5×10^{-11} to 1.5×10^{-10}
YP29	Levi-Civita transport along a Hopf fibre of the unit S^3 , read in the left-invariant frame, turns by 2π per circuit

Dimensions. $\alpha_\sigma, \xi, g, \theta, \theta_q, \bar{\theta}, k_{\text{top}}, c_\Delta, Y, T_3$ and Q (in units of e) are dimensionless; $\hbar\omega, E_{2,\text{min}}, \mu, \Lambda_{\text{YM}}$ and $\Delta = c_\Delta\Lambda_{\text{YM}}$ are energies; $\hbar\omega/c^2, h/(c^2T_{C,q})$ and Δ_m are masses; $d_{\text{min}}, \ell_P, L_Y$ and R_{box} are lengths; t_{min} is a time; F_{max} is a force and κ an inverse force; ς is a force (energy per length).

Provenance. FDT core: CORE-FDT-INVARIANTS.md, locked v6.3, author lock 2026-10-26, SHA-256 1989c0b4f7f1471f4eaa9305d231a06a9bbe4e6f07b4560b223f412e16b5463d. All core quotations are by section number.

References

- [1] Fundamental Density Theory, *Core FDT Invariants*, v6.3 (author lock 2026-10-26).
- [2] M. Alfaro, “Collective geometry and the Yang–Mills gap: the physical domain of the mass-gap problem in Fundamental Density Theory,” *Fundamental Density Theory* (16 September 2026; edition of 26 October 2026), densitytheory.com.
- [3] M. Alfaro, “Confinement and the mass gap: the common carrier, self-supported omnium patterns, and a background-dependent neutrino minimum,” *Fundamental Density Theory* (22 September 2026), on the locked core v5.4.
- [4] A. Jaffe and E. Witten, “Quantum Yang–Mills theory,” Clay Mathematics Institute, Millennium Prize problem description (2000), <https://www.claymath.org/wp-content/uploads/2022/06/yangmills.pdf>.
- [5] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That*, Benjamin, New York (1964).
- [6] K. Osterwalder and R. Schrader, “Axioms for Euclidean Green’s functions I, II,” *Communications in Mathematical Physics* **31** (1973) 83–112; **42** (1975) 281–305.
- [7] J. Milnor, “Link groups,” *Annals of Mathematics* **59** (1954) 177–195.
- [8] T. T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, “Three-quark potential in SU(3) lattice QCD,” *Physical Review Letters* **86** (2001) 18–21.
- [9] Y. Koma and M. Koma, “Precise determination of the three-quark potential in SU(3) lattice gauge theory,” *Physical Review D* **95** (2017) 094513.
- [10] C. Alexandrou, Ph. de Forcrand and A. Tsapalis, “The static three-quark SU(3) and four-quark SU(4) potentials,” *Physical Review D* **65** (2002) 054503.
- [11] Ph. de Forcrand and O. Jahn, “The baryon static potential from lattice QCD,” *Nuclear Physics A* **755** (2005) 475–480, arXiv:hep-ph/0502039.
- [12] K. G. Wilson, “Confinement of quarks,” *Physical Review D* **10** (1974) 2445–2459.
- [13] G. S. Bali, H. Neff, T. Düssel, T. Lippert and K. Schilling, “Observation of string breaking in QCD,” *Physical Review D* **71** (2005) 114513.
- [14] N. Brambilla, J. Ghiglieri and A. Vairo, “Three-quark static potential in perturbation theory,” *Physical Review D* **81** (2010) 054031.
- [15] S. Brendle, “Embedded minimal tori in S^3 and the Lawson conjecture,” *Acta Mathematica* **211** (2013) 177–190.

- [16] H. Georgi and S. L. Glashow, “Unity of all elementary-particle forces,” *Physical Review Letters* **32** (1974) 438–441.
- [17] H. Georgi, “The state of the art—gauge theories,” *AIP Conference Proceedings* **23** (1975) 575–582.
- [18] H. Fritzsch and P. Minkowski, “Unified interactions of leptons and hadrons,” *Annals of Physics* **93** (1975) 193–266.
- [19] J. C. Baez and J. Huerta, “The algebra of grand unified theories,” *Bulletin of the American Mathematical Society* **47** (2010) 483–552.
- [20] Super-Kamiokande Collaboration (A. Takenaka *et al.*), “Search for proton decay via $p \rightarrow e^+ \pi^0$ and $p \rightarrow \mu^+ \pi^0$ with an enlarged fiducial volume in Super-Kamiokande I–IV,” *Physical Review D* **102** (2020) 112011.
- [21] E. Witten, “An SU(2) anomaly,” *Physics Letters B* **117** (1982) 324–328.
- [22] Planck Collaboration, “Planck 2018 results. I. Overview and the cosmological legacy of Planck,” *Astronomy & Astrophysics* **641** (2020) A1.
- [23] D. J. Fixsen, “The temperature of the cosmic microwave background,” *The Astrophysical Journal* **707** (2009) 916–920.
- [24] S. Coleman and J. Mandula, “All possible symmetries of the S matrix,” *Physical Review* **159** (1967) 1251–1256.
- [25] V. N. Gribov, “Quantization of non-Abelian gauge theories,” *Nuclear Physics B* **139** (1978) 1–19.
- [26] I. M. Singer, “Some remarks on the Gribov ambiguity,” *Communications in Mathematical Physics* **60** (1978) 7–12.
- [27] R. Giles, “Reconstruction of gauge potentials from Wilson loops,” *Physical Review D* **24** (1981) 2160–2168.
- [28] C. Abel *et al.*, “Measurement of the permanent electric dipole moment of the neutron,” *Physical Review Letters* **124** (2020) 081803.
- [29] R. D. Peccei and H. R. Quinn, “CP conservation in the presence of pseudoparticles,” *Physical Review Letters* **38** (1977) 1440–1443.
- [30] M. Pospelov and A. Ritz, “Theta-induced electric dipole moment of the neutron via QCD sum rules,” *Physical Review Letters* **83** (1999) 2526–2529.
- [31] C. J. Morningstar and M. Peardon, “Glueball spectrum from an anisotropic lattice study,” *Physical Review D* **60** (1999) 034509.
- [32] Y. Chen *et al.*, “Glueball spectrum and matrix elements on anisotropic lattices,” *Physical Review D* **73** (2006) 014516.
- [33] M. L. Perl, E. R. Lee and D. Loomba, “Searches for fractionally charged particles,” *Annual Review of Nuclear and Particle Science* **59** (2009) 47–65.
- [34] S. Navas *et al.* (Particle Data Group), “Review of particle physics,” *Physical Review D* **110** (2024) 030001.